

A Study on Mixed Convective, MHD Flow from a Vertical Plate Embedded in Non-Newtonian Fluid Saturated Non-Darcy Porous Medium with Melting Effect

J. Siva Ram Prasad[†] and K. Hemalatha

V.R.Siddhartha Eng College, Dept of Mathematics, Vijayawada, A.P, 520006, India

[†]*Corresponding Author Email: nanijsrp@yahoo.co.in*

(Received March 12, 2014; accepted February 9, 2015)

ABSTRACT

We analyzed in this paper the problem of mixed convection along a vertical plate in a non-Newtonian fluid saturated non-Darcy porous medium in the presence of melting and thermal dispersion-radiation effects for aiding and opposing external flows. Similarity solution for the governing equations is obtained for the flow equations in steady state. The equations are numerically solved by using Runge-kutta fourth order method coupled with shooting technique. The effects of melting (M), thermal dispersion (D), radiation (R), magnetic field (MH), viscosity index (n) and mixed convection (Ra/Pe) on fluid velocity and temperature are examined for aiding and opposing external flows.

Keywords: Porous medium; non-Newtonian fluid; Melting; Thermal dispersion; Radiation; MHD.

1. INTRODUCTION

The study of an electrically conducting fluid in engineering applications is of considerable interest, especially in metallurgical and metal working processes or in the separation of molten metals from non-metallic inclusions by the application of a magnetic field. The phase change problem occurs in casting welding, purification of metals and in the formation of ice layers on the oceans as well as on aircraft surfaces. Because of its importance, accurate and robust methods of modelling phase change problems are of great interest. The velocity field in the liquid phase has a significant effect in determining the quality of the final product, and therefore, it is a great interest to study the fluid and solid phases. Recently, a significant number of studies have been conducted using computational fluid dynamics to enhance the physical and understand mathematical modelling capabilities in liquid – solid phase change as mentioned in introduction to convection in porous medium.

High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, magneto hydrodynamics (MHD) accelerators, and power generation systems are important applications for radiation heat transfer from a vertical wall to conductive grey fluids. Hossain and Takhar (1996) analyzed the effect of radiation using the Roseland

diffusion approximation on the forced and natural convection flow of an optically dense fluid from vertical surfaces. Hossain et.al (1999) studied the effect of radiation on natural convection heat transfer from a porous vertical plate.

Hydro magnetic flow and heat transfer problems are of great importance because of their industrial applications. As a result, many researchers have been conducted to study the effects of electrically conducting fluids such as liquid metals, water and others in the presence of magnetic fields on the flow and heat transfer.

There has been a renewed interest in studying magneto hydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the control and on the performance of many systems using electrically – conducting fluids. Among those, Kafoussias (1992) who studied the effect of magnetic field on free convection through a non homogeneous porous medium over an isothermal cone. Sparrow and Cess (1961) studied the effect of the magnetic field on the natural convection heat transfer. Romig (1964) studied the effect of electric and magnetic fields on the heat transfer of electrically conducting fluids. Riley (1964) analyzed the (MHD) free convection heat transfer. Garandet et.al (1992) analyzed the buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. Gulab

and Mishra (1977) analyzed the unsteady flow through magneto hydrodynamic porous media.

Non-Newtonian power law fluids are so wide spread in industrial process and in the environment. The melting phenomena on free convection from a vertical front in a non-Newtonian fluid saturated porous matrix are analyzed by Poulikakos and Spatz (1988). Nakayama and Koyama (1991) studied the more general case of free convection over a non-isothermal body of arbitrary shape embedded in a porous medium. Rastogi and Poulikakos (1995) examined the problem of double diffusive convection from a vertical plate in a porous medium saturated with a non-Newtonian power law fluid. Shenoy (1994) presented many interesting applications of non-Newtonian power law fluids with yield stress on convective heat transport in fluid saturated porous media. Considering geothermal and oil reservoir engineering applications, Nakayama and Shenoy (1992) studied a unified similarity transformation for Darcy and non-Darcy forced, free and mixed convection heat transfer in non-Newtonian inelastic fluid saturated porous media. Later Shenoy (1993) studied non-Darcy natural, forced and mixed convection heat transfer in non-Newtonian power law fluid saturated porous media. Bakier *et al.* (2009) studied Group method analysis of melting effect on MHD mixed convection flow from a radiative vertical plate embedded in saturated porous medium for Newtonian fluids. Recently, Melting and radiation effects on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing external flows was analyzed by Chamkha *et al.* (2010). Prasad *et al.* (2014) studied Melting and Thermal Dispersion-Radiation effects on mixed convection flow from a vertical plate embedded in Non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing external flows.

The present paper aims at analyzing the effect of melting and thermal dispersion-radiation under the influence of applied magnetic field on steady mixed convective heat transfer from a vertical plate embedded in a non-Newtonian power law fluid saturated non-Darcy porous medium for aiding and opposing external flows. Numerical results are obtained for velocity and temperature variation in melting region and presented in graphical form.

2. MATHEMATICAL FORMULATION

A mixed convective heat transfer in a non-Darcy porous medium saturated with a homogeneous non-Newtonian fluid adjacent to a vertical plate, with a uniform wall temperature is considered. This plate constitutes the interface between the liquid phase and the solid phase during melting inside the porous matrix at steady state. The plate is at a constant temperature T_m at which the material of the porous matrix melts. Figure.1 presents the geometry of the problem. The x-coordinate is measured along the plate and the y-coordinate normal to it. The solid phase is at temperature $T_0 < T_m$. A thin boundary

layer exists close to the right of vertical plate and temperature changes smoothly through this layer from T_m to T_∞ ($T_m < T_\infty$) which is the temperature of the fluid phase.

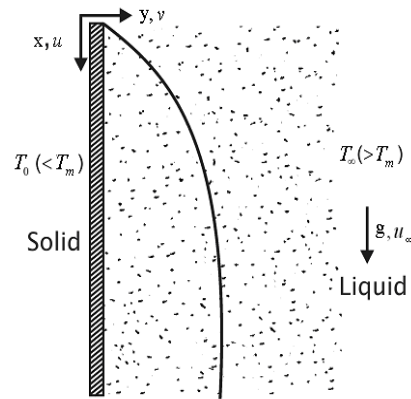


Fig. 1. Schematic diagram of the problem.

Taking into account the effect of thermal dispersion the governing equations for steady non-Darcy flow in a non-Newtonian fluid saturated porous medium can be written as follows.

$$\text{The continuity equation is } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

The momentum equation is

$$\left(\frac{\sigma B_0^2 K}{\rho v} + 1 \right) \frac{\partial u^n}{\partial y} + \frac{C\sqrt{K}}{v} \frac{\partial u^2}{\partial y} = \mp \frac{Kg\beta}{v} \frac{\partial T}{\partial y} \quad (2)$$

The energy equation is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left(\alpha \frac{\partial T}{\partial y} \right) - \frac{1}{\rho C_p} \frac{\partial q}{\partial y} \quad (3)$$

In the above equations, the term which represents the buoyancy force effect on the flow field has \mp signs. The plus sign indicates aiding buoyancy flow where as the negative sign stands for buoyancy opposed flow. Here u and v are the velocities along x and y directions respectively, n is the power-law fluid viscosity index, T is Temperature in the thermal boundary layer, K is Permeability, C is Forchheimer empirical constant, β is coefficient of thermal expansion, ν is Kinematics viscosity, ρ is Density, C_p is Specific heat at constant pressure, g is acceleration due to gravity, σ is electrical conductivity, B_0 is magnetic field intensity and thermal diffusivity $\alpha = \alpha_m + \alpha_d$, where α_m is the molecular diffusivity and α_d is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by Plumb (1983), the dispersion thermal diffusivity α_d is proportional to the velocity component i.e $\alpha_d = \gamma u d$, where γ is the dispersion coefficient and d is the meanparticle diameter. The radiative heat flux term q is written using the Rosseland approximation (Sparrow and Cess (1978), Raptis (1998) as

$$q = - \frac{4\sigma_R}{3a} \frac{\partial T^4}{\partial y} \quad (4)$$

Where σ_R is the Stefan - Boltzmann constant and 'a' is the mean absorption coefficient.

The physical boundary conditions for the present

problem are

$$y = 0, T = T_m, k \frac{\partial T}{\partial y} = \rho [h_{sf} + C_s(T_m - T_0)]v \quad (5)$$

and

$$y \rightarrow \infty, T \rightarrow T_\infty, u \rightarrow u_\infty \quad (6)$$

Where h_{sf} and C_s are latent heat of the solid and specific heat of the solid phases respectively and u_∞ is the assisting external flow velocity, $k = \alpha \rho C_p$ is the effective thermal conductivity of the porous medium. The boundary condition Eq. (5) means that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the heat of melting and the heat required for raising the temperature of solid to its melting temperature T_m .

Introducing the stream function ψ with

$$u = \frac{\partial \psi}{\partial y}, \text{ and } v = -\frac{\partial \psi}{\partial x}$$

The continuity Eq. (1) will be satisfied and the Eq.(2) and Eq.(3) transform to

$$\left(\frac{\sigma B_0^2 K}{\rho \nu} + 1\right) \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial y}\right)^n + \frac{C\sqrt{K}}{v} \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial y}\right)^2 = \mp \frac{Kg\beta}{v} \frac{\partial T}{\partial y} \quad (7)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[(\alpha_m + \gamma \frac{\partial \psi}{\partial y} d) \frac{\partial T}{\partial y} \right] + \frac{4\sigma_R}{3\rho C_p a} \frac{\partial}{\partial y} \left[\frac{\partial T^4}{\partial y} \right] \quad (8)$$

Introducing the similarity variables as

$$\psi = f(\eta)(\alpha_m u_\infty x)^{1/2}, \eta = \left(\frac{u_\infty x}{\alpha_m}\right)^{1/2} \left(\frac{y}{x}\right), \theta(\eta) = \frac{T - T_m}{T_\infty - T_m},$$

The momentum Eq. (7) and energy Eq. (8) are reduced to

$$n(1 + MH)f^{11}f^{n-1} + 2Ff^1f^{11} = \mp \left(\frac{Ra_x}{Pe_x}\right)^n \theta^1 \quad (9)$$

$$(1 + Df^1)\theta^{11} + \left(\frac{1}{2}f + Df^{11}\right)\theta^1 + \frac{4}{3}R\left[\left(\theta + C_r\right)^3\theta^{11} + 3\theta^{12}(\theta + C_r)^2\right] = 0 \quad (10)$$

Where the prime symbol denotes the differentiation with respect to the similarity variable η , $MH = \frac{\sigma B_0^2 K}{\rho \nu}$ is Magneto Hydro Dynamic parameter, Ra_x/Pe_x is the mixed convection parameter, $Ra_x = \frac{x}{\alpha} \left(\frac{g\beta k(T_\infty - T_m)}{v}\right)^{1/2}$ is the local Rayleigh number, $Pe_x = \frac{u_\infty x}{\alpha}$ is the local pecllet number. $F = f_0(Pe_d)^{2-n}$, $f_0 = \left(\frac{\alpha}{d}\right)^{2-n} \left(\frac{C\sqrt{K}}{v}\right)$ is the non-darcian parameter. Pe_d is the pore diameter dependent pecllet number. $D = \frac{\gamma d u_\infty}{\alpha_m}$ is the dispersion parameter. $C_r = \frac{T_m}{T_\infty - T_m}$ is the temperature ratio (taken as 0.1), $R = \frac{4\sigma_R(T_\infty - T_m)^3}{ka}$ is the radiation parameter.

Taking into consideration, the thermal dispersion effect together with melting, the boundary conditions Eq. (5) and Eq. (6) take the form

$$\eta = 0, \theta = 0, f(0) + \{1 + Df^1(0)\} 2M\theta^1(0) = 0. \quad (11)$$

$$\text{and } \eta \rightarrow \infty, \theta = 1, f^1 = 1. \quad (12)$$

Where $M = \frac{C_f(T_\infty - T_m)}{h_{sf} + C_s(T_m - T_0)}$ is the melting parameter. The local heat transfer rate from the surface of the plane is given by $q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0}$.

The Nusselt number is $Nu = \frac{hx}{k} = \frac{q_w x}{k(T_m - T_\infty)}$, where h is the local heat transfer coefficient and k is the effective thermal conductivity of the porous medium, which is the sum of the molecular thermal conductivity k_m and the dispersion thermal conductivity k_d .

The Nusselt number is obtained as

$$\frac{Nu_x}{(Pe_x)^{1/2}} = \left[1 + \frac{4}{3}R(\theta(0) + C_r)^3 + Df^1(0)\right] \theta^1(0) \quad (13)$$

3. SOLUTION PROCEDURE

The dimensionless equations Eq.(9) and Eq.(10) together with the boundary conditions Eq. (11) and Eq. (12) are solved numerically by means of the fourth order Runge-Kutta method coupled with double shooting technique. The solution thus obtained is matched with the given values of $f^1(\infty)$ and $\theta(0)$. In addition the boundary condition $\eta \rightarrow \infty$ is approximated by $\eta_{max} = 8$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. Numerical computations are carried out for $F = 0, 0.5, 1; D = 0, 0.5, 1; Ra/Pe = 1, 2, 5; M = 0, 0.8, 2; n = 0.5, 1, 1.5, 2.5, R = 0, 0.5, 1; MH = 0, 1, 5, 10$.

4. RESULTS AND DISCUSSION

In order to get clear insight on the physics of the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations. The parameters governing the physics of the present study are the melting (M), the mixed convection (Ra/Pe), Magneto Hydro Dynamic parameter (MH), the inertia (F), thermal dispersion (D), and Fluid viscosity index (n), radiation (R), temperature ratio (Cr). The numerical computations were carried out for the fixed value of buoyancy parameter $Ra/Pe = 1$ for both the aiding and opposing external flows. The results of the parametric study are shown in Fig. 2.1 to Fig. 17.

Figures 2.1, 2.2 and 2.3 show that the effect of imposing a magnetic field on the fluid velocity in the case of $\frac{Ra}{Pe} = 1$, dispersion parameter $D = 0.5$, radiation parameter $R = 0.5$, flow inertia parameter $F = 1$ when melting parameter $M = 2$ for different values of fluid viscosity index $n = 0.5, 1$ and 1.5 respectively for both aiding and opposing flows. The imposition of a magnetic field normal to the flow direction produces a resistive force that decelerates the motion of the fluid in the porous medium. It is obvious from the figures that the

fluid velocity decreases in aiding flow as the magnetic parameter MH is increased in the presence of melting, dispersion and radiation effects for all said viscosity index values. But in the opposing flow case the effect is found opposite.

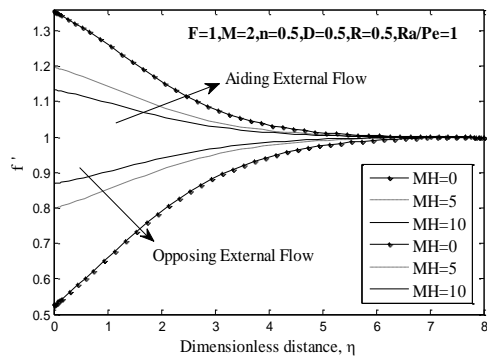


Fig. 2.1. The velocity profiles for different values of MH for $n=0.5$.

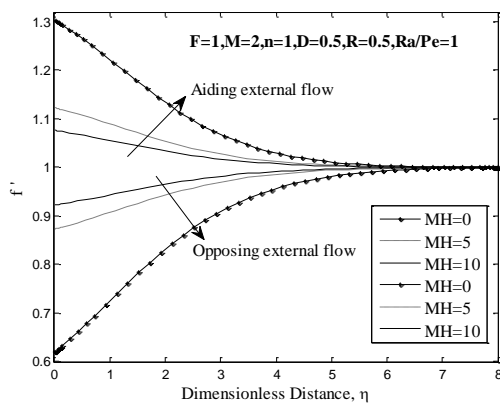


Fig. 2.2. The velocity profiles for different values of MH for $n=1$.

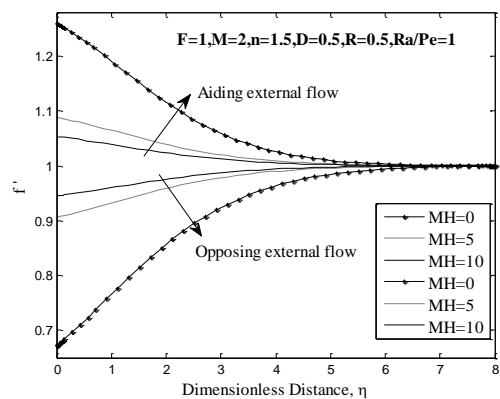


Fig. 2.3. The velocity profiles for different values of MH for $n=1.5$.

The dimensionless velocity is presented in the Fig. 3.1, 3.2 and 3.3 in both aiding and opposing flows, for the case of different buoyancy parameters $Ra/Pe=1, 2, 5$ when the buoyancy is aiding the flow and $Ra/Pe=1, 2, 3, 5$ when the buoyancy is opposing the flow for different values of fluid viscosity index $n=0.5, 1$ and 1.5 respectively. It can be seen from the figures that the fluid velocity increases as the buoyancy parameter increases in

the presence of melting, dispersion and radiation effects in aiding flow for $n=0.5, 1, 1.5$. In opposing flow as the buoyancy increases the fluid velocity decreases in the presence of melting, dispersion and radiation effects for all the said viscosity index values.

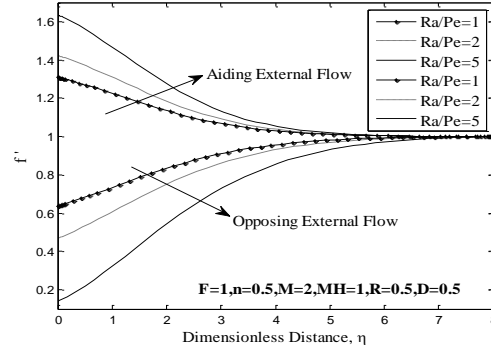


Fig. 3.1. The velocity profiles for different values of Ra/Pe for $n=0.5$.

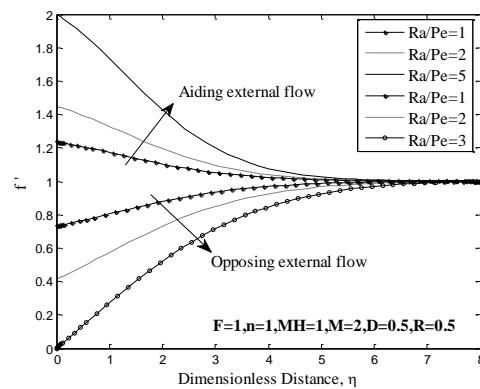


Fig. 3.2. The velocity profiles for different values of Ra/Pe for $n=1$.

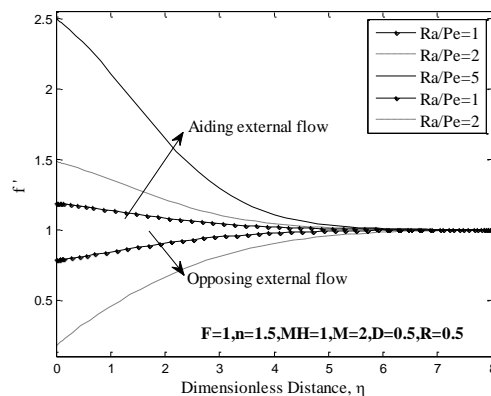


Fig. 3.3. The velocity profiles for different values of Ra/Pe for $n=1.5$.

The dimensionless velocity component is presented in Fig. 4.1, 4.2 and 4.3 in both aiding and opposing flows for different values of fluid viscosity index $n=0.5, 1$ and 1.5 respectively, for different values of dispersion parameter $D=0, 0.5, 1$. In aiding flow the fluid velocity increases with the increase of dispersion parameter and in the opposing flow the fluid velocity decreases with increase in dispersion parameter for all values of

viscosity index $n=0.5, 1$ and 1.5 .

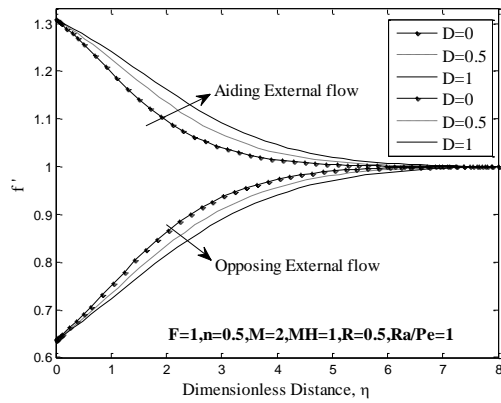


Fig.4.1. The velocity profiles for different values of D for $n=0.5$.

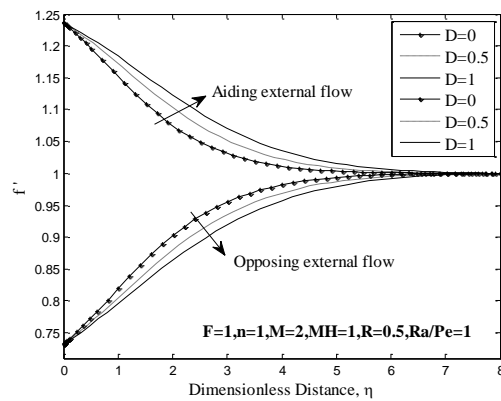


Fig.4.2. The velocity profiles for different values of D for $n=1$.

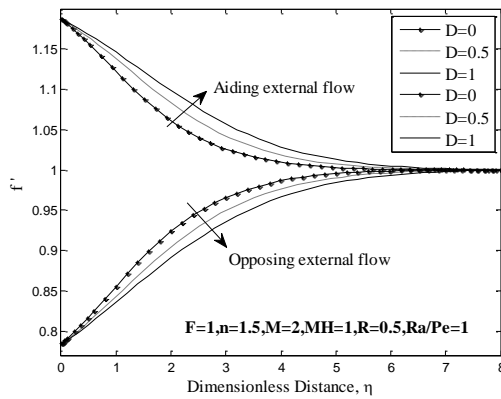


Fig.4.3. The velocity profiles for different values of D for $n=1.5$.

Figures 5.1, 5.2 and 5.3 show the effect of the melting parameter M on fluid velocity in both aiding and opposing flows for $n=0.5, 1$ and 1.5 respectively. The thickening of the boundary layer due to melting can also be seen in these figures. In aiding flow as the melting parameter increases, the fluid velocity increases. In opposing flow the fluid velocity decreases with increase in melting parameter for all $n=0.5, 1$ and 1.5 while the thickening of the boundary layer persists.

Figure 6 shows the variation of dimensionless flow

velocity for the non-Darcian medium characterized by the value of the inertial coefficient $F=1$ with varying values of power law fluid viscosity index n in its range both in aiding and opposing flows fixing $\frac{Ra}{Pe}=1$. The figure shows that the fluid velocity decreases with increase in fluid viscosity index in aiding flow. In opposing flow the fluid velocity increases with increase in fluid viscosity index.

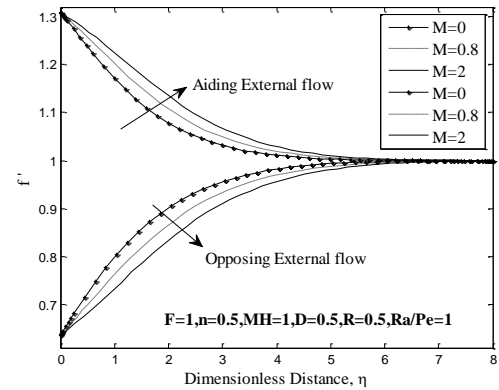


Fig. 5.1. The velocity Profiles for different values of M for $n=0.5$.

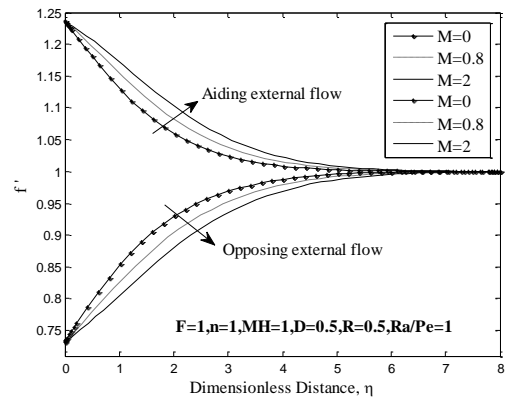


Fig. 5.2. The velocity Profiles for different values of M for $n=1$.

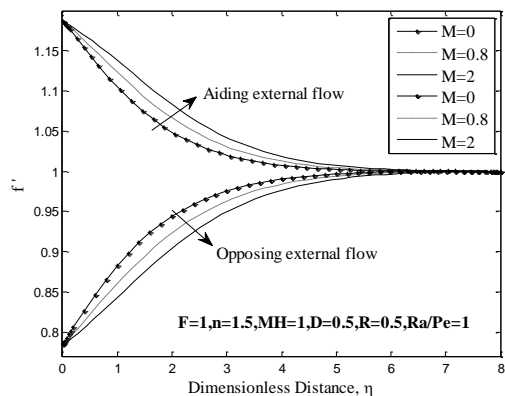


Fig. 5.3. The velocity Profiles for different values of M for $n=1.5$.

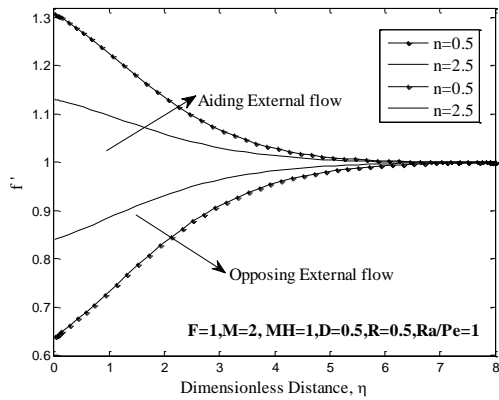


Fig. 6. The effect of viscosity index on velocity distribution in aiding and opposing flows.

The dimensionless velocity component is presented in Fig. 7.1, 7.2 and 7.3 in both aiding and opposing flows for different values of fluid viscosity index $n=0.5, 1$ and 1.5 respectively, for different values of radiation parameter $R=0, 0.5, 1$. In aiding flow the radiation parameter causes increase in the acceleration of the fluid motion which is observed by the increase in the fluid velocity. In the opposing flow, the presence of thermal radiation affects the activity of the fluid velocity inversely which causes the decrease in the fluid velocity as radiation parameter increases for all values of viscosity index $n = 0.5, 1$ and 1.5 .

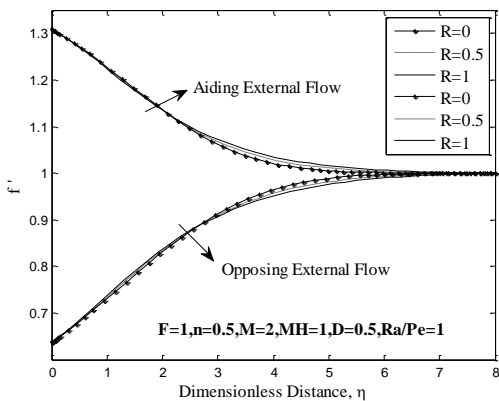


Fig. 7.1. The effect of radiation on velocity distribution for $n=0.5$.

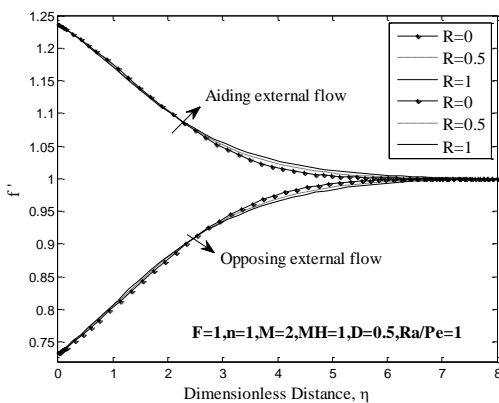


Fig. 7.2. The effect of radiation on velocity distribution for $n=1$.

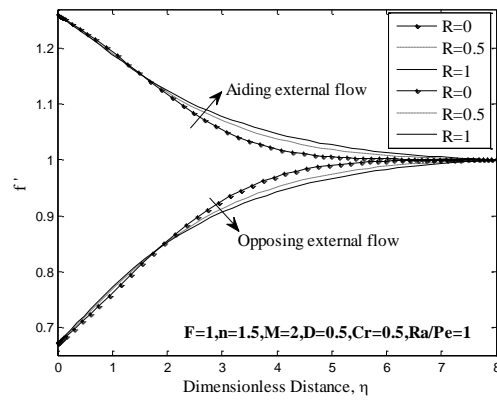


Fig. 7.3. The effect of radiation on velocity distribution for $n=1.5$.

The fluid temperature $\theta(\eta)$ is presented in Fig 8.1 and 8.2 in aiding and opposing flows respectively for the case of viscosity index $n = 1.5$, dispersion parameter $D=0.5$, melting parameter $M=2$, flow inertia parameter $F=1$, radiation parameter $R=0.5$ and different values of magnetic parameter $MH=0, 5, 10$ fixing $Ra/Pe=1$. It is observed that application of a magnetic field normal to the flow of an electrically conducting fluid gives rise to a resistive force that decelerates the motion of the fluid in the porous medium and causes decrease in its temperature for buoyancy aiding flow. In opposing flow the fluid temperature increases with increase in the magnetic parameter. From the figures it is observed that the magnetic effect is more effective in opposing flow than in aiding flow.

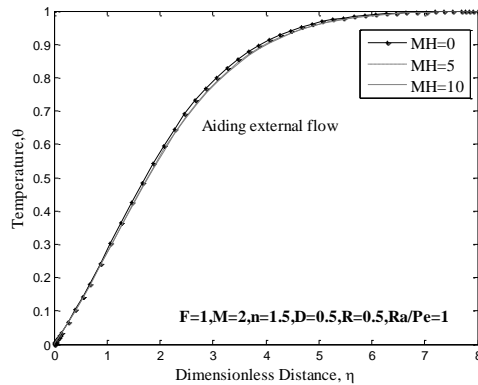


Fig. 8.1. The temperature profiles for different values of MH in aiding flow for $n=1.5$.

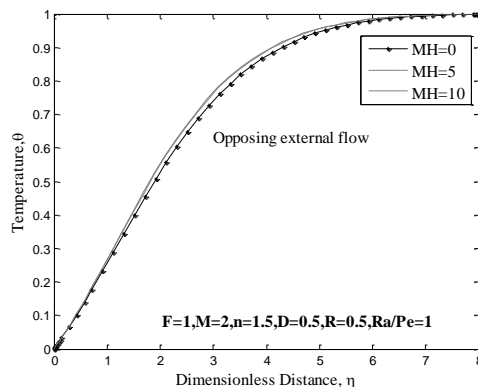


Fig. 8.2. The temperature profiles for different values of MH in opposing flow for $n=1.5$.

The fluid temperature $\theta(\eta)$ is presented in Fig 9.1, and 9.2 in aiding and opposing flows respectively for different buoyancy parameters $Ra/Pe = 1, 2, 5$. In aiding flow the fluid temperature increases with increase in the buoyancy parameter. In opposing flow the fluid temperature decreases with increase in the buoyancy parameter.

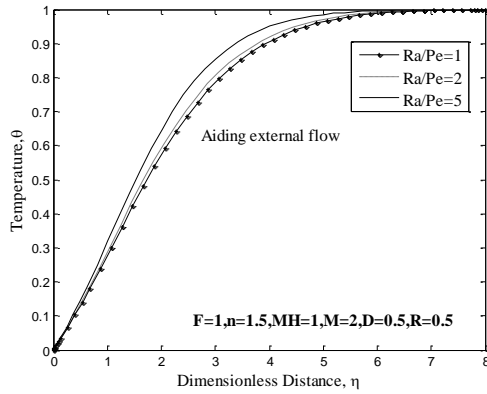


Fig. 9.1. The temperature profiles for different values of Ra/Pe in aiding flow for $n=1.5$.

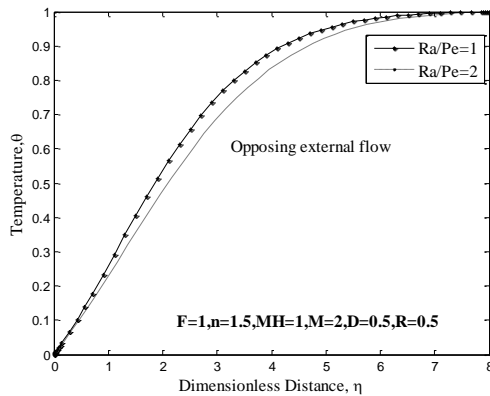


Fig. 9.2. The temperature profiles for different values of Ra/Pe in opposing flow for $n=1.5$.

Figures 10.1 and 10.2 show the effect of melting parameter M , on fluid temperature in aiding and opposing flows respectively for different values of melting parameter $M=0, 0.8, 2$. It is observed that the fluid temperature decreases with the increase in melting parameter in the thermal boundary in both flow cases.

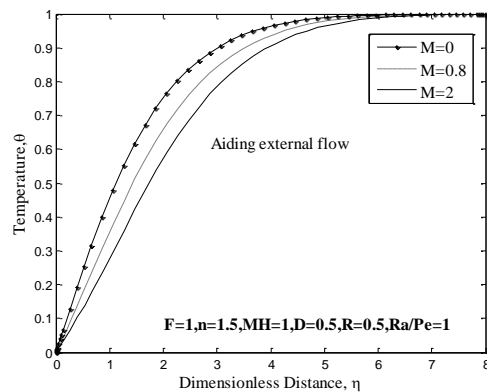


Fig. 10.1. The temperature profiles for different values of M in aiding flow for $n=1.5$

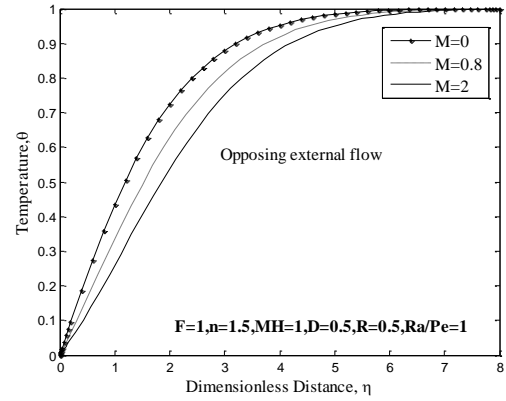


Fig. 10.2. The temperature profiles for different values of M in opposing flow for $n=1.5$.

The temperature profile $\theta(\eta)$ is presented in Fig. 11.1 and 11.2 in aiding and opposing flows respectively for different values of dispersion parameter $D=0, 0.5, 1$. The fluid temperature decreases with increase in dispersion parameter in both the flow cases.

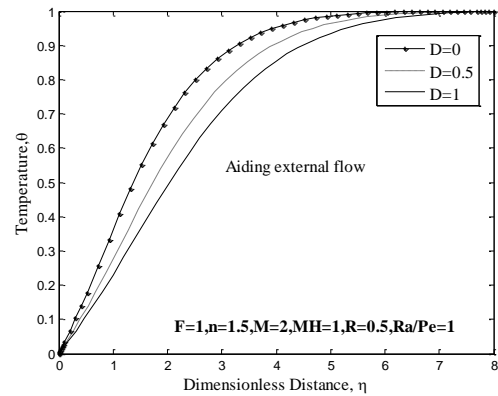


Fig. 11.1. The temperature profiles for different values of D in aiding flow for $n=1.5$.

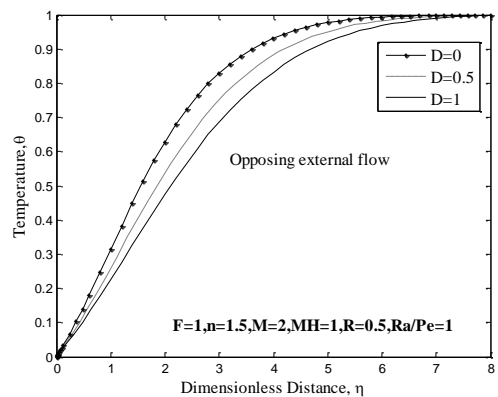


Fig. 11.2. The temperature profiles for different values of D in opposing flow for $n=1.5$.

Figure 12 shows the effect of magnetic parameter and melting strength on heat transfer rate in terms of Nusselt number given by eq. (13) for a fixed value of the mixed convection parameter $\frac{Ra}{Pe} = 1$. It can be seen that as the magnetic field strength increases, the heat transfer rate decrease/increase in aiding /opposing flows due to the fact that the fluid velocity becomes lower and the thermal boundary

layer increases while the heat transfer rate decreases with the increase of melting parameter in both flow cases.

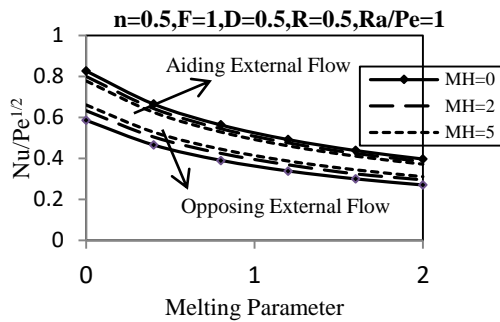


Fig. 12. Nusselt number variation with melting parameter M , for different MH values.

Figure 13 shows the effect of mixed convection parameter Ra/Pe on the average heat transfer coefficient as a function of melting parameter M . The heat transfer rate decreases with the increase of melting parameter in both flow cases while the average heat transfer coefficient increases with increasing aiding buoyancy and decreases with increasing opposing buoyancy.

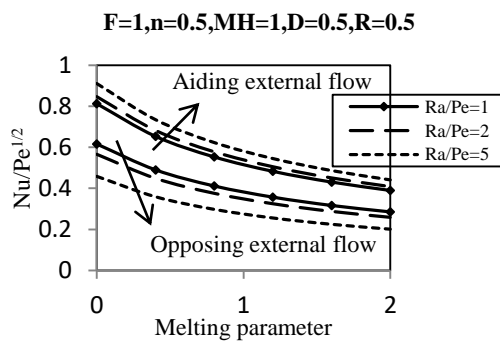


Fig. 13. Nusselt number variation with melting parameter M , for different mixed convection parameter (Ra/Pe) values.

The effect of radiation parameter and melting strength on heat transfer rate in terms of Nusselt number for a fixed value of the mixed convection parameter $\frac{Ra}{Pe}=1$ is shown in the Fig 14. It is noted that as the radiation parameter increases, the heat transfer rate increases in both aiding and opposing flows while the heat transfer rate decreases with the increase of melting parameter in both flow cases.

The effect of melting strength and thermal dispersion on heat transfer rate for a fixed value of the mixed convection parameter $\frac{Ra}{Pe}=1$ is shown in Fig. 15.1 and 15.2 for both aiding and opposing flows respectively, in terms of the Nusselt number. This shows that Nusselt number decreases significantly with increasing melting strength (M) for both flow cases. Further it is seen that the Nusselt number increases with increase in thermal dispersion parameter for both flow cases.

Figure 16 shows the effect of melting strength and inertia on heat transfer rate for a fixed value of the

mixed convection parameter $\frac{Ra}{Pe}=1$ for both aiding and opposing flows in terms of the Nusselt number. It is clear that the Nusselt number is decreasing with increasing inertia effect in aiding flow. Where as in the opposing flow the effect is found opposite. Further it is noted that Nusselt number decreases significantly with increasing melting strength (M) for both flow cases.

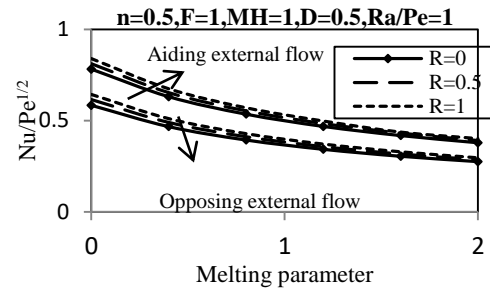


Fig. 14. Nusselt number variation with melting parameter M for different radiation parameter values in aiding and opposing flows.

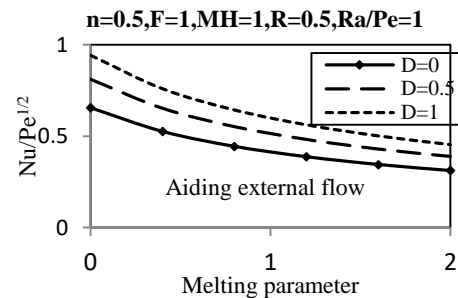


Fig. 15.1. Variation of the local Nusselt number with the melting parameter M for different values of D in aiding flow.

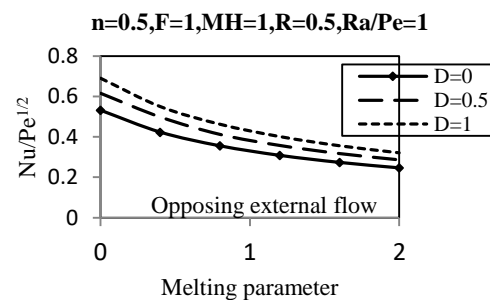


Fig. 15.2. Variation of the local Nusselt number with the melting parameter M for different values of D in opposing flow.

Figure 17 shows the effect of melting strength and fluid viscosity index for a fixed value of the mixed convection parameter $\frac{Ra}{Pe}=1$ on heat transfer rate for aiding and opposing flows respectively in terms of the Nusselt number. It is clear that the Nusselt number is decreasing with increasing the value of viscosity index in aiding flow. But in opposing flow the Nusselt number increases with the increase of fluid viscosity index while the heat transfer rate decreases with the increase of melting parameter in both flow cases.

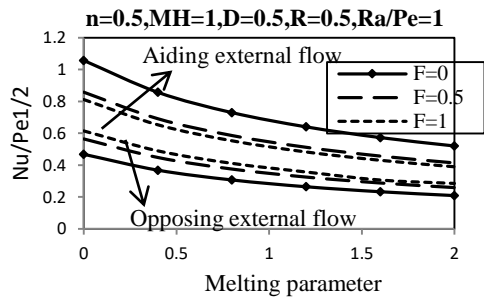


Fig. 16. Nusselt number variation with melting parameter for different values F in aiding and opposing flows.

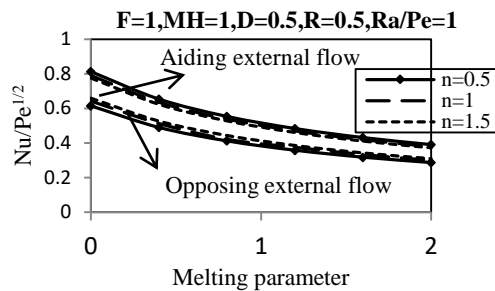


Fig. 17. Variation of the local Nusselt number with the melting parameter for different values of 'n' in both aiding and opposing flows.

5. CONCLUSION

The melting phenomenon has been analyzed in thin and thick fluids with the mixed convection flow and heat transfer in a non-Newtonian fluid saturated non-Darcy porous medium under the influence of applied magnetic field, considering the effects of thermal dispersion and thermal radiation by taking Forcheimer extension in the flow equations. Numerical results for the velocity and temperature profiles as well as the heat transfer rate in terms of Nusselt number are obtained for aiding and opposing flows and the same are presented graphically. Same results are obtained for thin and thick fluids. This study shows that thermal dispersion, radiation, mixed convection and melting tend to increase / decrease the velocity within the boundary in aiding / opposing flow. Further, the present model shows that the increase in MH parameter and viscosity index decreases / increases the velocity in aiding / opposing flow.

Within the boundary layer, the temperature decreases with increase in dispersion and melting in both flow cases. But the temperature decreases / increases with the increase in MH parameter in aiding / opposing flow. It is also noted that the temperature increases / decreases with the increase in mixed convection parameter in aiding / opposing flow.

Further it is found that the rate of heat transfer decreases with the increase in melting parameter while it increases with the increase in dispersion, radiation parameter values in both aiding and opposing flow cases. But heat transfer rate

decreases / increases as the magnetic parameter, non-Darcy parameter and viscosity index increases in aiding / opposing flow.

REFERENCES

- Bakier A. Y. and A. M. Rashad and M. A. Mansour (2009). Group method analysis of Melting effect on MHD Mixed convection flow from radiate vertical plate embedded in a saturated porous media. *Communications in Nonlinearscience and numerical Simulation* 14, 2160-2170.
- Chamkha.al, A. J. (2010). Melting and Radiation Effects on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing external flows. *International journal of the Physical sciences* 5(7), 1212-1224.
- Garandet, J. P., T. Alboussiere and R. Moreau (1992). Buoyancy driven convection in a Rectangular Enclosure with a Transverse magnetic Field. *Int. J. Heat Mass Transfer* 35, 741-749.
- Gulab, R. and Mishra (1977). Unsteady Flow Through Magnetohydrodynamic Porous Media. *Indian J. Pure. Appl. Math.* 8, 637-642.
- Hossain M. A. and H. S. Takhar (1996). Radiation Effect on Mixed Convection along a Vertical Plate with Uniform Surface Temperature. *Int. J. Heat Mass Transfer* 31, 243-248.
- Hossain M. A., M. A. Alim and D. A. Rees (1999). The Effect of Radiation on Free Convection from a Porous Vertical Plate. *Int. J. Heat Mass Transfer* 42, 181-191.
- Kafoussias, N. G. (1992). MHD Free Convection Flow through a Nonhomogenous Porous Medium over an Isothermal Cone Surface. *Mech. Res. Commun.* 19, 89-94.
- KRostagi, S. and D.Poulikakos (1995). Double-diffusion from a vertical surface in a porous region Saturated with a non-Newtonian fluid, *International journal of heat and mass transfer* 38, 935-946.
- Nakayama, A. and A. V. Shenoy (1992). A unified similarity transformation for Darcy and non-Darcy forced, free and mixed convection heat transfer in non-Newtonian inelastic fluid saturated porousmedia. *The Chemical Engineering Journal* 50, 33-45.
- Nakayama, A. and H. Koyama (1991). Buoyancy induced flow of non-Newtonian fluids over a non isothermal body of arbitrary shape in a fluid-saturated porous medium. *Applied Scientific Research* 48, 55-70.
- Plumb, O. A (1983). The Effect of Thermal Dispersion on Heat Transfer in packed Bed Boundary Layers, *Proceedings of 1st ASME/JSME. Thermal Engineering Joint*

- Conference 2*, 17-21.
- Poulikakos and T. L. Spatz (1988). Non-Newtonian natural convection at a melting front in a permeable solid matrix. *International communication in heat and mass transfer* 15, 593-603.
- Prasad. J. S. R, K. Hemalatha and B. D. C. N. Prasad (2014). Mixed Convection Flow from Vertical Plate Embedded in Non-Newtonian Fluid Saturated Non-Darcy Porous Medium with Thermal Dispersion-Radiation and Meltinf Effects. *Journal of Applied Fluid Mechanics* 7(3), 385-394.
- Raptis, A. (1998). Radiation and Free Convection Flow Through a Porous Medium, *Int.Comm. Heat Mass Transfer* 25, 289-295.
- Riley, N (1964). Magneto hydrodynamics free Convection. *J.Fluid Mech* 18, 577-586.
- Romig, M. (1964). The Influence of Electric and Magnetic Field on Heat Transfer to Electrically conducting Fluids. *Adv Heat Transfer* 1, 268-352.
- Shenoy, A. V. (1993). Darcy-Forchheimer natural, forced and mixed convection heat transfer in Non-Newtonian power law fluid saturated porous media. *Transport in porous media* 11, 219-241.
- Shenoy, A. V. (1994). Non-Newtonian fluid heat transfer in a porous media. *Advances in Heat transfer* 24, 101-190.
- Sparrow, E. M. and R. D. Cess (1961). Effect of Magnetic Field on Free Convection Heat Transfer. *Int.J.Heat Mass Transfer* 3, 267-274.
- Sparrow. E. M. and R. D. Cess (1978). *Radiation Heat Transfer, Washington, Hemisphere.*