

# Effects of Thermophoresis, Dufour, Hall and Radiation on an Unsteady MHD Flow past an Inclined Plate with Viscous Dissipation, Chemical Reaction and Heat Absorption and Generation

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## ABSTRACT

This paper investigates study of MHD(Magnetohydrodynamics) flow of viscous incompressible fluid past an inclined porous plate embedded in porous medium with the effects of Thermophoresis, Dufour, Hall, radiation, viscous dissipation, chemical reaction and heat generation or absorption. Non dimensional partial differential equations of governing equations of flow are solved numerically by applying Crank-Nicolson finite difference method for different values of parameters. Velocity, temperature, concentration profiles are discussed through graphs for different values of parameters and skin friction coefficients, Nusselt number and Sherwood number are discussed through tables.

**Keywords:** MHD, Dufour effect, Thermophoresis effect, Hall effect, radiation effect, Heat and Mass transfer, chemical reaction, Crank-Nicolson method.

## 1. INTRODUCTION

Large number of mathematicians have been attracted from last many decades toward investigation of unsteady MHD flow with effects of Thermophoresis, Dufour, radiation, chemical reaction, Hall parameter because there are number of avenues for research work. This investigation of such flow has application in MHD generators, soil sciences, nuclear power reactors, aeronautics, chemical engineering and so on.

Thermophoresis, Dufour effects and chemical reaction are significant if density difference exists and reaction level is not low respectively. If an electric field is applied at right angles to magnetic field, the whole current will not flow along the electric field, this nature of the electric current of flow across an electric field in presence of a magnetic field is called Hall effect.

Lee(Lee 2001) has analyzed a combined numerical and theoretical investigation of laminar natural convection heat and mass transfer in open vertical plates with unheated entry and unheated exist for various thermal and concentration boundary conditions. Sudha Karraiah et al.(Karraiah, Kesaraiah, and Bhavana 2013) studied Soret effect on unsteady MHD flow past a vertical plate with heat source. Postelnicu(Postelnicu 2004) studied numerically the influence of magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media with Soret and Dufour effects.

Sparrow et al.(Sparrow, Eichhorn, and Gregg 1959) has studied the combined effect of a steady free and forced convection laminar flow and heat transfer between two vertical parallel walls. Alam and Rahman(Alam and Rahman 2006) studied Soret-Dufour effects on unsteady MHD flow in porous medium. Olanrewaju(Oladapo 2010) analyzed Soret-Dufour effects of a transient free convective flow with radiative heat transfer past flat plate moving through a binary mixture and Dufour effects using both linear and non linear stability analysis.

Vempati et al.(Vempati and Gari 2010) has analyzed numerically Soret-Dufour effects on an unsteady MHD flow past an vertical porous plate with thermal radiation. Pandya and Shukla(Pandya and Shukla 2013) studied Soret-Dufour effects on unsteady MHD flow past an inclined porous plate embedded in porous medium. Ghosh(Ghosh 2002) investigated Hall effects on MHD flow in rotating system.

The objective of this work is to study combined effects of Thermophoresis, Dufour, Hall, radiation,

chemical reaction, viscous dissipation and heat generation or absorption on unsteady MHD flow past an inclined porous plate embedded in porous medium. Non dimensional form of partial differential equations of governing equations of flow has been solved by applying Crank-Nicolson implicit finite difference method. Velocity, temperature, concentration profiles are discussed through graphs for different values of parameters and skin friction coefficients, Nusselt number, Sherwood number have been discussed through tables.

## 2. MATHEMATICAL ANALYSIS

An unsteady MHD flow of viscous incompressible electrically conducting fluid past an inclined infinite porous plate with Thermophoresis effect, Dufour effect, Hall effect, radiation effect, viscous dissipation, chemical reaction and Heat generation or absorption are considered. x'-axis is taken along plate, y'-axis is normal to it and z'-axis is normal to x'y' plane. A uniform magnetic field  $B_0$  is considered along y'-axis and plate is electrically nonconducting. It is considered that radiation heat flux along x'-direction is negligible in comparison to y'direction. Initially fluid and plate are at same temperature  $T'_{\infty}$  and concentration  $C'_{\infty}$ . For t' > 0, the plate moves with velocity  $u_0$ , its temperature and concentration raise exponentially with time. Magnetic Reynolds number and transversely applied magnetic field are very small so induced magnetic field is negligible in comparison to applied magnetic field, Cowling(Cowling 1957).

$$J'_{x} = \frac{\sigma B_{0}}{1 + m^{2}} (mu' - w')$$
  
$$J'_{z} = \frac{\sigma B_{0}}{1 + m^{2}} (u' + mw')$$
(1)

where u' and w' are velocities,  $J'_x$  and  $J'_z$  are electric current density along x'-axis and z'-axis respectively, m is Hall parameter. Due to infinite length in x' direction, flow variables are function of t' and y' only. Under usual Boussinesq approximation, governing equations of this problem are given by:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0(constant) \tag{2}$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = v \frac{\partial^2 u'}{\partial y'^2} + g \gamma (T' - T'_{\infty}) \cos(\lambda) + g \gamma^* (C' - C'_{\infty}) \cos(\lambda) - \frac{\sigma B_0^2}{\rho (1 + m^2)} (u' + mw') - \frac{v u'}{K'}$$
(3)

$$\frac{\partial w'}{\partial t'} + v' \frac{\partial w'}{\partial y'} = v \frac{\partial^2 w'}{\partial y'^2} + \frac{\sigma B_0^2}{\rho(1+m^2)} (mu' - w') - \frac{vw'}{K'}$$
(4)

$$\rho c_p \left( \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + \frac{\rho D_m K_T}{c_s} \frac{\partial^2 C'}{\partial y'^2} + \mu \left( \frac{\partial u'}{\partial y'} \right)^2 - Q_0 (T' - T'_{\infty})$$
(5)

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T'}{\partial y'^2} -k_r (C' - C'_{\infty})$$
(6)

where  $\gamma^*$  is coefficient of volume expansion for mass transfer,  $\gamma$  is volumetric coefficient of thermal expansion, v' is velocity along y'-axis, K' is permeability of porous medium,  $\sigma$  is electrical conductivity,  $D_m$  is molecular diffusivity, g is acceleration due to gravity,  $K_T$  is thermal diffusion ratio,  $\mu$  is viscosity,  $\rho$  is fluid density, k is thermal conductivity of fluid, C' and T' are dimensional concentration and temperature,  $C'_{\infty}$  and  $T'_{\infty}$  are concentration and temperature of free stream,  $k_r$  is chemical reaction parameter; the term  $Q_0(T' - T'_{\infty})$ is considered to be amount of heat generated or absorbed per unit volume,  $Q_0$  is constant, for heat source  $Q_0 > 0$  and for heat sink  $Q_0 < 0$ ;  $c_p$  is specific heat at constant pressure,  $q_r$  is radiative heat along y'-axis, v is kinematic viscosity and  $T_m$ is mean fluid temperature.

Boundary and initial conditional for this model are given as:

$$t' \leq 0 \quad u' = 0 \quad w' = 0$$
$$T' = T'_{\infty} \quad C' = C'_{\infty} \quad \forall y'$$
$$t' > 0 \quad u' = u_0 \quad v' = -v_0 \quad w' = 0$$
$$T' = T'_{\infty} + (T'_{w} - T'_{\infty})e^{Bt'},$$
$$C' = C'_{\infty} + (C'_{w} - C'_{\infty})e^{Bt'} \quad at \ y' = 0$$
$$u' = 0 \quad w' = 0 \quad T' \to T'_{\infty}$$
$$C' \to C'_{\infty} \quad y' \to \infty \quad (7)$$

where  $T'_w$  and  $C'_w$  are concentration and temperature respectively of plate and  $B = \frac{v_0^2}{v}$ .

The radiative heat flux term by using the Roseland approximation is given by

$$q_r = -\frac{4\sigma}{3k_m} \frac{\partial T'^4}{\partial y'} \tag{8}$$

where  $\sigma$  and  $k_m$  are Stefan Boltzmann constant and mean absorption coefficient respectively. in this problem temperature difference within flow is very small, so that  $T'^4$  may be expressed linearly with temperature. It is observed by expanding in a Taylor's series about  $T'_{\infty}$  and considering negligible higher order term, hence

$$T^{\prime 4} \cong 4T_{\infty}^{\prime 3}T' - 3T_{\infty}^{\prime 4} \tag{9}$$

so, by equations 8 and 9, equation 5 is reduced

$$\rho c_p \left( \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = k \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_{\infty}'^3}{3k_m} \frac{\partial^2 T'}{\partial y'^2} + \frac{\rho D_m K_T}{c_s} \frac{\partial^2 C'}{\partial y'^2} + \mu \left( \frac{\partial u'}{\partial y'} \right)^2 - Q_0 (T' - T_{\infty}')$$
(10)

In order to obtain non dimensional form of governing equations, we introduce following quantities:

$$u = \frac{u'}{u_0}, t = \frac{t'v_0^2}{\nu}, \theta = \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}},$$

$$C = \frac{C' - C'_{\infty}}{C'_w - C'_{\infty}}, Gm = \frac{\nu g \gamma^* (C'_w - C'_{\infty})}{u_0 v_0^2},$$

$$Gr = \frac{\nu g \gamma (T'_w - T'_{\infty})}{u_0 v_0^2},$$

$$Du = \frac{D_m K_T (C'_w - C'_{\infty})}{c_s c_p \nu (T'_w - T'_{\infty})},$$

$$Sr = \frac{D_m K_T (T'_w - T'_{\infty})}{T_m \nu (C'_w - C'_{\infty})},$$

$$K = \frac{v_0^2 K'}{\nu^2}, Pr = \frac{\mu c_p}{k}, M = \frac{\sigma B_0^2 \nu}{\rho v_0^2},$$

$$R = \frac{4\sigma T'_{\infty}^3}{k_m k}, Sc = \frac{\nu}{D_m},$$

$$y = \frac{y' v_0}{\nu}, w = \frac{w'}{u_0}, Ec = \frac{u_0^2}{c_p (T'_w - T'_{\infty})}$$

$$K_r = \frac{k_r \nu}{v_0^2}, Q = \frac{Q_0 \nu}{\rho c_p v_0^2},$$
(11)

Using Substitutions of equation 11, we get nondimensional form of partial differential equations 3, 4, 10 and 6 respectively:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} =$$

$$\frac{\partial^2 u}{\partial y^2} + Gr\cos(\lambda)\theta + Gm\cos(\lambda)C$$

$$-\left(\frac{M}{1+m^2} + \frac{1}{K}\right)u - \left(\frac{mM}{1+m^2}\right)w \qquad (12)$$

$$\frac{\partial w}{\partial t} - \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} - \left(\frac{M}{1+m^2} + \frac{1}{K}\right)w + \left(\frac{mM}{1+m^2}\right)u \qquad (13)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left( 1 + \frac{4R}{3} \right) \frac{\partial^2 \theta}{\partial y^2} + Du \frac{\partial^2 C}{\partial y^2} + Ec \left( \frac{\partial u}{\partial y} \right)^2 - Q\theta \qquad (14)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} - K_r C \qquad (15)$$

non dimensional boundary and initial conditions are:

$$t \leq 0 \quad u = 0 \quad w = 0$$
  

$$\theta = 0 \quad C = 0 \quad \forall y$$
  

$$t > 0 \quad u = 1 \quad w = 0$$
  

$$\theta = e^{t} \quad C = e^{t} \quad at \ y = 0$$
  

$$u = 0 \quad w = 0 \quad \theta \to 0$$
  

$$C \to 0 \quad y \to \infty \qquad (16)$$

Now, many research workers have interest to calculate physical quantities skin-friction coefficients  $\tau_1$  and  $\tau_2$  along wall *x*-axis and *z*-axis respectively, Nusselt number *Nu* and Sherwood number *Sh*. Non-dimensional form of these physical quantities are:

$$\tau_{1} = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$
  

$$\tau_{2} = \left(\frac{\partial w}{\partial y}\right)_{y=0}$$
  

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$
  

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$
(17)

# 3. METHOD OF SOLUTION

Partial differential equations 12 to 15 are solved with initial and boundary conditions 16. To find exact solution of these partial differential equations are impossible. So, these are solved numerically applying Crank-Nicolson implicit finite difference method. First, equations 12, 13, 14 and 15 are expressed as:

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} - \frac{u_{i+1,j} - u_{i,j}}{\Delta y} = \left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i-1,j} - 2u_{i,j+1} + u_{i+1,j+1}}{2(\Delta y)^2}\right) + Gr\cos(\lambda) \left(\frac{\Theta_{i,j+1} - \Theta_{i,j}}{2}\right) + Gm\cos(\lambda) \left(\frac{Ci, j+1 - Ci, j}{2}\right) - \left(\frac{M}{1+m^2} + \frac{1}{K}\right) \left(\frac{u_{i,j+1} + u_{i,j}}{2}\right) - \left(\frac{mM}{1+m^2}\right) \left(\frac{w_{i,j+1} + w_{i,j}}{2}\right)$$
(18)

$$\frac{\frac{w_{i,j+1} - w_{i,j}}{\Delta t} - \frac{w_{i+1,j} - w_{i,j}}{\Delta y}}{\left(\frac{w_{i-1,j} - 2w_{i,j} + w_{i-1,j} - 2w_{i,j+1} + w_{i+1,j+1}}{2(\Delta y)^2}\right) - \left(\frac{M}{1 + m^2} + \frac{1}{K}\right) \left(\frac{uw_{i,j+1} + w_{i,j}}{2}\right) + \left(\frac{mM}{1 + m^2}\right) \left(\frac{u_{i,j+1} + u_{i,j}}{2}\right)$$
(19)

$$\begin{aligned} \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} - \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta y} &= \\ \frac{1}{Pr} \left( 1 + \frac{4R}{3} \right) \\ \left( \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i-1,j} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2} \right) + \\ Du \left( \frac{C_{i-1,j} - 2u_{i,j} + C_{i-1,j} - 2C_{i,j+1} + C_{i+1,j+1}}{2(\Delta y)^2} \right) \\ + Ec \left( \frac{u_{i+1,j} - u_{i,j}}{\Delta y} \right)^2 - Q \left( \frac{\theta_{i,j+1} + \theta_{i,j}}{2} \right) \end{aligned}$$
(20)

$$\frac{C_{i,j+1} - C_{i,j}}{\Delta t} - \frac{C_{i+1,j} - C_{i,j}}{\Delta y} = \frac{1}{Sc} \left( \frac{C_{i-1,j} - 2u_{i,j} + C_{i-1,j} - 2C_{i,j+1} + C_{i+1,j+1}}{2(\Delta y)^2} \right) + Sr \left( \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i-1,j} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2} \right) - K_r \left( \frac{C_{i,j+1} + C_{i,j}}{2} \right)$$
(21)

initial and boundary conditions are also expressed as:

$$u_{i,0} = 0 \quad w_{i,0} = 0 \quad \theta_{i,0} = 0 \quad C_{i,0} = 0 \quad \forall i$$
  
$$u_{0,j} = 1 \quad w_{0,j} = 0 \quad \theta_{0,j} = e^{j\Delta t} \quad C_{0,j} = e^{j\Delta t}$$
  
$$u_{L,j} = 0 \quad w_{L,J} = 0 \quad \theta_{L,j} \to 0 \quad C_{L,j} \to 0$$
  
(22)

where index *i* refers to *y* and *j* refers to time *t*,  $\Delta t = t_{j+1} - t_j$  and  $\Delta y = y_{i+1} - y_i$ . Knowing the values of *u*,  $\theta$  and *C* at time *t*, we can compute the values at time  $t + \Delta t$  as follows: we substitute i = 1, 2, ..., N - 1, where *N* correspond to  $\infty$ , equations 18 to 21 give tridiagonal system of equations with initial and boundary conditions in equation 22, are solved using Thomos algorithm as discussed in Carnahan et al. (Brice Carnahan and Wilkes 1969), we find values of  $\theta$  and *C* for all values of *y* at  $t + \Delta t$ . Equation 18 and 19 are solved by same to substitute these values of  $\theta$  and *C*, we get solution for *u* and *w* till desired time *t*.

The implicit Crank-Nicolson finite difference method is a second order method ( $o(\Delta t^2)$ ) in time and has no restriction on space and time steps, that is, the method is unconditionally stable. The computation is executed for  $\Delta y = 0.1$ ,  $\Delta t = 0.001$  and procedure is repeated till y = 4.

#### 4. **RESULT AND DISCUSSION**

In order to analysis, we would like to see numerical results for velocity profiles u, w along x-axis and zaxis, temperature profile  $\theta$  and concentration profile C with help of graphs by assigning numerical values of thermal Grashof number Gr, solutal Grashof number Gm, Schmidt number Sc, Prandtl number Pr, Thermophoresis number Sr, Dufour number Du, magnetic parameter M, Hall parameter m, permeability of porous medium K, radiation parameter R, Eckert number Ec, chemical reaction parameter Kr, heat absorption or generation constant Q, inclination angle  $\lambda$ . And coefficients of skin friction  $\tau_1$ and  $\tau_2$  along x-axis and z-axis, Nusselt number and Sherwood number are discussed through tables. Velocity profiles *u* and *w* in figure(1), temperature profile  $\theta$  in figure(3) and concentration profile C in figure(2) increase when Sr increases. On increasing Schmidt number Sc, it is analyzed that concentration profile C in figure(20) decreases. Figures (4), (5), (6) depict when Prandtl number Pr increases, velocity profiles u and w first increase after then at some distance from wall it decrease, temperature profile  $\theta$  decreases and concentration profile C first increases after then at some distance from wall it decreases. Figures (7), (8) and (9) depict that on increasing heat absorption or generation coefficient Q, velocity profiles u and w first increase after then decrease, temperature profile  $\theta$  decreases and concentration profile *C* first increases after then decreases. It is analyzed from figures(10) and (11) when Dufour number Du increases then temperature profile  $\theta$  increases and concentration profile *C* first decreases after then increases. Velocity profile *u* decreases while velocity profile *w* increases in figure(12) when magnetic parameter *M* increases.

It is studied that velocity profiles u and w first decrease after then increase in figure(13), temperature profile increases in figure(14) and concentration profile C increases in figure(15) as radiation parameter R increases. It is clear that concentration profile C decreases in figure(21) as chemical reaction parameter Kr increases. Figures (16) and (17) depict that temperature profile  $\theta$  increases and concentration profile C first decreases after then increases when Eckert number Ec increases. It is shown that on increasing inclination angle  $\lambda$ , Hall parameter m, velocity profiles u and w increase in figures (18), (19) respectively. It is analyzed from figures (22), (23), (24) that on increasing time t, velocity profiles u and w, temperature profile  $\theta$  and concentration profile C increase.

It is analyzed from table(1) that Du, Ec, Kr,  $\lambda$ , R and Sc increase then skin friction coefficients  $\tau_1$  and  $\tau_2$  decrease while Gm, Gr, m, Pr, Q, Sr and t increase then skin friction coefficients  $\tau_1$  and  $\tau_2$  increase and on increasing M,  $\tau_1$  decreases while  $\tau_2$  increases.

On the other hand it is analyzed from table(2) when Du, Ec, Kr,  $\lambda$ , M and R increase then Nusselt number Nu decreases and Sherwood number Sh increases and on increasing Gm, Gr, Pr, Q and t, Nusselt number Nu increases and Sherwood number Sh decreases while on increasing m, Sc and Sr Nusselt number Nu and Sherwood number Sh decreases.

## 5. CONCLUSION

In this work we have analyzed Thermophoresis-Dufour and radiation effects on unsteady MHD flow past an inclined porous plate embedded in porous medium in presence of Hall effect, chemical reaction effect, heat generation and absorption and viscous dissipation. From present numerical study, the following conclusions have been drawn:

- 1. On increasing Dufour number *Du*, radiation parameter *R*, Eckert number *Ec* concentration profile first decreases after then it increases.
- 2. Concentration profile first increases after then it decreases when prandtl number *Pr*, heat absorption coefficient *Q* increase.
- 3. On increasing prandtl number Pr, heat absorption coefficient Q, temperature profile decreases.
- 4. On increasing magnetic parameter M, veloc-



Fig. 1. Velocity Profile for Different Values of Sr



Fig. 2. Concentration Profile for Different Values of Sr



**Fig. 3. Temperature Profile for Different Values of** *Sr* 



Fig. 4. Velocity Profile for Different Values of Pr

ity profile *u* decreases and velocity profile *w* inreases while on

- 5. Velocity profiles, Temeperature profile and concentration profile increase when Soret number *Sr* increases.
- It has been seen that on increasing Hall parameter *m*, skin-friction coefficients τ and Nusselt number *Nu* increase while Sherwood number *Sh* decreases.



**Fig. 5. Temperature Profile for Different Values of** *Pr* 



Fig. 6. Concentration Profile for Different Values of *Pr* 



Fig. 7. Velocity Profile for Different Values of Q



Fig. 8. Temperature Profile for Different Values of *Q* 



Fig. 9. Concentration Profile for Different Values of Q



Fig. 10. Temperature Profile for Different Values of *Du* 



Fig. 11. Cocentration Profile for Different Values of *Du* 



Fig. 12. Velocity Profile for Different Values of M



Fig. 13. Velocity Profile for Different Values of *R* 



Fig. 14. Temperature Profile for Different Values of *R* 



Fig. 15. Concentration Profile for Different Values of *R* 



Fig. 16. Temperature Profile for Different Values of *Ec* 



Fig. 17. Concentration Profile for Different Values of *Ec* 



Fig. 18. Velocity Profile for Different Values of  $\lambda$ 



Fig. 19. Velocity Profile for Different Values of *m* 



**Fig. 20. Concentration Profile for Different Values of** *Sc* 



Fig. 21. Concentration Profile for Different Values of *Kr* 



Fig. 22. Velocity Profile for Different Values of t



Fig. 23. Temperature Profile for Different Values of *t* 



Fig. 24. Concentration Profile for Different Values of *t* 

Gr	Gm	K	M	m	Pr	Du	Sc	Sr	R	Q	Kr	Ec	λ	t	$\tau_1$	$\tau_2$
4	15	1	6	0.4	0.7	0.3	2	2	2	3	2	2	30	0.2	0.365933	0.475073
4	15	1	6	0.4	0.7	0.6	2	2	2	3	2	2	30	0.2	0.252774	0.469718
4	15	1	6	0.4	0.7	0.9	2	2	2	3	2	2	30	0.2	0.0986307	0.463457
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	6	30	0.2	0.174331	0.463616
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	10	30	0.2	-0.040225	0.451437
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	14	30	0.2	-0.235714	0.440629
4	1	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	-1.57878	0.33779
4	4	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	-1.15381	0.367813
4	9	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	-0.442202	0.417785
1	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	-0.286014	0.420261
9	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	1.57366	0.573396
15	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	2.97818	0.690603
4	15	1	6	0.4	0.7	0.15	2	2	2	3	6	2	30	0.2	0.0295669	0.450448
4	15	1	6	0.4	0.7	0.15	2	2	2	3	10	2	30	0.2	-0.238423	0.430839
4	15	1	6	0.4	0.7	0.15	2	2	2	3	15	2	30	0.2	-0.473261	0.413149
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	10	0.2	0.832599	0.508323
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	50	0.2	-0.377162	0.419417
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	75	0.2	-1.73075	0.319501
4	15	1	2	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	1.34901	0.205403
4	15	1	10	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	-0.33868	0.633676
4	15	1	13	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	-0.808624	0.706267
4	15	1	6	0.2	0.7	0.15	2	2	2	3	2	2	30	0.2	0.30089	0.25668
4	15	1	6	0.5	0.7	0.15	2	2	2	3	2	2	30	0.2	0.486815	0.567145
4	15	1	6	0.8	0.7	0.15	2	2	2	3	2	2	30	0.2	0.737974	0.746637
4	15	1	6	1	0.7	0.15	2	2	2	3	2	2	30	0.2	0.902002	0.802161
4	15	1	6	0.4	2	0.15	2	2	2	3	2	2	30	0.2	0.82657	0.504946
4	15	1	6	0.4	4	0.15	2	2	2	3	2	2	30	0.2	1.27385	0.527697
4	15	1	6	0.4	6	0.15	2	2	2	3	2	2	30	0.2	1.64794	0.540427
4	15	1	6	0.4	0.7	0.15	2	2	2	-1	2	2	30	0.2	0.246072	0.469568
4	15	1	6	0.4	0.7	0.15	2	2	2	-4	2	2	30	0.2	0.111319	0.463334
4	15	1	6	0.4	0.7	0.15	2	2	2	-7	2	2	30	0.2	-0.0317949	0.456905
4	15	1	6	0.4	0.7	0.15	2	2	2	7	2	2	30	0.2	0.565232	0.484863
4	15	1	6	0.4	0.7	0.15	2	2	2	10	2	2	30	0.2	0.671152	0.490059
4	15	1	6	0.4	0.7	0.15	2	2	2	14	2	2	30	0.2	0.802185	0.496543
4	15	1	6	0.4	0.7	0.15	2	2	5	3	2	2	30	0.2	0.266488	0.467202
4	15	1	6	0.4	0.7	0.15	2	2	7	3	2	2	30	0.2	0.231512	0.465092
4	15	1	6	0.4	0.7	0.15	2	2	9	3	2	2	30	0.2	0.211554	0.464127
4	15	1	6	0.4	0.7	0.15	5	2	2	3	2	2	30	0.2	-0.105512	0.444883
4	15	1	6	0.4	0.7	0.15	7	2	2	3	2	2	30	0.2	-0.269544	0.437092
4	15	1	6	0.4	0.7	0.15	9	2	2	3	2	2	30	0.2	-0.380888	0.432479
4	15	1	6	0.4	0.7	0.15	2	5	2	3	2	2	30	0.2	1.45291	0.588027
4	15	1	6	0.4	0.7	0.15	2	8	2	3	2	2	30	0.2	2.57766	0.703457
4	15	1	6	0.4	0.7	0.15	2	10	2	3	2	2	30	0.2	3.2865	0.779803
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.1	-0.735167	0.292596
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.3	1.1551	0.619508
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.4	1.77599	0.735828

Table 1. Skin friction coefficients  $\tau_1$  and  $\tau_2$  for different values of parameters

Gr	Gm	K	M	m	Pr	Du	Sc	Sr	R	Q	Kr	Ec	λ	t	Nu	Sh
4	15	1	6	0.4	0.7	0.3	2	2	2	3	2	2	30	0.2	0.922261	3.19397
4	15	1	6	0.4	0.7	0.6	2	2	2	3	2	2	30	0.2	0.733728	3.70869
4	15	1	6	0.4	0.7	0.9	2	2	2	3	2	2	30	0.2	0.429317	4.59048
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	6	30	0.2	0.708142	3.70483
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	10	30	0.2	0.449765	4.4023
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	14	30	0.2	0.205549	5.10646
4	1	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	0.950343	3.19691
4	4	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	0.96735	3.13988
4	9	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	0.986265	3.06478
1	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	0.972956	3.09194
9	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	0.991829	2.94609
15	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	0.925154	3.01295
4	15	1	6	0.4	0.7	0.15	2	2	2	3	6	2	30	0.2	0.95655	4.10817
4	15	1	6	0.4	0.7	0.15	2	2	2	3	10	2	30	0.2	0.926734	4.90377
4	15	1	6	0.4	0.7	0.15	2	2	2	3	15	2	30	0.2	0.898167	5.63625
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	10	0.2	0.993938	2.98542
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	50	0.2	0.981971	3.07297
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	75	0.2	0.924213	3.26652
4	15	1	2	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	0.987231	2.97189
4	15	1	10	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	0.982824	3.06806
4	15	1	13	0.4	0.7	0.15	2	2	2	3	2	2	30	0.2	0.970717	3.117
4	15	1	6	0.2	0.7	0.15	2	2	2	3	2	2	30	0.2	0.99382	3.01279
4	15	1	6	0.5	0.7	0.15	2	2	2	3	2	2	30	0.2	0.993837	3.0031
4	15	1	6	0.8	0.7	0.15	2	2	2	3	2	2	30	0.2	0.993253	2.9914
4	15	1	6	1	0.7	0.15	2	2	2	3	2	2	30	0.2	0.992224	2.98498
4	15	1	6	0.4	2	0.15	2	2	2	3	2	2	30	0.2	1.65275	1.65264
4	15	1	6	0.4	4	0.15	2	2	2	3	2	2	30	0.2	2.50537	-0.675988
4	15	1	6	0.4	6	0.15	2	2	2	3	2	2	30	0.2	3.65913	-4.16748
4	15	1	6	0.4	0.7	0.15	2	2	2	-1	2	2	30	0.2	0.527954	3.71902
4	15	1	6	0.4	0.7	0.15	2	2	2	-4	2	2	30	0.2	0.0802219	4.31804
4	15	1	6	0.4	0.7	0.15	2	2	2	-7	2	2	30	0.2	-0.493738	4.98819
4	15	1	6	0.4	0.7	0.15	2	2	2	7	2	2	30	0.2	1.36059	2.37218
4	15	1	6	0.4	0.7	0.15	2	2	2	10	2	2	30	0.2	1.59116	1.93674
4	15	1	6	0.4	0.7	0.15	2	2	2	14	2	2	30	0.2	1.85646	1.4004
4	15	1	6	0.4	0.7	0.15	2	2	5	3	2	2	30	0.2	0.708641	3.40214
4	15	1	6	0.4	0.7	0.15	2	2	7	3	2	2	30	0.2	0.619379	3.50003
4	15	1	6	0.4	0.7	0.15	2	2	9	3	2	2	30	0.2	0.560243	3.55939
4	15	1	6	0.4	0.7	0.15	5	2	2	3	2	2	30	0.2	0.949906	4.95705
4	15	1	6	0.4	0.7	0.15	7	2	2	3	2	2	30	0.2	0.928102	5.91417
4	15	1	6	0.4	0.7	0.15	9	2	2	3	2	2	30	0.2	0.90966	0.71033
4	15	1	6	0.4	0.7	0.15	2	5	2	3	2	2	30	0.2	1.06247	1.22139
4	15	1	6	0.4	0.7	0.15	2	8	2	3	2	2	30	0.2	1.03753	-0.535467
4	15	1	6	0.4	0.7	0.15	2	10	2	3	2	2	30	0.2	0.951732	-1.01559
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.1	0.942315	3.3287
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.3	1.08902	3.04364
4	15	1	6	0.4	0.7	0.15	2	2	2	3	2	2	30	0.4	1.19183	3.23992

Table 2. Nusselt number and Sherwood number Nu and Sh respectively for different values of parameters

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#### REFERENCES

- Alam, M. and M. Rahman (2006). Dufour and soret effects on mixed convection flow past a vertical porous flat plate with variable suction. *Non Linear Analysis Modelling and Control 11*(1), pp. 3–12.
- Brice Carnahan, H. L. and J. O. Wilkes (1969). *Applied Numerical Methods*. New York: John Wiley and Sons.
- Cowling, T. (1957). *Magneto hydrodynamics*. New York: Inter Science Publishers.
- Ghosh, S. (2002). Effects of hall current on mhd couttee flow in a rotating system with arbitrary magnetic field. *Czechoslovak Journal* of Physics 52(1), pp. 51–63.
- Karraiah, A. S., D. Kesaraiah, and M. Bhavana (2013). The soret effect on free convective unsteady mhd flow over a vertical plate with heat source. *International Journal of Innovative Research in Science Engineering and Technology* 2(5), pp. 1617–1628.
- Lee, K. (2001). Natural convection heat and

mass transfer in partially heated vertical parallel plates. *International Journal Heat and Mass Transfer 37*, pp. 67–76.

- Oladapo, O. P. (2010). Dufour and soret effects of a transient free convective flow with radiation heat transfer past a flat plate moving through a binary mixture. *Pasific Journal of Science and Technology* 11(1).
- Pandya, N. and A. K. Shukla (2013). Soretdufour and radiation effects on unsteady mhd flow past an impulsively started inclined porous plate with variable temperature and mass diffusion. *International Journal of Mathematics and Science Computing* 3(2), pp. 41–48.
- Postelnicu, A. (2004). Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering soret and dufour effects. *International Journal of Heat and Mass Transfer* 47(6), pp. 1467–1472.
- Sparrow, E., T. Eichhorn, and J. Gregg (1959). Combined forced and free convection in a boundary layer flow. *Physics of Fluids 2*, pp. 319–328.
- Vempati, S. and A. L. N. Gari (2010). Soret and dufour effects on an unsteady mhd flow past an infinite vertical porous plate with thermal radiation. *Applied Mathematics and Mechanics 31*(12), pp. 1481–1496.