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A Study on Inviscid Flow with a Free Surface over an Undulating Bottom

S. Panda

Department of Mathematics, Jadavpur University, Jadavpur, Kolkata, 700 032, India

Email: *srikumarp@iitrpr.ac.in, shree.iitg.mc@gmail.com*

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ABSTRACT

In this paper, the problem involving inviscid flow with a free surface over an undulating bottom is studied within the framework of linear theory. Applying perturbation analysis in conjunction with the Fourier transform technique, the boundary value problem arising from the flow problem is solved analytically. Behaviour of both interface and free-surface profiles, which are unknown at the outset, are analyzed. It is found that each profile (interface and free-surface) possesses a wave free region at the far upstream, followed by a modulated downstream wave. It also observed, for the first time, that the amplitude of the downstream wave is varying. Further, the effects of various system parameters are analyzed and demonstrated in graphical forms.

Keywords: Inviscid fluid; Irrotational flow; Linear theory; Perturbation analysis; Fourier transform technique.

1. INTRODUCTION

Problems involving free-surface inviscid flow over an obstacle are studied by many researchers to model various situations arising in oceanography and atmospheric sciences. The study of such flow problems is important to analyze the qualitative insight of the mechanism of wave generation by submerged bodies. Various mathematical techniques have been employed to study free-surface flows over different kinds of obstacles situated at the bottom of a channel. For example, Lamb (1932) studied the flow over a cylindrical obstruction lying on the bottom and calculated the drag force on the obstruction. Forbes and Schwartz (1982) considered the flow over a semi-circular obstruction and calculated the wave resistance offered by the semicircle. Vanden-Broeck (1987) solved numerically the problem of Forbes and Schwartz (1982), and discussed the existence of the supercritical solutions. Forbes (1988) presented a numerical solution for critical free-surface flow over a semicircular obstruction attached to the bottom of a running stream. Dias and Vanden-Broeck (1989) studied the problem involving free-surface flow past a submerged triangular obstacle at the bottom of a channel and solved the problem numerically by applying a series truncation method. Forbes (1985) studied the free-surface flow by considering a submerged point vortex in a single-layer fluid of infinite depth. Shen *et al.* (1989) obtained the numerical solution of the problem involving an inviscid fluid flow over a semi-circular as well as a semi-elliptical obstacles. Dias and Vanden-Broeck (2002) solved the steady free-surface flow problem numerically, and demonstrated that there exist supercritical flows with waves downstream only. A good review on the flow problems is provided by Gazdar (1973) and Yeung (1982). Recently, Belward *et al.* (2003) studied the inviscid flow over topography using a series solution method. Higgins *et al.* (2006) presented an analytical series method to obtain the solution of the problem involving flow of a fluid over a topography. They calculated the analytical series solutions for the supercritical, transcritical and subcritical flows. The above studies were focused on the solution of the problem involving steady flow of a fluid. For the problem involving unsteady flow, Grimshaw and Smyth (1986) presented a theoretical study of a stratified fluid which is flowing over bottom topography. They solved the problem by using weak nonlinear theory and pointed out that the flow can be described by a forced Korteweg-de Vries equation. Stokes *et al.* (2005) used numerical technique to analyze the unsteady flow with a submerged point sink beneath the free surface. Milewski and Vanden-Broeck (1999) considered the time dependent free-surface flow over a submerged moving obstacle in a single-layer channel and solved the problem using weak nonlinear theory. Generalized lattice Boltzmann method has been used by Rahmati and Ashrafizaadeh (2009) to study flow of an incompressible fluid in three-dimension. They have shown that their proposed model is very convenient for simulation as compared to the existing CFD simulation. Sheikholeslamia *et al.* (2014) used lattice Boltzmann method to study the magnetohydrodynamic flow in a concentric annulus. They have discussed the characteristics of flow and heat transfer for various values of system parameters. Based on perturbation method, Saghafian *et al.* (2015) investigated the slip-flow heat transfer in a two-dimensional incompressible flow. They have shown that their results are in good agreement with the available results. Using Lie group method, Abd-el-Malek and Amin (2014) studied the nonlinear flow over a horizontal bottom in a single-layer fluid. It is noticed that, the solutions of flow problems are determined, in most cases, for a specific bottom obstacle like a semi-circle (Forbes and Schwartz 1982; Forbes 1988; Vanden-Broeck 1987), a semi-ellipse (Forbes 1981; Forbes 1982), a step (King and Bloor 1987), a triangle (Dias and Vanden-Broeck 1989), etc. Therefore, the flow over an undulating bottom topography remained unsolved. The relevance of such study is of importance due to the fact that, the undulating bottom is a naturally occurring bottom formed in the sea due to sedimentation and ripples growth of sands. It is also noticed that, most of the above studies were carried out by assuming flow in a *single-layer* channel. However, the single-layer approximation becomes insufficient in oceanography and meteorology due to the continuous stratification of the fluid as pointed out by Belward and Forbes (1993). The availability of literature on two-layer flow problems is rather limited.

Therefore, in the present paper, inviscid flow over an undulating bottom is studied in a two-layer fluid system, where the upper layer is free to the atmosphere. The problem is formulated mathematically in terms of a mixed boundary value problem (BVP). This BPV is solved analytically with the help of perturbation analysis along with Fourier transform technique. The

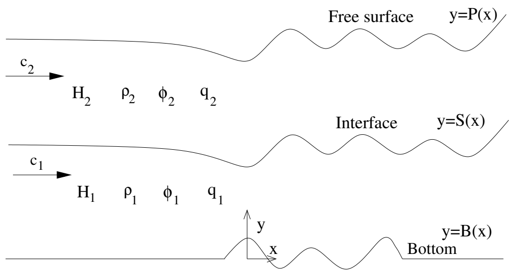


Fig. 1. Sketch of a two-layer flow with a free surface over an undulating bottom.

free-surface and the interface profiles, which are unknown at the outset, are determined to analyze their behaviour. The study also highlights the role of the Fourier transform technique in an elaborate way. It is found that the free-surface as well as the interface possess a train of waves downstream with varying amplitude. The varying nature, which is not observed in earlier studies, is noticed here for the first time. In addition, the effects of various system parameters are analyzed here.

2. Description and Mathematical Formulation

In this study, two-dimensional inviscid flow in a two-layer fluid over an undulating bottom, as shown in Fig. 1, is considered. The fluid is subject to the downward acceleration due to the gravity g, and it is flowing from the left to the right. The upper layer of the fluid system is free to the atmosphere and the lower layer is bounded by an irregular bottom $y = B(x)$, where the x-axis is chosen along the undisturbed bottom and the y-axis is measured vertically upward (see Fig. 1). The flow is assumed to be irrotational, and it is uniform at the far upstream. The upstream depth of the lower layer is H_1 and that of the upper layer is H_2. The upstream horizontal velocities in the lower and upper layer are, respectively, c_1 and c_2. Density, velocity and pressure in the lower layer are, respectively, ρ_1, $\overrightarrow{q_1}$ and p_1 and those in the upper layer are ρ_2 $(< \rho_1)$, $\overrightarrow{q_2}$ and p_2. Let $\phi_1(x,y)$ and $\phi_2(x,y)$ are the velocity potentials in the lower and upper layers respectively, so that $\overrightarrow{q_j} = (\phi_{j,x}, \phi_{j,y})$, $(j = 1,2)$ where $\phi_{j,x}$ and $\phi_{j,y}$ are, respectively, the partial derivative of ϕ_j with respect to x and y. The effect of the surface tension is ignored here. The free-surface, which is unknown at the outset, is given by $y = P(x)$. The interface (unknown at the outset) between two fluid layers is given by $y = S(x)$. Only waves that are stationary with respect to the bottom profile are taken into account, so that the partial derivatives with respect to time are taken to be equal to zero. The problem is made dimensionless using H_1 as the length scale and c_1 as the velocity scale. Therefore, the work proceeds purely with dimensionless variables.

Under the assumptions as mentioned above, the *equation of continuity* in each layer yields the Laplace equation:

$$\frac{\partial^2\phi_2}{\partial x^2}+\frac{\partial^2\phi_2}{\partial y^2} = 0 \quad \text{in the upper layer,} \quad (1)$$

$$\frac{\partial^2\phi_1}{\partial x^2}+\frac{\partial^2\phi_1}{\partial y^2} = 0 \quad \text{in the lower layer.} \quad (2)$$

As no fluid particle leaves the surface, the *kinematic condition* on the free surface $y = P(x)$ can be written as

$$\frac{\partial\phi_2}{\partial n} = 0 \quad \text{on} \quad y = P(x), \quad (3)$$

where $\partial/\partial n$ is the normal derivative at a point (x, y) on the surface.
The *dynamic condition* on the free surface, which is given below, is derived by using Bernoulli's equation:

$$\frac{1}{2}F^2\left(q_2^2-\gamma^2\right)+P(x) = 1+\lambda \quad \text{on} \quad y = P(x), (4)$$

where F $(= c_1/\sqrt{gH_1})$ is the Froude number, γ $(= c_2/c_1)$ is the ratio of the upstream fluid velocities, and λ $(= H_2/H_1)$ is the ratio of the upstream depths.

At the interface $y = S(x)$, the condition of no fluid exchange is

$$\frac{\partial\phi_2}{\partial n} = 0 \quad \text{on} \quad y = S(x), \quad (5)$$

$$\frac{\partial\phi_1}{\partial n} = 0 \quad \text{on} \quad y = S(x). \quad (6)$$

The continuity of pressure coupled with the Bernoulli's equation gives rise to the condition at the interface

$$\frac{1}{2}F^2\left(q_1^2-Dq_2^2\right)+(1-D)S(x) = \frac{1}{2}F^2\left(1-D\gamma^2\right) + (1-D) \quad \text{on} \quad y = S(x), \quad (7)$$

where $D = \rho_2/\rho_1 (< 1)$.

The condition of no penetration at the bottom gives rise to the bottom condition

$$\frac{\partial\phi_1}{\partial n} = 0 \quad \text{on} \quad y = B(x). \quad (8)$$

The conditions at the upstream are

$$\vec{q_2} \to \gamma\vec{i}, \ \vec{q_1} \to \vec{i}, \ S(x) \to 1, \ P(x) \to 1+\lambda \quad \text{as} \quad x \to -\infty. \quad (9)$$

The primary aim of the present work is to determine (analytically) the unknown functions $\phi_1(x,y)$ and $\phi_2(x,y)$ along with $S(x)$ and $P(x)$. These unknowns can be determined once the mixed coupled boundary value problem described in relations (1)-(9) is solved. In the following section, the above boundary value problem is solved using a similar kind of mathematical procedure, i.e., perturbation analysis along with Fourier transform technique, as described in Panda *et al.* (2015) .

3. SOLUTION OF THE PROBLEM

It is assumed that the bottom profile is given by $B(x) = \varepsilon f(x)$, where ε is the maximum height of the bottom profile and is a dimensionless quantity. When the height ε is small, then an approximate solution of the boundary value problem described in relations (1)-(9) can be derived by using the regular perturbation expansion in powers of ε, retaining only the first-order terms. The asymptotic expansions of the velocity potential, the interface profile and the free-surface profile can be expressed respectively as follows:

$$\left.\begin{array}{l} \phi_2(x,y) = \gamma x+\varepsilon\phi_{21}(x,y)+O(\varepsilon^2) \\ \phi_1(x,y) = x+\varepsilon\phi_{11}(x,y)+O(\varepsilon^2) \\ S(x) = 1+\varepsilon S_1(x)+O(\varepsilon^2) \\ P(x) = 1+\lambda+\varepsilon P_1(x)+O(\varepsilon^2) \end{array}\right\}, \quad (10)$$

where $\phi_{21}(x,y)$ and $\phi_{11}(x,y)$ are the first-order corrections of the velocity potentials; $S_1(x)$ and $P_1(x)$ are the first-order corrections of the interface and the free-surface, respectively.

Employing the asymptotic expansions (10) into relations (1)-(8); and then comparing the first order terms of ε on both the sides of all equations, the following BVP is obtained:

$$\frac{\partial^2\phi_{21}}{\partial x^2}+\frac{\partial^2\phi_{21}}{\partial y^2} = 0 \text{ in the upper layer,} \quad (11a)$$

$$\frac{\partial^2\phi_{11}}{\partial x^2}+\frac{\partial^2\phi_{11}}{\partial y^2} = 0 \text{ in the lower layer,} \quad (11b)$$

$$\frac{\partial\phi_{21}}{\partial y} = \gamma P_1'(x) \text{ on } y = 1+\lambda, \quad (11c)$$

$$F^2\gamma\frac{\partial\phi_{21}}{\partial x}+P_1(x) = 0 \quad \text{on} \quad y = 1+\lambda, \quad (11d)$$

$$\frac{\partial\phi_{21}}{\partial y} = \gamma S_1'(x) \quad \text{on} \quad y = 1, \quad (11e)$$

$$\frac{\partial\phi_{11}}{\partial y} = S_1'(x) \quad \text{on} \quad y = 1, \quad (11f)$$

$$F^2\left(\frac{\partial\phi_{11}}{\partial x}-D\gamma\frac{\partial\phi_{21}}{\partial x}\right)+(1-D)S_1(x)=0,\quad \text{on } y=1,\tag{11g}$$

$$\frac{\partial\phi_{11}}{\partial y}=f'(x)\quad\text{on}\quad y=0.\tag{11h}$$

The system defined in relations (11a)-(11h) gives rise to the linearized version of the original problem. To solve the above BVP (11a)-(11h), it is assumed that the Fourier transforms of the first-order potentials $\phi_{11}(x,y)$ and $\phi_{21}(x,y)$ exist, and are defined as:

$$\widehat{\phi}_{j1}(k,y)=\int_0^\infty \phi_{j1}(x,y)\sin(kx)dx,\ \ (j=1,2)\tag{12a}$$

with the inverse transforms

$$\phi_{j1}(x,y)=\frac{2}{\pi}\int_0^\infty \widehat{\phi}_{j1}(k,y)\sin(kx)dk,\ (j=1,2).\tag{12b}$$

Define $S_1(x)$, $P_1(x)$ and $f(x)$ as follows:

$$S_1(x)=\int_0^\infty a(k)\cos(kx)dk,\tag{12c}$$

$$P_1(x)=\int_0^\infty b(k)\cos(kx)dk,\tag{12d}$$

and

$$f(x)=\int_0^\infty M(k)\cos(kx)dk,\tag{12e}$$

where $a(k)$ and $b(k)$ are two unknowns will be determined here, and $M(k)$ can be determined from the shape of the undulating bottom.

Applying Fourier transform (12a) along with its inverse (12b) and using relations (12c)- (12e) in Eqs. (11a)-(11h), the velocity potentials (solution of the above BVP) are obtained as:

$$\phi_{11}(x,y)=\int_0^\infty\left[\frac{M(k)-a(k)\cosh k}{\sinh k}\cosh k(y-1)-a(k)\sinh k(y-1)\right]\sin(kx)dk,$$

and

$$\phi_{21}(x,y)=\int_0^\infty\left\{\frac{\gamma[a(k)-b(k)\cosh(k\lambda)]}{\sinh(k\lambda)}\cosh k(y-1-\lambda)-\gamma b(k)\sinh k(y-1-\lambda)\right\}\sin(kx)dk,$$

where

$$a(k)=\frac{E_1(k)F^2kM(k)\sinh(k\lambda)}{E_2(k)}\tag{13a}$$

and

$$b(k)=\frac{F^4\gamma^2kM(k)\sinh(k\lambda)}{E_2(k)}\tag{13b}$$

in which

$$E_1(k)=\left(F^2\gamma^2k\cosh k\lambda-\sinh k\lambda\right)/k$$

and

$$E_2(k)=\left\{F^2k\cosh k\sinh k\lambda+\left[F^2D\gamma^2k\cosh k\lambda-(1-D)\sinh k\lambda\right]\sinh k\right\}E_1(k)-\gamma^4DF^4k\sinh k.$$

It should be noted that the relation

$$E_2(k)=0\tag{14}$$

is known as the *dispersion relation*. It can be shown (discussed later in Section 4.) that, the relation (14) has two positive real roots, say k_0 and k_1. It is worth-mentioning here that the root of the dispersion relation signifies the wave number of the downstream waves. Note that, $-k_0$ and $-k_1$ are also real roots of the relation (14).

It is observed from relations (12c)- (12e) and (13) that, the interface as well as the free-surface profiles depend on the shape of the undulating bottom. Hence, the profiles $S(x)$ and $P(x)$ can be determined once the bottom profile is known. To evaluate these profiles, the following bottom topography is considered:

$$B(x)=\begin{cases}\frac{\varepsilon}{2}\left[1+\cos\left(\frac{\pi x}{L}\right)\right], & -L\le x\le L\\ 0, & \text{otherwise,}\end{cases}\tag{15}$$

where L is the half length of the obstacle.

From relations (12e), (13) and (15), $a(k)$ and $b(k)$ are determined as

$$a(k)=\frac{E_1(k)\pi F^2\sin(kL)\sinh(k\lambda)}{L^2\left(\frac{\pi^2}{L^2}-k^2\right)E_2(k)},\tag{16a}$$

$$b(k)=\frac{\pi F^4\gamma^2\sin(kL)\sinh(k\lambda)}{L^2\left(\frac{\pi^2}{L^2}-k^2\right)E_2(k)}.\tag{16b}$$

Now, substituting Eq. (16a) in Eq. (12c), the interface profile is obtained as:

$$S_1(x) = \frac{\pi F^2}{4L^2} \times \int_{-\infty}^{\infty} \frac{E_1(k)\sinh(k\lambda)\left[\sin k(x+L) - \sin k(x-L)\right]}{\left(\frac{\pi^2}{L^2} - k^2\right) E_2(k)} dk. \tag{17a}$$

In a similar way, from relations (16b) and (12d), the expression of the free-surface profile is determined as:

$$P_1(x) = \frac{\pi F^4 \gamma^2}{4L^2} \times \int_{-\infty}^{\infty} \frac{\sinh(k\lambda)\left[\sin k(x+L) - \sin k(x-L)\right]}{\left(\frac{\pi^2}{L^2} - k^2\right) E_2(k)} dk. \tag{17b}$$

It is worthy to note that, each integral in relations (17a) and (17b) is singular with poles on the real axis at $k = \pm k_0$ and $\pm k_1$. Therefore, the integrals in relations (17a) and (17b) have to be treated as a Cauchy principal value with an indentation below the singularities. Hence, using the residue theorem, the interface profile $S_1(x)$ and the free-surface profile $P_1(x)$ are obtained as:

$$S_1(x) = \begin{cases} \dfrac{-\pi^2 F^2}{L^2} \displaystyle\sum_{i=0}^{1} E_1(k_i) C(k_i), & \text{for } x > L \\ 0, & \text{for } x < -L, \end{cases} \tag{18a}$$

$$P_1(x) = \begin{cases} \dfrac{-\pi^2 F^4 \gamma^2}{L^2} \displaystyle\sum_{i=0}^{1} C(k_i), & \text{for } x > L \\ 0, & \text{for } x < -L, \end{cases} \tag{18b}$$

where

$$C(k) = \frac{\sinh(k\lambda)}{\left(\frac{\pi^2}{L^2} - k^2\right) E_2'(k)} \sin(kx)\sin(kL).$$

It can be noticed from relations (18a) and (18b) that, the interface and the free-surface profiles are oscillatory in nature. In addition, each profile possesses a wave-free region at the upstream of the obstacle, followed by a downstream wave. This is completely consistent with the result of Belward and Forbes (1993). The study further reveals that the amplitude of the downstream waves are varying, which is an important observation noticed for the first time. The varying nature occurs due to the presence of two wave numbers. The varying nature is important because wave free solution may occur at the downstream due to this varying nature. In addition, some other phenomena like beating behaviour, resonance, etc. also possible due to the varying nature for certain values of the system parameters. These phenomena were not noticed in earlier studies (Belward and Forbes 1993; Chakrabarti and Martha 2011).

4. Computational Results and Discussion

In this section, various computational results are demonstrated to analyze the behaviour of the free-surface as well as the interface for the subcritical type flow. Therefore, the value of the Froude number (F) is chosen, in this section, as $F < 1$. The effects of various system parameters are examined. In addition, this section also contains a discussion about the roots of the dispersion relation (14).

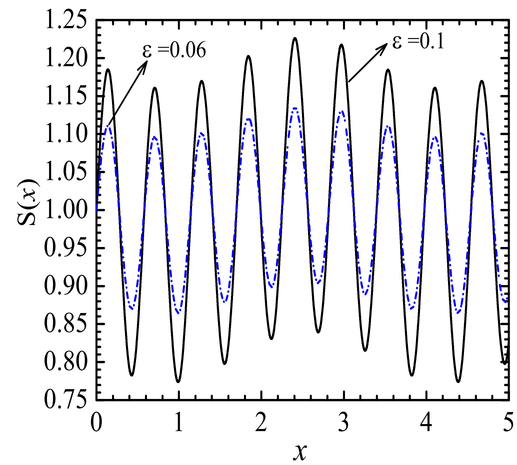
The positive real roots of the relation (14) are calculated using Newton's method for various values of the dimensionless parameters, and shown in Table 1. For this purpose, the ratio of densities is fixed at $D = 0.7$ and the ratio of the upstream fluid velocity is fixed at $\gamma = 1$. From Table 1, it is clear that the dispersion relation (14) has two non-zero real positive roots which affirm the remark made in Section 3. It is also noticed (*refer* first and second coloumns of Table 1) that the value of the wave number decreases as the Froude number F increases.

Table 1 Positive real roots of the dispersion relation (14) for $D = 0.7$ and $\gamma = 1$

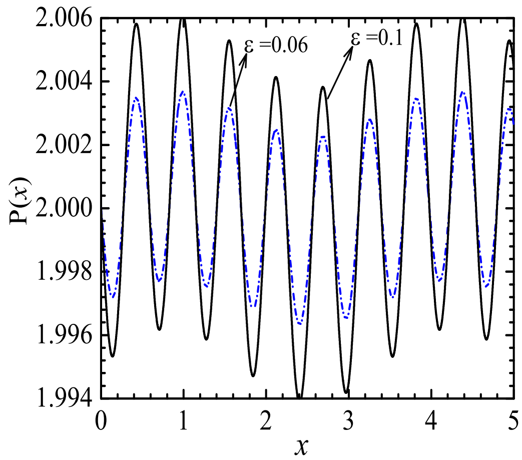
Values of the parameters	Positive real roots
$F = 0.2, \lambda = 1$	4.41023 24.4086
$F = 0.3, \lambda = 1$	1.85119 11.1012
$F = 0.3, \lambda = 2$	1.90928 11.1132
$F = 0.4, \lambda = 3$	0.88410 6.24986

The effect of height of the bottom obstruction is demonstrated in Figs. 2(a) and 2(b). The interface profile $S(x)$ and the free-surface profile $P(x)$ are shown in Figs. 2(a) and 2(b), respectively, for two different values of height of the obstacle $\varepsilon = 0.06$ and 0.1. From Fig. 2, it is found, for the first time, that $S(x)$ and $P(x)$ possess downstream waves with varying amplitude which confirms the observation made in Section 3. The amplitude is varying due to presence of waves having two different wave numbers. This phenomenon cannot be noticed in the case of a single-layer fluid with free surface or in the case of a two-layer fluid with a bounded upper layer (Belward and Forbes 1993; Chakrabarti and Martha 2011). Because, in these two cases, there exist wave with only one wave number, and hence, the amplitude becomes constant. It is also noticed that the amplitude of the downstream wave increases as the height of the obstacle increases. This agrees with the physical intuition, as high obstacle produces waves with higher amplitude.

The Froude number (F) plays a significant role in the behaviour of both free-surface and interface profiles. This important issue is not yet addressed in the literature to analyze the behaviour of the profiles. In the present work, the role of the Froude number (F) has been illustrated in Figs. 3(a) and 3(b). In Fig. 3(a), the interface profile $S(x)$ is depicted for three different values of the Froude number, $F = 0.3, 0.4$ and 0.5 with $\varepsilon = 0.1$, $\gamma = 1$, $\lambda = 1$, $L = 1$ and $D = 0.7$. Similarly, in Fig. 3(b), the free-surface profile $P(x)$ is depicted for the same set of values of the parameters as considered in Fig. 3(a). It is observed from Figs. 3(a) and 3(b) that, the amplitude of each profile increases as the Froude number increases. In addition, the variable na-

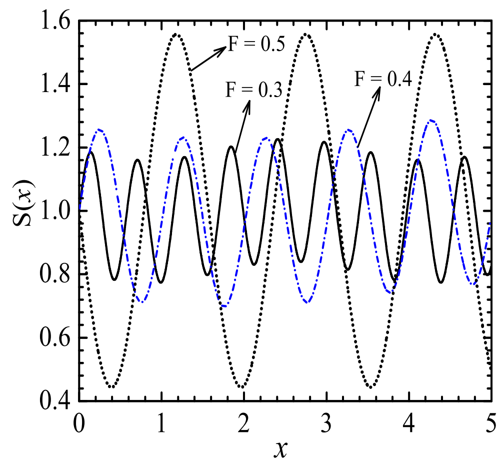


(a)

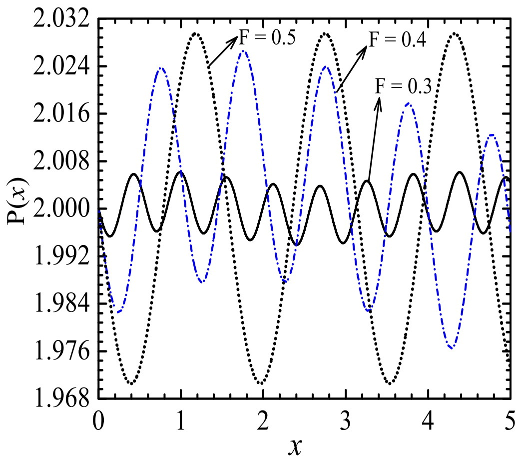


(b)

Fig. 2. Effect of height of the bottom obstruction. In (a) $S(x)$ and (b) $P(x)$ for $F = 0.3, \gamma = 1, \lambda = 1, L = 1$ and $D = 0.7$.



(a)



(b)

Fig. 3. Effect of the Froude number F. In (a) $S(x)$ and (b) $P(x)$ for $\varepsilon = 0.1, \gamma = 1, \lambda = 1$, $L = 1$ and $D = 0.7$.

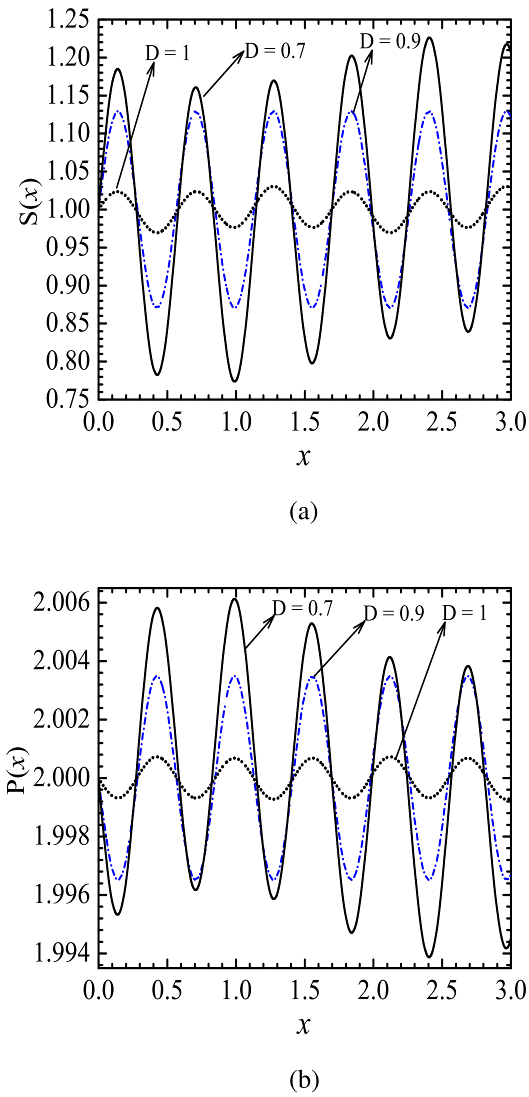


Fig. 4. Effect of the density ratio D. In (a) $S(x)$ and (b) $P(x)$ for $\varepsilon = 0.1$, $\gamma = 1$, $\lambda = 1$, $L = 1$ and $F = 0.3$.

ture of the profiles diminishes as the Froude number increases. As the varying nature diminishes, the downstream wave having varying amplitude become wave with constant amplitude. It is also noticed from Fig. 3, that the wavelength of the downstream wave increases with the Froude number. This is mainly due to the fact that the Froude number (F) is directly dependent on the wave number, which can be realized from the relation (14).

Effect of the density ratio is shown in Figs. 4(a) and 4(b). In Fig. 4(a) the interface profile $S(x)$ and in Fig. 4(b) the free-surface profile $P(x)$ are presented for three different values of the density ration $D = 0.7, 0.9$ and 1 with $\varepsilon = 0.1$, $\gamma = 1$, $\lambda = 1$, $L = 1$ and $F = 0.3$. It can be noticed from Fig. 4 that, the amplitudes of both profiles decrease as the density ratio of the fluids increases. Further, the varying nature of the amplitude diminishes for $D = 1$ and, therefore, it becomes a constant amplitude. This is due to the fact that, when the density ratio tends to 1, two-layer flow problem reduces to a single-layer flow problem. In the case of single-layer, there exists wave downstream with only one wave number, and as a result, the amplitude of the downstream becomes constant.

5. Conclusions

Two-dimensional inviscid flow over an undulating bottom in a two-layer fluid, in which the upper layer is free to the atmosphere, was considered to analyze the behaviour of the free-surface and the interface profiles. Based on linear theory, the problems are formulated mathematically in terms of mixed coupled boundary value problem (BVP). Applying perturbation analysis along with Fourier transform technique, the obtained BVP is solved to determine the free-surface and the interface profiles. The main findings of the present study are summarized as follows:

- It is found that, each profile possesses a wave-free region upstream of the obstacle, followed by a modulated downstream wave.
- The amplitude of the downstream waves is varying. This phenomenon can be not be noticed in case of a single-layer flow or a two-layer flow with a bounded surface.
- The wave number of the downstream wave decreases, i.e., wavelength increases, as the Froude number increases.

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