

Ohmic Heating and Viscous Dissipation Effects over a Vertical Plate in the Presence of Porous Medium

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ABSTRACT

An analysis is performed to investigate the ohmic heating and viscous dissipation effects on an unsteady natural convective flow over an impulsively started vertical plate in the presence of porous medium with radiation and chemical reaction. Numerical solutions for the governing boundary layer equations are presented by finite difference scheme of the Crank Nicolson type. The influence of various parameters on the velocity, the temperature, the concentration, the skin friction, the Nusselt number and the Sherwood number are discussed. It is observed that velocity and temperature increases with increasing values of permeability and increasing values of Eckert number, whereas it decreases with increasing values of magnetic parameter. An increase in ohmic heating and viscous heating increases the velocity boundary layer. An increase in ohmic heating decreases the temperature. An increase in magnetic field reduces the temperature profile. The velocity profile is highly influenced by the increasing values of permeability. It is observed that permeability has strong effect on velocity. An enhancement in ohmic heating increases the shear stress, decreases the rate of heat transfer and induces the rate of mass transfer.

Keywords: Ohmic heating; Viscous dissipation; Chemical reaction; Porous medium; Finite difference.

NOMENCLATURE

c_{∞}'	ambient concentration
c_w'	concentration near the wall
С	concentration
D	mass diffusivity
Ec	Eckert number
g	acceleration due to gravity
Gc	mass Grashof number
Gr	thermal Grashof number
k	thermal conductivity
kc	chemical reaction parameter
$\mathbf{k}_{\mathbf{m}}$	mean absorption coefficient
Μ	magnetic parameter
R	radiation parameter
Sc	Schmidt number

Т temperature T_{∞}' ambient temperature T_w' wall temperature t time п velocity in x- direction uniform velocity of the plate uo Prandtl number Pr velocity in y-direction v specific heat at constant pressure Cp kinematic viscosity υ permeability parame λ dynamic viscosity μ Stephan Boltzmann constant σ

1. INTRODUCTION

Heat and mass transfer effects on natural convection due to the combined buoyancy effects of thermal diffusion and diffusion of chemical species in the presence of porous medium has attracted many researches for the past few decades. This is because of its real time importance in the fields of geothermal and geophysical engineering such as migration of moisture through air contained in fibrous insulation, nuclear wastes disposal through underground, disposal of chemical contaminants through water saturated soil, food processing, chemical catalytic reactors and cooling of nuclear reactor. So far the existing work has been devoted to the phenomenon of natural convection with porous medium driven by combined buoyancy effect due to temperature and concentration variations, not to phenomenon driven by unsteady combined buoyancy effect due to temperature and concentration variations in the presence of porous medium with ohmic heating and viscous heating effects.

Ruan *et al.* (2001) discussed many aspects about ohmic heating. The principle of ohmic heating is based on the passage of alternating electrical current (AC) through a body such as a liquid-particulate food system which serves as an electrical resistance in which heat is generated. AC voltage is applied to the electrodes at both ends of the product body. The concept of ohmic heating of foods is not new. In the nineteenth century, several processes were patented that used electrical current for heating flowable materials. In the early twentieth century, 'electric' pasteurization of milk was achieved by passing milk between parallel plates with a voltage difference between them.

Bejan and Khair (1985) discussed the phenomenon of natural convection heat and mass transfer near a vertical surface embedded in fluid saturated porous medium. They are the first to introduce the combined buoyancy effects across the boundary layer in the presence of porous medium. Later in (1986) Trevisan and Bejan investigated the analytical and numerical study of natural convection heat and mass transfer through a vertical porous layer subjected to uniform fluxes of heat and mass from the side. Bejan and Neild (1992) analyzed the natural convection boundary layer flow in porous media owing to combined heat and mass transfer.

Raptis et al. (1987) presented the explicit finite difference scheme to study the unsteady free convective flow through a porous medium bounded by a semi-infinite vertical plate. Soundalgekar (1973) analyzed the viscous dissipative heat on the unsteady free convective oscillatory flow past an infinite vertical porous plate. Soundalgekar and Pop (1974) discussed the viscous dissipation effects on natural convective flow past an infinite vertical porous plate with variable suction. Effect of viscous dissipation on non-Darcy natural convection flow along a vertical wall embedded in a saturated porous medium was initiated by Murthy and Singh (1997). Viscous dissipation and radiation effects on hydromagnetic free convection flow past an impulsively started vertical plate with variable surface temperature and concentration is studied by Suneetha et al. (2008). Finite difference scheme is used to present the solution.

Poornima and Bhaskar reddy (2013) presented the approximate solution for the effects of thermal radiation and chemical reaction on magnetohydrodynamic natural convective flow past a semi-infinite vertical plate in the presence of moving porous plate. Palani and Srikanth (2009) studied the magnetohydrodynamic effects in a natural convective flow past a semi-infinite vertical plate with mass transfer. Gnaneswara Reddy (2012) presented an approximate solution to the unsteady laminar flow of a viscous incompressible micropolar fluid past a vertical porous plate in the presence of a transverse magnetic field and thermal

radiation with variable heat flux.

Unsteady laminar free convection boundary layer flow of a moving infinite vertical plate in a radiative and chemically reactive medium in the presence of transverse magnetic field was investigated by Reddy et al. (2013). Combined effects of viscous and ohmic heating in the transient, natural convective flow of a doubly stratified fluid past a vertical plate with radiation and chemical reaction was analyzed by Ganesan et al. (2013). MHD forced convective flow through porous medium over a fixed horizontal channel with thermal insulated and impermeable bottom wall in the presence of viscous dissipation and Joule heating was discussed by Raju at al. (2014). Combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to a vertical surface is analyzed, taking into account the effects of ohmic heating and viscous dissipation was illustrated by Chen (2004).

Based on the important factors discussed by various authors, the present article analyzes the viscous dissipation and ohmic heating effects on an unsteady natural convective flow over a impulsively started vertical plate in the presence of porous medium. Also, the MHD (magnetohydrodynamic), radiation and chemical reactions are taken into account. The governing boundary layer equations are solved by finite difference scheme of Crank-Nicolson scheme. The numerical results are compared with the previous study to ensure the accuracy. The effects of various parameters on velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are computed. The various results are illustrated graphically.

2. MATHEMATICAL ANALYSIS

An unsteady, two dimensional, laminar natural convective flow over an impulsively started vertical plate in the presence of porous medium is considered. Initially, it is assumed that the fluid and the plate are at the same temperature and concentration. As time increases, the plate is given an impulsive motion in the vertical upward direction with a constant velocity u_0 and at the same time the temperature of the plate is raised to T_w' and the concentration near the plate is raised to c_w' . All the fluid properties are assumed to be

constant except the body force term. The Rosseland approximation is used to describe the radiative heat flux and the viscous dissipation and ohmic heating are also considered in this analysis. The x-axis is taken along the direction of vertical moving plate and y-axis is taken normal to the plate. The flow model and the coordinate system are given in Fig.1. Under the above assumptions the boundary layer equations for the flow using Boussinesq's approximation are given by

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{y}} = \mathbf{0} \tag{1}$$



Fig. 1. Flow model and the coordinate system.

$$\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \beta (T' - T'_{\infty}) + g \beta^* (c' - c'_{\infty})$$

$$+ v \frac{\partial^2 u}{\partial y^2} - \frac{v}{\lambda'} u - \frac{\sigma B_0^2 u}{\rho}$$

$$\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y}$$

$$+ \frac{v}{c_p} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2 u^2}{\rho c_p}$$

$$(3)$$

$$\frac{\partial c'}{\partial t'} + u \frac{\partial c'}{\partial x} + v \frac{\partial c'}{\partial y} = D \frac{\partial^2 c'}{\partial y^2} - k_l (c' - c'_{\infty}) \tag{4}$$

Initial and boundary conditions are

$$t' \le 0$$
 $u = 0, v = 0, T' = T'_w, c' = c'_w$ for all x and y
 $t' > 0$ $u = u_0, v = 0, T' = T'_w, c' = c'_w$ at y = 0

$$u = 0, v = 0, T' = T_{\infty}', c' = c_{\infty}' \text{ at } \mathbf{x} = 0$$
 (5)

 $u \to 0, \ T' \to T_{\infty}', \ c' \to c_{\infty}' \text{ as } y \to \infty$

Now introducing the non-dimensional quantities

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$$X = \frac{xu_0}{\upsilon}, Y = \frac{yu_0}{\upsilon}, U = \frac{u}{u_0}, V = \frac{v}{u_0}, t = \frac{t'u_0^2}{\upsilon}, T = \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}}, C = \frac{c' - c_{\infty}'}{c'_w - c_{\infty}'}, Gr = \frac{\upsilon g \beta(T'_w - T'_{\infty})}{u_0^3}, Gc = \frac{\upsilon g \beta^*(c'_w - c_{\infty}')}{u_0^3}, Pr = \frac{\mu c_p}{k}, Sc = \frac{\upsilon}{D}, \lambda = \frac{\lambda' u_0^2}{\upsilon^2}, k_c = \frac{k_l \upsilon}{u_0^2}, R = \frac{kk_m}{4\sigma T'_{\infty}^3}, M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2} Ec = \frac{u_0^2}{c_p(T'_w - T'_{\infty})}$$
(6)

The governing equations in non-dimensional form is given by

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{7}$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = GrT + GcC + \frac{\partial^2 U}{\partial Y^2}$$
$$-\frac{U}{\lambda} - MU$$
(8)

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{\Pr} \left(1 + \frac{4}{3R} \right) \frac{\partial^2 T}{\partial Y^2} + Ec \left(\frac{\partial U}{\partial Y} \right)^2 + Ec M U^2$$
(9)

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - k_c C$$
(10)

The corresponding boundary conditions are $t \le 0$ U = 0, V = 0, T = 0, C = 0 for all X and Y t > 0 U = 1, V = 0, T = 1, C = 1 at Y = 0 U = 0, V = 0, T = 0, C = 0 at X = 0 (11) $U \rightarrow 0$, $T \rightarrow 0$, $C \rightarrow 0$ as $Y \rightarrow \infty$

3. NUMERICAL PROCEDURE

The two-dimensional, unsteady, coupled, non-linear partial differential equations (7) to (10) subject to the boundary conditions in equation (11) are discretized with Crank Nicolson implicit finite difference scheme which converges faster and is unconditionally stable. Based on the boundary conditions in equation (11), the region of integration is decided as Xmax=1 and Ymax=14, where Y_{max} corresponds to Y=∞. Here the subscript i designates the grid points in the direction of X, j designates the grid points in the direction of Y, and n along the t - direction. The equations at every internal nodal point for a particular i-level constitute a tridiagonal system. This system of tridiagonal matrix can be solved by Thomas algorithm (1969). Hence the values of U, V, T, and C are known at all nodal points in the region at $(n+1)^{th}$ time level. Computations are carried out for all the time levels until the steady state is reached. The scheme is proved to be unconditionally stable using the Von Neumann technique. The local truncation error for the scheme is $0(\Delta t^2 + \Delta Y^2 + \Delta X)$ and it approaches zero as Δt , ΔY and ΔX tends to zero. Stability and compatibility of the scheme ensures convergence.

4. METHOD OF SOLUTION

The Crank Nicolson finite difference scheme for the governing equations (7) to (10) are given by

$$\frac{1}{4\Delta X} \begin{bmatrix} U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n} + U_{i,j-1}^{n+1} - U_{i-1,j-1}^{n+1} \\ U_{i,j-1}^{n} - U_{i-1,j-1}^{n} \end{bmatrix} \\ + \frac{1}{2\Delta Y} \begin{bmatrix} V_{i,j}^{n+1} - V_{i,j-1}^{n+1} + V_{i,j}^{n} - V_{i,j-1}^{n} \end{bmatrix} = 0$$
(12)
$$\frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{\Delta t} + \frac{U_{i,j}^{n}}{2\Delta X} \begin{bmatrix} U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} \\ -U_{i-1,j}^{n} \end{bmatrix}$$

$$+\frac{V_{i,j}^{n}}{4\Delta Y}\left[U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}+U_{i,j+1}^{n}-U_{i,j-1}^{n}\right]$$
$$=Gr\left[\frac{T_{i,j}^{n+1}+T_{i,j}^{n}}{2}\right]+Gc\left[\frac{C_{i,j}^{n+1}+C_{i,j}^{n}}{2}\right]$$

$$+\frac{1}{2(\Delta Y)^{2}}\begin{bmatrix}U_{i,j+1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j-1}^{n+1}+U_{i,j+1}^{n}\\-2U_{i,j}^{n}+U_{i,j-1}^{n}\end{bmatrix}\\-\left(\frac{1}{\lambda}+M\right)\begin{bmatrix}U_{i,j}^{n+1}+U_{i,j}^{n}\\2\end{bmatrix}$$
(13)

$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} + \frac{U_{i,j}^{n}}{2\Delta X} \left[T_{i,j}^{n+1} - T_{i-1,j}^{n+1} + T_{i,j}^{n} - T_{i-1,j}^{n} \right] \\
+ \frac{V_{i,j}^{n}}{4\Delta Y} \left[T_{i,j+1}^{n+1} - T_{i,j-1}^{n+1} + T_{i,j+1}^{n} - T_{i,j-1}^{n} \right] \\
= \frac{1}{2 \operatorname{Pr}(\Delta Y)^{2}} \left(1 + \frac{4}{3R} \right) \left[T_{i,j+1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j-1}^{n+1} \right] \\
+ Ec \left(\frac{U_{i,j+1}^{n} - U_{i,j}^{n}}{\Delta Y} \right)^{2} + EcM \left(\frac{U_{i,j}^{n+1} + U_{i,j}^{n}}{2} \right)^{2} \tag{14}$$

$$\begin{aligned} \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} + \frac{U_{i,j}^{n}}{2\Delta X} \Big[C_{i,j}^{n+1} - C_{i-1,j}^{n+1} + C_{i,j}^{n} - C_{i-1,j}^{n} \Big] \\ + \frac{V_{i,j}^{n}}{4\Delta Y} \Big[C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1} + C_{i,j+1}^{n} - C_{i,j-1}^{n} \Big] \\ = \frac{1}{2Sc(\Delta Y)^{2}} \begin{bmatrix} C_{i,j+1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j-1}^{n+1} + C_{i,j+1}^{n} \\ - 2C_{i,j}^{n} + C_{i,j-1}^{n} \end{bmatrix} \\ -k_{c} \Bigg[\frac{C_{i,j}^{n+1} + C_{i,j}^{n}}{2} \Bigg] \end{aligned}$$

(15)

These equations (12) to (15) at every nodal point for a particular ith level constitute a tridiagonal system of equations. In order to get the physical insight into the problem the numerical values of U, V, T and C are computed for different values of ohmic heating and viscous dissipation, thermal Grashof number, mass Grashof number, Prandtl number Schmidt number, permeability parameter, magnetic parameter, radiation and chemical reaction parameter. Knowing the velocity, temperature and concentration it is interesting to calculate the skin friction coefficient, local Nusselt number, local Sherwood number, average skin friction, average Nusselt number and average Sherwood number. Five point approximations is used to approximate the derivatives in local skin friction, local Nusselt number and local Sherwood number. Newton Cote's formula is used to calculate the average skin friction, average Nusselt number and average Sherwood number.

5. **RESULTS AND DISCUSSIONS**

In order to get the physical insight into the problem, numerical computations are carried out for various values of parameters that describe the flow characteristics and the results are illustrated graphically. Finite difference scheme of Crank Nicolson type is used to obtain the results for the velocity, temperature, concentration, local skin friction, local Nusselt number, local Sherwood number, average skin friction, average Nusselt number and average Sherwood number. The fluids considered under study are air (Pr = 0.73) and water (Pr = 7). The Schmidt number is choosen in such a way that they represent Hydrogen (0.16), Oxygen (0.60), Ammonia (0.78), Carbon dioxide (0.94) and Ethyle benzene (2.0).

In order to ascertain the accuracy, the results from the present study are compared with those from previous study. In the absence of porous medium and taking the chemical reaction parameter as zero, the velocity and temperature profile for different values of the Eckert number are compared with that of suneetha *et al.* (2008) in Fig. 2. It is observed that the present results are in good agreement with that of the previous result.



Fig. 2. Comparison for velocity and temperature.

Fig. 3 illustrates the combined effects of viscous dissipation and ohmic heating in the presence of porous medium on velocity. The ohmic heating effect is characterized by the product of Eckert number and magnetic parameter. Ohmic heating is the generation of excess heat in the fluid either due to direct current or applied magnetic field. Eckert number expresses the relation between kinetic energy and the enthalphy.

The viscous dissipation is characterized by Eckert number. Both ohmic heating and viscous dissipation plays an important role in the thermal transport of the fluid. It is revealed from the figure that, for a particular value of magnetic parameter and increasing values of viscous dissipation increases the velocity boundary layer. Viscous dissipation increases the velocity boundary layer when the magnetic field is zero. But the viscous dissipation gradually reduces the velocity boundary layer with increasing values of magnetic field. An increase in ohmic heating and viscous heating increases the velocity boundary layer.



Fig. 4 represents the velocity profile for different permeability parameter. Permeability is the measure of the materials ability to permit liquid or gas through its pores or voids. Filters made of soil and earth dams are very much based upon the permeability of a saturated soil under load. Permeability is a part of the proportionality constant in Darcy's law. Darcy's law relates the flow rate and fluid properties to the pressure gradient applied to the porous medium. Hence for an increase in the permeability of the porous medium the velocity boundary layer increases. Thus the velocity profile is highly influenced by the enhancement of permeability.

Fig. 4. Effects on permeability on the velocity profile.

Fig. 5 depicts the combined effects of viscous dissipation and ohmic heating in the presence of porous medium on temperature. From the figure it is viewed that viscous dissipation increases the thermal boundary layer when the magnetic field is zero whereas the absence of viscous dissipation reduces the thermal boundary layer. For fixed values of Eckert number an increase in magnetic parameter reduces the temperature profile. Hence an increase in ohmic heating decreases the thermal boundary layer.



Fig. 5. Viscous and ohmic heating effects on temperature profile.

Figures (6) and (7) represent the temperature profile for Prandtl number and Radiation parameter respectively. In Fig. 6 the size of the thermal boundary layer increases with increasing values of Prandtl number. Prandtl number being the ratio of momentum diffusivity to the thermal diffusivity, together with ohmic heating induces the temperature. Hence, there is an increase in the thermal boundary layer. Fig. 7 depicts that increase in radiation parameter increases the thermal boundary layer. The increasing value of radiation corresponds to an increased dominance of conduction. This radiation parameter along with Eckert number and magnetic field enhances the thermal boundary layer thickness.





Fig. 8 shows that the concentration boundary layer increases with reducing values of chemical reaction parameter. The influence of chemical reaction parameter is analyzed for $k_c = 0, 0.1, 0.3, 0.5, 1$ and 2. In the present study a homogeneous first order chemical reaction is considered. A first order reaction is a reaction that proceeds at a rate that depends linearly on only one reactant concentration. In a homogeneous reaction the diffusion species can either be created or destroyed depending on the values of chemical reaction parameter. In the concentration profiles, a destructive reaction is considered. Hence it is observed that the concentration boundary layer decreases with increasing values of chemical reaction parameter.

Fig. 9 depicts that an increase in Schmidt number reduces the concentration boundary layer. Schmidt number is the ratio between the momentum diffusivity and the species diffusivity. The mass diffusion is based on the diffusion of the substance. It physically relates the relative thickness of the viscous boundary layer to the concentration boundary layer. Hence, the concentration boundary layer decreases with increasing values of Schmidt number.



Fig. 8. Effects of chemical reaction on concentration profile.



Figures (10) and (11) represent the viscous dissipation and ohmic heating effects on local skin friction and average skin friction respectively. An increase in ohmic heating increases the local shear stress and the average shear stress. Keeping all the other parameters fixed an increase in permeability and magnetic parameter decreases the local shear stress. An increase in viscous dissipation increases the local skin friction and average shear stress. This is due to the fact that an increase in the Ec raises the velocity which in turn increases the shear stress along the plate, and hence there is an increase in the skin friction. It is also viewed that an increase in permeability increases the average skin friction. This is because increase in permeability enhances the velocity thereby increasing the skin friction.



Figure (12) and (13) illustrates the effects of Ec, λ and M on Local Nusselt number and average Nusselt number respectively. An increase in Ec number decreases the local Nusselt number and average Nusselt number. An increase in permeability and magnetic field reduces the local Nusselt number and average Nusselt number and average Nusselt number. A decrease in ohmic heating effect increases the local and average Nusselt number. This is because increase in ohmic heating increases the temperature profile which inturn reduces the heat transfer and hence there is a decrease in the Nusselt number.



Nusselt number.



Fig. 14 represents the effects of chemical reaction and Schmidt number on local Sherwood number. It is viewed that an increase in chemical reaction for fixed values of Schmidt number the local Sherwood number increases. Similar type of behavior is seen for fixed chemical reaction parameter and increasing Schmidt number.



In Fig. 15 average Sherwood number is presented for Ec, λ and M. Average Sherwood number shows a slight influence for increasing values of Eckert number. An increase in the ohmic heating effect increases the average Sherwood number. Also increasing values of permeability and magnetic field increase the average Sherwood number.

6. CONCLUSION

In the present work, a numerical analysis is carried out to analyze the viscous and ohmic heating effects on natural convective flow over an impulsively started vertical plate in the presence of porous medium. The effects of radiation and chemical reaction are also considered. The non-dimensional governing boundary layer equations are solved numerically by finite difference scheme of Crank Nicolson type. The effects of Eckert number, Prandtl number, Schmidt number, permeability, radiation, magnetic field, chemical reaction, thermal Grashof number, mass Grashof number on velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are investigated. The results are illustrated graphically. The results are summarized as follows

- The velocity boundary layer increase with increasing values of permeability parameter.
- The velocity boundary layer and thermal boundary layer thickness reduces for increasing values of Eckert number and the magnetic field.
- The thermal boundary layer increases with increasing values of Prandtl number and also for increasing values of radiation parameter.
- The concentration boundary layer decreases with increasing values of chemical reaction and Schmidt number.
- 5) The local Sherwood number increases with increasing values of chemical reaction parameter and Schmidt number.
- 6) An increase in Eckert number and magnetic field increases the shear stress, reduces the heat transfer and induces the mass transfer.



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