Flow and Heat Transfer Behavior of MHD Dusty Nanofluid past a Porous Stretching/Shrinking Cylinder at Different Temperatures

C. Sulochana and N. Sandeep[†]

Department of Mathematics, Gulbarga University, Gulbarga-585106, India.

†*Corresponding Author Email:nsreddy.dr@gmail.com*

ABSTRACT

In this study we analyzed the momentum and heat transfer behavior of CuO-water and Al_2O_3-waternanofluids embedded with micrometer sized conducting dust particles towards a porous stretching/shrinking cylinder at different temperatures in presence of suction/injection, uniform magnetic field, shape of nano particles, volume fraction of micro and nano particles. The governing boundary layer equations are transformed to nonlinear ordinary differential equations by using similarity transformation. Numerical solutions of these equations can be obtained by using Runge-Kutta Felhberg technique. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs and tables. Results indicates that spherical shaped nano particles showed better thermal enhancement compared with cylindrical shaped nano particles, increase in volume fraction of nano particles helps to enhance the uniform thermal conductivity. But it does not happen by increase in volume fraction of dust particles. Enhancement in fluid particle interaction reduces the friction factor and improves the heat transfer rate.

Keywords: MHD; Nanofluid, Dusty fluid; Stretching/shrinking; Suction/injection; Convection.

1. INTRODUCTION

The fluid flow and heat transfer over a stretching/shrinking cylinder have wide range of applications in engineering and its allied areas. Now a days, low thermal conductivity in convectional fluids like water, ethylene glycol, oil etc. encountered variety of problems in engineering electronic devices. To overcome this drawback and enhance the thermal conductivity in the convectional fluids, past few decades many researchers concentrated on mixing of nano or micrometer sized particles in the base fluids. Mixing of nano meter sized particles in to base fluid is called nanofluid, which helps to enhance the thermal conductivity of the mixture fluid compared with base fluid. Mixing of milli or micrometer sized particles (dust particles) in the base fluids are also helps to improve the thermal conductivity of the base fluid and it is called dusty fluid. Till now, researchers concentrated on investigating the momentum and heat transfer characteristics of either dusty or nano fluids. In this study we are taking initiation to analyze the momentum and heat transfer characteristics of dusty nanofluids. That is the mixture of milli or micro meter sized conducting dust particles in to the nanofluid. This

also may help to enhance the thermal conductivity of the base fluid for some combination of dusty and nano mixtures.

Saffman (1962) was the first person who discussed about the laminar flow of the dusty gases. Choi (1995) introduced the concept of nanofluid by immersing the nano meter sized particles in to base fluids and he found the enhanced thermal conductivity in the base fluid due to the mixture of nano particles. Pulsatile flow and heat transfer behavior of dusty fluid over a long annular pipe was studied by Datta and Dalal (1995). Chamka (1998) discussed MHD flow and heat transfer over a non-isothermal stretching sheet immersed in a porous medium. Free convection of MHD flow over a cone with mixed thermal boundary conditions was discussed by Ece (2005). Buongiomo (2006) has given clear description on convective heat transfer in nanofluids. Das *et al*. (2007) written a book entitled "nanofluids science and technology" in this book they discussed the applications of nanofluids along with importance of convectional heat transfer in nanofluids. Palani and Ganeshan (2007) analyzed heat transfer behavior of dusty gases over infinite inclined plate.

Effect of thermal radiation on

magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model was discussed by Sheikholeslami *et al*. (2015). Nadeem *et al*. (2010) presented a homotopy analysis method to solve a boundary layer stagnation point flow over a stretching surface. Timofeeva and Dileep Singh (2009) studied effect of nano particle shape by considering Alumina based nanofluid. MHD effects on unsteady dusty viscous flow by considering volume fraction of dust particles was analyzed by Ibrahim Saidu *et al*. (2010). Makinde and Aziz (2011) analyzed boundary layer flow of a nanofluid by considering convective boundary conditions. Flow and heat transfer behavior over a nonlinearly stretching surface was studied by Cortell (2011). Akbar *et al*. (2014, 2014a) studied the stagnation point flow of a CNT suspended nanofluid and Prandtl fluid over a stretching and shrinking sheets. Partial slip effect on non-aligned stagnation point flow of a nanofluid over a stretching convective surface was discussed by Nadeem *et al*. (2015). Gireesha *et al*., (2012) discussed flow and heat transfer behavior of dusty fluid over a stretching surface. Remeli *et al*. (2012) discussed boundary layer flow of a nanofluid by considering suction/injection effects. Pavithra and Gireesha (2013) discussed unsteady flow and heat transfer behavior of the dust particles suspended flow over a stretching sheet. Ramana Reddy *et al*. (2014) studied aligned magnetic field, radiation and chemical reaction effects on unsteady dusty viscous flow with heat generation/absorption.

Swati Mukhopadhyay (2013) studied MHD boundary layer flow and heat transfer over an exponentially stretching sheet. Unsteady natural convection flow of a nanofluid over a vertical plate in presence of radiation was analyzed by Sandeep *et al*. (2013). Stagnation point flow of a Cu-water nanofluid over a stretching/shrinking sheet was discussed by Bachok (2013). Mohan Krishna *et al*. (2014) discussed influence of radiation on unsteady MHD nanofluid flow over infinitely vertical flat plate. Than *et al*. (2014) discussed mixed convection flow of a nanofluid over cylinder embedded in porous medium. Stagnation-point flow and mass transfer over stretching/shrinking cylinder in presence of chemical reaction was presented by Najib *et al*.(2014). Mishra and Singh (2014) illustrated dual solutions for mixed convection flow over a shrinking cylinder. Boundary layer analysis and convective heat transfer in nanofluids was discussed by MacDevette *et al*. (2014). Very recently Sulochana and Sandeep (2015) discussed the stagnation-point flow and heat transfer of Cu-water nanofluid towards horizontal and exponentially stretching or shrinking cylinders and concluded that heat transfer in a Cu-water nanofluid through horizontal and exponential cylinders is non-unique.

To the authors' knowledge no studies have been reported on the flow and heat transfer behavior of CuO-water and Al2O3-water nanofluids embedded with micrometer sized conducting dust particles towards a porous stretching/shrinking cylinder at different temperatures, in presence of suction/injection, uniform magnetic field, shape of nano particles, volume fraction of dust and nano particles. Numerical results are presented in this study. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs and tables.

2. MATHEMATICAL FORMULATION

Consider a steady two dimensional boundary layer flow of anano fluid embedded with conducting dust particles over a horizontal porous stretching/shrinking cylinder with radius R along the axial of the cylinder. The x-axis is measured along the axis of the cylinder and r-axis is measured in the radial direction as displayed in Fig.1. It is assumed that the stretching/shrinking velocity of the cylinder is $u_w = cx/L$, where cis a constant and L is the characteristic length. A uniform magnetic field of strength B_0 is applied in radial direction. Here magnetic Reynolds number is assumed to be small so that the induced magnetic field to be neglected. The dust particles are assumed as uniform in size, spherical and cylindrical shaped nano particles are considered and number density of dust particles along with volume fraction of dust and nano particles are taken into account.
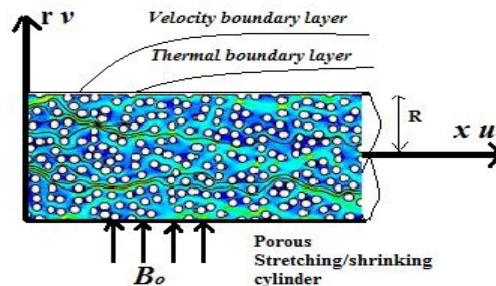

Fig. 1. Schematic representation of boundary layer flow.

The boundary layer equations as per above assumptions are given by (Gireesha *et al*. (2012), Than (2014))

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0, \quad (1)$$

$$\rho_{nf}(1-\phi_d)\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = (1-\phi_d)\mu_{nf}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$
$$+ KN(u_p - u) - \sigma B_0^2 u - \frac{\mu_f}{k}u, \quad (2)$$

$$u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial r} = \frac{K}{m}(u - u_p), \quad (3)$$

$$\frac{\partial}{\partial x}(ru_p) + \frac{\partial}{\partial r}(rv_p) = 0, \quad (4)$$

$$(\rho c_p)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=k_{nf}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)$$
$$+\frac{\rho_p}{\tau_v}(u_p-u)^2, \tag{5}$$

where r is the coordinate measured in the radial direction, (u,v) and (u_p,v_p) are the velocity components of the fluid and particle phase respectively in x and r directions, ϕ_d is the volume fraction of the dust particles, $K=6\pi\mu_f d$ is the stokes resistance with μ_f is the dynamic viscosity of the base fluid and d is the radius of the dust particle, N is the number density of the dust particles, m is the mass concentration of dust particles, σ is the electrical conductivity, ρ_{nf} and μ_{nf} are the density and the dynamic viscosity of the nano-fluid respectively, k is the permeability of the porous medium, T is the temperature in the boundary layer, $(\rho c_p)_{nf}$ is the heat capacitance of nanofluid, k_{nf} is the effective thermal conductivity of nanofluid, $\rho_p=Nm$ is the density of the particle phase and τ_v is the relaxation time of the dust particle

Boundary conditions for the proposed problem is

$$\left.\begin{array}{l}u=u_w,v=v_w,T=T_w=T_\infty+A\left(x/L\right)^2\ at\ r=R,\\u\to 0,u_p\to 0,v_p\to v,T\to T_\infty \qquad as\ r\to\infty,\end{array}\right\} \tag{6}$$

where v_w is the suction $(v_w<0)$ or injection $(v_w>0)$ velocity. The nanofluid constants are given by

$$\left.\begin{array}{l}\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\\(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s,\\\dfrac{k_{nf}}{k_f}=\left(\dfrac{k_s+(n-1)k_f-\phi(n-1)(k_f-k_s)}{k_s+(n-1)k_f+\phi(k_f-k_s)}\right),\\\mu_{nf}=\dfrac{\mu_f}{(1-\phi)^{2.5}},\end{array}\right\} \tag{7}$$

To measure the thermal conductivity of the nanofluid, k_{nf} for different shapes of nano particles, we adopted the formula, which is proposed by Hamilton and Crosser (1962). Where ϕ is the volume fraction of the nano particles, n is the nano particle shape, $n=3/2$ for cylindrical shaped nano particles and $n=3$ for spherical shaped nano particles. The subscripts $f\ and\ s$ refer to fluid and solid properties respectively.

For similarity solutions of equations (1)-(5) with respect to the boundary conditions (6), we are introducing the following similarity transformation

$$\left.\begin{array}{l}u=\dfrac{ax}{L}f'(\eta),v=-\dfrac{1}{r}\sqrt{\dfrac{v_f a}{L}}Rf(\eta),\eta=\dfrac{r^2-R^2}{2R}\sqrt{\dfrac{a}{v_f L}},\\u_p=\dfrac{ax}{L}G(\eta),v_p=-\dfrac{1}{r}\sqrt{\dfrac{v_f a}{L}}RF(\eta),\theta(\eta)=\dfrac{T-T_\infty}{T_w-T_\infty},\end{array}\right\} \tag{8}$$

where $T-T_\infty=A\left(x/L\right)^2\theta(\eta)$, η is the similarity variable, equation (8) identically satisfies equations (1) and (4), by defining η in this form, the boundary conditions at $r=R$ reducing to the boundary conditions at $\eta=0$ which is more convenient for numerical computations. Substituting (8) into equations (2), (3) and (5), we get the nonlinear ordinary differential equations of the form

$$\left.\begin{array}{l}\dfrac{(1-\phi_d)}{(1-\phi)^{2.5}}\left[(1+2\eta\delta)f'''+2\delta f''\right]+\\(1-\phi_d)\left(1-\phi+\phi\left(\dfrac{\rho_s}{\rho_f}\right)\right)\left(ff''-f'^2\right)+\\\qquad \alpha\beta(G-f')-(M+\gamma)f'=0,\end{array}\right\} \tag{9}$$

$$G'F-G^2+\beta(f'-G)=0, \tag{10}$$

$$\left.\begin{array}{l}\dfrac{k_{nf}}{k_f}\left[(1+2\eta\delta)\theta''+2\delta\theta'\right]+\Pr\alpha\beta Ec(G-f')^2+\\\Pr\left(1-\phi+\phi\left((\rho c_p)_s/(\rho c_p)_f\right)\right)(f\theta'-2f'\theta)=0,\end{array}\right\} \tag{11}$$

Subject to the boundary conditions

$$\left.\begin{array}{l}f(0)=S,f'(0)=\lambda,\theta(0)=1,\\f'(\eta)\to 0,G(\eta)\to 0,\\F(\eta)\to-f(\eta),\theta(\eta)\to 0,\ as\ \eta\to\infty\end{array}\right\} \tag{12}$$

where δ is the curvature parameter, α is the mass concentration of the dust particles, β is the fluid particle interaction parameter, τ_v is the relaxation time of the dust particles, M is the magnetic field parameter, γ is the porosity parameter, $\Pr$ is the Prandtl number, Ec is the Eckert number, S is the suction/injection parameter, here $S>0$ for suction and $S<0$ for injection and λ is the stretching/shrinking parameter, here $\lambda>0$ for stretching and $\lambda<0$ for shrinking, these are given by

$$\left.\begin{array}{l}\delta=\left(v_f L/aR^2\right)^{1/2},\alpha=\rho_p/\rho_f,\beta=1/a\tau_v,\\\gamma=v/ak,\tau_v=m/K,M=\sigma B_0^2/a\rho_f,\\\Pr=(\mu c_p)_f/k_f,v_w=-S\left(R^2 v_f a/r^2 L\right)^{1/2},\\\lambda=c/a,Ec=a^2 L/A(c_p)_f,\end{array}\right\} \tag{13}$$

The main physical quantities are interest the skin friction coefficient and Nusselt number are given by

$$C_f = \frac{\mu_{nf}}{\rho_f u_w^2}\left(\frac{\partial u}{\partial r}\right)_{r=R}, Nu_x = \frac{k_{nf}}{k_f(T_w - T_\infty)}\left(\frac{\partial T}{\partial r}\right)_{r=R} \quad (14)$$

By using non-dimensional variables, we have

$$C_f \operatorname{Re}_x^{1/2} = (1-\phi)^{-2.5} f''(0),$$
$$Nu_x \operatorname{Re}_x^{-1/2} = -(k_{nf}/k_f)\theta'(0), \quad (15)$$

Where $\operatorname{Re}_x = u_w x / v_f$ is the local Reynolds number.

3. NUMERICAL PROCEDURE

Equations (9) to (11), subject to the boundary conditions (12) are solved numerically by using Runge-Kutta Felhberg scheme. We considered

$$f = x_1, f' = x_2, f'' = x_3, F = x_4, G = x_5, G' = x_6,$$
$$\theta = x_7, \theta' = x_8.$$

Equations (9) to (11) are transformed into systems of first order differential equations. Subject to the following initial conditions

$$x_1(0) = S \quad x_2(0) = \lambda, \ x_3(0) = s_1$$
$$x_4(0) = -S, \ x_5(0) = s_2, \ x_7(0) = s_3 \quad (16)$$

We assumed the unspecified initial conditions in equation (16), transformed first order differential equations are integrated numerically as an initial valued problem to a given terminal point. We can check the accuracy of the assumed missing initial condition, by comparing the calculated value of the different variable at the terminal point with the given value by the existence of the difference in improved values so that the missing initial conditions must be obtained. The calculations are carried out by the program using MATLAB.

4. RESULTS AND DISCUSSION

Equations (9) to (11), subject to the boundary conditions (12) are solved numerically. For numerical results we considered $\eta = 10$, $\delta = \alpha = \beta = \gamma = 0.5$, $\lambda = M = 1, \phi = \phi_d = 0.1$, $n = 3$, $S = 0.5$ and $Ec = 0.1$. These values are kept as common in entire study except the varied values as displayed in respective figures and tables. Results shows the influence of non-dimensional governing parameters like curvature parameter δ, volume fraction of nano particles ϕ (in nanometers), volume fraction of dust particles ϕ_d (in micrometers), suction/injection parameter S, mass concentration of dust particles α, nano particle shape n, fluid particle interaction parameter β and stretching/shrinking parameter λ on velocity and temperature profiles along with skin friction coefficient and Nusselt number. In this study we analyzed the flow and heat transfer behavior of CuO-water and Al2O3-water nanofluids embedded with micro meter sized dust particles at different temperatures. Table 1 shows the thermophysical properties of nano particles and base fluidat different temperatures.

Table 1 Thermophysical properties of base fluid and nano particles at different temperatures

	ρ (Kgm^{-3})	c_p ($JKg^{-1}K^{-1}$)	k ($Wm^{-1}K^{-1}$)	Pr	Temp in ^0C
H_2O	999.6	4090	0.5884	8.80	10
H_2O	987.7	4066	0.6440	3.35	50
CuO	6320	531.8	76.5	--	--
Al_2O_3	3970	765	40	--	--

Figs. 2-4 illustrates the influence of magnetic field parameter on velocity profiles of fluid phase, velocity profiles of dust phase and temperature profiles of both CuO-water and Al2O3-water dusty nanofluids at different temperatures. It is evident from figures that an increase in magnetic field parameter depreciates the velocity profiles of the fluid and dust phases and enhances the temperature profiles. It is due the fact that a rise in magnetic field parameter develops the opposite force to the flow, is called Lorentz force this force declines the velocity profiles of the fluid phase. Since the dust particles are immersed in nanofluid, the velocity profiles of dust phase also follow the fluid phase. An increase in magnetic field parameter enhances the thermal boundary layer thickness and hence improves the temperature profiles of the flow. We observed an interesting result that there is no significant difference in the velocity profiles of the fluid and dust phases due to increase in temperature from 10^0C to 50^0C. But in temperature profiles we noticed hike in temperature profiles of the fluid due to the temperature difference and also observed that CuO-water dusty nanofluid showed better thermal enhancement compared with Al2O3-water dusty nanofluid.
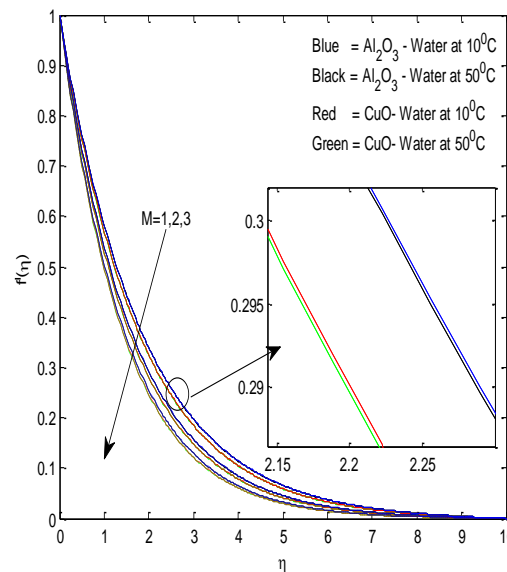

Fig. 2. Velocity profiles of fluid phase for different values of magnetic field parameter M.
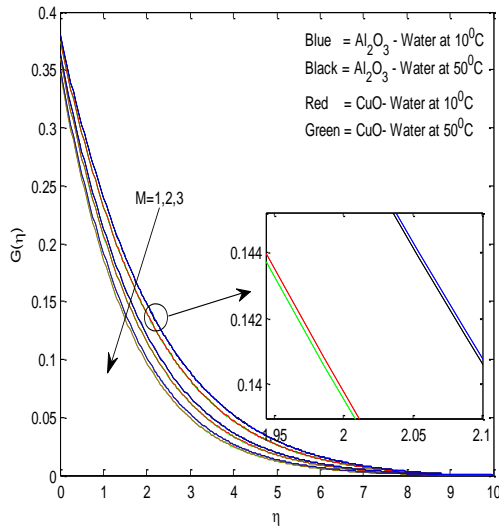
Fig. 3. Velocity profiles of dust phase for different values of magnetic field parameter M.
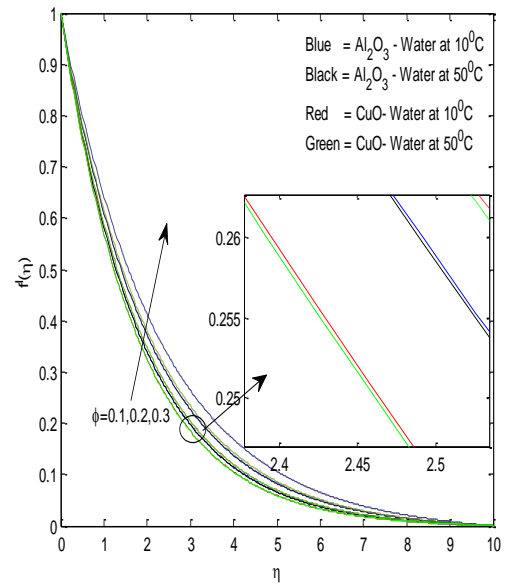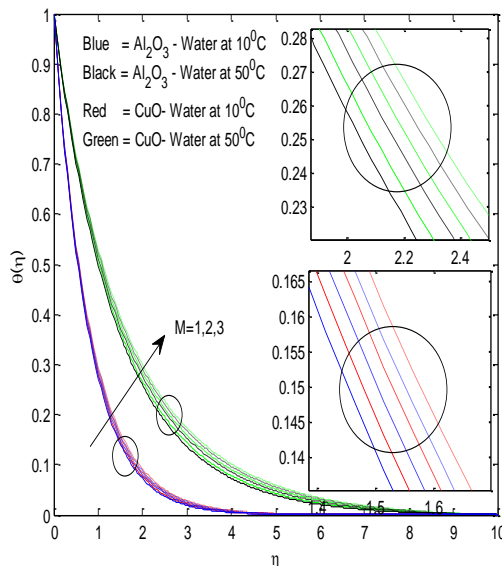

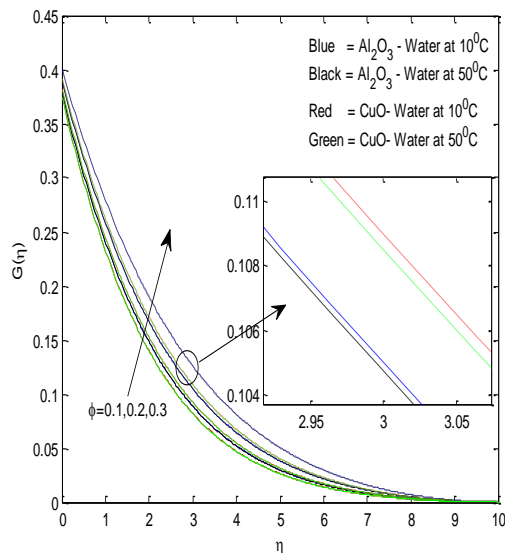**Fig. 4. Temperature profiles for different values of magnetic field parameter M.**

Figs. 5-7 depict the effect of volume fraction of nano particles on velocity profiles of fluid phase, velocity profiles of dust phase and temperature profiles of both CuO-water and Al2O3-water dusty nanofluids at different temperatures. It is clear from figures that enhancement in volume fraction of nano particles increases the velocity profiles of fluid and dust phases along with temperature profiles of the flow. Generally increase in volume faction of nano particles enhances the thermal conductivity of the flow. Due to this reason we observed hike in velocity and temperature profiles. Also, we noticed that the velocity profiles of Al2O3-water dusty nanofluid at 10^0C temperature showed better enhancement in velocity boundary layer thickness compared with CuO-water dusty nanofluid at the same temperature. Here we can conclude that at lower temperatures CuO-water dusty nanofluid improves the velocity boundary layer compared with CuO-water dusty nanofluid.

Fig. 5. Velocity profiles of fluid phase for different values of nano particle volume fraction ϕ.

Fig. 6. Velocity profiles of dust phase for different values of nano particle volume fraction ϕ.

Figs. 8-10 display the variations in velocity profiles of fluid phase, velocity profiles of dust phase and temperature profiles of both CuO-water and Al2O3-water dusty nanofluids at different temperatures for different values of volume fraction of dust particles. We observed an interesting result that increases in volume fraction of dust particles depreciates the velocity profiles of fluid and dust phases and improves the temperature profiles of the flow. This may happen due to the fact that an increase in volume fraction of the dust particles improves the conduction in particles. This causes to declines the velocity profiles of both fluid and dust phases. From fig.10 it is very clear that there exists a gradual enhancement in temperature profiles due to the temperature difference from 10^0C to 50^0C. Which

shows that after critical temperature level dust particles effectively enhances the temperature profiles of the flow. If we compare this result with fig.7 we have seen a uniform temperature distribution in fig.7 due to increase in volume fraction of nano particles. But in fig. 10 we noticed abnormal hike in temperature profiles for small temperature difference. From this we can say that dusty nanofluids help to gradual enhancement in temperature profiles after reaching critical temperature level. It is also noticed that CuO-water dusty nanofluid showed better heat transfer performance compared withAl2O3-water dusty nanofluid.
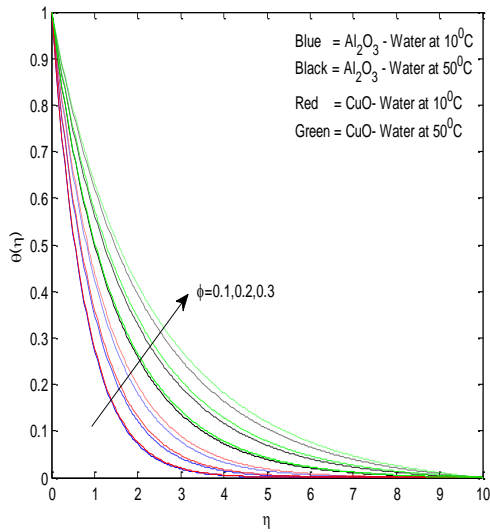
parameter on velocity and temperature profiles for both Al2O3-water and CuO-water dusty nanofluids at different temperatures. It is clear from figures that increases in curvature parameter enhance the velocity profiles of fluid and dust phases along with temperature profiles. This is due to the fact that increase in δ enhances the radius of the cylinder, this helps to reduce the contact area of the cylinder with the fluid and hence improves the velocity boundary layer thickness along with boundary layer thickness of temperature. Also, we observed that due to increase in curvature parameter we don't have significant differences in enhancement in temperature profiles of CuO-water and Al2O3-water dusty nanofluids.

Fig. 7. Temperature profiles for different values of nano particle volume fraction ϕ.

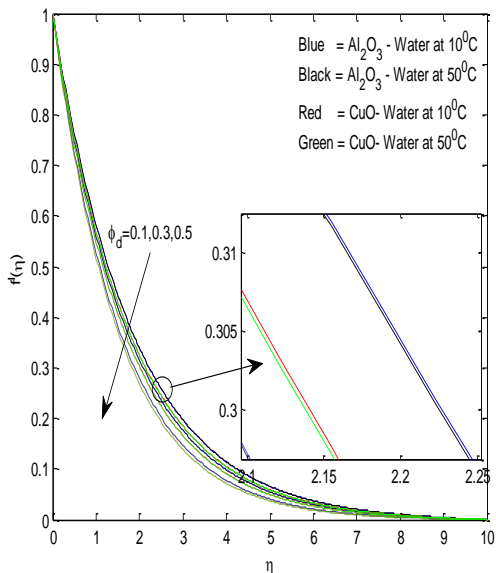**Fig. 9. Velocity profiles of dust phase for different values of volume fraction of dust particles ϕ_d.**
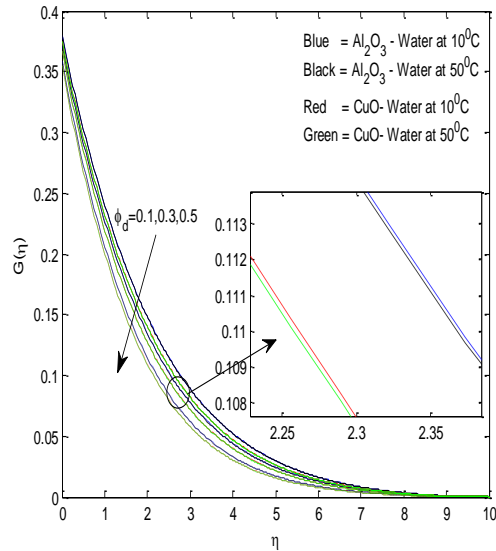

Fig. 8. Velocity profiles of fluid phase for different values of volume fraction of dust particles ϕ_d.
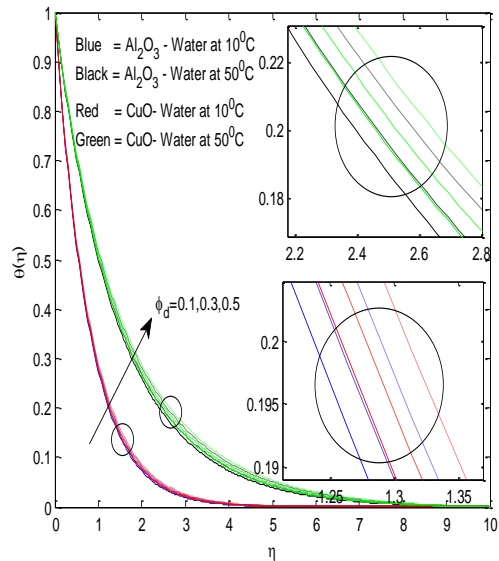

Fig. 10. Temperature profiles for different values of volume fraction of dust particles ϕ_d.

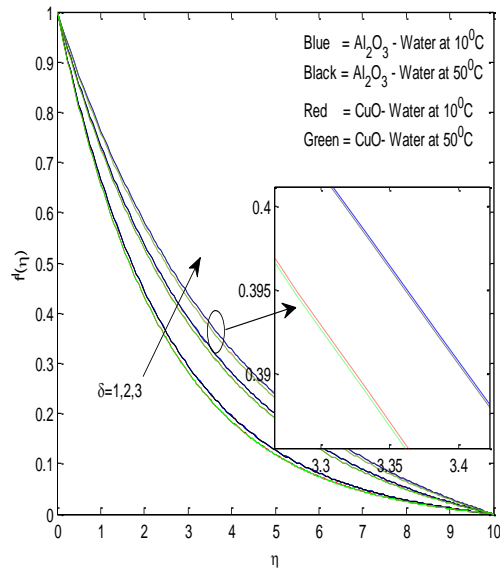Figs. 11-13 illustrate the influence of curvature

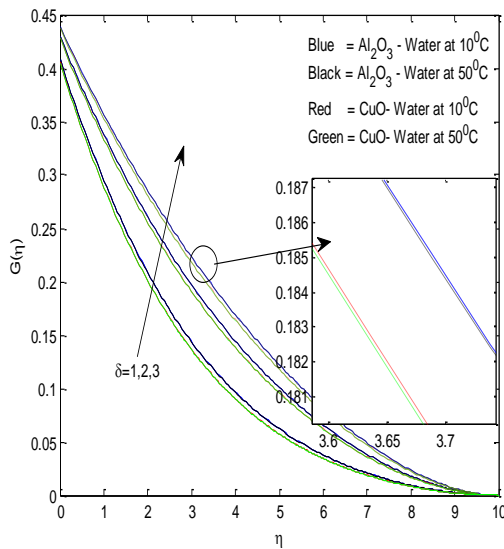**Fig. 11. Velocity profiles of fluid phase for different values of Curvature parameter δ.**

Fig. 12. Velocity profiles of dust phase for different values of Curvature parameter δ.

Figs. 14 present the effect of mass concentration of dust particles on velocity profiles of the flow. It is evident from figure that an increase in mass concentration of dust particles depreciates the velocity profiles of the fluid phase. We observed similar type of results as we explained in volume fraction of dust particles case. This agrees the general physical behavior of mass concentration of dust particles on the flow.

Fig.15 depicts the influence of shape of the nano particles on the temperature profiles of both Al_2O_3-water and CuO-water dusty nanofluids at different temperatures cylinder. It is clear that enhances in the value of n improves the temperature profiles of the flow. Here $n = 3/2$ indicates cylindrical shaped nano particles and $n = 3$ for spherical shaped nano particles. From this we conclude that spherical shaped nano particles have higher thermal

conductivity compared with cylindrical shaped nano particles.
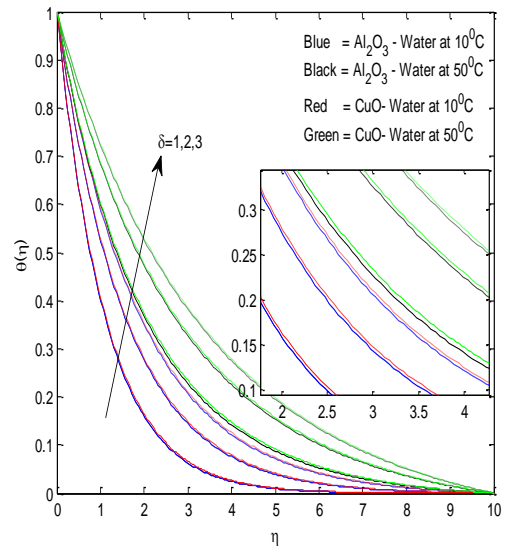
Fig. 13. Temperature profiles for different values of Curvature parameter δ.
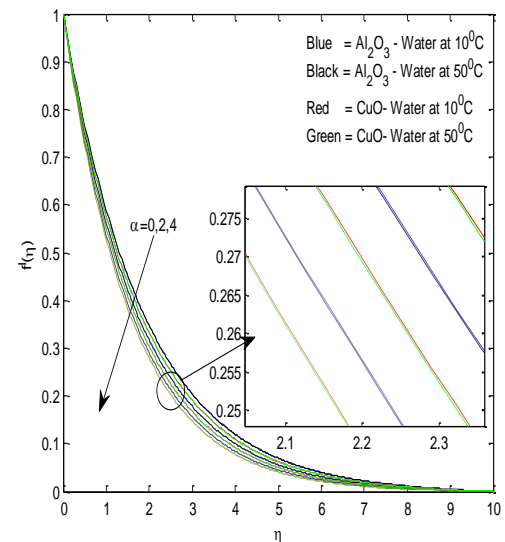
Fig. 14. Velocity profiles of fluid phase for different values of mass concentration of dust particles α.

Tables 2 and 3 depict the influence of non dimensional governing parameters on friction factor and Nusselt number for Al_2O_3-water and CuO-water dusty nanofluids at different temperatures. It is evident from the tables that increase in curvature parameter and volume fraction of nano particles increases the skin friction coefficient and reduces the Nusselt number. Enhancement in volume fraction of dust particles and mass concentration of dust particles depreciates the friction factor and heat transfer rate. Increase in suction/injection parameter depreciates the skin friction coefficient and enhances the Nusselt number. A raise in the value of fluid particle interaction parameter depreciates the friction factor and enhances the rate of heat transfer for both dusty nanofluids. Shape of the

nano particles does not influence the friction factor. But spherical shaped nano particles reduce the heat transfer rate. And these two tables agree the general behavior of the heat transfer that that at lower temperatures rate of heat transfer is more compared with higher temperatures.
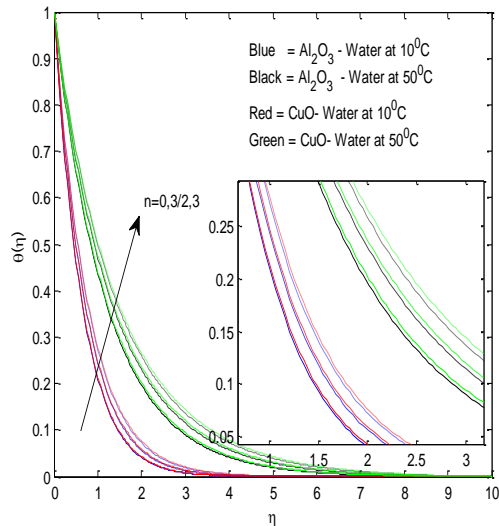
Fig.15. Temperature profiles for different nano particle shapes n .

5. CONCLUSIONS

In this study we are investigated the flow and heat transfer behavior of behavior of CuO-water and Al₂O₃-water nanofluids embedded with conducted dust particles towards a porous stretching/shrinking cylinder at different temperatures, in presence of suction/injection, uniform magnetic field, shape of nano particles, volume fraction of dust and nano particles. Numerical results are carried out. The conclusions are made as follows:

- Increase in volume fraction of dust particles and mass concentration of dust particles depreciates the heat transfer rate.

- Increase in fluid particle interaction parameter reduces the friction factor and improves the heat transfer rate.

- CuO-water dusty nanofluid shows better heat transfer performance compared with Al₂O₃-water dusty nanofluid.

- Increase in temperature does not shown significant difference in velocity profiles of CuO-water and Al₂O₃-water dusty nanofluids.

- A rise in the nano particle volume fraction improves the temperature profiles of the flow.

- Solutions exist only for certain range of stretching/shrinking parameter, for higher values of stretching/shrinking parameter velocity of the fluid is equal to free stream velocity.

- At lower temperaturesAl₂O₃-water dusty nanofluid improves the velocity boundary layer thickness compared with CuO-water dusty

nanofluid.

- Spherical shaped nano particles have better heat transfer performance compared with cylindrical shaped particles.

- A raise in the value of fluid particle interaction parameter enhances the heat transfer rate.

- Suction parameter have tendency to improve the heat transfer rate.

ACKNOWLEDGEMENTS

REFERENCES

Akbar, N.S., Z. H. KhanandS. Nadeem (2014). The combined effects of slip and convective boundary conditions on stagnation-point flow of CNT suspended nanofluid. *Journal of Molecular Liquids* 196, 21-25.

Akbar, N.S., Z. H. Khan, R. U.Haq and S. Nadeem (2014). Dual solutions in MHD stagnation-point flow of a Prandtl fluid impinging on a shrinking sheet.*Applied Mathematics and Mechanics* 35(7), 813-820.

Bachok, N.,A.Ishak,R. NazarandN. Senu(2013).Stagnation-point flow over a permeable stretching or shrinking sheet in a copper-water nanofluid.*Boundary value problems* 39.

Buongiomo, J. (2006). Convective transport in nanofluids.*J. Heat Transfer*128, 240-250.

Chamkha, A.J. (1998). Unsteady hydromagnetic flow and heat transfer from non-isothermal stretching sheet immersed in a porous medium.*Int.Comm. in Heat and Mass Transfer* 25, 899–906.

Choi, S.U.S. (1995).Enhancing thermal conductivity of fluids with nanoparticles.*DevelsAppls Non Newtonian flows* 66, 99-105.

Cortell, R. (2011). Heat and fluid flow due to non-linearly stretching surfaces. *Appl Math Comput*217, 7564-7572.

Das, S.K., Choi, S.U.S., Yu, Pradeep, T. (2007).*Nanofluids science and technology*.Willey New Jersey.

Datta, N.andD. C. Dalal (1995).Pulsatile flow of heat transfer of a dusty fluid through an infinitely long annular pipe.*Int. J. Multiphase flow* 21(3), 515-528.

Ece, M.C. (2005). Free convection flow about a cone under mixed thermal boundary conditions

and a magnetic field. *Appl.Math.Model.* 29,

Table 2 Variation in $f''(0)$ and $-\theta'(0)$ for Al₂O₃-water dusty nanofluid at different values of
$$S, \phi, \phi_d, \delta, \alpha, \beta \text{ and } n$$

Fluid Temperature	S	ϕ	ϕ_d	δ	α	β	n	$f''(0)$	$-\theta'(0)$
10^0C	-0.5	0.1	0.1	0.5	0.5	0.5	3	-0.490786	0.961668
10^0C	0	0.1	0.1	0.5	0.5	0.5	3	-0.515194	1.109492
10^0C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.540687	1.275818
50^0C	-0.5	0.1	0.1	0.5	0.5	0.5	3	-0.491056	0.597824
50^0C	0	0.1	0.1	0.5	0.5	0.5	3	-0.515551	0.655308
50^0C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.541138	0.717862
10^0C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.540687	1.275818
10^0C	0.5	0.2	0.1	0.5	0.5	0.5	3	-0.493743	1.044080
10^0C	0.5	0.3	0.1	0.5	0.5	0.5	3	-0.441364	0.858158
50^0C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.541138	0.717862
50^0C	0.5	0.2	0.1	0.5	0.5	0.5	3	-0.494470	0.592916
50^0C	0.5	0.3	0.1	0.5	0.5	0.5	3	-0.442225	0.493990
10^0C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.540687	1.275818
10^0C	0.5	0.1	0.3	0.5	0.5	0.5	3	-0.578037	1.266773
10^0C	0.5	0.1	0.5	0.5	0.5	0.5	3	-0.639403	1.251869
50^0C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.541138	0.717862
50^0C	0.5	0.1	0.3	0.5	0.5	0.5	3	-0.578462	0.708862
50^0C	0.5	0.1	0.5	0.5	0.5	0.5	3	-0.639792	0.694230
10^0C	0.5	0.1	0.1	1	0.5	0.5	3	-0.403701	0.903418
10^0C	0.5	0.1	0.1	2	0.5	0.5	3	-0.307137	0.641581
10^0C	0.5	0.1	0.1	3	0.5	0.5	3	-0.266463	0.527100
50^0C	0.5	0.1	0.1	1	0.5	0.5	3	-0.403701	0.903418
50^0C	0.5	0.1	0.1	2	0.5	0.5	3	-0.307137	0.641581
50^0C	0.5	0.1	0.1	3	0.5	0.5	3	-0.266463	0.527100
10^0C	0.5	0.1	0.1	0.5	0	0.5	3	-0.528451	1.281859
10^0C	0.5	0.1	0.1	0.5	2	0.5	3	-0.576355	1.258024
10^0C	0.5	0.1	0.1	0.5	4	0.5	3	-0.621766	1.234980
50^0C	0.5	0.1	0.1	0.5	0	0.5	3	-0.528909	0.722176
50^0C	0.5	0.1	0.1	0.5	2	0.5	3	-0.576787	0.705265
50^0C	0.5	0.1	0.1	0.5	4	0.5	3	-0.622175	0.689197
10^0C	0.5	0.1	0.1	0.5	0.5	1	3	-0.546874	1.273858
10^0C	0.5	0.1	0.1	0.5	0.5	4	3	-0.560930	1.274400
10^0C	0.5	0.1	0.1	0.5	0.5	8	3	-0.566683	1.275108
50^0C	0.5	0.1	0.1	0.5	0.5	1	3	-0.547323	0.716265
50^0C	0.5	0.1	0.1	0.5	0.5	4	3	-0.561372	0.717723
50^0C	0.5	0.1	0.1	0.5	0.5	8	3	-0.567124	0.718062
10^0C	0.5	0.1	0.1	0.5	0.5	0.5	0	-0.540687	1.505490
10^0C	0.5	0.1	0.1	0.5	0.5	0.5	1.5	-0.540687	1.375143
10^0C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.540687	1.275818
50^0C	0.5	0.1	0.1	0.5	0.5	0.5	0	-0.541138	0.845817
50^0C	0.5	0.1	0.1	0.5	0.5	0.5	1.5	-0.541138	0.773171
50^0C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.541138	0.717862

1121-1134.

Gireesha, B.J., G. S. Roopa,H. J. LokeshandC. S. Bagewadi(2012).MHD flow and heat transfer of a dusty fluid over a stretching sheet. *Int. J. physical and mathematical sciences* 3(1), 171-180.

Hamilton, R.L. and O. K. Crosser (1962).Thermal conductivity of heterogeneous two-component systems, *Journal of industrial &engineering chemistry fundamentals*1(3), 187-191.

Ibrahim, S., M. M. Waziri,A.RokoandM. Hamisu (2010).MHD effects on convective flow of dusty viscous fluid with volume fraction of dust particles.*ARPN J of Eng and applied sciences* 5, 86-91.

MacDevette, M.M., T. G. Myers andB. Wetton (2014). Boundary layer analysis and heat transfer of a nanofluid. *Microfluidics and Nanofluidics* 17, 401-412.

Makinde, O.D.andA.Aziz (2011).Boundary layer flow of a nanofluid past a stretching sheet with convective boundary condition.*Int.J.Ther. Sci. 50, 1326-1332.*

Mishra, U.andG.Singh (2014). Dual solutions of mixed convection flow with momentum and thermal slip flow over a permeable shrinking cylinder. *Comp Fluids* 93,107-115.

Mohankrishna, P., V. SugunammaandN. Sandeep (2014). Radiation and magneticfield effects on

unsteady natural convection flow of a

Table 3 Variation in $f''(0)$ and $-\theta'(0)$ for CuO-water dusty nanofluid at different values of

$$S, \phi, \phi_d, \delta, \alpha, \beta \text{ and } n$$

Fluid Temperature	S	ϕ	ϕ_d	δ	α	β	n	$f''(0)$	$-\theta'(0)$
10^0 C	-0.5	0.1	0.1	0.5	0.5	0.5	3	-0.503786	0.952694
10^0 C	0	0.1	0.1	0.5	0.5	0.5	3	-0.532431	1.097902
10^0 C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.562537	1.261488
50^0 C	-0.5	0.1	0.1	0.5	0.5	0.5	3	-0.504198	0.590453
50^0 C	0	0.1	0.1	0.5	0.5	0.5	3	-0.532980	0.646290
50^0 C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.563235	0.707217
10^0 C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.562537	1.261488
10^0 C	0.5	0.2	0.1	0.5	0.5	0.5	3	-0.528577	1.020499
10^0 C	0.5	0.3	0.1	0.5	0.5	0.5	3	-0.482229	0.828829
50^0 C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.563235	0.707217
50^0 C	0.5	0.2	0.1	0.5	0.5	0.5	3	-0.529675	0.575960
50^0 C	0.5	0.3	0.1	0.5	0.5	0.5	3	-0.483508	0.473963
10^0 C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.562537	1.261488
10^0 C	0.5	0.1	0.3	0.5	0.5	0.5	3	-0.598660	1.252726
10^0 C	0.5	0.1	0.5	0.5	0.5	0.5	3	-0.658330	1.238219
50^0 C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.563235	0.707217
50^0 C	0.5	0.1	0.3	0.5	0.5	0.5	3	-0.599320	0.698570
50^0 C	0.5	0.1	0.5	0.5	0.5	0.5	3	-0.658937	0.684457
10^0 C	0.5	0.1	0.1	1	0.5	0.5	3	-0.404006	0.523351
10^0 C	0.5	0.1	0.1	2	0.5	0.5	3	-0.307337	0.385989
10^0 C	0.5	0.1	0.1	3	0.5	0.5	3	-0.266615	0.326975
50^0 C	0.5	0.1	0.1	1	0.5	0.5	3	-0.418447	0.893695
50^0 C	0.5	0.1	0.1	2	0.5	0.5	3	-0.316820	0.635119
50^0 C	0.5	0.1	0.1	3	0.5	0.5	3	-0.273871	0.522113
10^0 C	0.5	0.1	0.1	0.5	0	0.5	3	-0.550635	1.267429
10^0 C	0.5	0.1	0.1	0.5	2	0.5	3	-0.597298	1.243967
10^0 C	0.5	0.1	0.1	0.5	4	0.5	3	-0.641678	1.221244
50^0 C	0.5	0.1	0.1	0.5	0	0.5	3	-0.551344	0.711418
50^0 C	0.5	0.1	0.1	0.5	2	0.5	3	-0.597968	0.694934
50^0 C	0.5	0.1	0.1	0.5	4	0.5	3	-0.642317	0.679245
10^0 C	0.5	0.1	0.1	0.5	0.5	1	3	-0.568591	1.256551
10^0 C	0.5	0.1	0.1	0.5	0.5	4	3	-0.582387	1.257134
10^0 C	0.5	0.1	0.1	0.5	0.5	8	3	-0.588057	1.259863
50^0 C	0.5	0.1	0.1	0.5	0.5	1	3	-0.569285	0.702656
50^0 C	0.5	0.1	0.1	0.5	0.5	4	3	-0.583073	0.703179
50^0 C	0.5	0.1	0.1	0.5	0.5	8	3	-0.588741	0.705543
10^0 C	0.5	0.1	0.1	0.5	0.5	0.5	0	-0.562537	1.494600
10^0 C	0.5	0.1	0.1	0.5	0.5	0.5	1.5	-0.562537	1.363365
10^0 C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.562537	1.261488
50^0 C	0.5	0.1	0.1	0.5	0.5	0.5	0	-0.563235	0.837206
50^0 C	0.5	0.1	0.1	0.5	0.5	0.5	1.5	-0.563235	0.764036
50^0 C	0.5	0.1	0.1	0.5	0.5	0.5	3	-0.563235	0.707217

nanofluid past an infinite vertical plate with heat source. *Chemical and Process Engineering Research*39-52.

Nadeem, S., A. HussainandM.Khan (2010). HAM solutions for boundary layer flow in the region of the stagnation point towards a stretching sheet. *Commun.Nonlinear Sci.NumerSimul.*15, 475–81.

Nadeem, S., R. MehmoodandN. S. Akbar (2015).Partial slip effect on non-aligned stagnation point nanofluid over a stretching

convective surface, *Journal of Magnetism and Magnetic Materials* 378, 457-462.

Najib, N., N. Bachok,M. D. Arifin, A. Ishak (2014).Stagnation point flow and mass transfer with chemical reaction past a stretching/shrinking cylinder.*Scientific Reports* 4, 4178.

Palani, G.andP.Ganesan(2007). Heat transfer effects on dusty gas flow past a semi-infinite inclined plate. *ForschungimIngenieurwesen* 71, 223–230.

Pavithra, G.M.andB. J. Gireesha (2013). Unsteady flow and heat transfer of a fluid particle suspension over an exponentially stretching sheet. *Journal of Heat Transfer* 5, 613-624.

Ramana Reddy, J.V., V. Sugunamma, P. Mohan Krishna, N. Sandeep (2014). Aligned Magnetic Field, Radiation and Chemical Reaction Effects on Unsteady Dusty Viscous Flow with Heat Generation/Absorption. *Chemical and Process Engineering Research* 27, 37-53.

Remeli, A., N. M. Arifin,F. IsmailandI. Pop (2012).Marangoni Driven Boundary layer flow in a nanofluid with suction/injection.*World applied sciences journal* 17, 21-26.

Saffman, P.G. (1962). On the stability of laminar flow of a dusty gas.*J.Fluid Mechanics,* 13,120-128.

Sandeep, N., V. SugunammaandP. Mohankrishna (2013). Effects of radiation on an unsteady natural convective flow of a EG-Nimonic 80a nanofluid past an infinite vertical plate. *Advances in Physics Theories and Applications* 23, 36-43.

Sheikholeslami,M.,D. D. Ganji, Y.M.JavedandR. Ellahi (2015).Effect of thermal radiation on magnetohydrodynamicsnanofluid flow and heat transfer by means of two phase model.*Journal of Magnetism and Magnetic Materials* 374, 36–43.

Sulochana,C.andN. Sandeep (2015). Stagnation-point flow and heat transfer behavior of Cu-water nanofluid towards horizontal and exponentially stretching or shrinking cylinders.*AppliedNanoscience* 5 (In Press).

Swati, M. (2013). MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium. *Alexandria Eng J*. 52, 259-265.

Than, L., R. NazarandI. Pop (2014). Mixed convection flow from a horizontal circular cylinder embedded in a porous medium filled by a Nanofluid: Buongiorne-Darcymodel.*Int.Journal of Thermal Sciences*84.

Timofeeva, E.V.andJ.L. R. Dileep Singh (2009).Particle shape effect on thermo physical properties of alumina nanofluids.*J.of Applied Physics* 106.