Journal of Applied Fluid Mechanics, Vol. 9, No. 1, pp. 419-430, 2016.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.68.224.24198

Applied Thermal Lattice Boltzmann Model for Fluid Flow of Free Convection in 2-D Enclosure with Localized Two Active Blocks: Heat Transfer Optimization

T. Naffouti [1,2†], J. Zinoubi [1,2], N. A. Che Sidik [3] and R. B. Maad [1]

[1] *Université Tunis El-Manar, Faculté des Sciences de Tunis, Département de Physique, Laboratoire d'Energétique et des Transferts Thermique et Massique, El Manar 2092, Tunis, Tunisia.*
[2] *Université Tunis El-Manar, Institut Préparatoire aux Études d'Ingénieurs El-Manar, El-Manar 2092, Tunis, Tunisia.*
[3] *Faculty of Mechanical Engineering, Universiti Teknologi Malaysia, 81310 UTM Skudai, Johor, Malaysia*

†*Corresponding Author Email: taoufiknaffouti@gmail.com*

(Received November 9, 2014; accepted January 21, 2015)

ABSTRACT

The aim of this paper is to analyze the laminar free convective flow generated by two identical hot blocks in two-dimensional enclosure cooled by the sides in order to optimize the heat transfer. The top wall and the flat surfaces on bottom wall are adiabatic except for the active sources located symmetrically. Each source of a rectangular form is heated at a uniform temperature while the Prandtl number is fixed at 0.71. Thermal Lattice Boltzmann model of D2Q4-D2Q9 is applied to solve the thermal flow problem. Numerical simulations have been conducted to reveal the effects of various parameters; Rayleigh number $10^3 \leq Ra \leq 10^6$, spacing between blocks $0.1 \leq D \leq 0.6$, block height $0.05 \leq H \leq 0.4$ and aspect ratio of the enclosure $1 \leq A \leq 4$ on fluid flow and heat transfer. The computational results by Lattice Boltzmann method have been found to be in good agreement with previous works. The results are presented in the form of isotherms and streamlines plots as well as the variation of the average Nusselt number along horizontal and vertical hot walls. It is found that increasing Rayleigh number and distance between active blocks enhance the heat transfer. The simulations show that the block height and aspect ratio are the most important parameters affecting dynamic and thermal fields and consequently the heat transfer efficiency in the enclosure.

Keywords: Free convection; 2-D enclosure; Active blocks; Lattice Boltzmann method; Optimum heat transfer.

NOMENCLATURE

A	aspect ratio of the enclosure
c_s	lattice sound speed
D	dimensionless spacing between sources S_1 and S_2
$\vec{e_k}$	discrete lattice velocity
f_k	discrete distribution function for the density
g_k	discrete distribution function for the temperature
$\vec{g}$	gravity field
H	dimensionless height of heat block
H_e	dimensionless height of the enclosure
l	dimensionless length of heat block
L_e	dimensionless length of the enclosure
$Nu_{h\,i}$	average Nusselt number along horizontal hot upper wall
Nu_{LV}	average Nusselt number along left vertical hot wall
Nu_{RV}	average Nusselt number along right vertical hot wall
p	pressure
Ra	Rayleigh number
T_h	temperature of hot source
T_c	temperature of cold vertical wall
$\vec{u}$	velocity vector (u,v)
w_k	weighting factors for f_k
w'_k	weighting factors for g_k
$\vec{x}$	lattice node in (x,y) coordinates
χ	thermal diffusivity
ΔT	temperature gradient
Δt	time step
Δx	lattice spacing units (=Δy)
β	thermal expansion coefficient
θ	dimensionless temperature field
τ_f	relaxation times for f_k
τ_g	relaxation times for g_k

ρ	fluid density
ρ_0	reference fluid density
ν	kinetic viscosity

Subscripts

c	cold
eq	equilibrium part
h	hot
hi	horizontal (i = 1,2)
k	discrete speed directions (k=0, , 8)
LV	left vertical
RV	right vertical

1. INTRODUCTION

Steady phenomenon of natural convection heat transfer in enclosure is motivated by its wide several areas of engineering, such as, heating/cooling of buildings and tunnels, cooling of electronic equipment, heat exchanger, performance of thermal system (Vinogradov *et al*., 2011; Rama Narasimha, *et al*., 2012 ; Aswatha *et al*., 2012 ; Naik *et al*., 2013 ; Naffouti *et al*. 2015), ect. In this active research area, several papers are substantially oriented toward the study of natural convection in enclosure with discrete heat sources by the reason of its fundamental interest and relevance to application in industry.

An investigation of the convection heat transfer due to protruded heat sources with glycol as the working fluid in an enclosure is conducted by Chen *et al*. (1991). Visualizations of flow patterns under several power inputs show a core flow within the enclosure as well as a recirculating cell in the gap between heaters. Numerical simulations and experimental data of natural convection air cooling of an array of two-dimensional discrete flush heaters on a vertical wall of a rectangular enclosure were performed by Ho and Chang (1994). The influences of the aspect ratio varying from 1 to 10 and the modified Rayleigh number ranging from 10^3 to 10^7 were obtained carefully. Heindel *et al*. (1995) studied the natural convection from an array of discrete heat sources in a cavity filled with water and a dielectric fluid. With increasing modified Rayleigh number, the cavity flow becomes more stratified along the vertical walls and multiple fluid cells develop in the central region. Papanicolaou and Gopalakrishna (1995) presented a two-dimensional computational investigation of natural convection in a shallow horizontal air layer driven by a single discrete source heated at a constant heat flux. Compared to case of a single source, the transition from conduction to convection was significantly delayed in the presence of adjacent sources. The rate of increase of Nusselt number with increasing Rayleigh number was higher in the case of multiple heat sources. Natural convection from an array of heat sources in a cavity was numerically analyzed and verified with experimental results by Heindel *et al*. (1995, 1996). Laminar, conjugate heat transfer was considered with water and FC-77 as fluids. An experimental work and a numerical simulation were studied by Wang *et al*. (1998) regarding natural convection in an inclined cube enclosure with multiple internal isolated plates. It was found that when all the three plates move up and down within the wide range of the enclosure, the effect of their position on heat transfer is very small. Numerical analysis on nine discrete heat sources mounted on one vertical wall of a rectangular enclosure filled with various liquids was done by Tou *et al*. (1999). The results reveal that the flow field is complex and the heat transfer from the discrete heaters is not uniform.

Barozzi *et al*. (1999, 2000) investigated the two dimensional buoyant flow in a closed cabinet containing two vertical heating plates with a time-accurate finite method. Time-dependent long-term solutions are predicted at Gr = 1×10^7. Madhavan and Sastri (2000) performed a numerical study on conjugate natural convection from protruding heat sources in an enclosure using Fluent. It is found that Rayleigh number, Prandtl number and enclosure boundary condition affects strongly the fluid flow and heat transfer characteristics. Deng *et al*. (2002) presented a two-dimensional numerical investigation of natural convection from two discrete heat sources mounted in a horizontal enclosure with insulating side walls at steady state in order to study the interaction between sources. The convection regime is ruled by the bottom heat source and the flow structure simply consists of two main cells circulating around a two inner-volumetric heat sources. Tou and Zhang (2003) investigated the heat transport in a liquid-filled vertical rectangular enclosure with a 3*3 array of discrete flush-mounted heaters along one vertical wall. They noticed that the heat transfer from discrete heaters is non-uniform and maximum Nusselt number occurs at the heater leading edge and decreases towards the trailing edge. Bae and Hyun (2004) investigated two dimensional laminar natural-convective air cooling in a vertical rectangular enclosure with three discrete flush mounted heaters on one side of the wall. The results show the influence of the time-dependent thermal condition of the lowest-elevation heater on the temperatures of the other heaters.

Tso *et al*. (2004) carried out experimental and numerical study for laminar natural-convection cooling of water in a rectangular cavity with nine heaters on one wall at various inclinations. The case of horizontal cavity heated from below reveals flow fields of the toroidal, bimodal or Rayleigh–Benard convection type, depending on the Rayleigh number. Da Silva *et al*. (2004) investigated the optimal distribution and sizes of three discrete heat sources in a vertical open channel cooled by natural convection. It is shown that the optimal distribution is not uniform and that as the Rayleigh number increases the heat sources placed near the tip of a boundary layer should have zero spacing. Behavior of thermal plumes from two heat sources in an enclosure was carried out by Ichimiya and Saiki (2005). Numerical results show that the upward motion is divided into three patterns depending on the pitch of two heated sections. Bhowmik *et al*. (2005) performed experiments to study convection heat transfer from discrete heat sources in a liquid cooled rectangular channel. The results indicate that the heat transfer coefficient is strongly affected by the Reynolds number. Convection heat transfer from an 8 by 4 array of heat sources maintained at uniform heat flux in a rectangular channel was experimentally investigated by Baskaya *et al*. (2005). The Nusselt number was lowest for fifth row and maximum for first row. Da Silva *et al*. (2005) investigated the distribution of heat sources in vertical open channels with natural convection. Numerical results show that for low Rayleigh numbers, heat sources select as optimal location the inlet plane of the channel. As the flow intensity increases, the optimal heat source size approaches the height of the wall.

Bhowmik *et al*. (2005) studied numerically and experimentally steady state convective heat transfer from in-line chips with water as a fluid. Effects of heat flux, flow rates, and chip number are investigated. Bazylak *et al*. (2006) made a computational analysis on natural convection heat transfer from flush mounted heat sources in a rectangular enclosure. Uniform heat flux was applied to heat sources and periodic boundary condition was used for the enclosure side walls. Chen *et al*. (2007) conducted numerical simulations of laminar, steady, natural convection flow in a square enclosure with discrete heat sources on the left and bottom walls. Numerical results indicate that the average Nusselt number increases as Rayleigh number increases. Banerjee *et al*. (2008) carried out a numerical study on natural convection in a horizontal planar square cavity with two discrete heat sources flush-mounted on its bottom wall. The maximum temperature of the right heater rises and that of the left heater decreases monotonically with increasing strength ratio. A numerical study of plumes behavior at high Rayleigh number has been conducted by Rosdzimin *et al*. (2010) using lattice Boltzmann method. It is shown the movement of thermal plumes is influenced by the distance between two heated cylinder and the cooled walls. Numerical investigation was conducted by Venkatachalapathy and Udayakumar (2010) on natural convection heat transfer from multiple heat sources in a square enclosure. The results show that the Nusselt number varies linearly with Grashof number and the heat transfer coefficients for inner heat sources in any row are lower compared with those near the enclosure walls.

From the available literature, it was found that none study has been carried out about heat transfer optimization of a natural convection flow in a confined medium with two active blocks. So, the objective of this numerical investigation is to optimize the heat transfer of a laminar free convection flow in an enclosure via two heated rectangular blocks placed on the bottom wall. It is a further effort to extend the previous works presented by Naffouti *et al*. (2012, 2013). Using lattice Boltzmann method, the effects of Rayleigh number (Ra = 10^3, 10^4, 10^5 and 10^6), distance between active blocks (D = 0.1, 0.2, 0.4 and 0.6), block height (H = 0.05, 0.1, 0.2 and 0.4) and aspect ratio of the enclosure (A = 1, 2, 3 and 4) on the fluid flow as well as heat transfer are presented and discussed.

2. PHYSICAL MODEL AND NUMERICAL PROCEDURE

2.1 Physical Model

The physical configuration and boundary conditions of the present numerical investigation are presented in Fig.1. The computational domain is a two-dimensional enclosure of height H_e and length L_e. It is heated with two identical heat blocks S_1 and S_2 in the form of a rectangular block located symmetrically at the bottom wall. Each active block having a fixed length l = 0.1 and a height H is maintained at a constant temperature T_h. The vertical walls of the enclosure are isothermally cooled at a constant temperature T_c and the horizontal walls are considered to be insulated except the two heat sources. Using thermal lattice Boltzmann model of D2Q4-D2Q9, we investigated the effects of Rayleigh number (Ra) from 10^3 to 10^6, spacing between heaters (D) from 0.1 to 0.6, source height (H) from 0.05 to 0.4 and aspect ratio (A) from 1 to 4 on flow pattern and heat transfer in enclosure.

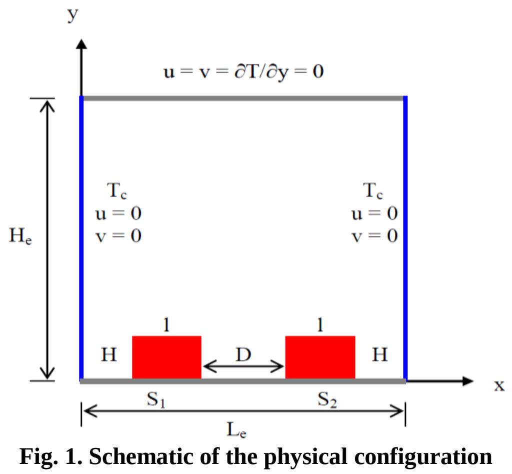

Fig. 1. Schematic of the physical configuration and coordinates.

2.2 Lattice Boltzmann Method

For two-dimensional flows, Huang *et al.* (2011) show that the simplest D2Q4 model is capable of obtaining results of equal accuracy as D2Q5 or D2Q9 models. In this investigation, we carried out a comparison between the accuracy and stability of these models in order to solve the thermal flow problem. It was found that the thermal field is the same with various models but the time of simulations with D2Q4 model is smaller than that of D2Q9 model. To predict the velocity of the flow, Mele (2013) shows that the D2Q4 or D2Q5 model is not appropriate because of insufficient lattice symmetry. He noted that D2Q9 model has been widely and successfully used for simulations of two-dimensional flows. It is a bit more demanding from a computational aspect than the hexagonal D2Q7 model, but it is more accurate. Hence, D2Q4-D2Q9 models are adopted in this study for the temperature and the flow (Fig. 2).

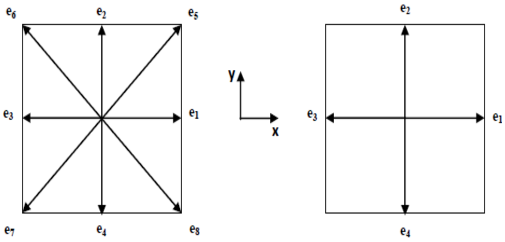

Fig. 2. Lattice structure for two dimensional with D2Q4-D2Q9 models.

In the LB method, the fluid is modelled by fictitious particle exchanging informations quantified by distribution functions that occupy nodes and transit to neighbouring nodes in a streaming phase. After introducing Bhatnagar-Gross-Krook approximation, double population function thermal lattice Boltzmann model introduced by He *et al.* (1998) is used.

For the flow field:

$$f_k(\vec{x}+\vec{e_k}\Delta t, t+\Delta t)-f_k(\vec{x},t)= -\frac{1}{\tau_f}[f_k(\vec{x},t)-f_k^{eq}(\vec{x},t)]+\Delta t\, F_k \quad (1)$$

For the temperature field:

$$g_k(\vec{x}+\vec{e_k}\Delta t, t+\Delta t)-g_k(\vec{x},t)= -\frac{1}{\tau_g}[g_k(\vec{x},t)-g_k^{eq}(\vec{x},t)] \quad (2)$$

where f_k and g_k are density distribution functions, τ_f and τ_g characterize the single relaxation times resulting from the BGK approximation for the collision operator, F_k is the external force in direction of lattice velocities $\vec{e_k}$ and the equilibrium density distribution functions can be formulated as

$$f_k^{eq}=w_k\,\rho[1+3\frac{\vec{e_k}.\vec{u}}{c^2}+\frac{9}{2}\frac{(\vec{e_k}.\vec{u})^2}{c^4}-\frac{3}{2}\frac{\vec{u}^2}{c^4}] \quad (3)$$

$$g_k^{eq}=w'_k\,\theta[1+\frac{\vec{e_k}.\vec{u}}{{c_s}^2}] \quad (4)$$

The weighting factors w_k for D2Q9 model are given as $w_0=\frac{4}{9}$, $w_{1-4}=\frac{1}{9}$, $w_{5-8}=\frac{1}{36}$ and $w'_k=0.25$ for D2Q4 model.

The discrete velocities $\vec{e_k}$ are defined as: $\vec{e_0}=(0,0)$, $\overrightarrow{e_{1-4}}=(\pm c,0)$ and $\overrightarrow{e_{5-8}}=(\pm c,\pm c)$.

$c=\frac{\Delta x}{\Delta t}$, Δx and Δt are the lattice space and the lattice time step size, respectively, which are set to unity.

For D2Q9 model, the macroscopic density ρ and fluid velocity vector $\vec{u}=(u,v)$ are calculated by summing the distribution functions over the nine-velocity directions, as:

$$[\rho,\rho\vec{u}]=\sum_{k=0}^{8}[f_k,\vec{e_k}\,f_k] \quad (5)$$

For D2Q4 model, the temperature θ is calculated by summing the distribution functions over the fore-velocity directions, as:

$$[\theta]=\sum_{k=1}^{4}[g_k] \quad (6)$$

The continuity, momentum and energy equations can be recovered through the Chapman-Enskog expansion (He and Luo., 1997) under incompressible limit assumption $Ma=\left|\vec{u}\right|/c_s<<1$ and without forcing term, as:

$$\begin{cases}\nabla.\vec{u}=0\\ \partial_t\vec{u}+\nabla.(\vec{u}\,\vec{u})=-(\nabla p)/\rho+\nu\nabla^2\vec{u}\\ \partial_t\theta+\nabla.(\vec{u}\,\theta)=\chi\nabla^2\theta\end{cases} \quad (7)$$

where $p=\rho c_s^2$ is the pressure from the equation of the state for the ideal gas, $c_s=\frac{c}{\sqrt{3}}$ is the sound speed. The kinetic viscosity and the thermal diffusivity are linked to the relaxation times:

$$\tau_f=3\nu+0.5 \quad (8)$$

$$\tau_g=2\chi+0.5 \quad (9)$$

In simulating natural convection problem the additional forcing term is modeled under the Boussinesq approximation; which considers that all fluid properties are constant, except the fluid density given by $\rho=\rho_0(1-\beta(T-T_r))$, where ρ_0 is a reference fluid density, then the external buoyant force $\rho_0\vec{G}=-\rho_0\beta(T-T_r)\vec{g}$ appearing in momentum equation will be expressed as

$$F_k=\frac{\vec{G}.(\vec{e_k}-\vec{u})}{c_s^2}f_k^{eq} \quad (10)$$

Following these considerations, $\left|\vec{u}\right|<<e_k$, $f_k^{eq}\approx w_k\,\rho(x,t)$ and T_r = 0, the final form of the external body force is $F_k=-3w_k\,\rho(\vec{x},t)\beta\theta(\vec{x},t)\vec{g}.\vec{e_k}$ (11)

2.3 Boundary Conditions and Dimensionless Parameters

Implementation of boundary condition for flow is applied with bounce-back boundary conditions on all walls, which means that incoming boundary populations equal to out-going populations after collision ($f_\alpha = f_\beta$, where the asterisk "α" and "β" denote opposite directions at the wall node). For example for flow field in the east boundary of the enclosure (Fig.3), the following conditions is used

$$f_{3,n} = f_{1,n}\ ,\ f_{6,n} = f_{8,n}\ ,\ f_{7,n} = f_{5,n} \tag{12}$$

where n is the lattice on the boundary.

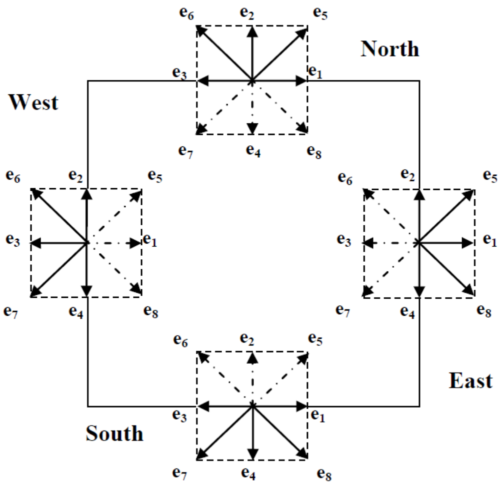

Fig. 3. Known and unknown distribution functions at the domain boundaries.

Furthermore, the boundary condition for the temperature distribution function at isothermal wall can be determinate by the following condition:

$$g_\alpha = (w_\alpha + w_\beta) T_{wall} - g_\beta \tag{13}$$

For superior hot wall of heater, boundary condition is evaluated as

$$g_2 = 0.5 - g_4 \tag{14}$$

The adiabatic boundary condition is transferred to Dirichlet-type condition using the conventional second-order finite difference approximation as:

$$g_{wall} = (4g_1 - g_2)/3 \tag{15}$$

To simulate the natural convection problems with the LBM, it is necessary to define the characteristic velocity $U = \sqrt{g\beta\Delta TL}$ used as a reference scale to check the compressibility limit; and for the sake of comparison with previous findings. The reference quantities used in the present investigation are presented as: L_0=L, $U_0=\chi/L$, $t_0=L^2/\chi$, $p_0=\rho_0 U_0^2$ and $\Delta T=T_h-T_c$ used for length, velocity, time, pressure and relative temperature respectively. Rayleigh and Prandtl numbers are defined as $Ra = \frac{g\beta\Delta T L^3}{\chi\nu}$ and $\Pr = \frac{\nu}{\chi}$, respectively. The average Nusselt number Nu_{h1} along horizontal hot upper wall of heater S_1 is calculated with a second order finite difference scheme as:

$$Nu_{h1} = \frac{1}{l}\sum \frac{3\theta_0 - 4\theta_1 + \theta_2}{2} \tag{16}$$

The following convergence criterion was adopted for all dependent variables at each point in the computation domain:

$$Error = \max\left|\Phi(t+1) - \Phi(t)\right| \le 10^{-5} \tag{17}$$

where Φ stands for a dependent variables θ, u and v; t refers to time.

3. RESULTS AND DISCUSSION

3.1 Grid Independence and Validation of LB Model

Grid independence is carried out to choose the accurate grid size for the numerical investigation using thermal LB model. Average Nusselt number along hot upper wall for each source is presented for various uniform grid size of lattice number ranging from 2500 to 62500 (Fig.4). It is showed that the grid size beyond 40000 elements for fixed Ra at 10^5 is adequate to describe the flow and heat transfer characteristics accurately. Therefore, for Ra < 10^5 and Ra = 10^6, grid sizes of 150*150 to 300*300 elements were used, respectively.

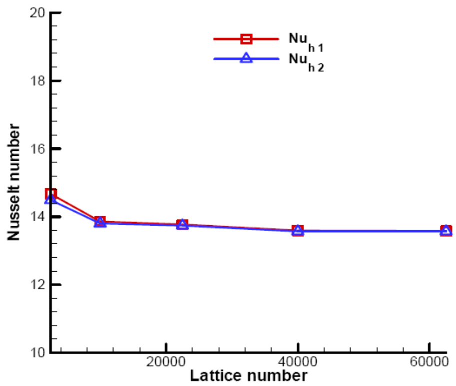

Fig.4. Effect of the lattice number on Nusselt number for A =1, Ra = 10^5, D = 0.4 and H = 0.05

The numerical thermal LB D2Q4-D2Q9 models for the problem of laminar natural convection in a square enclosure having differentially heated vertical wall is validated by comparing with studies of Kao *et al.* (2008) and de Vahl Davis (1983). Average Nusselt number for different computational approaches is presented in table 1. Good agreements between the predicted results with previous studies are shown.

The developed computational LB model is validated against with research of O.Aydin (2000) on natural convection in enclosure with centred heating from below and symmetrically cooling from sides. Fig. 5 shows a comparison of isotherms and streamlines contours of the flow for different dimensionless length ε of the active source and

Rayleigh number at 10^4 and 10^6. It is seen that the predicted results of the flow pattern is more accurate compared with previous investigation.

Table 1 Comparison of the present results with previous findings of the average Nusselt number along hot wall of the heater vs. Rayleigh number

Ra	Present (LBM)	Kao *et al.* [2008] (LBM)	de Vahl Davids [1983] (DF)
10^3	1.121	1.113	1.118
10^4	2.241	2.231	2.245
10^5	4.509	4.488	4.510
10^6	8.769	8.696	8.806

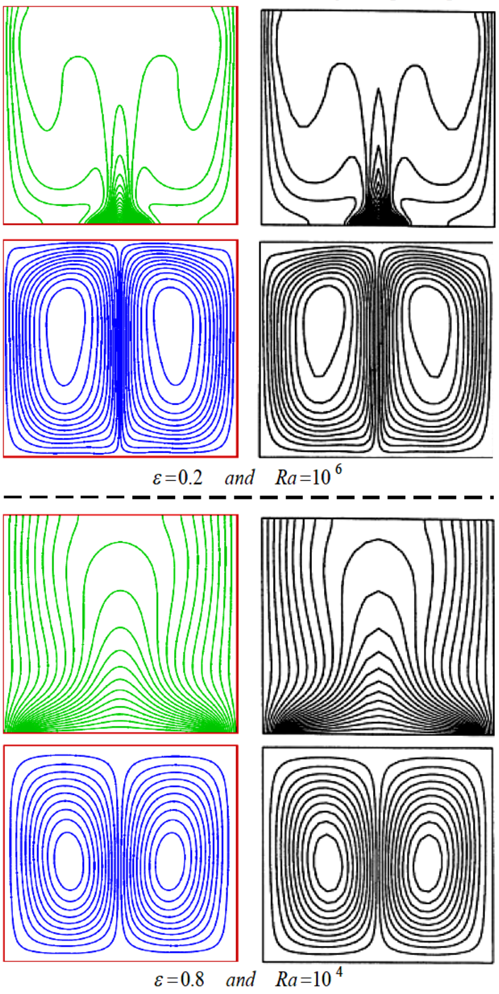

Fig. 5. Comparison of the present results with O.Aydin [2000] for a centred hot source: isotherms (top) and streamlines (bottom).

Fig. 6 depicts the average Nusselt number along hot source vs Rayleigh number ranging from 10^3 to 10^6 for two length ε = 0.2 and 0.8. This comparison reveals excellent agreements with finding of O.Aydin (2000) and the problem of natural convection flow and heat transfer in square enclosure asymmetrically heated from below using thermal D2Q9-D2Q9 models studied by Naffouti and Djebali (2012).

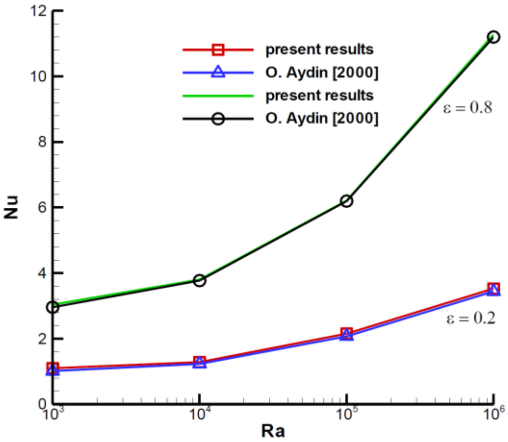

Fig. 6. Comparison of the average Nusselt number versus Rayleigh along hot wall for a centred hot source of length ε = 0.2 and 0.8 with previous findings.

After these favorable comparisons, it is found that the lattice Boltzmann method is a powerful numerical technique for simulating fluid flow and modeling the physics in fluids. In the present prediction, we use the D2Q4-D2Q9 models in order to analyze laminar natural convection heat transfer in a 2-D enclosure with two isothermal rectangular heat blocks located on the bottom wall. For a fixed Prandtl number at 0.71, numerical results have been conducted to elucidate the effects of Rayleigh number Ra = 10^3, 10^4, 10^5 and 10^6, spacing between heat sources D = 0.1, 0.2, 0.4 and 0.6, source height H = 0.05, 0.1, 0.2 and 0.4 and aspect ratio A = 1, 2, 3 and 4 on thermal and dynamic repartitions and heat transfer rate.

3.2 Behaviour of Dynamic and Thermal Fields of the Flow

3.2.1 Effect of Rayleigh Number

In order to point out the effect of Rayleigh number on natural convection throughout the square enclosure (A = 1), streamline and isotherm contour plots are presented in Fig.7 for various values of Ra ranging from 10^3 to 10^6 with H = 0.05 and D = 0.6. It may be seen that the behaviours of the flow structure and the temperature distributions are symmetrical about the vertical mid plane of the enclosure due to the symmetrical boundary conditions. For Ra $\leq$ 10^4, the flow pattern is characterized by two counter-rotating cells where the left one turns in counter-clockwise and the right one clockwise direction. In fact, the ascending flow blocked by upper adiabatic wall moves horizontally toward the corresponding cold wall under effect of cooling then it descends vertically to supply the active blocks in fresh air by the low. Moreover, flow circulation is so weak owing to dominance of viscous forces over the buoyancy force. This shows that the conduction dominates heat transfer inside the enclosure. As the Rayleigh number increases to 10^6, isotherms deviate toward cold walls where the degree of stratification becomes significant causing the propagation of thermal plume resulting from the interaction between heaters toward adiabatic upper wall. This interaction phenomenon is signalled previously by several investigations in free and confined mediums (Brahimi *et al*., 1989; Ichimiya and Saiki., 2005). It is related to a vertical elongation of main cells and an intensification of flow circulation inside the enclosure. This leads the convection heat transfer becomes dominant to conduction with the growth of Rayleigh number. Beyond Ra = 10^4, it is shown the appearance of two secondary small cells between hot sources due to blocking of hot air by the main cells.

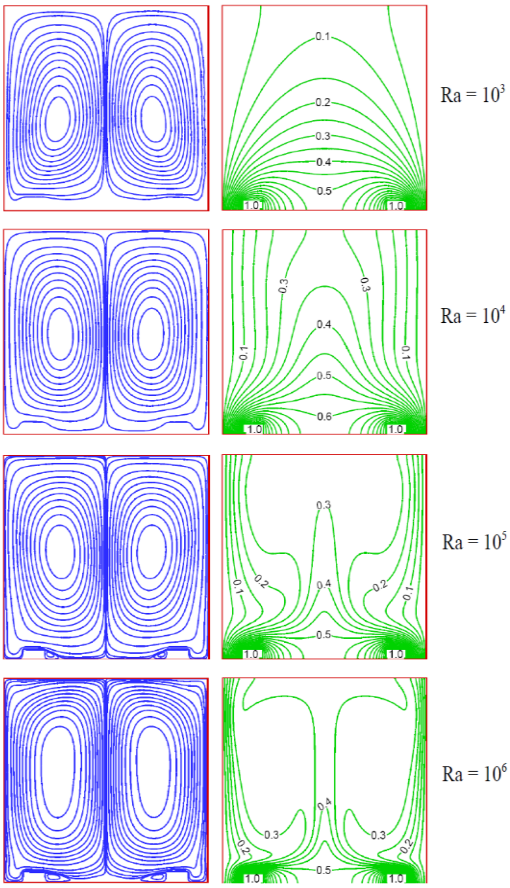

Fig. 7. Streamlines (left) and isotherms (right) vs. Rayleigh number with A = 1, H = 0.05 and D = 0.6.

3.2.2 Effect of Spacing between Heat Blocks

Study of spacing D between rectangular hot sources S_1 and S_2 is significant parameter to give more information about dynamic and thermal fields. Fig.8 illustrates the streamlines and isotherms of the flow in enclosure for D = 0.1, 0.2, 0.4 and 0.6 while A, Ra and H are fixed at 1, 10^5 and 0.05 respectively. It can be seen that contours plots are symmetric for different spacing and the convection heat transfer is dominant over conduction owing to higher values of Rayleigh number. When heat sources are closer to each other (D = 0.1), thermal structure is almost identical to a thermal plume generated by a single hot block (Paroncini and Corvaro., 2009; Naffouti *et al*., 2013) while the flow field is described by two big counter-rotating cells. However, the ascending hot air is strongly entrained by the cold air which comes by the low from the sides causing a strong interaction of the resulting flow. With increasing spacing D to 0.6, interaction between two thermal plume produced by the heaters becomes so weak owing to location of heat sources near the vertical cold walls. Same result is noted by the former work (Rosdzimin *et al*., 2010) on the interaction of two thermal plumes from two heated cylinders inside an enclosure. Beyond D = 0.1, streamlines show the formation of two small secondary cells near hot sources characterized by a weak circulation.

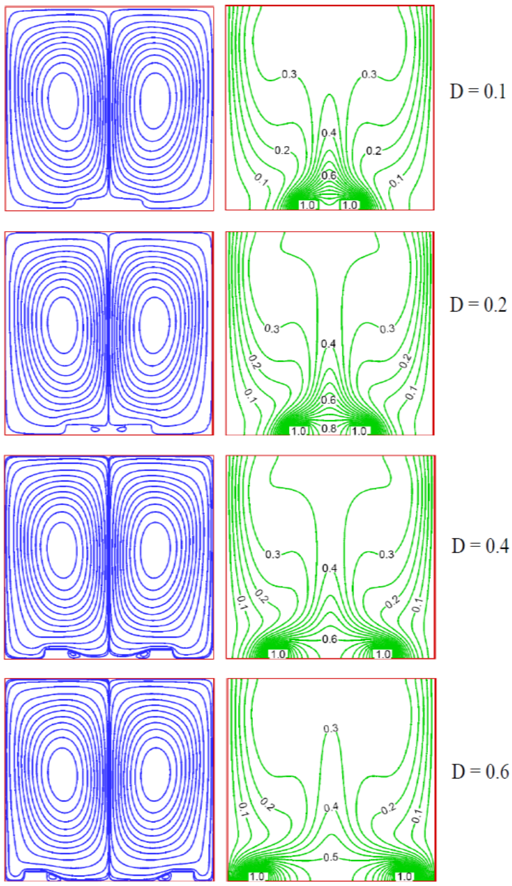

Fig.8. Streamlines (left) and isotherms (right) vs. spacing between heat blocks with A = 1, Ra = 10^5 and H = 0.05.

3.2.3 Effect of Block Height

Next, attention is focused on the effect of height H of both heat sources ranging from 0.05 to 0.4 on the natural convection inside enclosure for A = 1, D = 0.6 and Ra = 10^5. Streamlines and isotherms distribution of the flow obtained through the present numerical investigation for various heights are demonstrated in Fig. 9.

From the figure, it may be observed that the flow pattern and thermal contours are affected by the variation of block height. Similar conclusion is mentioned by Paroncini and Corvaro (2009). Through plots, the symmetric behaviour of the fluid flow is observed. For H = 0.05, dynamic structure consists of four counter-rotating cells; two small cells between heaters with two big cells characterized by a strong circulation of the flow in enclosure. From the graphs, it is shown that the circulation intensity increases with increasing H due to strong effect of the buoyancy force. Moreover, as H increases the size of big cells decrease while the corresponding of small cells increases. For the largest height of heat sources at 0.4, structure of main cells is destroyed and a completely different global flow pattern is formed having six cells. Furthermore, stratification of the isotherms increases as H increases in the region between heaters and vertical cold walls in particularly for H = 0.4 thus induces a thinner boundary layer development.

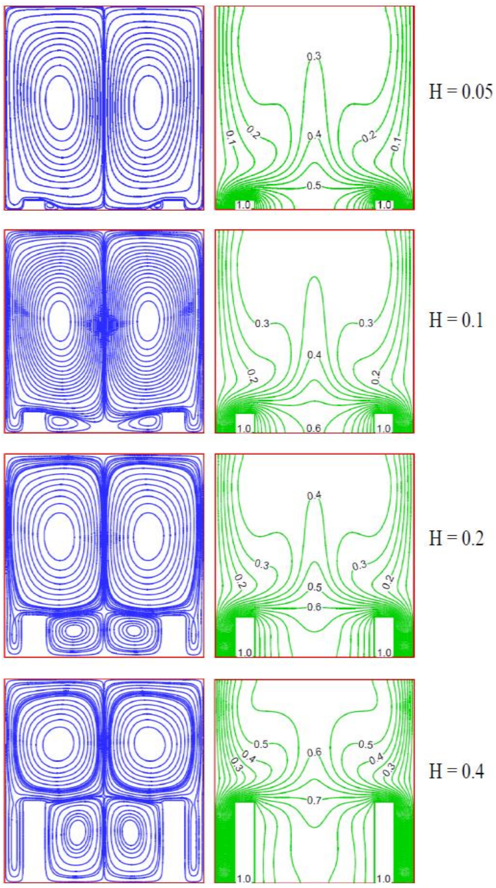

Fig. 9. Streamlines (left) and isotherms (right) vs. height of heat sources with A = 1, Ra = 10^5 and D = 0.6

3.2.4 Effect of Aspect Ratio of the Enclosure

The flow and thermal fields versus aspect ratio of the enclosure A = 1, 2, 3 and 4 with Ra = 10^5, D = 0.6 and H = 0.05 are illustrated in Fig.10.

For various aspect ratios, streamlines consist of two symmetric convection cells with clockwise and anticlockwise rotations in enclosure. From the graphs, it can be seen that the cores of the cells flow pattern moves toward cooled sidewalls as increasing A. This is related to an intensification of longitudinal elongation of the cells with the growth of enclosure volume which reduces the intensity of flow recirculation. However, in the convection region around the active blocks, the density of the isotherms becomes weaker with increasing A causing a production of feeble temperature gradients. It is due to cooling effect of the active blocks which becomes significant as increasing aspect ratio.

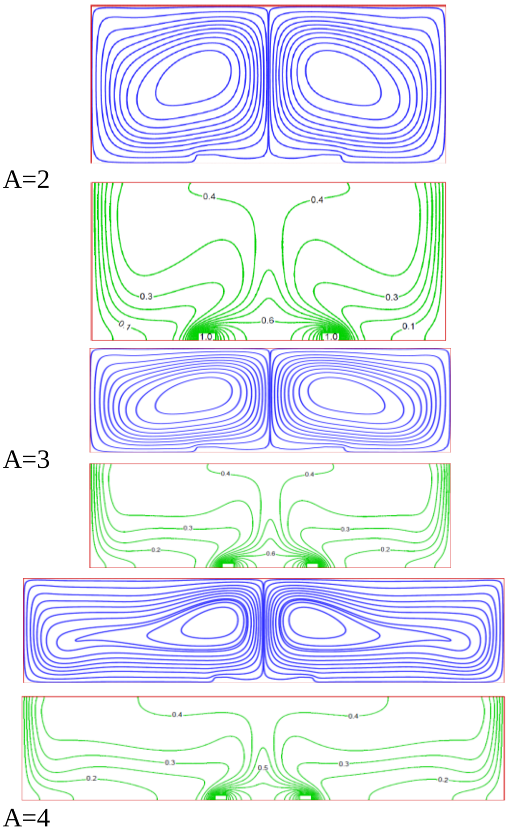

Fig. 10. Streamlines (top) and isotherms (bottom) vs. aspect ratio of the enclosure With Ra = 10^5,D = 0.6 and H = 0.05

3.3 Heat Transfer Optimization of the Flow

In this section, the effects of Rayleigh number, spacing between blocks, block height and aspect ratio of the enclosure on heat transfer are discussed. The average Nusselt number along horizontal hot wall of both heaters S_1 and S_2 noted respectively Nu_{h1} and Nu_{h2} vs. various pertinent parameters is revealed in Figs. 11-14. As seen from the plots, heat transfer rates are the same on upper hot surfaces by the reason of symmetrical phenomenon in enclosure. For A = 1, D = 0.6 and H = 0.05, Fig.11 shows a fast growth rate of the average Nusselt number with increasing Ra thus indicating the enhancement of convection mode. These results consolidate the view of the resulting isotherms given in Fig. 7.

From Fig.12 with A = 1, Ra = 10^5 and H = 0.05, it is found that the heat transfer rate slightly increases from 11.54 to 14.12 as D increases from 0.1 to 0.6 and consequently the improving of heat transfer especially for the largest spacing. From Fig.13 related to A = 1, Ra = 10^5 and D = 0.6, Nu_{h1} and Nu_{h2} practically remain invariant

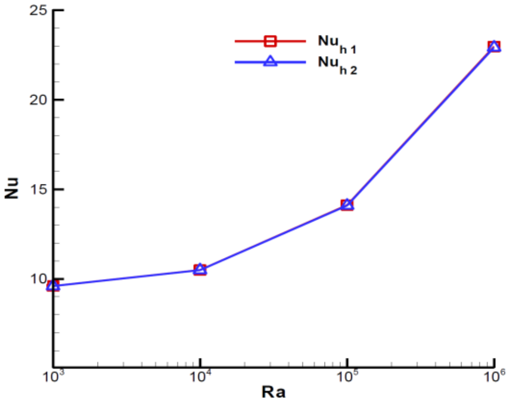

Fig. 11. Variation of the average Nusselt number along horizontal hot upper wall of heat blocks S1 and S2 for various Rayleigh number with A = 1, D = 0.6 and H = 0.05.

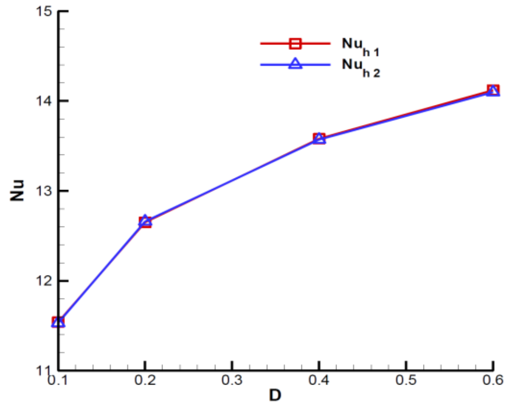

Fig.12. Variation of the average Nusselt number along horizontal hot upper wall of heat blocks S1 and S2 for various blocks spacing with A = 1, Ra = 10^5 and H = 0.05

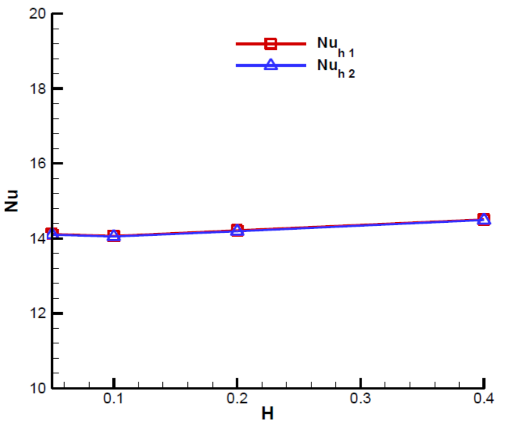

Fig. 13. Variation of the average Nusselt number along horizontal hot upper wall of heat blocks S1 and S2 for various block height with A = 1, Ra = 10^5 and D = 0.6.

with different height H. This shows that heat transfer is not affected by the variation of the parameter H along horizontal hot surface of heater. For fixed Ra , D and H at 10^5, 0.6 and 0.05, respectively, plots of Fig.14 demonstrate that increasing aspect ratio from 1 to 4 tends to decrease the heat transfer rate about of 17 % owing to reduction of thermal gradients in the proximity of the active blocks which become almost independent of the thermal boundary conditions.

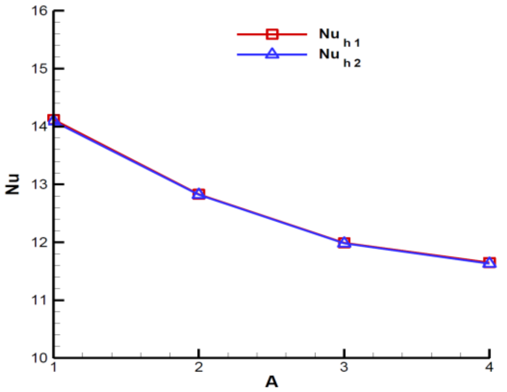

Fig. 14. Variation of the average Nusselt number along horizontal hot upper wall of heat blocks S1 and S2 for various aspect ratio of the enclosure with Ra = 10^5, D = 0.6 and H = 0.05.

In order to give more information about the behaviour of heat transfer in enclosure, a comparison between average Nusselt number on the left and the right vertical hot walls of heat source S1 noted respectively Nu_{RV1} and Nu_{LV1} for different parameters is illustrated in Figs. 15-18. It is clear that Nu_{LV1} is bigger than Nu_{RV1} for all configurations due to strong thermal gradients in the region between blocks and cooled sidewalls. It can be concluded that the heat transfer is more enhanced along external lateral walls of the hot sources. For various Rayleigh number with A = 1, D = 0.6 and H = 0.05 (Fig.15), rate of Nu_{LV1} increases to 25.77 with increasing of Ra than enhancing the convective heat transfer in the region at the left of hot source S1.

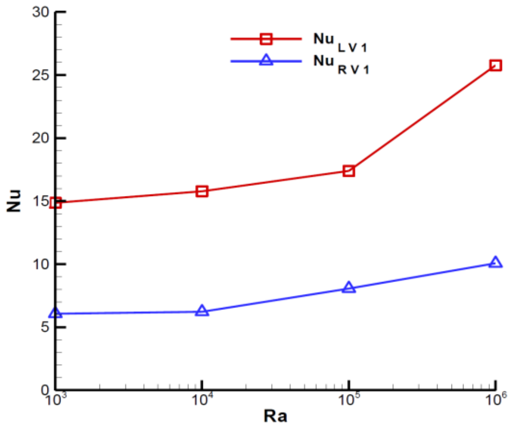

Fig.15. Variation of the average Nusselt number along vertical hot walls of heat block S1 for various Rayleigh number with A = 1, D = 0.6 and H = 0.05.

For different spacing with fixed A at 1, Ra at 10^5 and H at 0.05, Fig.16 shows that Nu_{LV1} is rather uniform. With increasing D a slight growth rate of Nu_{RV1} is showed to attain its higher value at 8.06. For A = 1, Ra =10^5 and D = 0.6, plots of Fig.17 reveal the decrease of Nu_{LV1} and Nu_{RV1} with increasing of block height causing a reduction of the convective heat transfer. As seen from the Fig.18 (Ra = 10^5, D = 0.6 and H = 0.05), Nu_{LV1} and Nu_{RV1} rate is reduces with increasing of aspect ratio from 1 to 4.

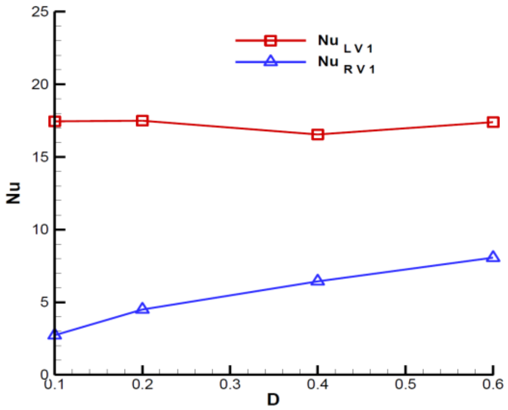

Fig. 16. Variation of the average Nusselt number along vertical hot walls of heat block S_1 for various blocks spacing with A = 1, Ra = 10^5 and H =0.05.

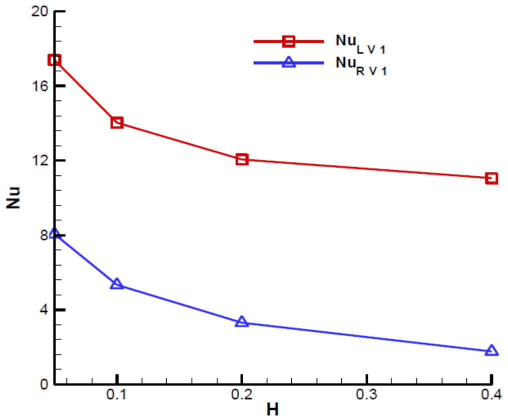

Fig. 17. Variation of the average Nusselt number along vertical hot walls of heat block S_1 for various block height with A = 1, Ra = 10^5 and D = 0.6

4. CONCLUSION

In this paper, a numerical method of thermal lattice Boltzmann model D2Q4-D2Q9 was employed to simulate free convection heat transfer and fluid flow inside a 2-D enclosure cooled by the sides and heated from below with two rectangular heat blocks. This investigation with LBM has been presented to unveil mainly the effects of pertinent parameters in the following ranges: Rayleigh number $10^3 \leq Ra \leq 10^6$, spacing between hot blocks $0.1 \leq D \leq 0.6$, block height $0.05 \leq H \leq 0.4$ and aspect ratio of the enclosure $1 \leq A \leq 4$, on the thermo-fluid characteristics of the laminar flow. Graphical results for the streamline and temperature contours as well as the average Nusselt number along each hot wall were analysed. The main results drawn from this study are illustrated as follow:

- Results of lattice Boltzmann method (LBM) were validated with former studies and consequently a good agreement is demonstrated. This shows that LBM is a powerful numerical technique for solving fluid flows and heat transfer problems in mechanical engineering.

- Contours of streamline and isotherm are symmetrical about vertical mid-plane of the enclosure for all studied configurations hence average Nusselt number on hot wall for both heat sources is the same owing to the symmetrical boundary conditions.

- As increasing Rayleigh number, flow circulation and stratification of isotherms become significant causing the propagation of thermal plume flow to upper adiabatic wall. This indicates that the natural convection is more dominated to conduction hence enhancing heat transfer rates in enclosure especially for high values of Ra.

- As increases spacing D, the interaction between two thermal plumes produced by the heaters becomes so weak while the average Nusselt number increases especially for the largest spacing.

- Dynamic structure of the flow is more and more destroyed with increasing source height and consequently six cells are formed for bigger height. Furthermore, heat transfer rates decrease as block height increases hence weakening the convective mode in enclosure.

- Optimum of heat transfer with fixed Rayleigh number at 10^5 is obtained for D = 0.6, H = 0.05 and smaller A corresponding to a square enclosure.

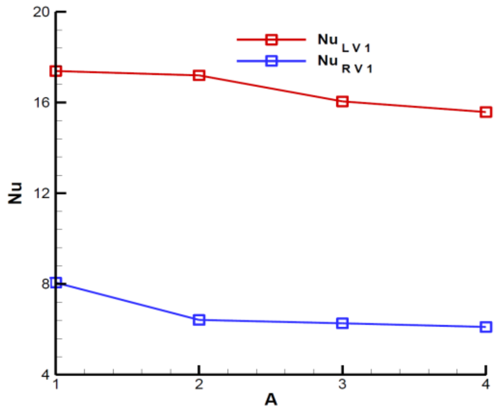

Fig.18. Variation of the average Nusselt number along vertical hot walls of heat block S_1 for various aspect ratio of the enclosure with Ra = 10^5, D = 0.6 and H = 0.05.

REFERENCES

Aswatha, C., J. Gangadhara Gowda, S. N. Sridhara and K. N. Seetharamu (2012). Buoyancy Driven

Heat Transfer in Cavities Subjected to Thermal Boundary Conditions at Bottom Wall. *Journal of Applied Fluid Mechanics* 5(2), 43-53.

Aydin, O. and J. Yang (2000). Natural convection in enclosure with localized heating from below and symmetrically cooling from sides. *International Journal of Numerical Methods for Heat and Fluid Flow* 10(5), 518-529.

Bae, J. H. and J. M. Hyun (2004). Time dependent buoyant convection in an enclosure with discrete heat sources. *International Journal of Thermal Science* 43(1), 3-11.

Banerjee, S., A. Mukhopadhyay, S. Sen and R. Ganguly (2008). Numerical study on natural convection in a bi-heater configuration of passive electronic cooling. *International Journal of Thermal Science* 47(11), 1516-1527.

Barozzi, G. S. and M. A. Corticelli (2000). Natural convection in cavities containing internal sources. *Journal of Heat and Mass Transfer* 36 (6), 473-480.

Barozzi, G. S., M. A. Corticelli and E. Nobile (1999). Numerical simulation of time-dependent buoyant flows in an enclosed vertical channel. *Journal of Heat and Mass Transfer* 35(2), 89-99.

Baskaya, S., U. Erturhan and M. Sivrioglu (2005). An experimental study on convection heat transfer from an array of discrete heat sources. *International Communications in Heat and Mass Transfer* 32(1-2), 248-257.

Baskaya, S., U. Erturhan and M. Sivrioglu (2005). Experimental investigation of mixed convection from an array of discrete heat sources at the bottom of a horizontal channel. *Heat and Mass Transfer* 42(1), 56-63.

Bazylak, A., N. Djilali and D. Sinton (2006). Natural convection in an enclosure with distributed heat sources. *Numerical Heat Transfer, Part A* 49, 655–667.

Bhowmik, H., C. P. Tso and K. W. Tou (2005). Analysis of convection heat transfer from discrete heat sources in a vertical rectangular channel. *Journal of Electronic Packaging* 127 (3), 215-222.

Bhowmik, H., C. P. Tso, K. W. Tou and F. L. Tan (2005). Convection heat transfer from discrete heat sources in a liquid cooled rectangular channel. *Applied Thermal Engineering* 25(16), 2532–2542.

Brahimi, M., L. Dehmani and D. K. Son (1989). Structure turbulente de l'écoulement d'interaction de deux panaches thermiques. *International Journal of Heat and Mass Transfer* 32(8), 1551-1559.

Chen, L., D. Keyhani and R. Pitts (1991). Convection heat transfer due to protruded heat sources in an enclosure. *Journal of Thermophysics and Heat Transfer* 5(2), 217-223.

Chen, T. H. and L. Y. Chen (2007). Study of buoyancy-induced flows subjected to partially heated sources on the left and bottom walls in a square enclosure. *International Journal of Thermal Science* 46(12), 1219-1231.

Da Silva, A. K., G. Lorenzini and A. Bejan (2005). Distribution of heat sources in vertical open channels with natural convection. *International Journal of Heat and Mass Transfer* 48(8), 1462-1469.

Da Silva, A. K., S. Lorente and A. Bejan (2004). Optimal distribution of discrete heat Sources on a wall with natural convection. *International Journal of Heat and Mass Transfer* 47(2), 203-214.

De Vahl, D., G. (1983). Natural convection of air in a square cavity: a bench mark numerical solution. *International Journal for Numerical Methods in Fluids* 3(3), 249-264.

Deng, Q. H., G. F. Tang and Y. G. Li (2002). Interaction between Discrete Heat Source in Horizontal Natural Convection Enclosures. *International Journal of Heat and Mass Transfer* 45(26), 5117- 5132.

Deng, Q. H., G. F. Tang and Y. Li (2002). A combined temperature scale for analyzing natural convection in rectangular enclosures with discrete wall heat sources. *International Journal of Heat and Mass Transfer* 45(16), 3437–3446.

He, X. and L. S. Luo (1997). Lattice Boltzmann Model for the incompressible Navier-Stokes equation. *Journal of Statistical Physics* 88(3-4), 927-944.

He, X., S. Chen and G. D. Doolen (1998), A novel thermal model for the lattice Boltzmann method in incompressible limit. *Journal of Computational Physics* 146(1), 282-300.

Heindel T. J., S. Ramadhyani and F. P. Incropera (1995). Conjugate Natural Convection from an Array of Discrete Heat Sources; Part 1-Two- and Three-Dimensional Model Validation. *International Journal of Heat and Fluid Flow* 16(6), 501-510.

Heindel, T. J., F. P. Incropera and S. Ramadhyani (1995). Conjugate Natural Convection from an Array of Discrete Heat Sources: Part 2-A Numerical Parametric Study. *International Journal of Heat and Fluid Flow* 16(6), 511-518.

Heindel, T. J., F. P. Incropera and S. Ramadhyani (1996). Enhancement of Natural Convection Heat Transfer from an Array of Discrete Heat Sources. *International Journal of Heat and Mass Transfer* 39(3), 479-490.

Ho, C. J. and J. Y. Chang (1994). A Study of Natural Convection Heat Transfer in a Vertical Rectangular with Two-dimensional Discrete Heating: Effect of Aspect Ratio. *International Journal of Heat and Mass Transfer* 37(6), 917–926.

Huang, H. B, X-Y Lu and M. C. Sukop (2011). Numerical study of lattice Boltzmann methods for a convection-diffusion equation coupled with Navier-Stokes equations. *Journal of Physics A: Mathematical and Theorethical* 44, 1-18.

Ichimiya, K. and H. Saiki (2005). Behavior of thermal plumes from two heat sources in an enclosure. *International Journal of Heat and Mass Transfer* 48(16), 3461–3468.

Ichimiya, K. and H. Saiki (2005). Behavior of thermal plumes from two-heat sources in an enclosure. *International Journal of Heat and Mass Transfer* 48(16), 3461–3468.

Kao, P. H., Y. H. Chen and R. J. Yang (2008). Simulations of the macroscopic and mesoscopic natural convection flows within rectangular cavities. *International Journal of Heat and Mass Transfer* 51(15-16), 3776-3793.

Madhavan, P. N. and V. M. K. Sastri (2000). Conjugate natural convection cooling of protruding heat sources mounted on a substrate placed inside an enclosure: A parametric study. *Computer Methods in Applied Mechanics and Engineering* 188*(1-3), 187-202.*

Mele, I. (2013). *Lattice Boltzmann method*, Seminar. Univerza v Ljubljani Fakulteta za matematiko in Fiziko.

Naffouti, T., J. Zinoubi and R. B. Maad (2015). Experimental Investigation of the Effect of Spacing between Vertical Plates on the Development of a Thermal Plume from an Active Block. *Journal of Applied Fluid Mechanics* 8(1), 75-84.

Naffouti,T. and R. Djebali (2012). Natural convection flow and heat transfer in square enclosure asymetrically heated from below: A lattice Boltzmann comprehensive study. *Computer Modeling in Engineering and Sciences* 88(3), 211-227.

Naffouti,T., J. Zinoubi and R. Ben Maad (2013). Lattice Boltzmann Analysis of 2-D Natural Convection Flow and Heat Transfer within Square Enclosure including an Isothermal Hot Block, *International Journal of Thermal Technologies* 3(4), 146-154.

Naik, R., V. Varadarajan, G. Pundarika and K. R. Narasimha (2013). Experimental Investigation and Performance Evaluation of a Closed Loop Pulsating Heat Pipe. *Journal of Applied Fluid Mechanics* 6(2), 267-275.

Papanicolaou, E. and S. Gopalakrishna (1995). Natural Convection in Shallow Horizontal Air Layers Encountered in Electronic Cooling. *Journal of Electronic Packaging* 117(4), 307-316.

Paroncini, M. and F. Corvaro (2009). Natural convection in a square enclosure with a hot source. *International journal of Thermal Sciences* 48(9) 1683-1695.

Rama N., K., S. N. Sridhara, M. S. Rajagopal and K. N. Seetharamu (2012). Influence of Heat Input, Working Fluid and Evacuation Level on the Performance of Pulsating Heat Pipe. *Journal of Applied Fluid Mechanics* 5(2), 33-42.

Rosdzimin, A. R. M., N. A. Che Sidik and S. M. Zuhairi (2010). Study of plume behavior two heated cylinders at high Rayleigh number using Lattice Boltzmann method. *The 10th Asian International Conference on Fluid Machinery 1225, 780-787.*

Tou, S. K. W. and X. F. Zhang (2003). Three-dimensional numerical simulation of natural convection in an inclined liquid-filled enclosure with an array of discrete heaters. *International Journal of Heat and Mass Transfer* 46(1), 127-138.

Tou, S. K. W., C. P Tso and X. Zhang (1999). 3-D Numerical Analysis of Natural Convective Liquid Cooling of 3x3 Heater Array in Rectangular Enclosure. *International Journal of Heat and Mass Transfer* 42(17), 3231-3244.

Tso, C. P., L. F. Jin, S. K. W. Tou and X. F. Zhang (2004). Flow pattern in natural convection cooling from an array of discrete heat sources in a rectangular cavity at various orientations. *International Journal of Heat and Mass Transfer* 47(19-20), 4061-4073.

Venkatachalapathy, S. and M. Udayakumar (2010). Numerical investigation of natural convection heat transfer from multiple heat sources in a square enclosure. *Journal of Engineering and Applied Sciences* 5 (1), 56-63.

Vinogradov, I., L. Khezzar and D. Siginer (2011). Heat Transfer of Non-Newtonian Dilatant Power Law Fluids in Square and Rectangular Cavities. *Journal of Applied Fluid Mechanics* 4 (2), 37-42.

Wang, Q., Y. Bo, M. Yaling and T. Wenquan. (1998). Natural convection in an inclined cube enclosure with multiple internal isolated plates. *Journal of Engineering Thermophysics* 19(2), 204-208.