



Electrohydrodynamic Dispersion with Interphase Mass Transfer in a Poorly Conducting Couple Stress Fluid Bounded by Porous Layers

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ABSTRACT

Exact analysis of miscible dispersion of solute with interphase mass transfer in a poorly conducting couple stress fluid flowing through a rectangular channel bounded by porous layers is considered because of its application in many practical situations. The generalized dispersion model of Sankarasubramanian and Gill is used, which brings into focus the exchange coefficient, the convective coefficient and the dispersion coefficient. The exchange coefficient comes into picture due to the interphase mass transfer and independent of solvent fluid viscosity. It is observed that the convective coefficient increases with an increase in the porous parameter while it decreases with an increase in the couple stress parameter. The dispersion coefficient is plotted against wall reaction parameter for different values of porous parameter and couple stress parameter. It is noted that the dispersion coefficient decreases with an increase in the value of couple stress parameter but increases with porous parameter.

Keywords: Poorly conducting fluid; Generalised dispersion; Interphase mass transfer; Couple stress fluid.

NOMENCLATURE

<p>a couple stress parameter</p> <p>C concentration of solute</p> <p>D mass diffusion</p> <p>\vec{E} electric field.</p> <p>h width of the channel</p> <p>J_i current density</p> <p>K ratio of mass diffusion to kinematic viscosity</p> <p>k permeability of porous media</p> <p>k_s reaction rate constant</p> <p>p pressure</p> <p>Pe Peclet number</p> <p>T absolute temperature</p> <p>T_b conduction temperature</p> <p>U non dimensional velocity</p> <p>u velocity component in the x direction</p> <p>u_b slip velocity at permeable surface</p> <p>u_p Darcy velocity in the porous layer</p> <p>\bar{u} average velocity of the flow</p> <p>0 reference quantity</p>	<p>X non dimensional x co-ordinate</p> <p>λ couple stress coefficient in the free flow</p> <p>σ porous parameter</p> <p>ρ_e distribution of electric charge density</p> <p>α_p slip parameter</p> <p>α conductivity variation parameter</p> <p>α_h volumetric expansion coefficient of σ</p> <p>γ kinematic viscosity</p> <p>σ_c electrical conductivity</p> <p>ϵ_0 dielectric constant for free space</p> <p>ϵ dielectric constant</p> <p>μ viscosity of fluid</p> <p>ϕ electric potential</p> <p>ρ density of fluid</p> <p>β reaction rate parameter</p> <p>β_1 couple stress coefficient</p> <p>η non dimensional y co-ordinate</p>
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V applied uniform electric potential
 We Electric number

θ non dimensional concentration
 θ_m dimensionless cross sectional average concentration
 τ dimensionless time

1. INTRODUCTION

In our recent paper (2011) we have developed an exact analysis of generalized dispersion of unsteady convective diffusion in a couple stress poorly conducting fluid bounded by porous layers in the presence of an electric field with no interphase mass transfer. But in many biomedical problems, the interphase mass transfer plays a significant role. Hence, it is necessary to develop a technique to handle physiological problems, which involve interphase mass transport. Early works on dispersion were mainly concerned with Taylor's (1953) dispersion, which is valid asymptotically for large time. Physiological fluid flow problems have been mainly concerned with transient phenomena where Taylor's model is not valid. However, Sankarasubramanian and Gill (1973) have developed an analytical method to analyse the transient dispersion of a non-uniform initial distribution, called generalized dispersion in laminar flow in a tube with a first order chemical reaction occurring at the tube wall. This method can be applied to physiological problems where a first order chemical reaction occurs at the tube wall. One such situation is the transport of oxygen and nutrients to tissue cells and removal of metabolic waste products from tissue cells. Interphase mass transfer also takes place in pulmonary capillaries where the carbon dioxide is removed from the blood and oxygen is taken up by the blood.

Rudraiah *et al.* (1989) have studied the effect of couple stress and electric field on the dispersion of erythrocytes in a channel bounded by rigid walls and showed that the couple stress augments haemolysis. Rudraiah *et al.* (2005, 2006) have shown that self-generated electric field reduces the concentration of RBC's and hence increases dispersion. In bioengineering problems, particularly in the mechanism of controlling haemolysis, the assumption of "micro-capillaries bounded by rigid walls" is unrealistic, because there is transport of oxygen, proteins and other nutrients from micro capillaries to the permeable tissues of cartilages, endothelium of arteries and so on. The physiological fluids flowing in the micro capillaries slip at the boundaries of the permeable tissues. Therefore, in a study involving the control of haemolysis, it is important that the combined effects of couple stress and a slip at the boundaries of the micro capillaries have to be taken into account.

Blood mainly consists of plasma in which micron sized white blood cells, red blood cells and platelets are suspended. The suspension of these particles has spin relative to the plasma and this necessitates one to consider conservation of angular momentum in addition to the conservation of linear momentum. Stokes (1966) introduced a special type of such fluid called couple stress fluid, in which the spin matches with the natural vorticity. Therefore, the

objective of this paper is to consider these effects in the study of the unsteady convective diffusion of RBCs in the physiological fluid modeled as a poorly conducting couple stress fluid.

Rudraiah *et al.*, (1986) have studied the dispersion in a Stokes's couple-stress fluid flow by using the generalized dispersion model of Gill and Sankarasubramanian (1970). The corresponding problems for plane-poiseuille flows of micropolar, casson and Ostwald-de-Waele fluids have been investigated by Siddheshwar and Manjunath (2000, 2005). Siddheshwar and Manjunath (1999) have studied the effects of buoyancy and homogeneous chemical reaction on unsteady convective diffusion of solute in a Boussinesq fluid flow. Siddheshwar and Vasanthi Moses (1989) have studied the effects of couple stress and magnetic field on unsteady convective diffusion in a rectangular channel. Their work is silent about considering solute reaction at the channel walls in their all time analysis of dispersion.

The problem for Plane-Poiseuille flow of a power-law fluid with interphase mass transfer has been investigated by Siddheshwar *et al.* (2000). An exact analysis of miscible dispersion of solute with interphase mass transfer in a couple stress fluid flow has been investigated by Indira *et al.* (1996). Shashikala and Ranganatha (2008) have studied the effect of interphase mass transfer on unsteady convective diffusion in a simplified cross model fluid. Manjula (2008) have investigated the unsteady convective diffusion with interphase mass transfer in a couple stress fluid bounded by porous beds. Reaction at the walls is of practical interest and in the simplest case, a first order chemical reaction at the walls is considered by them in carrying out an exact analysis of unsteady convection in couple stress fluid flows. In this paper we present an exact analysis of miscible dispersion of solute with interphase mass transfer in a couple stress poorly conducting fluid bounded by porous beds. The generalized dispersion model of Sankarasubramanian and Gill (1973) has been used which brings into focus the exchange coefficient K_0 , convective coefficient K_1 and dispersion coefficient K_2 . Only the last two coefficients (K_1, K_2) are influenced by the porous parameter and couple stress parameter arising due to suspension in the fluid. The exchange coefficient arises mainly due to the interphase mass transfer and is independent of the solvent fluid velocity. The interphase mass transfer also influences the convection and dispersion coefficients. Asymptotically, the large time evaluations have been carried out for all the three coefficients to get a feel of the nature of these coefficients.

2. MATHEMATICAL FORMULATION

The physical configuration of the problem

considered in this paper is shown in Fig. 1. It consists of a poorly conducting couple stress fluid flowing in a rectangular channel (Region 1) bounded by porous layers (Region 2) and separated by a distance $2h$ apart. A Cartesian co-ordinate system is considered such that the origin is at the middle of the channel.

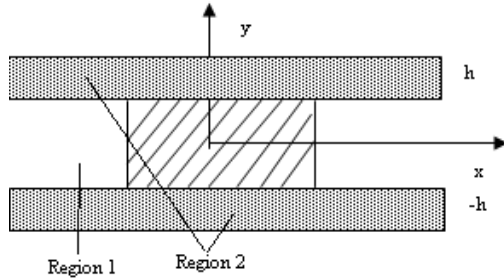


Fig. 1. Physical configuration.

We assume the flow of a poorly conducting couple stress fluid to be laminar, fully developed and unidirectional flowing with an uniform axial pressure gradient. In the presence of couple stress and electric field, the basic equations are given below

For Region 1:

$$0 = \frac{-\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \lambda \frac{d^4 u}{dy^4} + \rho_e E_x \quad (1)$$

$$0 = \frac{-\partial p}{\partial y} + \rho_e E_y \quad (2)$$

For Region 2:

$$0 = -\frac{\partial p}{\partial x} - \frac{\mu}{k} (1 + \beta_1) u_p + \rho_e E_x \quad (3)$$

$$0 = \frac{-\partial p}{\partial y} + \rho_e E_y \quad (4)$$

where equation (3) is the modified Darcy equation with β_1 a couple stress coefficient given by $\beta_1 = \lambda/\mu k$ in the porous media, u is the velocity in the x-direction in the free flow, u_p the Darcy velocity in the porous layer, p the pressure, μ the viscosity of couple stress fluid, λ the couple stress coefficient in the free flow and k is the permeability of porous media.

The boundary conditions on the velocity are

$$\frac{\partial u}{\partial y} = \frac{-\alpha_p}{\sqrt{k}} (u_b - u_p) \quad \text{at } y = h \quad (5a)$$

$$\frac{\partial u}{\partial y} = \frac{\alpha_p}{\sqrt{k}} (u_b - u_p) \quad \text{at } y = -h \quad (5b)$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at } y = \pm h \quad (5c)$$

where α_p is the slip parameter. Equation (5a) and Eq. (5b) are well known Beavers and Joseph (1967) slip conditions at the upper and lower permeable surfaces respectively and Eq. (5c) specifies vanishing of couple stress.

To find electric force $\rho_e E_x$, we now consider

The conservation of charges

$$\frac{\partial \rho_e}{\partial t} + \frac{\partial J_i}{\partial x_j} = 0 \quad (6a)$$

where ρ_e is the distribution of charge density

$$J_i = \rho_e q_i + \sigma_c E_i \quad (6b)$$

J_i the current density, which is the sum of convective current, $\rho_e q_i$, and conduction current, $\sigma_c E_i$, σ_c the electrical conductivity, E_i , the electric field. These are supplemented with the Maxwell's Field equations for a conducting medium.

Gauss law

$$\frac{\partial E_i}{\partial x_i} = \frac{\rho_e}{\epsilon_0} \quad (6c)$$

where ϵ_0 is the dielectric constant for free space.

In a poorly conducting fluid, the induced magnetic field is negligible and there is no applied magnetic field, hence the Faraday's law become

$$\frac{\partial E_i}{\partial x_j} - \frac{\partial E_j}{\partial x_i} = 0 \quad (6d)$$

That is, the electric field is conservative, so that

$$E_i = -\frac{\partial \phi}{\partial x_i} \quad (6e)$$

where ϕ is the electric potential.

Equation (6a), using Eq. (6b) and $\rho_e q_i \ll \sigma_c E_i$, takes the form

$$\frac{D \rho_e}{Dt} + \frac{\partial (\sigma_c E_i)}{\partial x_j} = 0 \quad (6f)$$

$$\text{where } \frac{D}{Dt} = \frac{\partial}{\partial t} + q_j \frac{\partial}{\partial x_j}.$$

We note that in a poorly conducting fluid $\sigma_c \ll 1$ and hence any perturbation on it is assumed to be negligible and increases with conduction temperature, T_b such that

$$\sigma_c = \sigma_0 [1 + \alpha_h (T_b - T_0)] \quad (6g)$$

Here σ_0 is that of σ_c at $T_b = T_0$, α_h is the volumetric expansion coefficient of σ_c .

Further, T_b in Eq. (6g) is the solution of

$$\frac{d^2 T_b}{dy^2} = 0 \quad (6h)$$

satisfying the conditions

$$T_b = T_0 \quad \text{at } y = -h \quad (6i)$$

$$T_b = T_1 \quad \text{at } y = h \quad (6j)$$

Non dimensionalising Eq. 6h and Eqs. 6i, 6j using

$$T_b^* = \frac{T_b - T_0}{\Delta T}, \eta = \frac{y}{h}$$

we have

$$\frac{d^2 T_b^*}{d\eta^2} = 0 \quad (6k)$$

satisfying the conditions

$$T_b = T_0 \quad \text{at} \quad \eta = -1 \quad (6l)$$

$$T_b = T_1 \quad \text{at} \quad \eta = 1 \quad (6m)$$

Solution of Eq. (6k), satisfying the conditions (6l, m), is

$$T_b = \frac{\Delta T}{2} \eta + \frac{\Delta T}{2} + T_0 \quad (6n)$$

where $\Delta T = T_1 - T_0$.

Substituting the solution given by Eq. (6n) into Eq. (6g), we get

$$\sigma_c = \sigma_0 [1 + \alpha(\eta + 1)] \approx \sigma_0 e^{\alpha(\eta + 1)} \quad (\because \alpha \ll 1) \quad (6o)$$

where $\alpha = \alpha_h (\Delta T / 2)$ is the conductivity variation parameter.

In a poorly conducting fluid, the frequency of charge distribution is smaller than the corresponding relaxation frequency of the electric field, so that $D\rho_e/Dt$ in Eq. (6f) is negligible compared to $\partial(\sigma_c E_i)/\partial x_j$. Then, from Eq. (6f) after neglecting $D\rho_e/Dt$ and using Eqs. (6e) and (6o), we get

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{1}{\sigma_c} \frac{\partial \phi}{\partial y} \frac{\partial \sigma_c}{\partial y} = 0 \quad (6p)$$

subject to the boundary conditions

$$\phi = Vx/h \quad \text{at} \quad y = -h \quad (6q)$$

$$\phi = V(x - x_0)/h \quad \text{at} \quad y = h \quad (6r)$$

where V is the applied uniform electric potential.

We make quantities in Eqs. (6p) and (6q, r) dimensionless, using

$$\phi^* = \frac{\phi}{V}, \quad X = \frac{x}{hPe}, \quad \eta = \frac{y}{h} \quad (6s)$$

where the asterisks (*) denote the dimensionless quantities. Substituting Eq. (6s) into Eqs. (6p) and (6q,r) and for simplicity neglecting the asterisk, we get

$$\frac{\partial^2 \phi}{\partial \eta^2} + \frac{1}{\sigma_c} \frac{\partial \phi}{\partial \eta} \frac{\partial \sigma_c}{\partial \eta} = 0 \quad (6t)$$

satisfying boundary conditions

$$\phi = XPe \quad \text{at} \quad \eta = -1 \quad (6u)$$

$$\phi = Pe(X - X_0) \quad \text{at} \quad \eta = 1 \quad (6v)$$

The solution of Eq. (6t), satisfying the boundary conditions (6u,v), is

$$\phi = Pe \left[X - \frac{X_0(1 - e^{-\alpha y})}{(e^\alpha - e^{-\alpha})} \right] \quad (6w)$$

The expression for ρ_e can be obtained, from Eq. (6c), using Eq. (6w), as

$$\rho_e = -\frac{PeX_0\alpha^2 e^{-\alpha y}}{(e^\alpha - e^{-\alpha})} \quad (6x)$$

Eq. (6e), using Eq. (6w), becomes

$$E_x = -1, \quad E_y = \frac{PeX_0\alpha e^{-\alpha y}}{(e^\alpha - e^{-\alpha})} \quad (6y)$$

Hence

$$\rho_e E_x = \frac{PeX_0\alpha^2 e^{-\alpha \eta}}{(e^\alpha - e^{-\alpha})} \approx \frac{PeX_0\alpha(1 - \alpha \eta)}{2} \quad (\because \alpha \ll 1) \quad (6z)$$

We now make the equations (1) (3) and (5) dimensionless using

$$U = \frac{u}{\bar{u}}, \quad p^* = \frac{p}{\rho \bar{u}^2}, \quad \eta = \frac{y}{h}, \quad X = \frac{x}{hPe}, \quad Pe = \frac{\bar{u}h}{D}$$

$$, \quad \rho_e^* = \frac{\rho_e}{\left(\frac{\epsilon_0 V}{h^2}\right)}, \quad E_x^* = \frac{E_x}{\left(\frac{V}{h}\right)}, \quad E_y^* = \frac{E_y}{\left(\frac{V}{h}\right)}, \quad u_p^* = \frac{u_p}{\bar{u}}$$

where \bar{u} is the average velocity of the flow. Equation (1) in non-dimensional form after replacing for electric force can be written as

$$\frac{d^4 U}{d\eta^4} - a^2 \frac{d^2 U}{d\eta^2} = -Ka^2(B_1 - B_2 + B_2\alpha\eta) \quad (7)$$

where

$$B_1 = \frac{dp}{dX}, \quad B_2 = \frac{WePe^2\alpha X_0}{2}, \quad l = \sqrt{\frac{\lambda}{\mu}}, \quad a = \frac{h}{l}$$

is the couple stress parameter, $K = D/\gamma$ is the ratio of mass

diffusion to kinematic viscosity, $\gamma = \mu/\rho$,

$We = \epsilon_0(V/h^2)/\rho\bar{u}^2$ is the electric number,

$Pe = \bar{u}h/D$ is the Peclet number.

Equation (5) in non dimensional form is

$$\frac{\partial U}{\partial \eta} = -\alpha_p \sigma(u_b - u_p) \quad \text{at} \quad \eta = 1 \quad (8a)$$

$$\frac{\partial U}{\partial \eta} = \alpha_p \sigma(u_b - u_p) \quad \text{at} \quad \eta = -1 \quad (8b)$$

$$\frac{\partial^2 u}{\partial \eta^2} = 0 \quad \text{at} \quad \eta = \pm 1 \quad (8c)$$

where $\sigma = h/\sqrt{k}$ is the porous parameter.

The solution of (7) satisfying the conditions (8) is

$$U = \frac{-K(B_1 - B_2)}{2} [1 - \eta^2] - \frac{2}{a^2} \left(1 - \frac{\cosh a\eta}{\cosh a} \right) + A_0 \left[1 - \frac{KB_2\alpha \sinh a\eta}{a^2 \sinh a} + A_1 + A_2\eta + A_3\eta^2 \right] \quad (9)$$

where

$$A_0 = \frac{2}{\sigma^2(1 + \beta_1)} - \frac{2}{\sigma\alpha_p} \left(\frac{\tanh a}{a} - 1 \right)$$

$$A_1 = KB_2\alpha \left\{ \frac{1}{a^2} - \frac{\coth a}{a} + \frac{1}{3} \right\}, \quad A_3 = \frac{KB_2\alpha}{6}$$

$$A_2 = KB_2\alpha \left\{ \frac{\coth a}{a} - \frac{1}{2} - \frac{1}{\sigma^2(1 + \beta_1)} \right\} \quad (10)$$

3. GENERALIZED DISPERSION MODEL

We consider the dispersion of reactive solute in the fully developed flow through a parallel channel bounded by porous beds and introduce a slug of concentration $C = C_0 f(x, y)$ into this flow. The mass balance equation in a fully developed flow considering the solute concentration C undergoing heterogeneous chemical reaction is

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \quad (11)$$

with the initial condition

$$C(0, x, y) = C_0 f(x, y) \quad (12a)$$

where C_0 is a reference concentration.

The heterogeneous reaction conditions are

$$-D \frac{\partial C}{\partial y}(t, x, h) = k_s C(t, x, h) \quad (12b)$$

$$D \frac{\partial C}{\partial y}(t, x, -h) = k_s C(t, x, -h) \quad (12c)$$

where k_s is the reaction rate constant catalyzed by the walls.

The away boundary conditions are

$$C(t, \infty, y) = \frac{\partial C}{\partial x}(t, \infty, y) = 0 \quad (12d)$$

and

$$C(t, x, y) = \text{finite} \quad (12e)$$

On introducing the following non-dimensional quantities

$$U = \frac{u}{\bar{u}}, \quad \eta = \frac{y}{h}, \quad X = \frac{x}{hPe}, \quad Pe = \frac{\bar{u}h}{D}, \quad \theta = \frac{C}{C_0},$$

$$\tau = \frac{tD}{h^2} \quad \text{and} \quad \beta = \frac{k_s h}{D} \quad (13)$$

equations (11) and (12) become

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} = \frac{1}{Pe^2} \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial \eta^2} \quad (14)$$

and

$$\theta(0, X, \eta) = \varphi(x)\psi(\eta) \quad (15a)$$

$$\frac{\partial \theta}{\partial \eta}(\tau, X, 1) = -\beta \theta(\tau, X, 1) \quad (15b)$$

$$\frac{\partial \theta}{\partial \eta}(\tau, X, -1) = \beta \theta(\tau, X, -1) \quad (15c)$$

$$\theta(\tau, \infty, \eta) = \frac{\partial \theta}{\partial X}(\tau, \infty, \eta) = 0 \quad (15d)$$

$$\theta(\tau, X, \eta) = \text{finite} \quad (15e)$$

Here the right hand side of (15a) is the assumed form of the non-dimensional form of $f(x, y)$. The solution of (14), subject to the conditions (15) following Gill and Sankarasubramanian (1970), is

$$\theta(\tau, X, \eta) = f_0(\tau, \eta) \theta_m(\tau, X) + f_1(\tau, \eta) \quad (16)$$

$$\frac{\partial \theta_m}{\partial X}(\tau, X) + f_2(\tau, \eta) \frac{\partial^2 \theta_m}{\partial X^2}(\tau, X) + \dots \quad (16)$$

where θ_m is the dimensionless cross sectional average concentration and is given by

$$\theta_m = \frac{1}{2} \int_{-1}^1 \theta \, d\eta \quad (17)$$

Equation (16) signifies that the difference between θ and its mean θ_m can be accounted by the convective and diffusive contributions. This is based on an observation by Taylor (1953).

Integrating equation (14) with respect to η in

$[-1, 1]$ and using the definition of θ_m , we get

$$\frac{\partial \theta_m}{\partial \tau} = \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial X^2} - \frac{1}{2} \left[\frac{\partial \theta}{\partial \eta} \right]_{-1}^1 - \frac{\partial}{\partial X} \int_{-1}^1 U \theta \, d\eta \quad (18)$$

On using (16) in (18), we get the dispersion model for θ_m as

$$\frac{\partial \theta_m}{\partial \tau} = K_0 \theta_m + K_1 \frac{\partial \theta_m}{\partial X} + K_2 \frac{\partial^2 \theta_m}{\partial X^2} + K_3 \frac{\partial^3 \theta_m}{\partial X^3} + \dots \quad (19)$$

where K_i 's are given by

$$K_i(\tau) = \frac{\delta_{i2}}{Pe^2} - 2 \frac{\partial f_i(\tau, 1)}{\partial \eta} - \frac{1}{2} \int_{-1}^1 U f_{i-1}(\tau, \eta) \, d\eta \quad (20)$$

($i = 1, 2, 3, \dots$)

Here $f_{i1} = 0$ and δ_{i2} is the Kronecker delta defined by

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

The exchange coefficient $K_0(\tau)$ accounts for the non-zero solute flux at the channel wall and negative sign indicates the depletion of solute in the system with time caused by the irreversible reaction, which occurs at the channel wall. The presence of non-zero solute flux at the walls of the channel also affects the higher order K_i due to the

explicit appearance of $\frac{\partial f_i}{\partial \eta}(\tau, 1)$ in (20). Equation

(19) can be truncated after the terms involving K_2 without causing serious error because K_3, K_4 etc. become negligibly small compared to K_2 using Gill and Sankarasubramanian (1970). The resulting model for the mean concentration is

$$\frac{\partial \theta_m}{\partial \tau} = K_0(\tau) \theta_m + K_1(\tau) \frac{\partial \theta_m}{\partial X} + K_2(\tau) \frac{\partial^2 \theta_m}{\partial X^2} \quad (21)$$

Substituting (16) in (14) and using the generalized dispersion model of Sankarasubramanian and Gill (1973) in the resulting equation, we get the equation for f_0, f_1 and f_2 from the general equation of the form:

$$\frac{\partial f_k}{\partial \tau} = \frac{\partial^2 f_k}{\partial \eta^2} - U f_{k-1} + \frac{1}{Pe^2} f_{k-2} - \sum_{i=0}^k K_i f_{k-i}, \quad (k = 0, 1, 2) \quad (22)$$

where $f_{-1} = f_{-2} = 0$.

We note that to evaluate K_i 's we need to know the f_k 's, which are obtained by solving (22) for f_k 's subject to the boundary conditions

$$f_k(\tau, 0) = \text{finite} \tag{23a}$$

$$\frac{\partial f_k}{\partial \eta}(\tau, 1) = -\beta f_k(\tau, 1) \tag{23b}$$

$$\frac{\partial f_k}{\partial \eta}(\tau, 0) = 0 \tag{23c}$$

$$\frac{1}{2} \int_{-1}^1 f_k(\tau, \eta) d\eta = \delta_{k0}, \quad (k = 0, 1, 2) \tag{23d}$$

The function f_0 and the exchange coefficient K_0 are independent of the velocity field and can be solved easily. It should be pointed out here that a simultaneous solution has to be obtained for these two quantities since K_0 which can be obtained from (20) as

$$K_0(\tau) = \frac{1}{2} \left[\frac{\partial f_0}{\partial \eta} \right]_{-1}^1 \tag{24}$$

appears in the defining differential equation for f_0 which may be written from (20) for $k=0$ as

$$\frac{\partial f_0}{\partial \tau} = \frac{\partial^2 f_0}{\partial \eta^2} - f_0 K_0 \tag{25}$$

We now derive an initial condition for f_0 using (17) by taking $\tau = 0$ in that equation to get

$$\theta_m(0, X) = \frac{1}{2} \int_{-1}^1 \theta(0, X, \eta) d\eta \tag{26}$$

Substituting $\tau = 0$ in (16) and setting $f_k(y) = 0$ ($k = 1, 2, 3, \dots$) gives us the initial condition for f_0 as

$$f_0(0, \eta) = \frac{\theta(0, X, \eta)}{\theta_m(0, X)} \tag{27}$$

We note that the left hand side of (27) is a function of θ_m only and the right hand side is a function of both X and η . Thus clearly the initial concentration distribution must be a separable function of X and η . This is the justification for the chosen form of $\theta(0, X, \eta)$ in (27). Substituting (15a) into (27), we get

$$f_0(0, \eta) = \frac{\psi(\eta)}{\frac{1}{2} \int_{-1}^1 \psi(\eta) d\eta} \tag{28}$$

The solution of the reaction diffusion equation (25) with these conditions may be formulated as

$$f_0(\tau, \eta) = g_0(\tau, \eta) \exp\left[-\int_0^\tau K_0(\eta) d\eta\right] \tag{29}$$

from which it follows that $g_0(\tau, \eta)$ has to satisfy

$$\frac{\partial g_0}{\partial \tau} = \frac{\partial^2 g_0}{\partial \eta^2} \tag{30}$$

along with the conditions

$$f_0(0, \eta) = g_0(0, \eta) = \frac{\psi(\eta)}{\frac{1}{2} \int_{-1}^1 \psi(\eta) d\eta} \tag{31a}$$

$$g_k(\tau, 0) = \text{finite} \tag{31b}$$

$$\frac{\partial g_0}{\partial \eta}(\tau, 1) = -\beta g_0(\tau, 1) \tag{31c}$$

The solution of (30) subject to conditions (31) is

$$g_0(\tau, \eta) = \sum_{n=0}^{\infty} A_n \cos(\mu_n \eta) \exp[-\mu_n^2 \tau] \tag{32}$$

where μ_n 's are the roots of

$$\mu_n \tan \mu_n = \beta, \quad n = 0, 1, 2, \dots \tag{33}$$

and A_n 's are given by

$$A_n = \frac{2 \int_{-1}^1 \psi(\eta) \cos(\mu_n \eta) d\eta}{\left(1 + \frac{\sin(2\mu_n)}{2\mu_n}\right) \int_{-1}^1 \psi(\eta) d\eta} \tag{34}$$

Now from (29) it follows that

$$f_0(\tau, \eta) = \frac{2g_0(\tau, \eta)}{\int_{-1}^1 g_0(\tau, \eta) d\eta} = \frac{\sum_{n=0}^9 A_n \exp[-\mu_n^2 \tau] \cos(\mu_n \eta)}{\sum_{n=0}^9 \frac{A_n}{\mu_n} \exp[-\mu_n^2 \tau] \sin(\mu_n)} \tag{35}$$

The first ten roots of the transcendental equation (33) are obtained using mathematica and are given in Table 1. We find that these ten roots ensured convergence of the series seen in the expansions for f_0 and K_0 . Having obtained f_0 , we get K_0 from (24) in the form

$$K_0(\tau) = -\frac{\sum_{n=0}^9 A_n \mu_n \exp[-\mu_n^2 \tau] \sin(\mu_n)}{\sum_{n=0}^9 \frac{A_n}{\mu_n} \exp[-\mu_n^2 \tau] \sin(\mu_n)} \tag{36}$$

Table 1 Roots of the equation $\mu_n \tan \mu_n = \beta$

β	μ_0	μ_1	μ_2	μ_3	μ_4
0.01	0.0998	3.1447	6.2847	9.4258	12.567
0.05	0.2217	3.1574	6.2911	9.4300	12.570
0.1	0.3110	3.1731	6.2990	9.4353	12.574
0.5	0.6532	3.2923	6.3616	9.4774	12.606
1	0.8603	3.4256	6.4373	9.5293	12.645
5	1.3138	4.0335	6.9096	9.8927	12.935
10	1.4288	4.3058	7.2281	10.200	13.214
100	1.5552	4.6657	7.7763	10.887	13.998
β	μ_5	μ_6	μ_7	μ_8	μ_9
0.01	15.708	18.850	21.991	25.133	28.274
0.05	15.711	18.852	21.993	25.134	28.276
0.1	15.714	18.854	21.995	25.136	28.277
0.5	15.739	18.876	22.013	25.152	28.292
1	15.771	18.902	22.212	25.172	28.309
5	16.010	19.105	22.212	25.327	28.448
10	16.259	19.327	22.410	25.506	28.610
100	17.109	20.220	23.332	26.445	29.557

By considering the simplest case of the initial concentration occupying the entire cross section of the channel, we take $\psi(\eta) = 1$ and then $K_0(\tau)$ for this case becomes

$$K_0(\tau) = -\frac{\sum_{n=0}^9 \frac{1}{(\mu_n^2 + \beta^2 + \beta)} \exp[-\mu_n^2 \tau]}{\sum_{n=0}^9 \frac{1}{\mu_n^2 (\mu_n^2 + \beta^2 + \beta)} \exp[-\mu_n^2 \tau]}, n = 0 \quad (1) \quad (37)$$

We now proceed and do only long time analysis of K_0, K_1, K_2, \dots . As $\tau \rightarrow \infty$, we get the asymptotic solution for K_0 from (37) as

$$K_0(\infty) = -\mu_0^2 \quad (38)$$

where μ_0 is the first root of the equation (33). Physically this represents first order chemical reaction coefficient. Having obtained $K_0(\infty)$, we can now get $K_1(\infty)$ from (20) (with $i = 1$) knowing $f_0(\infty, \eta)$ and $f_1(\infty, \eta)$. Likewise, $K_2(\infty), K_3(\infty) \dots$ require the knowledge of K_0, K_1, f_0, f_1 and f_2 . Equation (35) in the limit $\tau \rightarrow \infty$ reduces to

$$f_0(\infty, \eta) = \frac{\mu_0}{\sin \mu_0} \cos(\mu_0 \eta) \quad (39)$$

We then find f_1, K_1, f_2 and K_2 . For asymptotically long times, i.e., $\tau \rightarrow \infty$, (20) and (22) give us K_i 's and f_k 's as

$$K_i(\infty) = \frac{\delta_{i2}}{Pe^2} - \beta f_i(\infty, 1) - \int_{-1}^1 f_{i-1}(\infty, \eta) d\eta \quad (40)$$

($i = 1, 2, 3, \dots$)

$$\frac{\partial^2 f_k}{\partial \eta^2} + \mu_0^2 f_k = K_1 f_{k-1} - \left(\frac{1}{Pe^2} - K_2\right) f_{k-2} \quad (41)$$

($k = 1, 2$)

The f_k 's must satisfy the conditions (23) and this permits the eigenfunction expansion in the form

$$f_k(\infty, \eta) = \sum_{j=0}^9 B_{j,k} \cos(\mu_j \eta), k = 1, 2, 3, \dots \quad (42)$$

Substituting (42) in (41) and multiplying the resulting equation by $\cos(\mu_j \eta)$ and integrating with respect to η from -1 to 1, we get after simplification

$$B_{j,k} = \frac{1}{\mu_j^2 - \mu_0^2} \left[\frac{1}{Pe^2} B_{j,k-2} - \sum_{i=1}^k B_{j,k-i} K_i - \left(1 + \frac{\sin 2\mu_j}{2\mu_j}\right)^{-1} \sum_{l=0}^9 B_{l,k-1} I(j,l) \right] \quad (43)$$

($k = 1, 2$)

where

$$I(j,l) = \int_{-1}^1 U \cos(\mu_l \eta) \cos(\mu_j \eta) d\eta \quad (44)$$

$$B_{j,-1} = 0, B_{j,0} = 0 \text{ for } j = 1 \text{ (1) } 9 \quad (45)$$

The first expansion coefficient $B_{0,k}$ in (42) can be expressed in terms of $B_{j,k}$ ($j = 1 \text{ (1) } 9$) by using the conditions (23) as

$$B_{0,k} = -\left(\frac{\mu_0}{\sin \mu_0}\right) = \sum_{j=1}^9 B_{j,k} \frac{\sin \mu_j}{\mu_j} \quad (46)$$

($k = 1, 2, 3, 4, \dots$)

Further, from (42) and (39) we find that

$$B_{0,0} = -\frac{\mu_0}{\sin \mu_0} \quad (47)$$

Substituting $i = 1$ in (40) and using (44), (45) and (47) in the resulting equation, we get

$$K_1(\infty) = -\frac{I(0,0)}{\left(1 + \frac{\sin 2\mu_0}{2\mu_0}\right)} \quad (48)$$

Substituting $i = 2$ in (40) and using (43), (44) and (47) in the resulting equation, we get

$$K_2(\infty) = \frac{1}{Pe^2} - \frac{\sin \mu_0}{\mu_0 \left(1 + \frac{\sin 2\mu_0}{2\mu_0}\right)} \sum_{l=1}^9 B_{l,1} I(0,l) \quad (49)$$

Using the asymptotic coefficients $K_0(\infty)$, $K_1(\infty)$ and $K_2(\infty)$ in (21) one can determine the mean concentration distribution as a function of X, τ and the parameters of the problem a, Pe, σ and β . This distribution is valid only for long time and is a gross approximation at short and moderate times.

The initial conditions for solving (21) can be obtained from (15a) by taking cross-sectional average. Since we are making long time evaluations of the coefficients, we note that the side effect is independent of θ_m on the initial concentration distribution. In view of this, the solution to (21) with asymptotic coefficients can be written as

$$\theta_m(\tau, X) = \frac{1}{2Pe\sqrt{\pi K_2(\infty) \tau}} \exp\left\{K_0(\infty) \tau - \frac{[X + K_1(\infty) \tau]^2}{4K_2(\infty) \tau}\right\} \quad (50)$$

where

$$\theta_m(\tau, \infty) = 0, \frac{\partial \theta_m}{\partial X}(\tau, \infty) = 0 \quad (51)$$

Equation (51) is obtained from (15d) and $K_0(\infty)$, $K_1(\infty)$ and $K_2(\infty)$ are given by the Eqs. (38), (48) and (49).

4. RESULTS AND DISCUSSION

An exact analysis of unsteady convective diffusion of solute with interphase mass transfer in a couple stress

poorly conducting fluid flow through a rectangular channel bounded by porous beds has been considered using the generalized dispersion model of Sankarasubramanian and Gill (1973). The walls of the channel act as catalysts to the reaction. The problem brings into focus three important dispersion coefficients namely the exchange coefficient $-K_0$ which arises essentially due to the wall reaction, the classical convective coefficient $-K_1$, and the diffusion coefficient K_2 . One of the major purposes of this work is to study the effect of interfacial mass transfer on K_0 , K_1 and K_2 . With this aim, the asymptotic values of these three coefficients are plotted in figures 2 to 6 for various values of couple stress parameter a , porous parameter σ and reaction rate parameter β . From these figures we predict the following.

From Fig. 2, it is evident that $-K_0$ increases with an increase in the value of the wall reaction parameter β but it is unaffected by the porous parameter and the couple stress parameter.

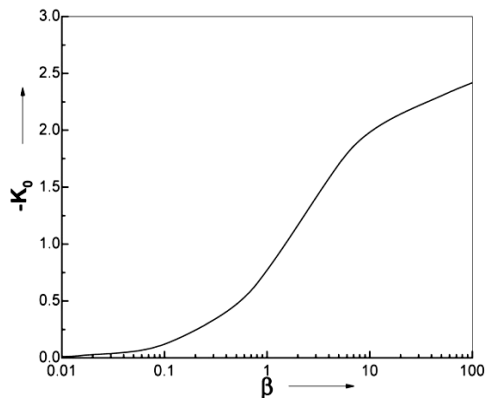


Fig. 2. Plots of exchange coefficient(-K) versus reaction rate parameter(beta).

The classical convective coefficient $-K_1$ is plotted in figures 3 and 4 versus wall reaction parameter β for different values of porous parameter σ and couple stress parameter a respectively for a fixed value of slip parameter $\alpha_p = 0.1$.

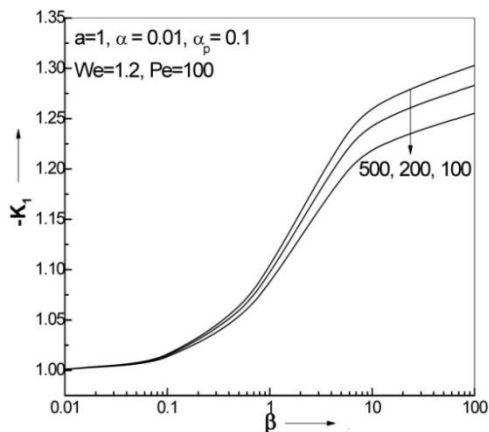


Fig. 3. plots of the convective coefficient- K_1 versus wall reaction parameter β for different values of σ .

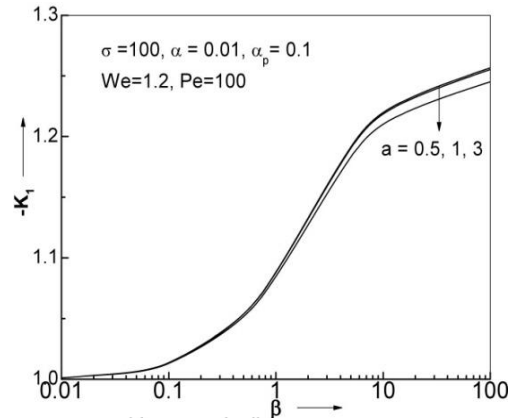


Fig. 4. plots of convective coefficient $-K_1$ versus wall reaction parameter β for different values of couple stress parameter a .

From these figures we conclude that increase in β and σ as well as decrease in a is to increase $-K_1$. This is advantageous in maintaining the laminar flow. The scaled dispersion coefficient $K_2 - Pe^{-2}$ is plotted against β in Fig. 5 for different values of σ and for fixed values of $a=1$ and $\alpha_p = 0.1$ and in Fig. 6 for different values of a for fixed values of $\alpha_p = 0.1$ and $\sigma=100$. From these figures, it is clear that the increase in σ and a is to increase and decrease the effective dispersion coefficient respectively. These are useful in the control of dispersion of a solute.

The cross sectional average concentration θ_m is plotted versus X in figures 7 to 9 respectively for different values of σ, β, a and for fixed values of the other parameters given in these figures. It is clear that the increase in σ and β decreases θ_m , while an increase in a increases θ_m as expected on the physical grounds. This may be attributed to the fact that an increase in σ and β is to reduce the velocity and hence decrease θ_m .

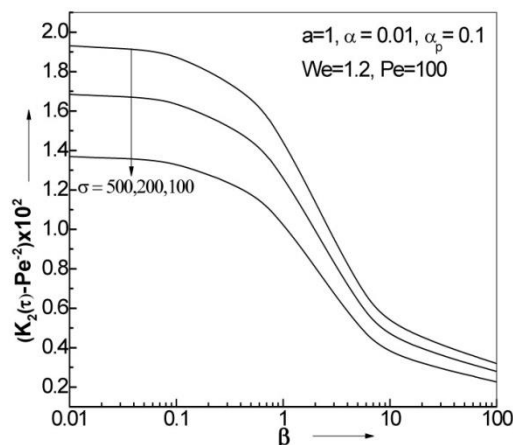


Fig. 5. Plots of scaled dispersion coefficient $K(\tau) - Pe^{-2}$ versus β for different values of σ .

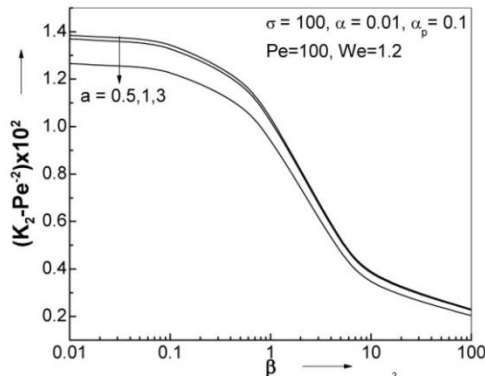


Fig. 6. Plots of scaled dispersion $K(\tau)-Pe^{-2}$ versus β for different values of a .

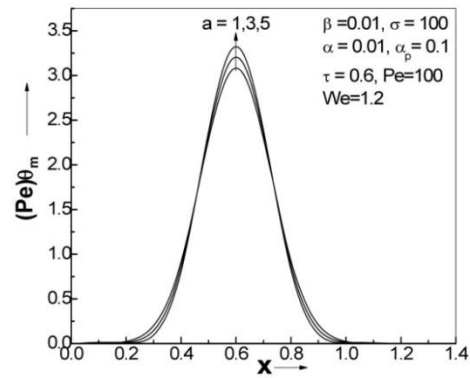


Fig. 9. Plots of mean concentration θ_m versus X for different values of couple stress parameter a .

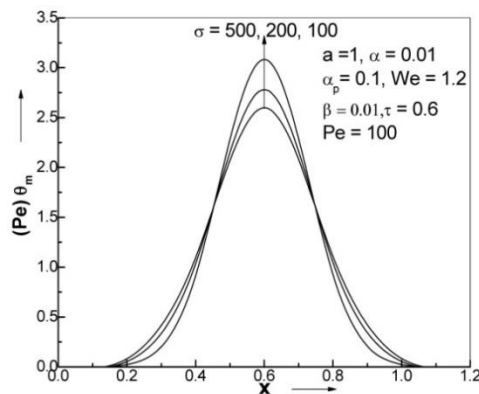


Fig. 7. Plots of mean concentration θ_m versus X for different values of porous parameter σ .

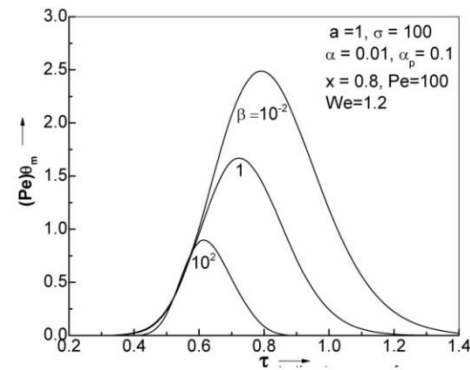


Fig. 10. Plots of mean concentration θ_m versus τ for different values of reaction rate parameter at the wall β .

This θ_m is also plotted in figures 10 to 12 respectively against the time τ for different values of σ, β and a for fixed values of the other parameters given in these figures. We note that the peak of θ_m decreases with an increase in β occurring at the lower interval of time τ . We also note that, although the peak decreases with an increase in σ and increases with an increase in a but occurs at almost at the same interval of time τ . These information are useful to understand the transport of solute at different times.

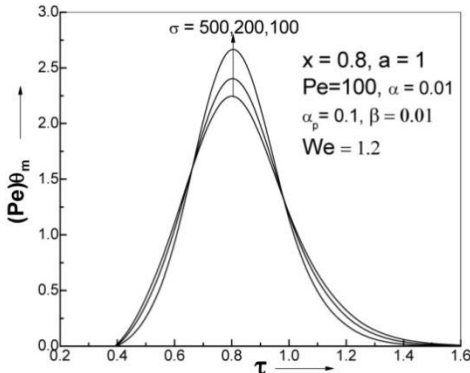


Fig. 11. Plots of mean concentration θ_m versus τ for different values of porous parameter σ .

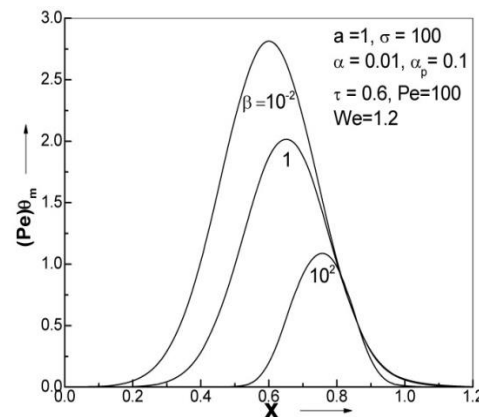


Fig. 8. Plots of mean concentration θ_m versus X for different values of reaction rate parameter at the wall β .

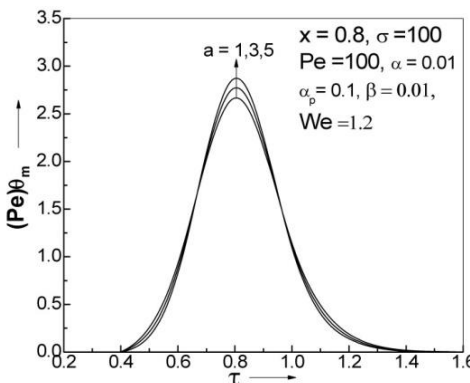


Fig. 12. Plots of mean concentration θ_m versus τ for different values of couple stress parameter a .

5. CONCLUSIONS

This paper brings into focus three important dispersion coefficients namely the exchange coefficient $-K_0$ which arises essentially due to the wall reaction, the classical convective coefficient $-K_1$, and the diffusion coefficient K_2 . We study the effect of interfacial mass transfer on K_0 , K_1 and K_2 . Wide range of parametric study has been done to understand the underlying physics and draws the following conclusions:

- Increase in the value of the wall reaction parameter β , increases the exchange coefficient ($-K_0$) but it is unaffected by the porous parameter and the couple stress parameter.
- Increase in β and σ as well as decrease in a is to increase the convective coefficient ($-K_1$).
- Increase in σ is to increase the effective dispersion coefficient (K_2).
- Increase in a is to decrease the effective dispersion coefficient (K_2).
- Increase in σ and β decreases mean concentration (θ_m), while an increase in a increases θ_m .
- The peak of θ_m decreases with an increase in β occurring at the lower interval of time τ .
- We also note that although the peak decreases with an increase in σ and increases with an increase in a but occurs at almost at the same interval of time τ .

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