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Heat and Mass Transfer Effects on Unsteady MHD Natural Convection Flow of a Chemically Reactive and Radiating Fluid through a Porous Medium Past a Moving Vertical Plate with Arbitrary Ramped Temperature

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ABSTRACT

Investigation of unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, chemically reactive and optically thin radiating fluid past an exponentially accelerated moving vertical plate with arbitrary ramped temperature embedded in a fluid saturated porous medium is carried out. Exact solutions of momentum, energy and concentration equations are obtained in closed form by Laplace transform technique. The expressions for the shear stress, rate of heat transfer and rate of mass transfer at the plate for both ramped temperature and isothermal plates are derived. The numerical values of fluid velocity, fluid temperature and species concentration are displayed graphically whereas those of shear stress, rate of heat transfer and rate of mass transfer at the plate are presented in tabular form for various values of pertinent flow parameters. It is found that, for isothermal plate, the fluid temperature approaches steady state when $t > 1.5$. Consequently, the rate of heat transfer at isothermal plate approaches steady state when $t > 1.5$.

Keywords: Natural convection; Magnetic field; Chemical reaction; Radiation; Arbitrary ramped temperature.

NOMENCLATURE

a	surface acceleration parameter
a^*	absorption coefficient
B_0	uniform magnetic field
C'	species concentration
c_p	specific heat at constant pressure
D_M	chemical molecular diffusivity
g	acceleration due to gravity
G_r	thermal Grashof number
G_c	solutal Grashof number
k	thermal conductivity of the fluid
K_1'	permeability of porous medium
K_1	permeability parameter
K_2'	chemical reaction coefficient
K_2	chemical reaction parameter
M	magnetic parameter
N	radiation parameter
P_r	Prandtl number
q_r'	radiating flux vector
T'	fluid temperature
t_0	critical time for rampedness
t_1	non-dimensional critical time for rampedness
U_0	characteristic velocity
u'	fluid velocity in $x'-$direction
S_c	Schmidt number
β'	coefficient of thermal expansion
β^*	coefficient of expansion for species concentration
σ	electrical conductivity
ρ	fluid density
σ^*	Stefan-Boltzmann constant
ν	kinematic coefficient of viscosity

1. INTRODUCTION

Theoretical/experimental investigation of problems of unsteady hydromagnetic natural convection flow of an electrically conducting fluid within porous and non-porous media has received considerable attention of several researchers during past few decades due to its overwhelming and important applications in many areas of science and engineering which includes geophysics, astrophysics, electronics, aeronautics, metallurgy, chemical and petroleum engineering etc. Keeping in view the importance of this fluid flow, several researchers investigated unsteady hydromagnetic natural convection flow of an electrically conducting fluid past bodies with different geometries under different initial and boundary conditions. Mention may be made of the research studies of Gupta (1960), Pop (1969), Chamkha (1997, 2000), Helmy (1998), Kim (2000), Raptis *et al*. (2003), Makinde and Tshela (2014) and Seth *et al*. (2013, 2014).

Hydromagnetic natural convection flow of radiating and non-radiating fluid with heat and mass transfer in porous and non-porous media is studied by several researchers due to its varied and wide applications in astrophysics, geophysics, aeronautics, electronics, metallurgy, chemical and petroleum industries. Hydromagnetic natural convection flow of an electrically conducting fluid in a fluid saturated porous medium has also been successfully exploited in crystal formation.

Oreper and Szekely (1983) have found that the presence of a magnetic field can suppress natural convection currents and the strength of magnetic field is one of the important factors in reducing non-uniform composition thereby enhancing quality of the crystal. In addition to it, hydromagnetic problems with heat and mass transfer is of much significance in MHD flow-meters, MHD energy generators, MHD pumps, controlled thermo-nuclear reactors, MHD accelerators etc. Hossain and Mandal (1985) investigated mass transfer effects on unsteady hydromagnetic free convection flow past an accelerated vertical porous plate. Jha (1991) studied hydromagnetic free convection and mass transfer flow past a uniformly accelerated vertical plate through a porous medium when magnetic field is fixed with the moving plate.

Elbashbeshy (1997) discussed heat and mass transfer along a vertical plate in the presence of magnetic field. Chamkha and Khaled (2000) investigated coupled heat and mass transfer by natural convection from a vertical, semi-infinite flat plate embedded in a porous medium in the presence of an external magnetic field.

Chen (2004) analyzed hydromagnetic natural convection flow with heat and mass transfer over a permeable inclined surface with variable wall temperature and concentration. Ibrahim *et al*. (2004) discussed unsteady magnetohydrodynamic flow of micro-polar fluid and heat transfer past a vertical porous plate through a porous medium in the presence of thermal and mass diffusions with a constant heat source. Prasad and Reddy (2007) analyzed an unsteady, two-dimensional, hydromagnetic, laminar free convective boundary-layer flow of an incompressible, Newtonian, electrically-conducting and radiating fluid past an infinite heated vertical porous plate with heat and mass transfer by taking into account the effect of viscous dissipation. Chaudhary and Jain (2007) studied hydromagnetic natural convection flow with heat and mass transfer past an infinite vertical oscillating plate embedded in a fluid saturated porous medium. Makinde and Sibanda (2008) studied MHD mixed convection heat and mass transfer flow past a vertical porous plate embedded in a porous medium with constant heat flux. Rajesh and Varma (2009) investigated the effects of thermal radiation on unsteady free convection flow past an exponentially accelerated infinite vertical plate with mass transfer in the presence of magnetic field. Sangapatnam *et al*. (2009) studied the effects of thermal radiation on the natural convective heat and mass transfer of a viscous, incompressible, gray absorbing, emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation.

Eldabe *et al*.(2011) discussed unsteady MHD flow of a viscous and incompressible fluid with heat and mass transfer in a porous medium near a moving vertical plate with time-dependent velocity. Prakash *et al*. (2013) investigated diffusion thermo and radiation effect on unsteady MHD free convection flow through porous medium past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion.

In most of the chemical engineering processes, chemical reaction occurs between a foreign mass and the fluid. Chemical reactions can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. These processes take place in numerous industrial applications viz. polymer production, manufacturing of ceramics or glassware, food processing etc.

Chamkha (2003) investigated MHD flow over a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. Afify (2004) studied the effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field. Ibrahim *et al*. (2008) analyzed the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Bakr (2011) discussed the effects of chemical reaction on MHD free convection and mass transfer flow of a micro polar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference.

Chamkha *et al*. (2011) discussed the effects of Joule heating, chemical reaction and thermal radiation on unsteady hydromagnetic natural convection boundary layer flow with heat and mass transfer of a micro polar fluid from a semi-infinite heated vertical porous plate in the presence of a uniform transverse magnetic field. Bhattacharya and Layek (2012) obtained similarity solution of MHD boundary layer flow with mass diffusion and chemical reaction over a porous flat plate with suction/blowing. Kishore *et al*. (2013) studied the effects of radiation and chemical reaction on unsteady hydromagnetic natural convection flow of a viscous fluid past over an exponentially accelerated vertical plate.

Mohamed et al. (2013) investigated unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible and electrically conducting and radiating fluid past through a porous medium near an impulsively moving hot vertical plate in the presence of homogeneous chemical reaction of first order and temperature dependent heat sink.

In fact, natural convection flows are generally modelled by the researchers under the consideration of uniform surface temperature or uniform surface heat flux. But, in many physical situations, the temperature of the bounding surface may require non-uniform or arbitrary wall conditions. Moreover, there may be step discontinuities in the surface temperature or ramped surface temperature. Keeping in view this fact, several researchers investigated natural convection from a vertical plate with step discontinuities in the surface temperature considering various aspects of the problem. Mention may be made of the research investigations of Hayday *et al.* (1967), Kelleher (1971), Kao (1975) and Lee and Yovanovich (1991).

In recent years, several researchers investigated unsteady hydromagnetic free convection flow past a vertical plate with ramped temperature considering various variations in the problem. Mention may be made of the research studies of Rajesh (2010), Samiulhaq *et al*. (2012), Das (2012), Nandkeolyar and Das (2013, 2014), Ahmed and Das (2013a, b), Nandkeolyar *et al*. (2013a, 2013b), Narahari and Sulaiman (2013), Das *et al*. (2014), Kundu *et al*. (2014). Recently, Seth *et al*. (2014, 2015a, b) considered hydromagnetic natural convection heat and mass transfer flow with Hall current past an infinite moving vertical plate with ramped temperature in a rotating medium considering different aspects of the problems.

However, in all these investigations, researchers have considered the time interval for rampedness in the plate temperature as fixed i.e. $0 < t' \le t_0$, (t' and t_0 being time and characterisrtic time respectively). Recently, Ahmed and Dutta (2014) investigated magnetohydrodynamic transient flow of a viscous, incompressible, electrically conducting and optically thin radiating fluid past an impulsively moving infinite vertical plate with temporary ramped wall temperature. They considered time t_0 as an arbitrary constant in place of characteristic time. But they have non-dimensionalized physical variables, namely, y' and t' in their problem using t_0 . This is possible only when t_0 is considered as a characteristic time. With this non-dimensionalization process they have defined Reynolds number $R_e = U_0^2 t_0/\nu$, (U_0 and ν being the uniform velocity of the plate at time $t' > 0$ and kinematic coefficient of viscosity respectively) which is not correct. If t_0 is characteristic time then its dimension in terms of characteristic length L is $\frac{L^2}{\nu}$ then $\frac{U_0^2 t_0}{\nu}$ is equal to $\frac{U_0^2 L^2}{\nu^2}$ which is equal to R_e^2. Due to this reason, the definition of Reynolds numberis also not taken correctly. Also interval of ramped profile reduces to $0 < t \le 1$ (t being non-dimensional time) only when t_0 is considered characteristic time. In our opinion $U_0^2 t_0/\nu$ is a non-dimensional arbitrary time constant which we have named as critical time for rampedness. This is true if t_0 is not a characteristic time. It is to be noted that time interval for ramped profiles varies from material to material depending upon the specific heat capacity of the material. Arbitrary ramped profiles appear in real world situation in building air-conditioning systems, fabrication of thin-film photovoltaic devices, phase transition in processing of materials, turbine blade heat transfer, heat exchangers etc.

Purpose of present investigation is to study unsteady hydromagnetic natural convection heat and mass transfer flow of an electrically conducting, viscous, incompressible, chemically reactive and optically thin heat radiating fluid past an exponentially accelerated moving vertical plate through fluid saturated porous medium with arbitrary ramped wall temperature. This problem is totally different from the research paper investigated by Ahmed and Dutta (2014).

In our paper, we have considered medium as porous, chemically reactive and optically thin radiating fluid when fluid flow is induced due to exponentially accelerated moving vertical plate along with thermal and concentration buoyancy forces. We have approximated gradient of radiating flux vector i.e. $\frac{\partial q_r'}{\partial y'}$ by approximation suggested by Raptis (2011) in place of Cogley *et al*. (1968) which was adopted by Ahmed and Dutta (2014).

This research study may have strong bearings on numerous problems of practical interest where initial temperature profiles are of much significance in designing of so many hydromagnetic devices and in several industrial processes occurring at high temperatures where the effects of thermal radiation play a vital role in the fluid flow characteristics. We have compared numerical values of shear stress at the plate with those of Das *et al*. (2011) as a special case of our results which are in excellent agreement with the numerical values of Das *et al*. (2011).

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider unsteady magnetohydrodynamic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reactive and optically thin heat radiating fluid through fluid saturated porous medium past an infinite moving vertical plate with arbitrary ramped temperature. Choose the coordinate system in such a way that x'-axis is along the length of the plate in the upward direction and y'- axis normal to plane of the plate in the fluid. The fluid is permeated by a uniform transverse magnetic field B_0 applied in a direction parallel to y'-axis. Initially i.e. at time $t' \le 0$, both the fluid and plate are at rest and maintained at a uniform temperature T'_∞. Also species concentration at the surface of the plate as well as at every point within the fluid is maintained at uniform concentration C'_∞.

At time $t' > 0$, plate is exponentially accelerated with velocity $U_0 e^{a't'}$ in x'- direction (U_0 being characteristic velocity). Temperature of the plate is raised or lowered to $T'_\infty + (T'_w - T'_\infty)t'/t_0$ when $0 < t' \le t_0$ (t_0 being critical time for rampedness) and uniform temperature T'_w when $t' > t_0$. Also at time $t' > 0$, species concentration at the surface of the plate is raised to uniform species concentration C'_w and is maintained thereafter. The fluid is considered as a gray, emitting-absorbing radiation but non scattering medium. It is assumed that there exists a homogeneous chemical reaction of first order with constant rate K'_2 between diffusing species and the fluid. Physical model of the problem is presented in Fig. 1.

Fluid is metallic liquid or partially ionized whose magnetic Reynolds number is very small. Hence, the induced magnetic field generated by fluid motion is negligible in comparison to the applied one (Crammer and Pai, 1973). Thus, the magnetic field $\vec{B} = (0, B_0, 0)$. Since, no external electric field is applied into the flow-field, the effect of polarization of fluid becomes negligible (Meyer, 1958). With these assumptions, the governing equations for unsteady magnetohydrodynamic natural convection flow of an electrically conducting, viscous, incompressible, chemically reactive and optically thin heat radiating fluid through fluid saturated porous medium are given by

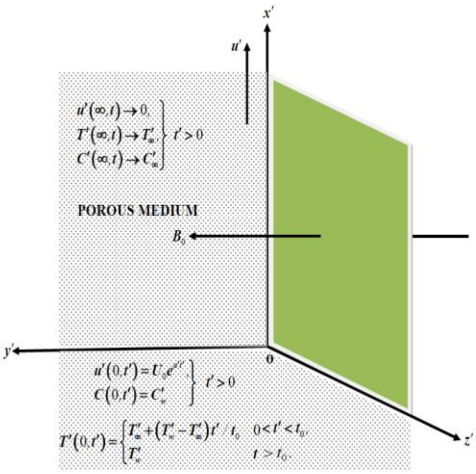

Fig. 1. Physical model of the problem.

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_0^{\,2}}{\rho}\right) u' - \nu \frac{u'}{K'_1} + g\beta'(T' - T'_\infty)$$
$$+ g\beta^*(C' - C'_\infty), \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q'_r}{\partial y'}, \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} - K'_2(C' - C'_\infty), \tag{3}$$

where u', T', K_1', K_2', k, c_p, C', D_M, q_r', ν, σ, ρ, g, β' and β^* are, respectively, fluid velocity in x'-direction, fluid temperature, permeability of porous medium, chemical reaction coefficient, thermal conductivity, specific heat at constant pressure, species concentration, chemical molecular diffusivity, radiating flux vector, kinematic coefficient of viscosity, electrical conductivity, fluid density, acceleration due to gravity, coefficient of thermal expansion and coefficient of expansion for species concentration.

Appropriate initial and boundary conditions for the fluid flow problem are given by

$$u' = 0,\ T' = T'_\infty,\ C' = C'_\infty \quad \text{for } y' > 0 \text{ and } t' \le 0, \tag{4a}$$

$$u' = U_0 e^{a't'},\ C' = C'_w \quad \text{at } y' = 0 \text{ for } t' > 0, \tag{4b}$$

$$T' = T'_\infty + (T'_w - T'_\infty)t'/t_0 \quad \text{at } y' = 0 \text{ for } 0 < t' \le t'_0, \tag{4c}$$

$$T' = T'_w \quad \text{at } y' = 0 \text{ for } t > t_0, \tag{4d}$$

$$u' \to 0,\ w' \to 0,\ T' \to T'_\infty,$$
$$C' \to C'_\infty \quad \text{as } y' \to \infty \text{ for } t' > 0. \tag{4e}$$

For an optically thin gray fluid the local radiant absorption (Raptis, 2011) is expressed as

$$\frac{\partial q'_r}{\partial y'} = -4a^*\sigma^*(T'^4_\infty - T'^4), \tag{5}$$

where a^* is absorption coefficient and σ^* is Stefan-Boltzmann constant.

It is assumed that the temperature difference between the fluid in the boundary layer and free stream is sufficiently small so that T'^4 may be expressed as a linear function of T'. This is accomplished by expanding T'^4 in a Taylor series about free stream temperature T'_∞. Neglecting second and higher order terms, T'^4 is expressed as

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty. \tag{6}$$

Using equations (5) and (6), in equation (2), we obtain

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{16a^*\sigma^* T'^3_\infty}{\rho c_p}\left(T' - T'_\infty\right). \tag{7}$$

In order to represent equations (1), (3) and (7) along with initial and boundary conditions (4a) to (4e) in non-dimensional form, we are introducing the following non-dimensional variables and parameters

$$\left.\begin{aligned} & y=\frac{U_0}{\nu}y',\ t=\frac{U_0^2}{\nu}t',\ u=\frac{u'}{U_0},\ T=\frac{T^{'}-T'_\infty}{T'_w-T'_\infty}, \\ & C=\frac{C'-C'_\infty}{C'_w-C'_\infty}, a=a'\frac{\nu}{U_0^2},\ G_r=\frac{\nu g\beta'\left(T'_w-T'_\infty\right)}{U_0^3}, \\ & G_c=\frac{\nu g\beta^*\left(C'_w-C'_\infty\right)}{U_0^3}, K_1=K'_1\frac{U_0^2}{\nu^2},\ K_2=\frac{\nu K'_2}{U_0^2}, \\ & M=\frac{\sigma B_0^2\nu}{\rho U_0^2},\ N=\frac{16a^*\sigma\nu T'_\infty}{U_0^2\rho c_p}, P_r=\frac{\nu\rho c_p}{k}, \\ & S_c=\frac{\nu}{D_M}\ \text{and}\ t_1=\frac{U_0^2 t_0}{\nu}, \end{aligned}\right\} \tag{8}$$

where $a,\ G_r,\ G_c,\ K_1,\ K_2,\ M,\ N,\ P_r,\ S_c$ and t_1 are, respectively, surface acceleration parameter, thermal Grashof number, solutal Grashof number, permeability parameter, chemical reaction parameter, magnetic parameter, radiation parameter, Prandtl number, Schmidt number and non-dimensional fixed time which varies from material to material of plate depending on specific capacity of material which we have named as non-dimensional critical time for rampedness.

The equations (1), (3) and (7), in non-dimensional form, reduce to

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-Mu-\frac{u}{K_1}+G_rT+G_cC, \tag{9}$$

$$\frac{\partial T}{\partial t}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}-NT, \tag{10}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}-K_2\,\mathrm{C}, \tag{11}$$

Initial and boundary conditions (4a) to (4e) in non-dimensional form, are presented below

$$u=0,\ T=0,\ C=0 \qquad \text{for } y\ge 0 \text{ and } t\le 0, \tag{12a}$$

$$u=e^{at},\ C=1 \qquad \text{at } y=0 \text{ for } t>0, \tag{12b}$$

$$T=\frac{t}{t_1} \qquad \text{at } y=0 \text{ for } 0<t\le t_1, \tag{12c}$$

$$T=1 \qquad \text{at } y=0 \text{ for } t>t_1, \tag{12d}$$

$$u\to 0,\ T\to 0,\ C\to 0 \qquad \text{as } y\to\infty,\ t>0. \tag{12e}$$

Exact solution for fluid velocity $u(y,t)$, fluid temperature $T(y,t)$ and species concentration $C(y,t)$ is obtained by solving equations (9) to (11) subject to the initial and boundary conditions (12a) to (12e) using Laplace transform technique and are presented in the following form after simplification:

$$\begin{aligned} u(\mathrm{y,t}) &= \frac{e^{at}}{2}f_1(y,\lambda_1,a,t)-\frac{y}{2\lambda_2}\left[e^{\lambda_2 t}\left\{f_1(\mathrm{y},\lambda_1,\lambda_2,\mathrm{t})-\right.\right. \\ & \left. f_2(y,\lambda_2,K_2,S_c,t)\right\}-\left\{f_1(y,\lambda_1,0,t)+\right. \\ & \left.\left. f_2(y,0,K_2,S_c,t)\right]+G(y,t)+\right. \\ & H(t-t_1)G(y,t-t_1), \end{aligned} \tag{13}$$

$$T(y,t)=\frac{1}{t_1}\left[P(y,t)-H(t-t_1)P(y,t-t_1)\right], \tag{14}$$

$$\begin{aligned} C(y,t) &= \frac{1}{2}\left[e^{y\sqrt{S_cK_2}}\,erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}+\sqrt{K_2t}\right)+\right. \\ & \left. e^{-y\sqrt{S_cK_2}}\,erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}-\sqrt{K_2t}\right)\right], \end{aligned} \tag{15}$$

where

$$\begin{aligned} G(y,t) &= \frac{\alpha}{t_1}\left(\frac{1}{2\lambda_3{}^2}\right)\left[e^{\lambda_3 t}\left\{f_3(y,P_r,N,\lambda_3,t)\right.\right. \\ & \left. -f_3(y,1,\lambda_1,\lambda_3,t)\right\}-\lambda_3\left\{f_4(y,P_r,N,\lambda_3,t)+\right. \\ & \left.\left. f_4(y,1,\lambda_1,\lambda_3,t)\right\}\right], \end{aligned}$$

$$\begin{aligned} P(y,t) &= \frac{1}{2}\left[\left(t+\frac{y}{2}\sqrt{\frac{P_r}{N}}\right)e^{y\sqrt{P_rN}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{Nt}\right)\right. \\ & \left.\left(t-\frac{y}{2}\sqrt{\frac{P_r}{N}}\right)e^{-y\sqrt{P_rN}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{Nt}\right)\right], \end{aligned}$$

where $\alpha=\dfrac{G_r}{1-P_r},\ \gamma=\dfrac{G_c}{1-S_c},\ \lambda_1=M+\dfrac{1}{K_1},$

$\lambda_2=\dfrac{K_2S_c-\lambda_1}{1-S_c}$ and $\lambda_3=\dfrac{\mathrm{P}_rN-\lambda_1}{1-P_r}.$

The expressions for $f_i\,(\mathrm{i}=1,2,3,4)$ are provided in Appendix.

$H(t-t_1)$ and $erfc(x)$ are unit step function and complementary error function respectively.

2.1 Solution for Unit Prandtl and Unit Schmidt Number

Solution (13) for fluid velocity $u(y,t)$ is not valid for unit Prandtl number and unit Schmidt number. Since Prandtl number is a measure of relative strength of viscosity to thermal diffusivity of fluid and Schmidt number is a measure of relative strength of viscosity to molecular (mass) diffusivity of fluid, the case $P_r=1$ and $S_c=1$ corresponds to those fluids for which the viscous, thermal and concentration boundary layer thicknesses are same order of magnitude. There are some fluids of practical significance which belong to this category (Chen, 2004). Setting $P_r=1$ and $S_c=1$ in equations (10) and (11) and following the same procedure adopted earlier, exact solution for fluid velocity $u(y,t)$, fluid temperature $T(y,t)$ and fluid concentration $C(y,t)$ is obtained and presented in the following simplified form :

$$\begin{aligned} u(y,t) &= \frac{1}{2}e^{at}f_1(y,\lambda_1,a,t)+\frac{\gamma_1}{2}\left[f_1(y,\lambda_1,0,t)\right. \\ & \left. -f_1(y,K_2,0,t)\right]+G_1(y,t)-H(t-t_1) \\ & \times G_1(y,t-t_1), \end{aligned} \tag{16}$$

$$T(y,t)=\frac{1}{t_1}\left[P_1(y,t)-H(t-t_1)P_1(y,t-t_1)\right], \tag{17}$$

$$C(y,t)=\frac{1}{2}\left[e^{y\sqrt{K_2}}erfc\left(\frac{y}{2}\sqrt{\frac{1}{t}}-\sqrt{K_2t}\right)+e^{-y\sqrt{K_2}}\right.$$

$$\left.\times erfc\left(\frac{y}{2}\sqrt{\frac{1}{t}}-\sqrt{K_2t}\right)\right], \quad (18)$$

where

$$G_1(y,t)=\frac{\alpha_1}{2t_1}\left[f_5(y,\lambda_1,t)-f_5(y,N,t)\right],$$

$$P_1(\text{y},\text{t})=\frac{1}{2}\left[\left(t+\frac{y}{2}\sqrt{\frac{1}{N}}\right)e^{y\sqrt{N}}erfc\left(\frac{y}{2}\sqrt{\frac{1}{t}}+\sqrt{Nt}\right)\right.$$

$$\left.+\left(t-\frac{y}{2}\sqrt{\frac{1}{N}}\right)e^{-y\sqrt{N}}erfc\left(\frac{y}{2}\sqrt{\frac{1}{t}}+\sqrt{Nt}\right)\right],$$

where $\alpha_1=\frac{G_r}{(N-\lambda_1)}$ and $\gamma_1=\frac{G_c}{K_2-\lambda_1}$ The expression for f_5 is provided in Appendix.

2.2 Solution when Fluid is in Contact to Isothermal Plate

Solution (13) to (15) is the solution for fluid velocity, fluid temperature and species concentration for natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reactive and optically thin radiating fluid past an exponentially accelerated moving vertical plate with arbitrary ramped temperature. In order to emphasize the effects of ramped temperature distribution within the plate on fluid flow, it may be worthwhile to compare such flow with the one near an exponentially accelerated moving vertical plate with uniform temperature. Keeping in view the assumptions made earlier, the solutions for fluid velocity and fluid temperature for the flow past an exponentially accelerated moving isothermal vertical plate is obtained and presented in the following form

$$u(y,t)=\left(\frac{e^{at}}{2}\right)f_1(y,\lambda_1,a,t)-\left(\frac{\alpha}{2\lambda_3}\right)\left[e^{\lambda_3t}\left\{f_1(y,\lambda_1,\lambda_3,t)\right.\right.$$

$$\left.-f_2(y,\lambda_3,N,P_r,t)\right\}-\left\{f_1(y,\lambda_1,0,t)+\right.$$

$$\left.\left.f_2(y,0,N,P_r,t)\right\}\right]-\left(\frac{\gamma}{2\lambda_2}\right)\left[e^{\lambda_2t}\times\right.$$

$$\left\{f_1(y,\lambda_1,\lambda_2,t)-f_2(y,\lambda_2,K_2,P_r,t)\right\}$$

$$\left.-\left\{f_1(y,\lambda_1,0,t)-f_2(y,0,K_2,S_c,t)\right\}\right], \quad (19)$$

$$T(y,t)=\left(\frac{1}{2}\right)\left[e^{y\sqrt{P_rN}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{Nt}\right)+e^{-y\sqrt{P_rN}}\right.$$

$$\left.\times erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{Nt}\right)\right]. \quad (20)$$

2.3 Shear Stress and Rate of Heat Transfer at the Plate

The expressions for shear stress τ and rate of heat transfer N_u at the plate are obtained and presented in the following form for both ramped temperature and isothermal plates.

(i) For ramped temperature plate:

$$\tau=\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$=e^{at}f_6(\lambda_1,a,t)-\left(\frac{\gamma}{\lambda_2}\right)e^{\lambda_2t}\left[f_7(\lambda_1,\lambda_2,t)-f_8(K_2,\lambda_2,S_c,t)\right]$$

$$+G_2(0,t)+H(t-t_1)G_2(0,t-t_1), \quad (21)$$

$$N_u=\left.\frac{\partial T}{\partial y}\right|_{y=0}=P_2(0,t)-H(t-t_1)P_2(0,t-t_1), \quad (22)$$

where

$$G_2(0,t)=\left(\frac{\alpha}{\lambda_3{}^2t_1}\right)\left[-f_9(\lambda_3,P_r,N,t)+f_{10}(\lambda_1,\lambda_3,P_r,N,t)\right],$$

$$P_2(0,t)=\left(\frac{1}{2t_1}\right)\left[-2e^{-Nt}\sqrt{\frac{P_rt}{\pi}}+\left(\sqrt{\frac{\text{Pr}}{N}}+2t\sqrt{P_rN}\right)\right.$$

$$\left.\left(erfc(\sqrt{Nt})-1\right)\right].$$

(ii) For isothermol plate:

$$\tau=\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$=-f_{11}(\lambda_1,a,t)+\left(\frac{\alpha}{\lambda_3}\right)\left[f_{12}(\lambda_1,\lambda_3,P_r,N,t)+\right.$$

$$\left.f_{12}(\lambda_1,0,P_r,N,t)\right]+\left(\frac{\gamma}{\lambda_2}\right)\times$$

$$\left[f_{12}(\lambda_1,\lambda_2,S_c,K_2,t)+f_{12}(\lambda_1,0,S_c,K_2,t)\right], \quad (23)$$

$$N_u=\left.\frac{\partial T}{\partial y}\right|_{y=0}=-e^{-Nt}\sqrt{\frac{P_r}{t\pi}}+\sqrt{P_rN}\left(erfc\left(\sqrt{Nt}\right)-1\right), \quad (24)$$

where the expressions $f_i(\text{i}=6,7,.....,12)$ are provided in Appendix.

2.4 Rate of Mass Transfer

The expressions for rate of mass transfer S_h at the plate is obtained and presented in the following form

$$S_h=\left.\frac{\partial C}{\partial y}\right|_{y=0}=-e^{-K_2t}\sqrt{\frac{S_c}{t\pi}}+\sqrt{K_2S_c}\left(erfc\left(\sqrt{(K_2t)}\right)-1\right) \quad (25)$$

3. VALIDATION OF RESULT

In order to validate our numerical results, we have compared, the numerical values of shear stress at the plate of a special case of our numerical results with those of Das *et al*. (2011), for various values of K_1 and N, taking $M=0,$ $t_1=1,$ $a=0,$ $t=1,$ $G_r=10,$ $G_c=0,$ $K_2=0$ and $P_r=0.71$. Our numerical results are in excellent agreement with those of Das *et al* (2011) which is clearly evident from Tables 1 and 2.

Table 1 Shear stress τ at the ramped temperature plate when $M=0,\ t_1=1,\ a=0,$ $t=1$, $K_2=0,\ G_r=10,\ G_c=0$ and $P_r=0.71$.

$K_1 \downarrow\ N \rightarrow$		25	30	35
Das *et al.*(2011)	0.040	3.93632	3.97912	4.01576
	0.045	3.61771	3.66324	3.70195
	0.050	3.34675	3.39462	3.43524
Present Value	0.040	3.93632	3.97912	4.01576
	0.045	3.61771	3.66324	3.70195
	0.050	3.34675	3.39462	3.43524

Table 2 Shear stress τ at isothermal plate when $M=0,\ t_1=1,\ a=0,\ t=1$, $K_2=0,\ G_r=10,$ $G_c=0$ and $P_r=0.71$.

$K_1 \downarrow\ N \rightarrow$		25	30	35
Das *et al.*(2011)	0.040	3.91459	3.95998	3.99849
	0.045	3.59386	3.64215	3.68301
	0.050	3.32075	3.37170	3.41473
Present Value	0.040	3.91459	3.95998	3.99849
	0.045	3.59386	3.64215	3.68301
	0.050	3.32075	3.37170	3.41473

4. RESULTS AND DISCUSSION

In order to analyze the effects of critical time for rampedness, surface acceleration parameter, time, radiation, permeability of porous medium, mass diffusion, solutal buoyancy force and chemical reaction on the flow-field, the numerical values of fluid velocity within boundary layer region, computed from the analytical solutions (13) and (19), are displayed graphically versus boundary layer coordinate y for various values of dimensionless fixed time t_1, a, time t, radiation parameter N, permeability parameter K_1 ,Schimidth number S_c, solutal Grashof number G_c and chemical reaction parameter K_2 in figures 2 to 9 taking magnetic parameter $M=4$, Prandtl number $P_r=0.71$ (ionized air) and thermal Grashof number $G_r=20$. It is observed from figures 2 to 9 that, for both ramped temperature and isothermal plates, fluid velocity u attains a distinctive maximum value in the vicinity of the plate and then decreases properly on increasing boundary layer co-ordinate y to approach free stream value.

This is due to fact that thermal and solutal buoyancy forces have significant role on the fluid flow in the region near the plate and its effect is nullified in the free stream. Also, it is evident from figures 3 to 9 that fluid velocity is faster in case of isothermal plate than that of ramped temperature plate.This may be due to reason that in the case of ramped temperature plate, plate temperature T increases with respect to time t and attains value $T=1$ when $t>t_1$ whereas in case of isothermal plate, plate temperature is $T=1$ for every value of time t i.e. $t>0$. This means that plate temperature is cooler up to the critical time for rampedness t_1 in case of ramped temperature plate than that for isothermal plate. Fig. 2 depicts the effect of critical time for rampedness on fluid flow in the boundary layer region for ramped temperature plate. It is noticed from Fig. 2 that u decreases on increasing t_1. This implies that, for ramped temperature plate, fluid flow is getting decelerated in the boundary layer region with the increase in critical time for rampedness.

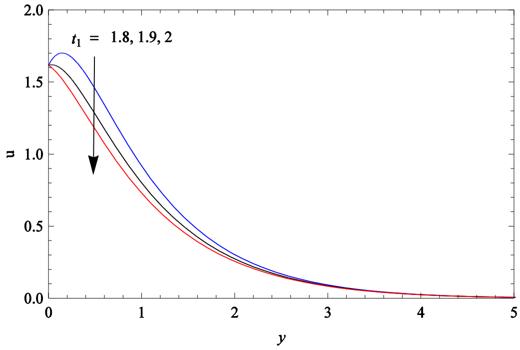

Fig. 2. Velocity profiles when $a=0.4$, $t=1.2$, $N=2$, $K_1=0.2$, $S_c=0.6$, $G_c=7$ and $K_2=0.5$.

Fig. 3 illustartes the effect of surface acceleration parameter on fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 3 that u increases on increasing a for both ramped temperature and isothermal plates. This implies that, for both ramped temperature and isothermal plates, fluid flow is getting accelerated in the boundary layer region with the increase in surface acceleration parameter.

Fig. 4 depicts the effect of time on fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 4 that u increases on increasing t for both ramped temperature and isothermal plates. This implies that fluid flow in the boundary layer region is getting accelerated with the progress of time for both ramped temperature and isothermal plates.

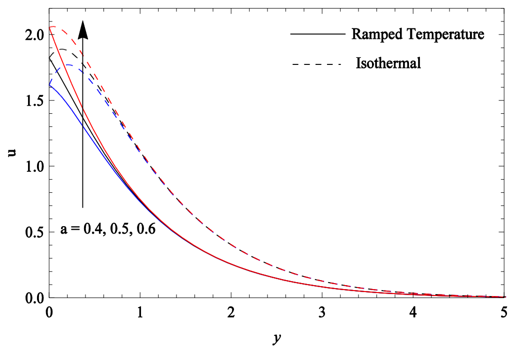

Fig. 3. Velocity profiles when $t_1=2$, $t=1.2$, $N=2$, $K_1=0.2$, $S_c=0.6$, $G_c=7$ and $K_2=0.5$.

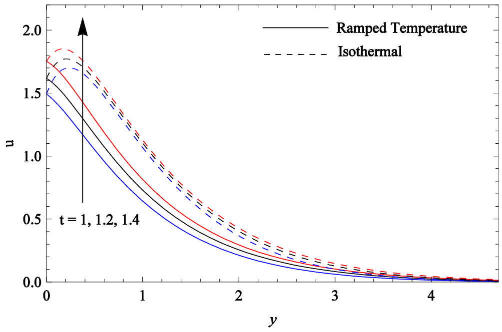

Fig. 4. Velocity profiles when $t_1 = 2$, $a = 0.4$, $N = 2$, $K_1 = 0.2$, $S_c = 0.6$, $G_c = 7$ and $K_2 = 0.5$.

Fig. 5 illustrates the effect of radiation on fluid velocity for both ramped temperature and isothermal plates.

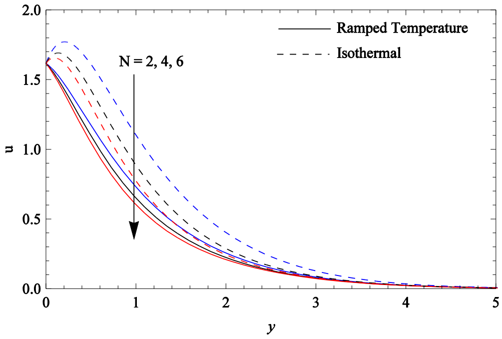

Fig. 5. Velocity profiles when $t_1 = 2$, $a = 0.4$, $t = 1.2$, $K_1 = 0.2$, $S_c = 0.6$, $G_c = 7$ and $K_2 = 0.5$.

It is noticed from Fig. 5 that, for both ramped temperature and isothermal plates, u decreases on increasing N. This implies that radiation has a tendency to decelerate the fluid flow in the boundary layer region for both ramped temperature and isothermal plates for optically thin radiating fluid. This is due to fact that fluid temperature is getting reduced due to thermal radiation and fluid flow in the boundary layer region is getting retarded.

Fig. 6 reveals the effect of permeability of porous medium on fluid flow in boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 6 that, for both ramped temperature and isothermal plates, u increases on increasing K_1. An increase in permeability of medium implies that there is a decrease in the resistance of the porous medium which in turn accelerate fluid flow in boundary layer region for both ramped temperature and isothermal plates.

Fig. 7 depicts the effects of mass diffusion on the fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 7 that, for both ramped temperature and isothermal plates, fluid velocity u decreases on increasing S_c. S_c represents the ratio of momentum diffusivity and molecular (mass) diffusivity. S_c decreases on increasing mass diffusivity. This implies that mass diffusion tends to accelerate the fluid flow in the boundary layer region for both ramped temperature and isothermal plates.

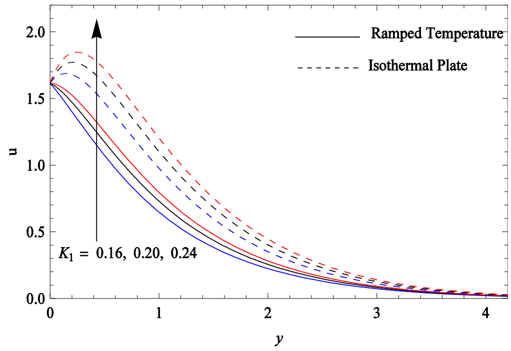

Fig. 6. Velocity profiles when $t_1 = 2$, $a = 0.4$, $t = 1.2$, $N = 2$, $S_c = 0.6$, $G_c = 7$ and $K_2 = 0.5$.

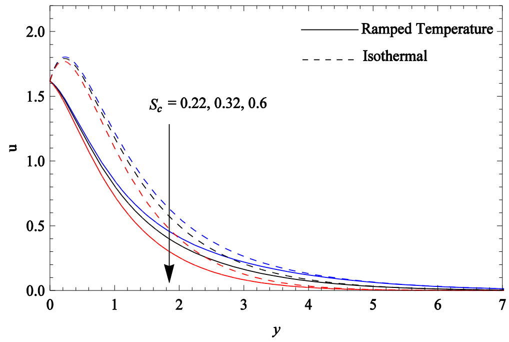

Fig. 7. Velocity profiles when $t_1 = 2$, $a = 0.4$, $t = 1.2$, $N = 2$, $K_1 = 0.2$, $G_c = 7$ and $K_2 = 0.5$.

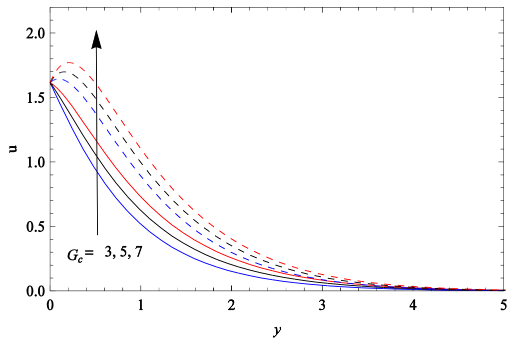

Fig. 8. Velocity profiles when $t_1 = 2$, $a = 0.4$, $t = 1.2$, $N = 2$, $K_1 = 0.2$, $S_c = 0.6$ and $K_2 = 0.5$.

Fig. 8 reveals the effects of solutal buoyancy force on the fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 8 that, for both ramped temperature and isothermal plates, fluid velocity u increases on increasing G_c. G_c signifies the relative strength of solutal buoyancy force to viscous force. G_c increases when solutal buoyancy force increases. This implies that solutal buoyancy force tends to accelerate the fluid flow in the boundary layer region for both ramped temperature and isothermal plates.

Fig. 9 exhibits the effect of chemical reaction on the fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 9 that u decreases on increasing K_2 for both ramped temperature and isothermal plates. This implies that chemical reaction tends to decelerate fluid flow in the boundary layer region for both ramped temperature and isothermal plates.

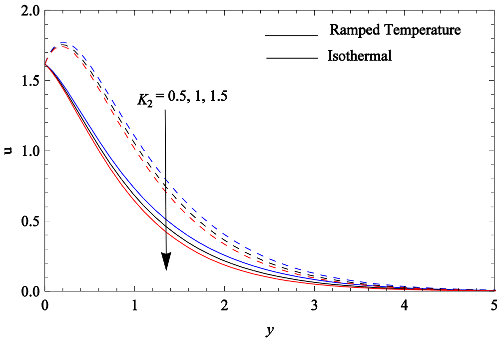

Fig. 9. Velocity profiles when $t_1 = 2$, $a = 0.4$, $t = 1.2$, $N = 2$, $K_1 = 0.2$, $S_c = 0.6$ and $G_c = 7$.

The numerical values of fluid temperature T, computed form analytical solutions (14) and (20), are depicted graphically in Figs. 10 to 12 for various values of critical time for rampedness t_1, time t and radiation parameter N taking $P_r = 0.71$. It is observed from Figs. 10 to 12 that fluid temperature T is maximum at the surface of the plate and decreases properly on increasing boundary layer co-ordinate y to approach free stream value. It is also noticed from Figs. 11 and 12 that fluid temperature is higher in case of isothermal plate than that of ramped temperature plate. Fig. 10 depicts the effect of critical time for rampedness on fluid temperature for ramped temperature plate. It is noticed from Fig. 10 that T decreases on increasing t_1 for ramped temperature plate. An increase in critical time for rampedness implies that there is a decrease in fluid temperature in the boundary layer region for ramped temperature plate.

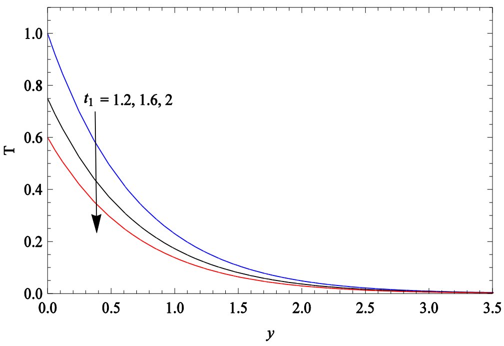

Fig. 10. Temperature profiles when $t = 1.2$ and $N = 2$.

Figs. 11 and 12 demonstrate the influence of time and radiation on fluid temperature T for both ramped temperature and isothermal plates. It is noticed from Figs. 11 and 12 that fluid temperature T increases on increasing t whereas it decreases on increasing N for both ramped temperature and isothermal plates. This implies that radiation has a tendency to reduce fluid temperature in the boundary layer region for both ramped temperature and isothermal plates. Fluid temperature is getting enhanced in the boundary layer region with the progress of time for both ramped temperature and isothermal plates. It is interesting to note from Fig.11 that fluid temperature at isothermal plate attains steady state when $t > 1.5$.

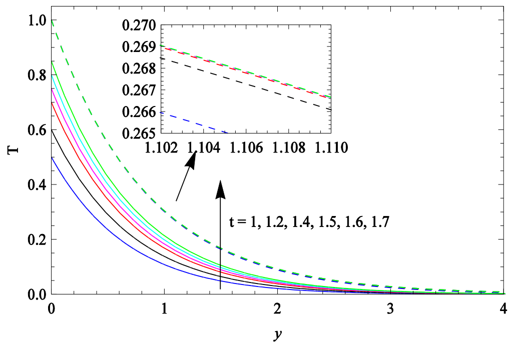

Fig. 11. Temperature profiles when $t_1 = 2a$ and $N = 2$.

The numerical values of species concentration C, computed form analytical solution (15), is depicted graphically in Figs. 13 and 14 for various values of K_2 and S_c taking $t = 1.2$. It is observed from Figs. 13 and 14 that species concentration C is maximum at the surface of the plate and it decreases properly on increasing boundary layer co-ordinate y to approach free stream value. It is noticed from Figs. 13 and 14 that C decreases on increasing either K_2 or S_c. This implies that chemical reaction tends to reduce species concentration whereas mass diffusion has a reverse effect on it.

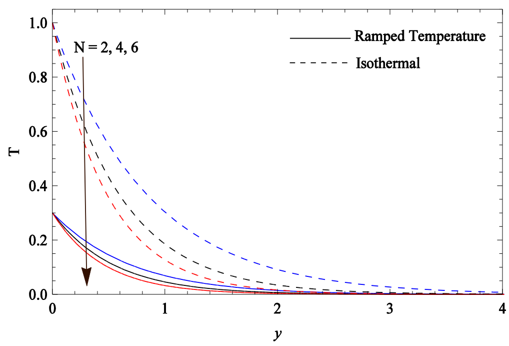

Fig. 12. Temperature profiles when $t_1 = 2$ and $t = 1.2$.

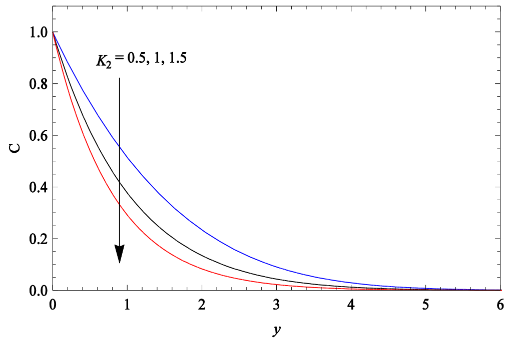

Fig. 13. Concentration profiles when S_c = 0.6.

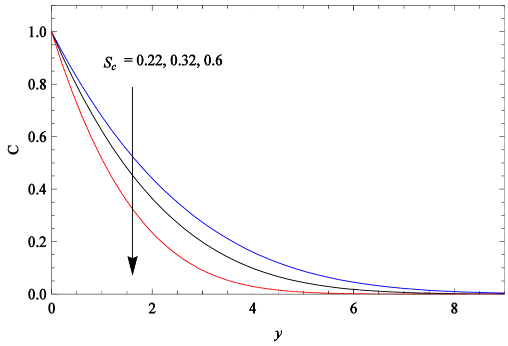

Fig. 14. Concentration profiles when K_2 = 0.5.

The numerical values of shear stress τ at the plate, computed from analytical expressions (21) and (23), are presented in tabular form in Tables 3 to 6 for various values of S_c, t_1, a, t, N, K_2, K_1 and G_c taking $M = 4, G_r = 20$ and $P_r = 0.71$ (ionized air).

Table 3 Shear stress τ at the plate when a = 0.4, t = 1.2, N = 2, K_2 = 0.5, K_1 = 0.2 and G_c = 7.

$S_c \downarrow$ $t_1 \rightarrow$		1.8	1.9	2
Ramped Temperature $(-\tau)$	0.22	0.00101	0.15313	0.29004
	0.32	0.05043	0.20259	0.33946
	0.6	0.15077	0.30289	0.43980

It is observed from Table 3 that shear stress τ at the plate increases on increasing either S_c or t_1 for ramped temperature plate. This implies that mass difusion tends to reduce shear stress at ramped temperature plate. An increase in critical time for rampedness implies that there is an enhancement in shear stress at ramped temperature plate. It is noticed from Table 4 that τ increases on increasing either a or t for ramped temperature plate whereas it decreases on increasing either a or t for isothermal plate. An increase in surface acceleration parameter and time there is an enhancement in shear stress at ramped temperature plate whereas there is a reduction in the shear stress at isothermal plate.

Table 4 Shear stress τ at the plate when S_c = 0.6, t_1 = 2, N = 2, K_2 = 0.5, K_1 = 0.2 and G_c = 7.

$a \downarrow$ $t \rightarrow$		1	1.2	1.4
Ramped Temperature $(-\tau)$	0.4	0.36303	0.43980	0.55320
	0.5	1.06106	1.10116	1.20237
	0.6	1.62500	1.85044	2.17257
Isothermal plate (τ)	0.4	2.06136	1.71403	1.32218
	0.5	1.55350	1.05268	0.482837
	0.6	0.98958	0.30340	0.29452

It is analyzed from Table 5 that, for ramped temperature plate, τ increases on increasing either N or K_2 whereas it decreases on increasing for isothermal plate. This implies that radiation and chemical reaction tend to enhance the shear stress at ramped temperature plate whereas it have reverse effect on the shear stress at isothermal plate. It is revealed from Table 6 that τ decreases on increasing either K_1 or G_c for ramped temperature plate whereas it increases on increasing either K_1 or G_c for isothermal plate. This implies that permeability of porous medium and solutal buoyancy force tend to reduce the shear stress at ramped temperature plate whereas these agencies have reverse effect on the shear stress at isothermal plate.

Table 5 Shear stress τ at the plate when S_c = 0.6, t_1 = 2, a = 0.4, t = 1.2, K_1 = 0.2 and G_c = 7.

$N \downarrow$ $K_2 \rightarrow$		0.5	1	1.5
Ramped Temperature $(-\tau)$	2	0.43980	0.52305	0.59174
	4	0.65161	0.73485	0.80355
	6	0.80346	0.88671	0.95540
Isothermal plate (τ)	2	1.71403	1.63079	1.56209
	4	1.22640	1.14315	1.07446
	6	0.90824	0.82500	0.69898

Table 6 Shear stress τ at the plate when S_c = 0.6, t_1 = 2, a = 0.4, t = 1.2, N = 2 and K_2 = 0.5.

$K_1 \downarrow$ $G_c \rightarrow$		3	5	7
Ramped Temperature $(-\tau)$	0.16	1.99723	1.47851	0.95979
	0.20	1.53334	0.98657	0.43980
	0.24	1.19918	0.63079	0.06240
Isothermal plate (τ)	0.16	0.04290	0.56162	1.08034
	0.20	0.62049	1.16726	1.71403
	0.24	1.04295	1.61134	2.17973

The numerical values of rate of heat transfer at the plate i.e. N_u, computed from the analytical expressions (22) and (24), are presented in tabular form in Tables 7 and 8 for various values of N, t, P_r and t_1 whereas those of rate of mass transfer at the plate i.e. S_h, computed from the analytical expression (25), is presented in tabular form in Table 9 for different values of K_2 and S_c taking $t = 1.2$.

It is noticed form Table 7 that, for both ramped temperature and isothermal plates, N_u increases on increasing N. N_u increases on increasing t for ramped temperature plate whereas it decreases on increasing t for isothermal plate. This implies that radiation tends to enhance the rate of heat transfer at both the ramped temperature and isothermal plates. As time progresses, rate of heat transfer is getting enhanced at ramped temperature plate whereas it is getting reduce at isothermal plate.

Table 7 Rate of heat transfer N_u at the plate when P_r = 0.71 and t_1 = 2.

$N \rightarrow$ / $t \downarrow$		2	4	6
Ramped Temperature	1	0.48301	0.61614	0.72672
	1.2	0.56096	0.72571	0.86089
	1.4	0.63868	0.83527	0.99505
	1.5	0.67750	0.89005	1.06213
	1.6	0.716292	0.94482	1.12921
	1.7	0.75507	0.99959	1.19630
Isothermal plate	1	1.19477	1.68523	2.06398
	1.2	1.19218	1.68523	2.06398
	1.4	1.19171	1.68523	2.06398
	1.5	1.19166	1.68523	2.06398
	1.6	1.19164	1.68523	2.06398
	1.7	1.19164	1.68523	2.06398

Table 8 Rate of heat transfer $N_u a$ at the plate when N = 2 and t = 1.2.

$P_r \downarrow$ $t_1 \rightarrow$	1.8	1.9	2
0.3	0.62328	0.59048	0.56096
0.5	0.80466	0.76231	0.72419
0.71	0.95886	0.90839	0.86297

Table 9 Rate of mass transfer at the plate.

$K_2 \downarrow$ $S_c \rightarrow$	0.22	0.32	0.6
0.5	0.37359	0.45057	0.61696
1.0	0.48489	0.58480	0.80077
1.5	0.58120	0.70095	0.95981

It is interesting to note from Table 7 that rate of heat transfer at isothermal plate attains steady state when $t > 1.5$. It is evident from Table 8 that, for ramped temperature plate, N_u increases on increasing P_r whereas it decreases on increasing t_1. Since P_r is a measure of the relative strength of momentum diffusivity to thermal diffusivity of fluid, P_r decreases on increasing thermal diffusivity of fluid. This implies that thermal diffusivity tends to reduce rate of heat transfer at ramped temperature plate. An increase in critical time for rampedness there is a reduction in rate of heat transfer at ramped temperature plate. It is revealed from Table 9 that S_h increases on increasing either K_2 or S_c. This implies that chemical reaction tends to enhance rate of mass transfer at the plate whereas mass diffusion has a reverse effect on it.

5. CONCLUSIONS

Investigation of unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, chemically reactive and optically thin radiating fluid past an exponentially accelerated moving vertical plate with variable ramped temperature embedded in a fluid saturated porous medium is carried out. Significant finding are as follows:

- Critical time for rampedness tends to reduce fluid fluid flow as well as fluid temperature in boundary layer region.
- For both ramped temperature and isothermal plates:
  Surface acceleration parameter, permeability of porous medium, mass diffusion, solutal buoyancy force and time tend to accelerate fluid flow in boundary layer region whereas radiation and chemical reaction have reverse effect on it. Radiation tends to reduce fluid temperature and fluid temperature is getting enhanced with progress of time. It approaches steady state when $t > 1.5$ for isothermal plate.
- Chemical reaction tends to reduce species concentration whereas mass diffusion has a reverse effect on it.
- We have compared numerical values of shear stress at the plate of Das *et al*. (2011) as a special case of our result which are in excellent agreement with the numerical values of Das *et al*. (2011).
- Critical time for rampedness tends to enhance shear stress at the plate whereas mass diffusion has a reverse effect on it for ramped temperature plate.
- Chemical reaction tends to enhance rate of mass transfer at the plate whereas mass diffusion has a reverse effect on it.

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Appendix

$$f_1(c_1,c_2,c_3,c_4)=e^{c_1\sqrt{(c_2+c_3)}}erfc\left(\frac{c_1}{2\sqrt{c_4}}+\sqrt{(c_2+c_3)c_4}\right)$$
$$+e^{-c_1\sqrt{(c_2+c_3)}}erfc\left(\frac{c_1}{2\sqrt{c_4}}-\sqrt{(c_2+c_3)c_4}\right)$$

$$f_2(c_1,c_2,c_3,c_4,c_5)=e^{c_1\sqrt{c_4(c_2+c_3)}}erfc\left(\frac{c_1}{2}\sqrt{\frac{c_4}{c_5}}+\sqrt{(c_2+c_3)c_5}\right)+$$
$$e^{-c_1\sqrt{c_4(c_2+c_3)}}erfc\left(\frac{c_1}{2}\sqrt{\frac{c_4}{c_5}}-\sqrt{(c_2+c_3)c_5}\right)$$

$$f_3(c_1,c_2,c_3,c_4,c_5)=e^{c_1\sqrt{c_2(c_3+c_4)}}erfc\left(\frac{c_1}{2}\sqrt{\frac{c_2}{c_5}}+\sqrt{(c_3+c_4)c_5}\right)+$$
$$e^{-c_1\sqrt{c_2(c_3+c_4)}}erfc\left(\frac{c_1}{2}\sqrt{\frac{c_2}{c_5}}-\sqrt{(c_3+c_4)c_5}\right)$$

$$f_4(c_1,c_2,c_3,c_4,c_5)=\left(c_5+\frac{1}{c_4}+\frac{c_1}{2}\sqrt{\frac{c_2}{c_3}}\right)e^{c_1\sqrt{c_2c_3}}erfc\left(\frac{c_1}{2}\sqrt{\frac{c_2}{c_5}}\right.$$
$$\left.+\sqrt{c_3c_5}\right)+\left(c_5+\frac{1}{c_4}-\frac{c_1}{2}\sqrt{\frac{c_2}{c_3}}\right)e^{-c_1\sqrt{c_2c_3}}$$

$$\times erfc\left(\frac{c_1}{2}\sqrt{\frac{c_2}{c_5}}-\sqrt{c_3c_5}\right)$$

$$f_5(c_1,c_2,c_3)=\left(c_3+\frac{c_1}{2\sqrt{c_2}}\right)e^{c_1\sqrt{c_2}}erfc\left(\frac{c_1}{2\sqrt{c_2}}+\sqrt{c_2c_3}\right)$$
$$+\left(c_3-\frac{c_1}{2\sqrt{c_2}}\right)e^{-c_1\sqrt{c_2}}erfc\left(\frac{c_1}{2\sqrt{c_2}}-\sqrt{c_2c_3}\right)$$

$$f_6(c_1,c_2,c_3)=-\frac{e^{-(c_1+c_2)c_3}}{\sqrt{\pi c_3}}+\sqrt{(c_1+c_2)}\left(erfc\left(\sqrt{(c_1+c_2)c_3}\right)-1\right)$$

$$f_7(c_1,c_2,c_3)=\sqrt{(c_1+c_2)}\left(erfc\left(\sqrt{(c_1+c_2)c_3}\right)-1\right)$$
$$-\sqrt{c_1}\left(erfc\left(\sqrt{c_1c_3}\right)-1\right)$$

$$f_8(c_1,c_2,c_3,c_4)=\sqrt{(c_1+c_2)c_3}\left(erfc\left(\sqrt{(c_1+c_2)c_4}\right)-1\right)$$
$$-\sqrt{c_1c_3}\left(erfc\left(\sqrt{c_1c_4}\right)-1\right)$$

$$f_9(c_1,c_2,c_3,c_4)=\left\{\left(\frac{c_1}{c_2}\right)\sqrt{\frac{c_2}{c_3}}+\sqrt{c_2c_3}+c_1c_4\sqrt{c_2c_3}\right\}$$
$$\times\left(erfc\left(\sqrt{c_3c_4}\right)-1\right)$$

$$f_{10}(c_1,c_2,c_3,c_4,c_5)=\frac{e^{-c_4c_5}c_2\sqrt{c_3c_5}}{\sqrt{\pi}}e^{c_2c_5}\sqrt{c_3(c_4+c_2)}\times$$
$$\left(erfc\left(\sqrt{(c_2+c_4)c_5}\right)-1\right)+\frac{1}{c_1}$$
$$\left[\left(\frac{1}{2}\right)\left(c_2+2c_1(1+c_2t)\right)\times\right.$$

$$\left(erfc\left(\sqrt{c_1 c_5}\right)-1\right)-\sqrt{c_1}\left\{c_2\sqrt{\frac{c_5}{\pi}}\right.$$
$$+\sqrt{(c_1+c_2)}e^{c_2 c_5}\times$$
$$\left.\left(erfc\left(\sqrt{(c_1+c_2)c_5}\right)-1\right)\right\}\Bigg]$$

$$f_{11}(c_1,c_2,c_3)=e^{c_2 c_3}\left[\frac{e^{-(c_1+c_2)c_3}}{\sqrt{c_3\pi}}+\sqrt{(c_1+c_2)}\times\right.$$
$$\left.\left(erfc\left(\sqrt{(c_1+c_2)c_3}\right)-1\right)\right]$$

$$f_{12}(c_1,c_2,c_3,c_4,c_5)=e^{c_2 c_5}\left[-\sqrt{(c_1+c_2)}\left(erfc\left(\sqrt{(c_1+c_2)c_5}\right)-1\right)\right.$$
$$\left.+\sqrt{c_3(c_2+c_4)}\left(erfc\left(\sqrt{(c_2+c_4)c_5}\right)-1\right)\right]$$