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A Study on Steady Natural Convective Heat Transfer inside a Square Cavity for Different Values of Rayleigh and Nusselt Numbers

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ABSTRACT

The aim of the present numerical study is to understand the steady natural convection flow and heat transfer in a Square cavity with heated left wall. The top and bottom walls of the cavity are kept to be adiabatic. The finite volume approach for the range of Rayleigh Number as $10^3 \le Ra \le 10^7$and Pr=0.71 is used to solve the governing equations, in which buoyancy is modeled via the Boussinesq approximation in FLUENT. The computed flow patterns and temperature fields are shown by means of streamlines and isotherms respectively. The influence of Rayleigh numbers on the hot wall of the cavity are analyzed. Change in Velocity with different Rayleigh Number near the top wall of the enclosure are investigated here. Variations of the maximum value of the dimensionless stream function and Nusselt Number were also presented. The computed result indicated that Nusselt number increases along the length of the hot wall and decreases near the end of the wall.

Keywords: Heat transfer; FLUENT; Rayleigh number; Square cavity.

NOMENCLATURE

Cp	specific heat of the body (J/kgK)	α	thermal diffusivity ($m^2 s^{-1}$)
g	acceleration due to gravity (ms^{-2})	β	volume expansion coefficient (K^{-1})
L	side of the square cavity (m)	ΔT	change in temperature (K)
Pr	Prandtl number	κ	thermal conductivity ($Wm^{-1}K^{-1}$)
Ra	Rayleigh number	μ	dynamic viscosity 1(Ns/m^2)
T	temperature (K)	ν	kinematic viscosity ($m^2 s^{-1}$)
T_c	temperature of cold vertical wall(K)	ρ	density (kg m^{-3})
T_h	temperature of hot bottom wall(K)	ψ	stream function (kg/s)
u	x Component of velocity		
v	y Component of Velocity		

1. INTRODUCTION

The curiosity of natural convection in enclosures filled with air having cold vertical walls and adiabatic horizontal walls has been theme of research over the past years. The natural convection issue is of concern in various engineering and technology uses such as solar energy collectors, cooling of electronics components etc. A. Dalal *et al.* (2008) states that natural convection occur in the vicinity of tilted square cylinder in the range of $(0^0 \le \theta \le 45^0)$ inside an enclosure having horizontal adiabatic wall and cold vertical wall figure out by cell-centered finite volume method, which is used to reckoned two dimension Navier strokes equation for incompressible laminar flow. And taken the value of Rayleigh number is $= 10^5$, Pr= 0.71. G. De Vahl Davis (1983) expressed that differentially heated side walls of square cavity are figure out for precise solution of the equations. It has taken the Rayleigh numbers in the range of $10^3 \le Ra \le 10^6$. N. C. Markatos (1984) deal with Buoyancy-driven laminar and turbulent flow is reckoned by computational method. It has taken the

Rayleigh numbers in the range of $10^3 \leq Ra \leq 10^{16}$. Natural Convection problem involving buoyancy driven flow in a cavity, was first suggested as a suitable validation test case for CFD codes by Jones (1979). A comprehensive review of the early works on this subject is found in T. H. Kuehn and Goldstein (1976). In particular, internal flow problems are considerably more complex than external ones (Basak *et al.* 2006). In the literature, investigations on natural convection heat transfer reported that the heat transfer occurs in an enclosure due to the temperature differences across the walls. The natural convection of air in enclosures or channels either uniformly heated/cooled or discretely heated have received much attention (Chadwick *et al.* 1991; Refai *et al.* 1991). Basak *et al.* (2006) have reported the effect of temperature boundary conditions (Constant temperature and sinusoidally varying) on the bottom wall for Ra varying from 10^3 to 10^5 for both the Prandtl numbers of 0.7 and 10. The temperature of side walls as well as bottom wall affects the stratification rates and flow patterns (Dixit and Babu 2006). It has been observed from the literature that most of the study on natural convection in a cavity is extended up to Ra = 107 and considered air as a working fluid have studied for Ra 103 to 105 only for the cases of constant temperature and sinusoidal varying temperature at the bottom wall. However, in the present investigation the studies are extended up to Ra = 10^7 and linearly varying temperature bottom wall is included for the range of Ra studied. Recently, Basak *et al.* (2006) have used Galerkin finite element method to study effect of thermal boundary conditions on natural convection flows within a square cavity. A numerical two-dimensional simulations were conducted for the free convective flow of a low-Prandtl number fluid Pr. 0:0321. With internal heat generation in a square cavity having adiabatic top and bottom walls and isothermal side walls by Salvatore Arcidiacono *et al.* (2001). A numerical study has been performed by P. K. Bose *et al.* (2013) to analyze the flow and temperature fields as well as the heat transfer rate on laminar natural convection in quadrentral cavity filled with water having finned hot vertical wall and cold bottom wall in the range $10^4 \leq Ra \leq 10^7$.In this paper our aim is to analyze the heat flow inside a square cavity subjected to heated left wall and cold right wall with uniform temperature and adiabatic top and bottom walls for the range of Ra from 10^3 to 10^7.

2. PROBLEM FORMULATION

2.1 Physical Description

The problem under consideration is a two-dimensional Square enclosure. The schematic of the geometry and coordinate system is depicted in Fig. 1. The square cavity of length L has a hot left wall temperature T_h and cold right wall temperature T_c. The walls are kept at uniform constant temperatures while the top and bottom walls are adiabatic. The closed cavity is filled with air.

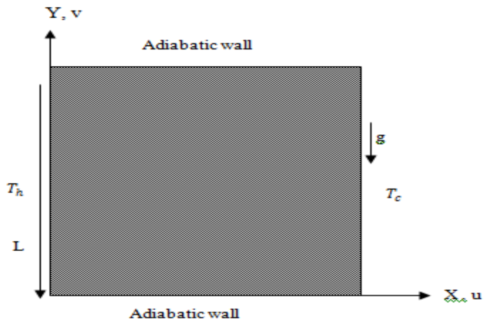


Fig. 1. Schematic of the problem and the coordinate system

2.2 Mathematical Formulation

The system was considered to be incompressible, steady-state, Newtonian and the Boussinesq approximation was applied for fluid with constant physical properties. It is assumed that the radiation effect can be taken to be negligible. The gravitational acceleration acts in the negative y-directive. The continuity, momentum and energy equations for two-dimensional Cartesian coordinate system are solved using the Boussinesq approximation. The governing equations are defined as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum Equations:

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_c) \tag{3}$$

Energy Equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

These variables have their common meaning in fluid dynamics and heat transfer as listed in the nomenclature. The thermo physical properties of air are evaluated at the reference temperature that is the average of the two wall temperatures.in this study Pr = 0.71.the effects of the Rayleigh number which is varied from $10^3\ to\ 10^7$, are of great interest in this study.

The boundary conditions may be taken as:

At left wall T =T_h at x=0; 0≤y≤L

At right wall T =T_c, at x= L ; 0≤y≤L

At the top and bottom walls $\frac{\partial T}{\partial y} = 0$ at y=0 , L ; 0≤x≤L

At the walls $\Psi = 0$ (No-slip boundary conditions)

3. NUMERICAL SCHEME

In the present investigation, the set of governing equations are integrated over the control volumes, which produces a set of algebraic equations. The SIMPLE algorithm is used to solve the coupled system of governing equations. Discretization of the momentum and energy equations is performed by a second order upwind scheme and pressure interpolation is provided by PRESTO scheme. Convergence criterion considered as residuals is admitted 10^{-3} for momentum and continuity equations and for the energy equation it is lower than 10^{-6}. The calculations are carried out using the FLUENT 6.3 commercial code. The grid system of 110×110 is adopted here shown in Fig. 2 below.

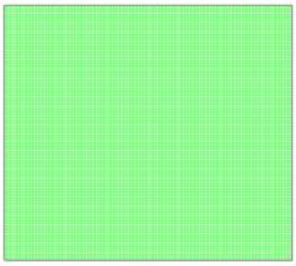
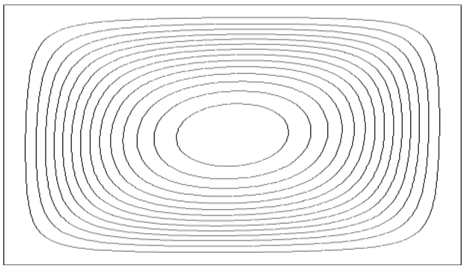


Fig. 2. Uniform grid structure used of the square cavity

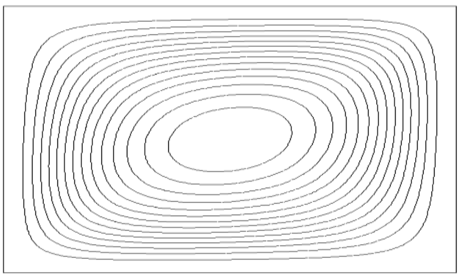
4. RESULTS AND DISCUSSIONS

Cases with the hot surface of the square cavity were first studied. The effect of Nusselt number on the hot surface of cavity at different Rayleigh numbers ($Ra = 10^3, 10^4, 10^5, 10^6 and\ 10^7$) are systematically investigated. Comparisons of the flow fields and velocity profile near the top wall are presented.

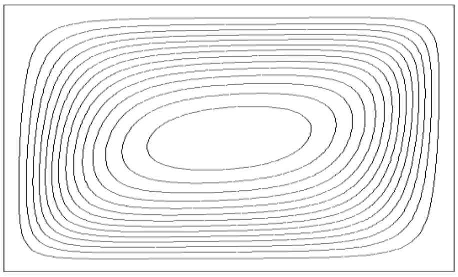
4.1 Flow patterns Streamlines



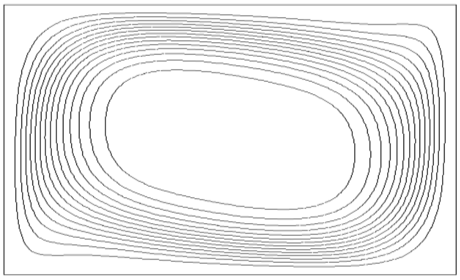
$\mathbf{Ra = 10^3}$



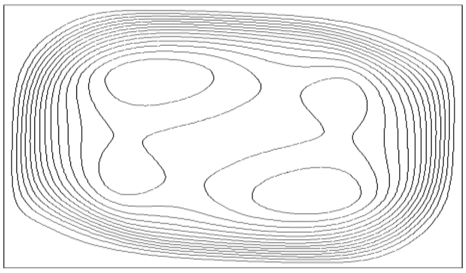
$\mathbf{Ra = 10^4}$



$\mathbf{Ra = 10^5}$



$\mathbf{Ra = 10^6}$

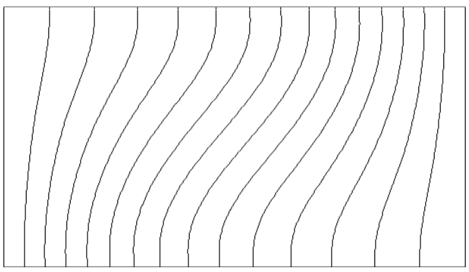


$\mathbf{Ra = 10^7}$

Fig. 3. Comparision of the Streamlines at Different values of Ra

The flow fields are shown by means of streamlines in Fig. 3 due to the buoyant effects caused by the temperature difference between the left and right wall of the cavity, recirculating vortices are formed which are clearly demonstrated by the close streamlines. As Rayleigh number increases the central vortices expand .At low Rayleigh number the flow fields are symmetric and as Ra increases the shape changes from circle to ellipse but at high Rayleigh number the central vortex split into two new vortices. Increase in Ra results in higher intensity of streamlines near the hot and cold wall.

4.2 Heat Transfer Isotherms



$\mathbf{Ra = 10^3}$

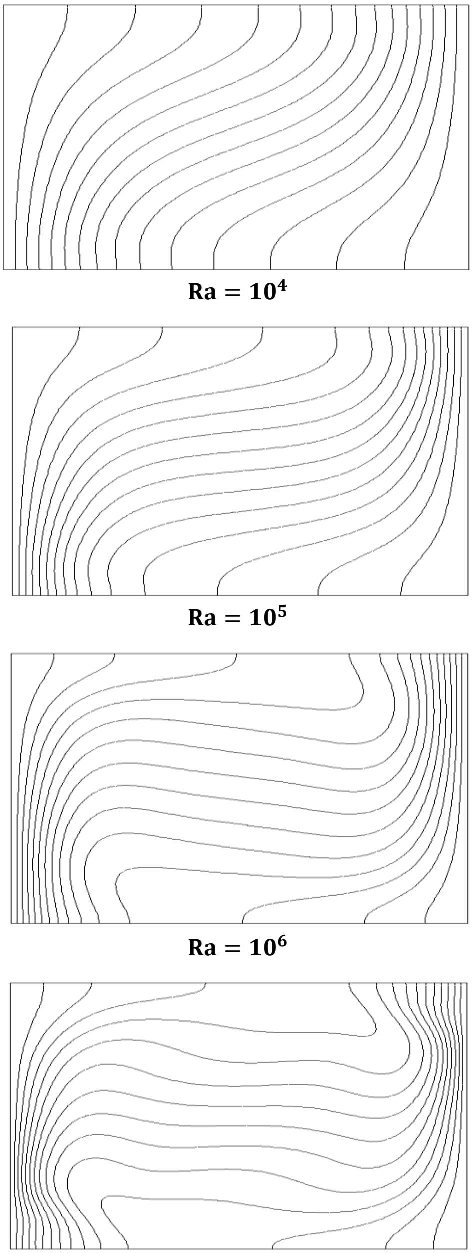


Fig. 4. Comparisons of the isotherms at Different values of Ra

The thermal fields are presented in the form of isotherms in Fig. 4 with same arrangement as in Fig. 3. As Rayleigh number increases heat transfer rate increases, velocity increases and bending of the isotherms in the core region such that they are no longer orthogonal to the gravitational field. At low Rayleigh number isotherms are perpendicular since the intensity of the buoyancy driven convection is very weak and the thermal field is thus nearly unaffected as Rayleigh number increases, the isotherms are closely packed and buoyancy driven convection is very strong. The flow intensity is examined through the maximum value of the dimensionless stream function, Ψ_{max} whose variation versus the Rayleigh number is shown in Fig. 5

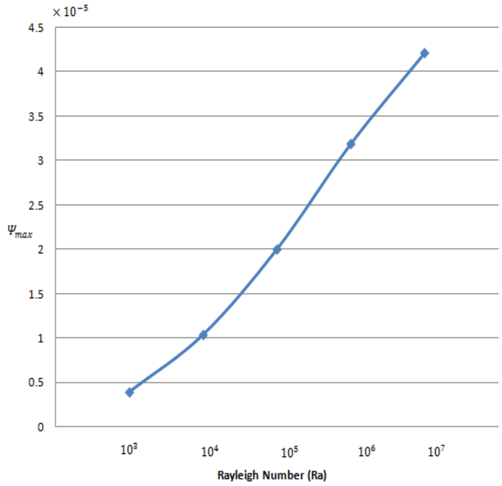


Fig. 5. The maximum value of dimentionless stream function for different R values

In this Fig. 5 the flow intensity is examined through the maximum value of the dimensionless stream function Ψ_{max}, whose variation versus the Rayleigh number is shown here. As Ra increases flow intensity increases.

4.3 Variation of Nusselt Number

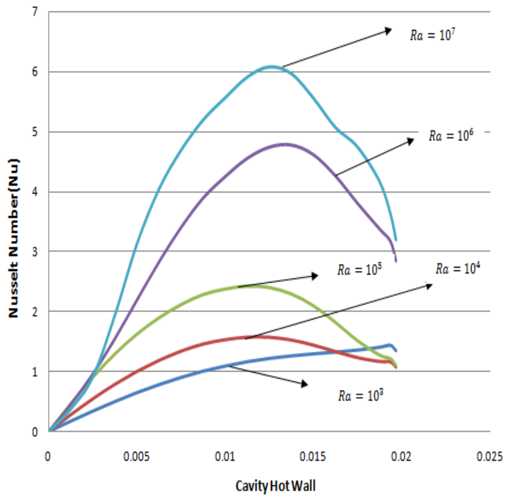


Fig. 6. Variation of the Nusselt number along length of the hot wall for different Ra values

Variation of Nusselt number along the hot wall of the cavity are plotted in Fig. 6. As heat flux along the length increases the Nusselt number increases gradually up to the middle of the wall and then decreases gradually. The high peak shows in the middle portion at high value of Ra the Nusselt increases three times than at low value of Ra.

4.4 Variation of Velocity Magnitude

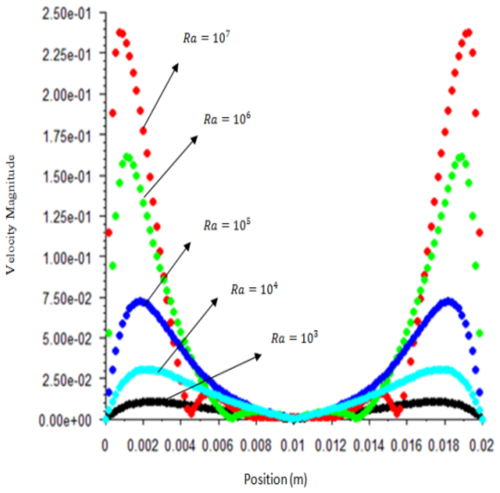


Fig. 7. Variation of the Velocity magnitude at midsection of the cavity for different Ra.

In the above Fig. 7 the vertical velocity is plotted at the middle of the heated wall (at the isosurface y=0.01) for different Ra.

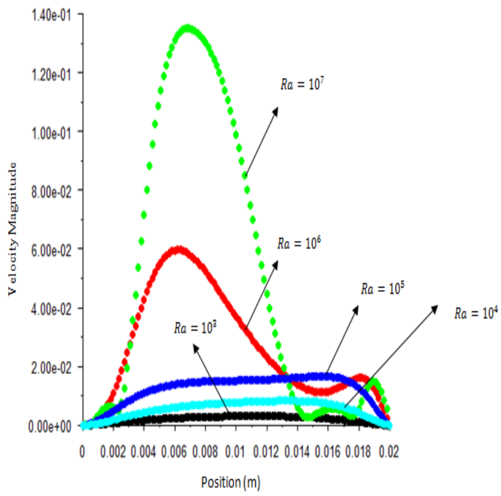


Fig. 8. Variation of the Velocity magnitude near the top wall of the cavity for different Ra

This Fig. 8 shows that near the top wall velocity increases, where heat is flowing from left heated wall to the right wall, at different values of Ra.

5. CONCLUSION

A numerical investigation of steady natural convective heat transfer inside a square cavity is carried out .The effect of Rayleigh numbers ($Ra = 10^3,\ 10^4,\ 10^5,\ 10^6\ and\ 10^7$), Nusselt number on the flow inside the cavity and heat transfer are studied here with Pr=0.71.From the result and discussion we conclude that as Rayleigh Number increases the central vortices of streamlines expand and isotherms are closely packed near the walls. Velocity increases near the top wall as hot air flows from left wall to the right wall. Result indicates that as Ra increases velocity of the fluid increases and heat transfer rate increases.

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