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Flow of Two Immiscible Couple Stress Fluids between Two Permeable Beds

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ABSTRACT

The paper deals with the flow of two immiscible couple stress fluids between two homogeneous permeable beds. The flow is considered in two zones: zone I and II contain free flow of two immiscible couple stress fluids between two permeable porous beds at the bottom and top. The flow in the free channel bounded by two permeable beds is assumed to be governed by Stokes's couple stress fluid flow equations and that in the permeable beds by Darcy's law. The continuity of velocity, vorticity, shear stress and couple stress are imposed at the fluid-fluid interface and Beavers-Joseph (BJ) slip boundary conditions are employed at the fluid-porous interface. The equations are solved analytically and the expressions for velocity, skin friction and volumetric flow rate are obtained. The effects of the physical governing parameters on velocity are investigated.

Keywords: Immiscible fluids; Couple stress fluid; Permeable beds; Darcy's law; Beavers-Joseph (BJ) slip boundary condition.

1. INTRODUCTION

The analysis of flow properties of non-Newtonian fluids are very important in the fields of fluid dynamics because of their engineering applications. Due to the complex stress-strain relationships of non-Newtonian fluids, not many investigators have studied the flow behavior of the fluids in various flow fields. It is well known that the properties of many rheological complex fluids, such as polymer solutions, liquid crystals, lubricants, soaps, blood, and greases are not well described by the Navier-Stokes equations. For this reason, many non-Newtonian fluid theories have been proposed. Couple stress fluid introduced by V. K. Stokes (1966) is one among the category of non-Newtonian fluids which allows for polar effects such as sustenance of couple stresses and body couples in the fluid medium. It is a simple generalization of the classical theory of viscous fluids that shows all important features and effects of couple stresses and its governing equations are similar to the Navier-Stokes equations. This theory has attracted the attention of several researchers due to its widespread applications in lubrication theory, blood flows, liquid crystals etc. Ariman and Cakmak (1967) discussed couple stresses in fluids. Stokes discussed the hydromagnetic steady flow of a fluid with couple stress effects. A review of couple stress (polar) fluid dynamics was reported by V. K. Stokes (1984). Soundalgekar and Aranake (1978) discussed the effects of couple stresses on MHD Couette flow. Rudraiah and Chandrashekara (2010) investigated the effects of couple stress fluid on the control of Rayleigh-Taylor instability at the interface between a dense fluid accelerated by a lighter fluid. Shivakumara et al. (2012) investigated the effect of different forms of basic temperature gradients on the criterion for the onset of convection in a layer of couple stress fluid saturated porous medium. Rani and Reddy (2013) examined the influence of Soret and Dufour effects on double diffusive transient free convective flow of a couple stress fluid over a semi-infinite vertical cylinder.

The flow of fluids through a porous medium, especially, is a topic of current interest in many engineering applications. Examples of these applications are in the fields of agricultural engineering to study the underground water resources; in studies of water in river beds; and in petroleum technology to study the movement of natural gas, oil and water through oil reservoirs. Prasad and Amit Kumar (2001) discussed the flow of a hydromagnetic fluid between permeable beds under exponentially decaying pressure gradient. Vajravelu et al. (2003) analyzed the pulsatile flow of a viscous fluid between two permeable beds. Harmindar and Singh (2005) have studied free convection flow of two immiscible viscous liquids through parallel permeable beds. Malathy and Srinivas (2008) have investigated the pulsating flow of a hydromagnetic fluid between permeable beds. Iyengar and Punnamchandar (2013) studied the flow of an incompressible micropolar fluid between permeable beds with an imposed uniform magnetic field. Kumar and Prasad (2014) studied the analytical solution for the MHD pulsatile flow between permeable beds of a viscous Newtonian fluid saturated porous medium. In recent years, the fluid flow in two immiscible fluids in a channel has received considerable attention by researchers. Vajravelu et al. (1995) studied the hydromagnetic unsteady flow of two conducting immiscible fluids between two permeable beds.

Many problems in the fields of hydrology, geophysics, biology and petroleum industry in which the systems involving two or more immiscible fluids of different densities/viscosities flowing in the same channel or through porous media are encountered. Typical fluid flow examples of these systems are: oil-water, gas-oil, air-water and gas-oil-water systems (Hochmuth and Sunada (1985)). These are referred to as multi-phase flows in literature. Blood flow in arteries has been studied by many researchers considering blood as a two phase flow. Several investigations on multiphase flows are reported by various researchers such as Chaturani and Samy (1985), Rao and Usha (1995), Sharan and Popel (2001), Garcia and Riahi (2014) etc. In view of the above discussion, an attempt has been made in this paper, to the study the flow of two immiscible couple stress fluids between two permeable beds. The non-dimensional velocity profiles are displayed graphically for different values of couple stress parameter and other parameters.

2. FORMULATION OF THE PROBLEM

Consider the flow of two immiscible couple stress fluids in a channel of height $2h$ bounded by two permeable beds of infinite thickness with different permeabilities. The permeabilities of lower and upper beds are K_1 and K_2, respectively. The flow geometry is described in Fig. 1. X and Y are the axial (horizontal) and vertical coordinates, respectively with the origin at the center of the channel. Fluid flow is generated due to a constant pressure gradient which acts at the mouth of the channel. The fluid in the lower zone (viscosity μ_1 and density ρ_1) occupies the region $(-h \leq Y \leq 0)$ comprising the lower half of the channel and this region is named as zone I. The fluid in the upper zone (viscosity μ_2 and density $\rho_2(<\rho_1)$) is assumed to occupy the upper half of the channel $(0 \leq Y \leq h)$, and this region is called zone II. In the present case, fluid in zone I is denser than the fluid in zone II. The flow in the upper and lower permeable beds is assumed to be governed by Darcy's law. The Beavers-Joseph (BJ) slip boundary conditions are used at the interfaces of the permeable beds. The flow in the zone I and II $(-h \leq Y \leq h)$ is assumed to be governed by couple stress fluid flow equations of V. K. Stokes (1984).

The equations of motion that characterize a couple stress fluid flow are similar to the Navier-Stokes equations and are given by

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \bar{q}) = 0 \tag{1}$$

$$\rho \frac{d\bar{q}}{dt} = \rho \bar{f} + \frac{1}{2}\mathrm{curl}(\rho \bar{\ell}) - \mathrm{grad}(P) + \mu \mathrm{curl}(\mathrm{curl}\bar{q})) - \eta \mathrm{curl}(\mathrm{curl}(\mathrm{curl}(\mathrm{curl}\bar{q}))) + (\lambda + 2\mu)\mathrm{grad}(\mathrm{div}\bar{q}) \tag{2}$$

The scalar quantity ρ is the density and P is the pressure at any point in the fluid. The vectors $\bar{q}$, $\bar{\omega}$, $\bar{f}$ and $\bar{\ell}$ are the velocity, rotation (vorticity), body force per unit mass and body couple per unit mass, respectively. The material constants λ and μ are the viscosity coefficients and η and η' are the couple stress viscosity coefficients satisfying the constraints $\mu \geq 0$; $3\lambda + 2\mu \geq 0$; $\eta \geq 0$, $|\eta'| \leq \eta$. There is a length parameter $l = \sqrt{\eta/\mu}$ which is a characteristic measure of the polarity of the couple stress fluid and this parameter is identically zero in the case of non-polar fluids.

The force stress tensor τ_{ij} (V. K. Stokes (1984)) and the couple stress tensor m_{ij} that arises in the theory of couple stress fluids are given by

$$\tau_{ij} = (-P + \lambda \mathrm{div}(\bar{q}))\,\delta_{ij} + 2\mu d_{ij} + \frac{1}{2}\varepsilon_{ijk}\left[m_{,k} + 4\eta w_{k,rr} + \rho c_k\right] \tag{3}$$

$$m_{ij} = \frac{1}{3} m \delta_{ij} + 4\eta \omega_{j,i} + 4\eta' \omega_{i,j} \tag{4}$$

In the above $\omega_{i,j}$ is the spin tensor and ρc_k is the body couple vector, d_{ij} is the components of rate of shear strain, δ_{ij} is the Kronecker symbol, ε_{ijk} is the Levi-Civita symbol and comma denotes covariant differentiation.

Darcy's law forms a basis for modeling fluid transport in porous media. In applications where fluid velocities are low, such as movements of groundwater and petroleum etc. Darcy's law well describes the fluid transport in porous media. It states

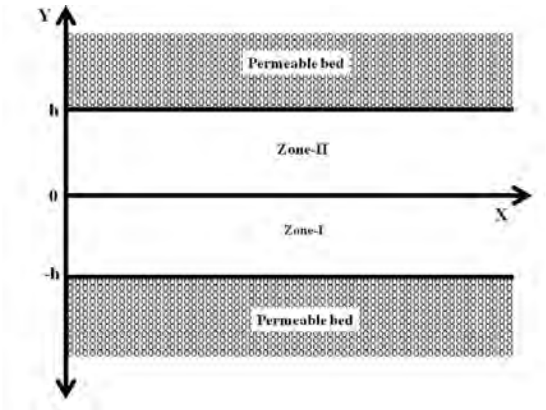

Fig. 1. Geometry of the problem.

that the filtration velocity of the fluid is proportional to the difference between the body force and the pressure gradient.

The flow in the infinite permeable bed is governed by Darcy law

$$\bar{q} = \frac{K}{\mu}(f - \nabla P) \tag{5}$$

where the constant K is called the permeability of the porous medium.

The following assumptions are made in the analysis of the problem:

1. The flow is assumed to be one-dimensional, steady, laminar and incompressible.
2. The permeable beds are homogeneous.
3. The thickness of the permeable beds is infinitely large, so that Darcy law can be applied with Beavers and Joseph condition at the fluid-porous interface of the channel.

$$\left.\begin{aligned} U &= U_{s_1} \quad \text{at} \quad Y = -h \\ \frac{dU}{dY} &= \frac{\alpha^*}{\sqrt{K_1}}(U_{s_1} - Q_1) \quad \text{at} \quad Y = -h \end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned} U &= U_{s_2} \quad \text{at} \quad Y = h \\ \frac{dU}{dY} &= -\frac{\alpha^*}{\sqrt{K_2}}(U_{s_2} - Q_2) \quad \text{at} \quad Y = h \end{aligned}\right\} \tag{7}$$

where U_{s_1} and U_{s_2} are the slip velocities at the interfaces of the lower and upper permeable beds, respectively. Q_1 and Q_2 are the Darcy's velocities in the lower and upper permeable beds, respectively. K_1 and K_2 are the permeabilities of the lower and upper permeable beds, respectively. α^* is the slip parameter.

Since the flow is one-dimensional, we assume the velocity of the fluid to be $\bar{q} = (U(Y), 0, 0)$. We introduce the non-dimensional variables $x = \frac{X}{h}$, $y = \frac{Y}{h}$, $u = \frac{U}{U_o}$, $p = \frac{P}{\rho_1 U_0^2}$ where U_o is the maximum velocity of the fluid in the channel.

Equation of continuity (1) is satisfied identically for the assumed form of velocity and neglecting body forces and body couples from equation (2), we get the following sets of non-dimensional form of governing equations (neglecting body couples) and boundary conditions corresponding to the flow in the two zones.
zone I: $(-1 \le y \le 0)$
The governing equation in zone I is:

$$\frac{d^4 u_1}{dy^4} - s_1 \frac{d^2 u_1}{dy^2} = -Re\, s_1 \frac{dp}{dx} \tag{8}$$

zone II: $(0 \le y \le 1)$
The governing equation in zone II is:

$$\frac{d^4 u_2}{dy^4} - s_2 \frac{d^2 u_2}{dy^2} = -Re\, s_2 \frac{1}{n_\mu} \frac{dp}{dx} \tag{9}$$

where $Re = \frac{\rho_1 U_o h}{\mu_1}$ is the Reynolds number, $s_i = \frac{\mu_i h^2}{\eta_i}$ is the couple stress parameter and $n_\mu = \frac{\mu_2}{\mu_1}$ is the viscosity ratio (i=1,2).

2.1 Boundary and Interface conditions

A characteristic feature of the two-layer flow problem is the coupling across fluid/fluid interfaces. The fluid layers are mechanically coupled via transfer of momentum across the interface. Transfer of momentum results from the continuity of interface tangential velocity and from a stress balance across the interface.

To determine the velocity distributions $u_1(y)$ and $u_2(y)$ in the zones I and II described above, we adopt the following boundary and interface conditions.
(i) at the upper fluid-porous boundary, couple stresses vanish (no-spin) and Beavers - Joseph (BJ) slip condition is taken i.e.

$$\frac{du}{dy} = -\frac{\alpha^* h}{\sqrt{K_2}}(u_{s_2} - u_{p_2})$$
$$\text{where } u_{p_2} = -\frac{1}{n_\mu} n_K\, Da\, Re \frac{dp}{dx} \tag{10}$$

$$u(1) = u_{s_2} \text{ and } \frac{d^2 u_2}{dy^2} = 0 \text{ at } y = 1 \tag{11}$$

(ii) at the fluid-fluid interface, velocity, rotation, shear stress and couple stress are continuous:

$$u_1 = u_2,\ \frac{du_1}{dy} = \frac{du_2}{dy},\ \tau_{xy}|_1 = \tau_{xy}|_2 \text{ and } \frac{d^2u_1}{dy^2} = n_\eta \frac{d^2u_2}{dy^2} \text{ at } y = 0 \quad (12)$$

(iii) at the lower fluid-porous boundary couple stresses vanish (no-spin) and Beavers-Joseph (BJ) slip condition is taken i.e.

$$\frac{du}{dy} = \frac{\alpha^* h}{\sqrt{K_1}}(u_{s_1} - u_{p_1}) \text{ where } u_{p_1} = -Da\,Re\,\frac{dp}{dx} \quad (13)$$

$$u(-1) = u_{s_1} \text{ and } \frac{d^2u_1}{dy^2} = 0 \text{ at } y = -1 \quad (14)$$

where u_{p_1} and u_{p_2} are dimensionless Darcy's velocities in the lower and upper permeable beds, respectively.

2.2 Velocity distributions

zone I: $(-1 \le y \le 0)$
Solving Eqns. (8) and (9), we see that the velocity component of zone I is given by

$$u_1(y) = c_{11} + c_{12}y + c_{13}\cosh s_1 y + c_{14}\sinh s_1 y + \frac{1}{2}\,Re\,B\,y^2 \quad (15)$$

zone II: $(0 \le y \le 1)$
and that of zone II is given by

$$u_2(y) = c_{21} + c_{22}y + c_{23}\cosh s_2 y + c_{24}\sinh s_2 y + \frac{1}{2}\frac{1}{n_\mu}\,Re\,B\,y^2 \quad (16)$$

The solutions $u_1(y)$ and $u_2(y)$ involve eight constants c_{11}, c_{12}, c_{13}, c_{14}, c_{21}, c_{22}, c_{23} and c_{24}. These constants are found from the eight boundary conditions given in (10)-(14) and are solved using MATHEMATICA. As the expressions are cumbersome, they are not presented here.

2.3 Skin friction

The dimensionless skin friction at the lower and upper boundaries are given by

$$\tau_{xy}|_1 = \left[\frac{\partial^3 u_1}{\partial y^3} - s_1\frac{\partial u_1}{\partial y}\right]_{y=-1} = -c_{12}s_1 - c_{13}s_1^2(s_1-1)\sinh s_1 + c_{14}s_1^2(s_1-1)\cosh s_1 + s_1\,Re\,B \quad (17)$$

$$\tau_{xy}|_2 = \left[\frac{\partial^3 u_2}{\partial y^3} - s_2\frac{\partial u_2}{\partial y}\right]_{y=1} = -c_{22}s_2 - c_{23}s_2^2(s_2-1)\sinh s_2 + c_{24}s_2^2(s_2-1)\cosh s_2 + \frac{1}{n_\mu}\,s_2 ReB \quad (18)$$

2.4 Volumetric flow rate

The non-dimensional volumetric flow rate of the channel is given by $q = q_1 + q_2$ where

$$q_1 = \int_{-1}^{0} u(y)dy = c_{11} + \frac{c_{12}}{2} + c_{13}\frac{\sin s_1}{s_1} + c_{14}\frac{(1-\cos s_1)}{s_1} + \frac{1}{6}\,Re\,B \quad (19)$$

$$q_2 = \int_{0}^{1} u(y)dy = c_{21} + \frac{c_{22}}{2} + c_{23}\frac{\sin s_2}{s_2} + c_{24}\frac{(\cos s_2 - 1)}{s_2} + \frac{1}{6}\frac{1}{n_\mu}\,Re\,B \quad (20)$$

3. Results and Discussion

The analytical solutions for the flow of two immiscible couple stress fluids between two permeable porous beds are obtained. The solutions are evaluated numerically and depicted graphically. The variations of velocity for different values of parameters are shown through figures.

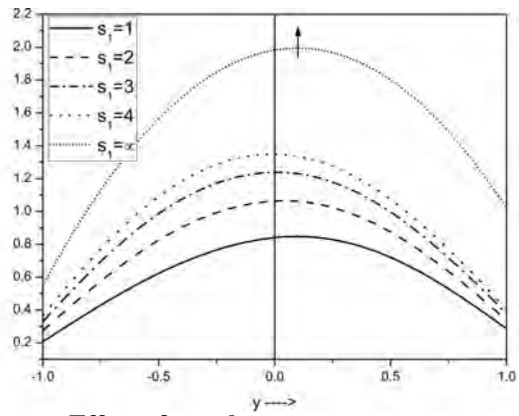

Fig. 2. Effect of couple stress parameter on velocity for $\alpha^* = 0.5, B = -1.2, Da = 0.01, n_\beta = 0.8, n_K = 0.8, n_\mu = 0.8, Re = 2, s_2 = 2$.

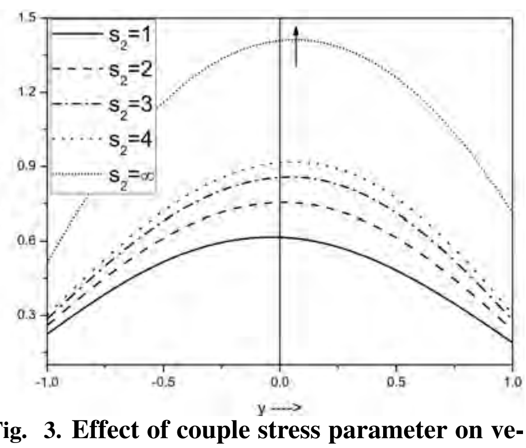

Fig. 3. Effect of couple stress parameter on velocity for $\alpha^* = 0.5, B = -0.8, Da = 0.02, n_\beta = 0.8, n_\eta = 0.8, n_K = 0.8, n_\mu = 0.8, Re = 2, s_1 = 2$.

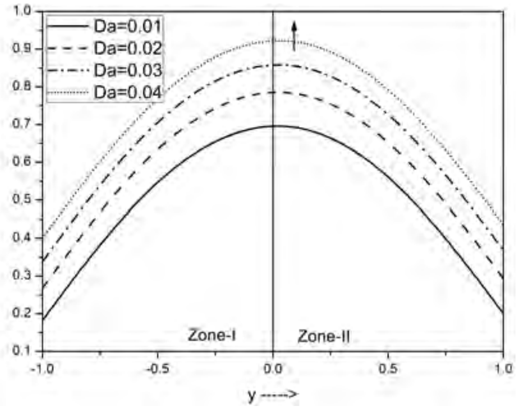

Fig. 4. Effect of Darcy number on velocity for $\alpha^* = 0.5, B = -1, n_\beta = 0.8, n_K = 0.8, n_\mu = 0.8, n_\eta = 0.8, Re = 1.5, s_1 = 2, s_2 = 2.$

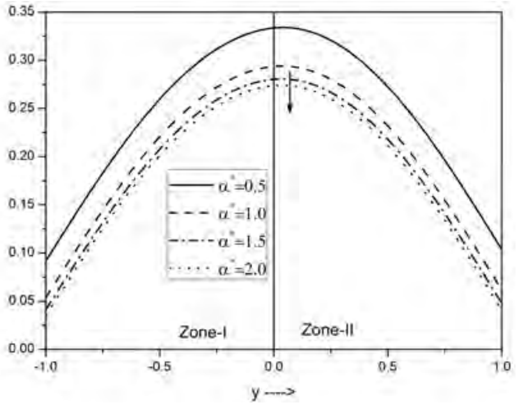

Fig. 5. Effect of slip parameter on velocity for $B = -2, Da = 0.01, n_\beta = 0.8, n_K = 1.2, n_\mu = 0.8, n_\eta = 0.8, Re = 0.8, s_1 = 1, s_2 = 1.$

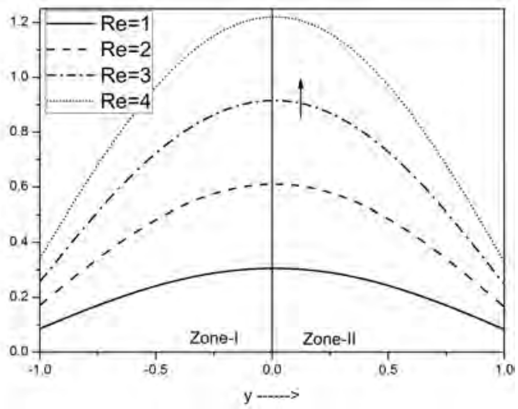

Fig. 6. Effect of Reynolds number on velocity for $\alpha^* = 0.6, B = -0.5, Da = 0.02, n_\beta = 0.8, n_K = 0.8, n_\mu = 0.8, n_\eta = 0.8, s_1 = 2, s_2 = 2.$

The effect of the couple stress parameter s_1 on the velocity distribution is shown in Fig. 2. As s_1 increases, the velocity increases. Fig. 3 depicts the effect of the couple stress parameter s_2 on the velocity distribution. As s_2 increases, the velocity increases. It is seen that as s_i (i=1,2) increases, the velocity increases in both zones of the channel. As

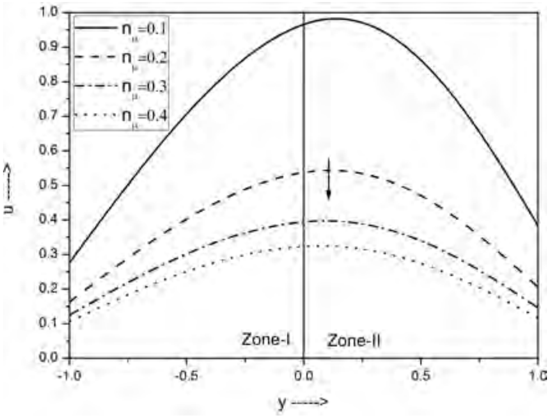

Fig. 7. Effect of viscosity ratio on velocity for $\alpha^* = 0.5, B = -0.5, Da = 0.02, n_\beta = 0.8, n_K = 0.8, n_\eta = 0.8, Re = 1.5, s_1 = 1.2, s_2 = 1.2.$

$s_i \to \infty$ (i.e. $\eta \to 0$), we get the case of Newtonian (viscous) fluid. Hence, from Figs. 2 and 3, we conclude that the velocity in case of couple stress fluid is less than that of a Newtonian fluid case. Thus, the presence of couple stresses in the fluid decreases the velocity. This may be due to the fact that the couple stresses spend some energy to rotate the particles, thereby decreasing velocity of the particles.

Fig. 4 shows the effect of the Darcy number Da on the velocity distribution. As Da is increasing, the velocity is increasing. Fig. 5 depicts the effects of slip parameter α^* on the velocity field. As α^* increases, the velocity decreases. The effect of the Reynolds number (Re) on the velocity distribution is shown in Fig. 6. As the Reynolds number increases, velocity increases. Fig.7 shows the velocity profiles for different values of the viscosity ratio n_μ. As the viscosity ratio n_μ increases and offers more resistance to the flow. Hence, velocity decreases.

The variation of skin friction is presented numerically through table. From table 1, we notice that for fixed values of $\alpha^* = 0.5$, $c_1 = c_2 = 0.1$, $Da = 0.01$, $n_K = 1.2$, $n_\beta = 0.8$, $n_\mu = 0.8$, $s_1 = 2$ and $s_2 = 2$ as the Reynolds number Re increases from 1 to 4 for B=-0.5 to -2.0 at the lower bed the skin friction increases while at the upper bed it decreases.

The variation of skin friction is presented numerically through table. From table 2, we notice that for fixed values of $B = -0.5$, $c_1 = c_2 = 0.1$, $Da = 0.01$, $n_K = 1.2$, $n_\beta = 0.8$, $n_\mu = 0.8$, $s_1 = 2$ and $s_2 = 2$ as the α^* increases from 0.1 to 0.4 for Re=0.5 to 2.0 at the lower bed the skin friction increases while at the upper bed it decreases.

4. CONCLUSION

The flow of two immiscible incompressible couple stress fluids between two permeable beds flowing

Table 1. Variation of skin friction with Re at the interface of Lower Permeable Bed (LPB) and Upper Permeable Bed (UPB):$\alpha^* = 0.5, c_1 = c_2 = 0.1, Da = 0.01, n_K = 1.2, n_\beta = 0.8, n_\mu = 0.8, s_1 = 2$ and $s_2 = 2$.

τ	$Re = 1$	$Re = 2$	$Re = 3$	$Re = 4$
B=-0.5				
LPB	0.45343	0.90687	1.3603	1.81374
UPB	0.44588	0.89176	1.3376	1.78353
B=-1.0				
LPB	0.90687	1.81374	2.72061	3.62748
UPB	0.89176	1.78353	2.67529	3.56705
B=-1.5				
LPB	1.36031	2.72061	4.08092	5.44122
UPB	1.33765	2.67529	4.01294	5.35058
B=-2.0				
LPB	1.81374	3.62748	5.44122	7.25497
UPB	1.78353	3.56705	5.35058	7.13411

Table 2. Variation of skin friction with α^* at the interface of Lower Permeable Bed (LPB) and Upper Permeable Bed (UPB):$B = -0.5, c_1 = c_2 = 0.1, Da = 0.01, n_K = 1.2, n_\beta = 0.8, n_\mu = 0.8, s_1 = 2$ and $s_2 = 2$.

τ	$\alpha^* = 0.1$	$\alpha^* = 0.2$	$\alpha^* = 0.3$	$\alpha^* = 0.4$
Re=0.5				
LPB	0.22977	0.22825	0.22749	0.22702
UPB	0.21915	0.22103	0.22198	0.22255
Re=1.0				
LPB	0.45955	0.45651	0.45498	0.45405
UPB	0.43830	0.44206	0.44396	0.44511
Re=1.5				
LPB	0.68932	0.68477	0.68247	0.68108
UPB	0.65745	0.66309	0.66594	0.66767
Re=2.0				
LPB	0.91910	0.91303	0.90996	0.90811
UPB	0.87660	0.88412	0.88793	0.89022

in axial direction under the influence of a constant pressure gradient has been analyzed. It is observed that

1. The presence of couple stresses in the fluid decreases the velocity.
2. The velocity of the fluid increases by the increase of the Reynolds number Re and Darcy number Da.
3. The velocity of the fluid is decreased by the increase of the slip parameter α^*.
4. Flow rate is high when the permeability of the permeable beds is low.

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