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Experimental Study of the Laminar-Turbulent Transition in a Tilted Taylor-Couette System Subject to Free Surface Effect

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Abstract

An experimental study of the laminar-turbulent transition between two coaxial rotating cylinders with the inner cylinder rotates and outer one stationary is presented in this paper. Special attention is given to the onset of various flow modes in tilted and partially filled system. The effect of the inclination angle was investigated on the different flow regimes occurring at fully and/ or partially filled space between two rotating cylinders. The flow behavior, the transitional phenomena and the features of various flow modes are discussed for different inclination angles, filling ratio and Taylor numbers. It is established that the different filling ratio and inclination angles of the Taylor-Couette system deeply affect the flow patterns. Furthermore, the most significant result concerns the relaminarization of the flow when the aspect ratio is decreased and inclination angle is increased for a given value of Taylor number.

Keywords: Laminar-turbulent transition; Free surface; Filling ratio; Tilted system; Relaminarization phenomenon; Taylor-Couette system; Flow visualization.

Nomenclature

d	gap width= R_2-R_1 (mm)	LCF	laminar Couette Flow
H	height of cylinder (mm)	*TVF*	taylor Vortex Flow
R_1	radius of inner cylinder (mm)	TCF	taylor Couette Flow
R_2	radius of outer cylinder (mm)	*WVF*	wavy Vortex Flow
T	temperature (°C)	*MWVF*	modulated Wavy Vortex Flow
Ta	Taylor number $Ta = \frac{\rho \Omega_1 R_1 d}{\mu}\sqrt{\frac{d}{R_1}}$	*CHAOS*	chaotic Mode
		g	gravity (m/s^2)
		Ta_{c1}	first instability (*TVF*)
μ	dynamic viscosity (Pa.s)	Ta_{c2}	second instability (W*VF*)
ρ	density (Kg/m^3)	Ta_{c3}	third instability (*MWVF*)
η	radius ratio=R_1/R_2	Ta_{c4}	fourth instability (*CHAOS*)
Ω_1	angular velocity of inner cylinder (rad/s)		
α	inclination angle (°)		
Γ=H/d	aspect ratio		

1. Introduction

The fluid flowing in annulus between two concentric rotating cylinders with one cylinder can be set to rotate or both cylinders rotate, termed Taylor- Couette flow (TCF), has been intensively investigated, both fundamental and applied. Since the seminal works of Couette (1890) and Taylor (1923), this flow has received potential interests because of its importance in the hydrodynamic stability theory as well as the laminar turbulent transition.

Further, The Taylor-Couette flow has been the subject of numerous investigations as it is crucial to many technical applications; tribology, filtration, rotating machinery, catalytic reactions.

Due to the various flow regimes existing between laminar turbulent transitions, a large body of literature focused on this flow for over a century by physicist and mathematicians.

Couette (1890) has the first who measured the dynamic viscosity of liquid contained between two rotating cylinders where outer cylinder rotates and inner one held stationary.

However, the stability of the flow between two concentric cylinders has been determined by Taylor (1923), both analytically and experimentally, for a narrow gap. Their work showed excellent agreement between theoretical analysis and experimental measurement. Later, Chandrasekhar (1954 and 1958) presented an extensive study of the flow stability problem in Taylor-Couette system for a small and wide gap.

The first experimental investigation demonstrating the existence of more than one pattern in Taylor-Couette flow has been made by Coles (1965). Moreover, he was the first that observed and reported the regime of wavy vortex flow.

Gollub and Swinney 1975 and Fenstermacher *et al* (1979) have presented several experimental investigations of the laminar turbulent transition in the Taylor-Couette system. They showed that successions of instabilities are enough to drive the flow from laminar state to chaotic regime with increasing the angular velocity of the inner cylinder.

In addition, Burkhalter and Koschmieder (1973) measured the wavelength of axisymmetric vortices from their initiation up to values as large as 80Tac1. They have reported that the wavelength of the vortices in the annular gap can be varied by changing the initial conditions.

Bouabdallah (1980) presented interesting analytical and experimental studies to determine the various flow regimes existing between laminar and turbulent flow by using the chaos theory and polarographyic technique.

So far, a substantial experimental measurements and theoretical works have been carried out regarding the transition zones between laminar Couette flow and turbulent flow. In addition, the flow behavior and various bifurcations have been analyzed and detailed by numerous authors for different boundary conditions, among these are Donnelly (1991), Marcus (1984), Jones, (1982), Andereck *et al* 1986 ,Takeda *et al* (1993), Wereley and Lueptow 1999, Mullin *et al* (2000), Abcha *et al* 2008, Borrero and Schatz (2010), Sobolik *et al,* (2011)Martinez-Arias *et al*.(2014).

On the other hand, the presence of the free surface in the Taylor Couette system profoundly affects the flow patterns and the occurrence of the various instabilities.

Mahamdia *et al* (2003 and 2007) used the polarographic technique to present the experimental results of the effects of the free surface and aspect ratio on the behavior of flow between two concentric cylinders. They showed the existence of a critical height of the column of liquid for which the laminar turbulent transition occurs without the wavy mode.

Watanabe *et a.l* (2007) invistigated, both numerically and experimentally, the flow patterns in the finite Taylor Couette geometry (Γ=1.6). They have shown that the flow has multiple patterns depending on the cylinder lengths and the Reynolds numbers.

Oualli *et al* (2013) conducted a numerical study of flow between two rotating cylinders by varying the outer cylinder cross-section. They found that the onset of the first instability is considerably retarded Recently, Watanabe and Toya (2012) and Watanabe *et al.* (2014) have investigated, both numerically and experimentally, the effect of the free surface on the occurrence of the Taylor vortices in the annulus between to coaxial rotating cylinders. They showed that anomalous modes with outer flow near the bottom end wall or inner flow at the free surface appear in some conditions.

From the foregoing discussion, it is clear that substantial experimental and theoretical works have been published for the transition regimes. However, only few studies have been reported on the flow modes with free surface and inclination of the flow system. In the present work, an experimental attempt has been made to investigate the combined effect of inclination of the system and the free surface on the flow patterns for a wide range of α and Γ.

2. EXPERIMENTAL SETUP, VISUALI -ZATION TECHNIQUES AND PROCEDURES

2.1 Taylor–Couette Apparatus

The experimental device consisting of two coaxial rotating cylinders are made of the plexiglass. The outer cylinder is kept stationary while the inner one rotates with angular velocity Ω_1. The system is planned for the use of several interchangeable cylinders and designed for to facilitate the operations of assembling and disassembling, as shown in fig. 1. The geometry is characterized by the following parameters:

- The radius ratio $\eta=R_1/R_2=0.909$ ($R_1=50\pm0.2$mm and $R_2=55\pm0.2$mm).
- The aspect ratio Γ=H/d which is variable.

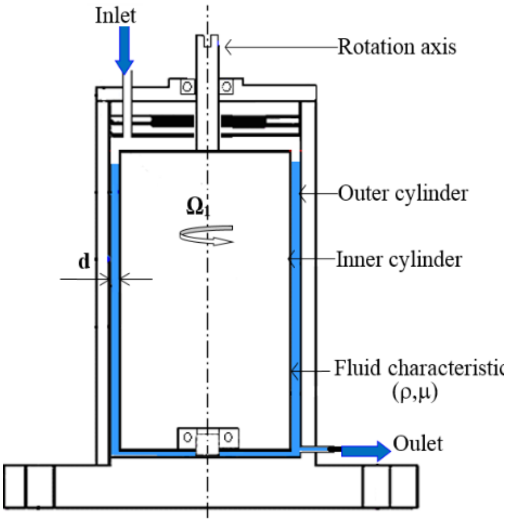
The control parameter that defines the various flow regimes is Taylor number, Ta.

The Taylor number was increased stepwise by a quasi-static increase of the rotating velocity of inner cylinder following the relationship:

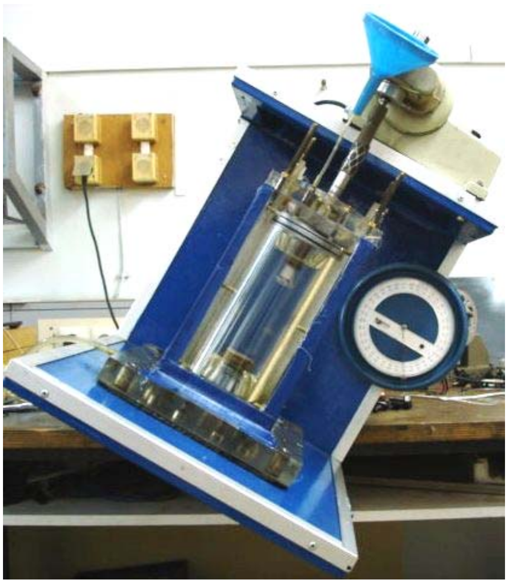
$$\frac{\Delta\Omega}{\Omega} \leq 1 \qquad (1)$$

Where $\Delta\Omega_1/\Omega_1$ is the rate of increase.

The working liquid is composed of 20% of Vaseline oil with 80% of ether petroleum. The flow was made visible by aluminum flakes as a light-diffusing impurity added in a small amount (1 g/l). The density and dynamic viscosity are, respectively, ρ=785 kg/m3 and μ=4.9×10-3 Pa.s measured at T=22° C. The temperature is measured by an electronic captor (−50 to +150°C) with accuracy less than 1/10°C.



a) Sketch of the Taylor-Couette geometry



b) Tilted Taylor-Couette system

Fig. 1. Taylor-Couette apparatus.

2.2 Visualization Techniques

The visualization techniques are based on the control of luminous flux; the observations are made in a darkroom to eliminate stray light, as follow:

- Visualization by the reflexion of light, which is proceeded by reflexion of a beam of light diffused by front an outside source on the flow showing the mode and structure associated to the movement, as illustrated in Fig. 2.

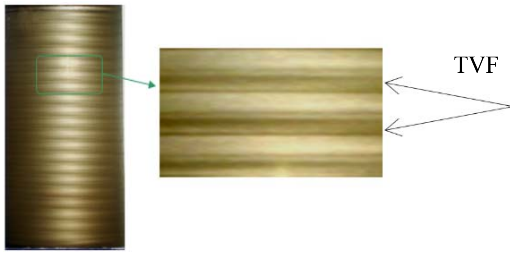


Fig. 2. Visualization by optical reflexion.

- Visualization by transverse optic transmission, which is based on the optical transmission of a beam luminous coming from a source placed at the opposite of the observer and crossing the whole of the flow. This mode of lighting makes possible to visualize the in-depth structure of the movement related to the shape of the cell, as illustrated in fig. 3.

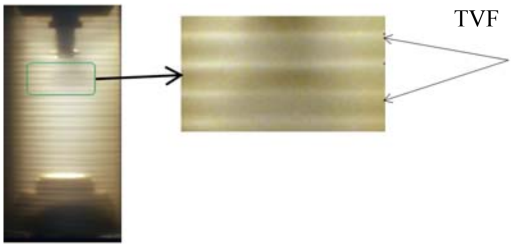


Fig. 3. Visualization by optical transmission.

Photographs of the flow were taken by a digital camera Sony Cyber-Shot (DSCP100) that have five-mega pixel and 3X optical zoom. The camera shoot pictures JPEGs at five resolution settings, ranging from 640-by-480 to 2592-by-1944 and shoots MPEG-VX video at up to 640-by-480 at 30 fps (frames per second).

The photometric data processing is performed using appropriate software on Windows XP or Vista (Office Picture Manager). In addition, image-processing software can perform filtering and improved photometric quality (filter photo, Xn view).

2.3 Procedure

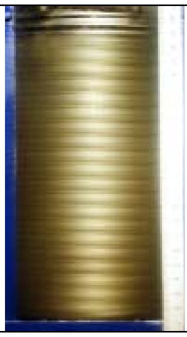
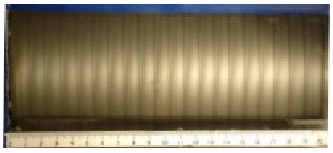
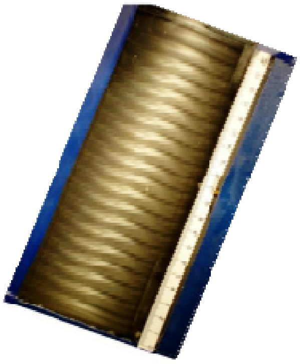
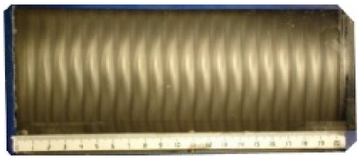
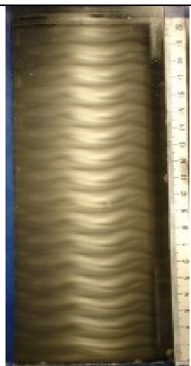
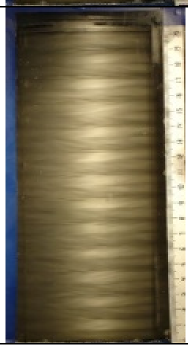
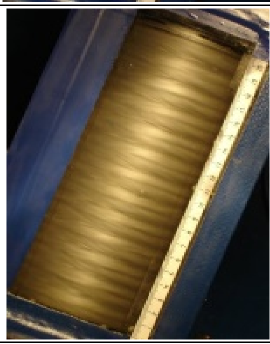
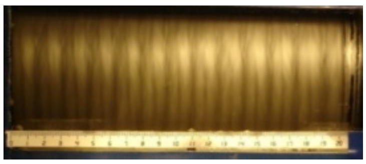
2. 3. 1. First Procedure

For the flow system completely filled, we fixed the Taylor numbers corresponding to the onset of different flow modes, i.e., TVF, WVF, MWVF and chaos and then we varied the inclination angles to studying the flow behavior. On the other hand, when the system is partially filled, we have examined only the first and second instabilities (TVF and WVF).

2. 3. 2. Second Procedure

In this procedure, we fixed the inclination angle and filling ratio and we varied the angular velocity of inner cylinder in order to determine critical values of Taylor number ,i.e., Ta_{c1}, Ta_{c2}, Ta_{c3} and Ta_{c4} corresponding to the occurrence of various flow regimes.

Table 1 Effect of inclination on the onset of various regimes for completely filled system ($\Gamma = \Gamma_{max} = 40$)

	α=0	α=30°	α=90°
TVF Ta_{c1}=42±2			
WVF Ta_{c2}=49±3			
MWVF Ta_{c3}=390±7			
Chaos Ta_{c4}=710±10			

3. Results and Discussion

3.1 System Completely Filled

The various flow regimes obtained in this study are compared with experimental data reported previously in the literature for a similar geometry (Coles 1965, Fenstermacher *et al.* 1979, Bouabdallah and Cognet 1980, Mehel *et al.* 2007)

The observations carried out show that if the system is completely filled $\Gamma=\Gamma_{max}=40$, the inclination angles have no effect on the flow behavior for the various instabilities (TVF, WVF, MWVF and Chaos), as shown in table 1.

3. 2. System Partially Filled

3. 2. 1 Taylor Vortex Flow

When the system is partially filled $\Gamma<\Gamma max$, we have examined the inclination of system on the flow behavior for two regimes; TVF and WVF respectively. We increase slowly the angular velocity of inner cylinder until the appearance of the TVF at Ta_{c1}=44±3, then we proceed to the inclination of the system from 0 up to 90°. As can be seen in fig.4, the disappearance of the Taylor vortices is made gradually near the free surface with increasing of the inclination angles. The flow relaminarization phenomenon appears for a given critical value of $\alpha=\alpha_c$.

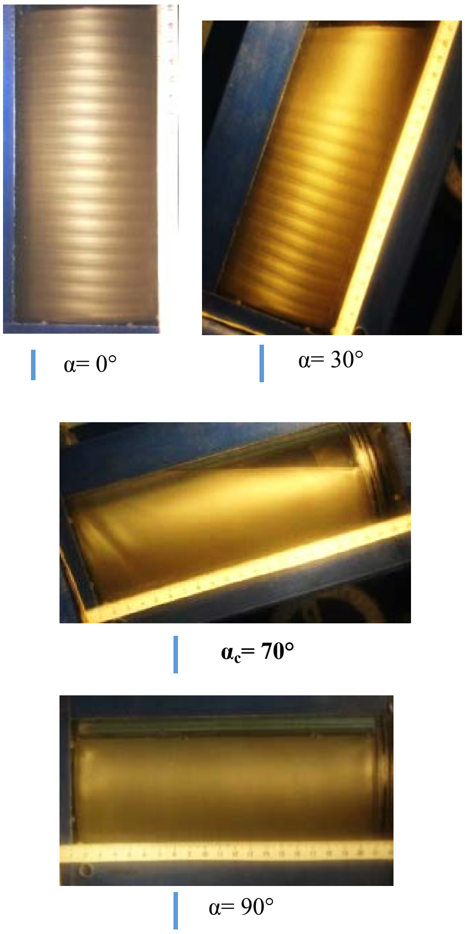


Fig. 4. Effect of inclination on Taylor vortex flow at Tac1=44±2 for partially filled system (Γ = 37). Destruction of Taylor vortices and relaminarization of the flow at α_c = 70°.

We note that for a system partially filled, this experimental study showed up the coexistence of three flow modes in the Taylor–Couette system, as follow:

Zone 1: Laminar Couette Flow (LCF)

Zone 2: Mode of the Taylor Vortices with warping (compression and expansion of the waves).

Zone 3: Mode of the Taylor vortices without warping similar to the vertical Taylor-Couette flow.

In addition, we observed the appearance of a laminar zone at the free surface, which is due to the disappearance of Taylor vortices near the higher part of the system. Further, we noted that the zone adjacent the laminar zone consists of Taylor vortex inclined. This flow is characterized by a size of the vortices, which is variable in the same circumference. This situation is physically remarkable with a compression of waves on a side and an expansion of waves at the opposite side for the same value of Ta, as shown in fig 8.

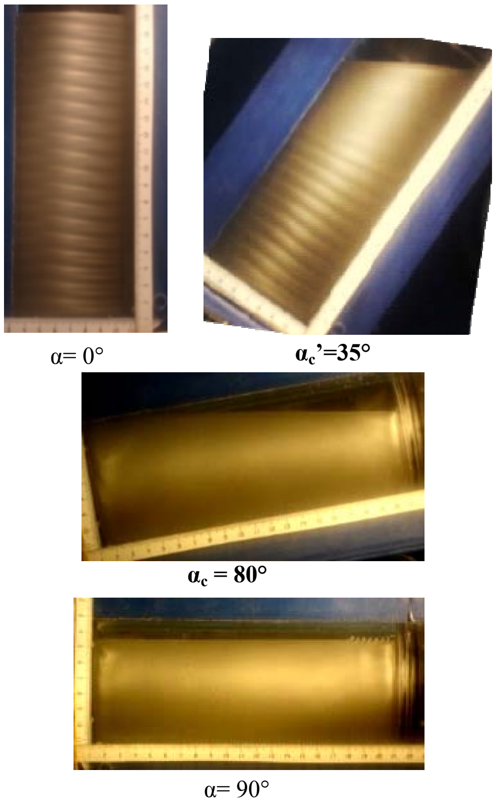


Fig. 5. Effect of inclination on the wavy vortex flow at Ta=56±3 for partially filled system (Γ = 37). Disappearance of Wavy mode at α_c'=35° and flow relaminarization at α_c = 80°.

3. 2. 2 Wavy Vortex Flow

The figure 5 depicts the influence of combined effects of the filling ratio and inclination angle on the wavy vortex flow. We observe that the WVF is progressively attenuated according to inclination angle. For $\alpha=\alpha_c$', the travelling waves disappear gradually and the WVF returns to the TVF. With more increased the inclination angle up to $\alpha = \alpha_c$ the flow becomes completely laminar (relaminarization phenomenon).

For other filling ratio, the critical angles of relaminarization α_c and critical angles of disappearance of wavy mode α'_c are shown in the following figure 6 and 7.

In the forgoing discussion, we have examined the effects of α and Γ on the flow patterns for the case where the Taylor number is fixed in advance (Ta=44 and Ta=56). For example, to given the value of Ta_{c1} at fixed Γ, we vary α until to reach $\alpha=\alpha_c$ that is to say an angular critical value which characterizes the relaminarization of the flow.

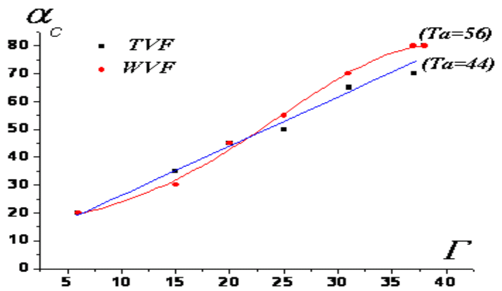


Fig. 6. Critical inclination angles of the relaminarization α_c versus Γ.

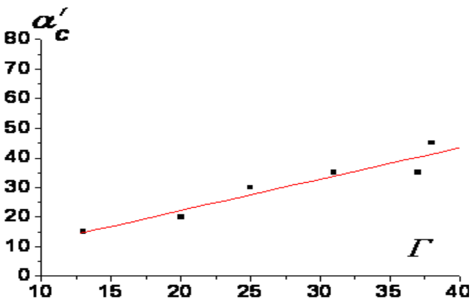


Fig. 7. Critical angle inclination angles α'_c of the disappearance of WVF versus Γ at Ta_{c2}=56±3.

In otherwise, we fixed the inclination angle and filling ratio with varying the angular velocity in order to determine critical values of Taylor number for different flow regimes (Ta_{c1}, Ta_{c2}, Ta_{c3} and Ta_{c4})

The examination of the experimental results allowed us to analyze the effect of the filling ratio and the inclination angle which leading to determine the phenomenological laws as shown in the relationships 2 and 3 :

Linear low: Relation valid for Γ_{max} = 40 (completely filled)

$$Ta_c(\alpha^*) = A \tag{2}$$

For the flow system completely filled, the experimental test allows us to find the following critical values: Ta_{c1} = 42±2, Ta_{c2} = 48±4, Ta_{c3} = 390±8 and Ta_{c4} = 740. In addition, we have found that the various flow regimes are independent of the inclination angle. We note also that the curves showing the evolution of critical Taylor numbers versus the inclination angle are straight without slope. This result is probably due to the confinement of the flow by the end walls.

Exponential low

$$Ta_c(\alpha^*) = A + B\exp(\alpha^*/C) \tag{3}$$

Relation valid for Γ □ Γ_{max} (partially filled). The constants A, B and C are given in table 2, which are determined from the curves in fig. 9.

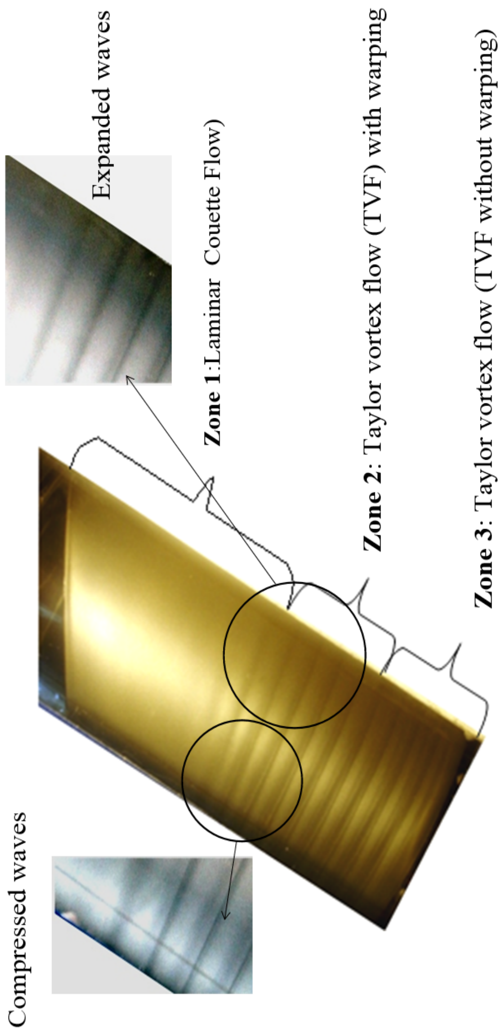


Fig. 8. Coexistence of three flow modes (Ta = 44±3, α=30±0.5°).

For the system partially filled, the critical Taylor numbers gradually increase for TVF, WVF and Chaos regimes if α increases. In the other hand, for the MWVF the evolution of the critical Taylor number versus inclination angle decreases.

Table 3 reports the critical values of inclination angle, α^*_c, of the no-occurrence of TVF and WVF for different filling ratio. We note that both regimes cited above have the same α^*_c for Γ=37 and 31 but not for Γ=20.

4. CONCLUSION

The present experimental study made it possible to show the effect of tilted Taylor-Couette flow system on the onset of different flow regimes such as TVF, WVF, MWVF, and CHAOS respectively.

Table 2: Value A, B and C for different filling ratio at Ta_{c1}, Ta_{c2}, Ta_{c3} and Ta_{c4}

Γ \ Tac	Tac1	Tac2	Tac3	Tac4
40	linear Laws A=41.62±1	linear Laws A=48.89±3	linear Laws A=387.27±8	linear Laws A=732.03±15
37	Exponential law A=42.54±2 B=0.025±0.003 C=11.59±0.5	Exponential law A=39.85±3 B=8.08±0.01 C=44.77±2	Exponential law A=86.05±4 B=317.64±9 C= – 34.87±1	Exponential law A=754.13±20 B =1.98±0.02 C=21.91±0.3
31	Exponential law A=42.09±2 B=0.86±0.01 C=25.34±0.8	Exponential law A=44.61±3 B=3.67±0.05 C=26.85±0.3	Exponential law A= – 35.96±1 B=411.27±8 C=– 56.11±3	Exponential law A=660.9±20 B=59.01±3 C=55.73±3
20	Exponential law A=42.12±2 B=0.82±0.01 C=17.12±1	Exponential law A=45.79±3 B=6.56±0.7 C=41.03±2	Exponential law A=53.39±3 B=247.54±7 C=–30.93±0.5	Exponential law A=637.42±20 B=71.96±3 C=47.83±3

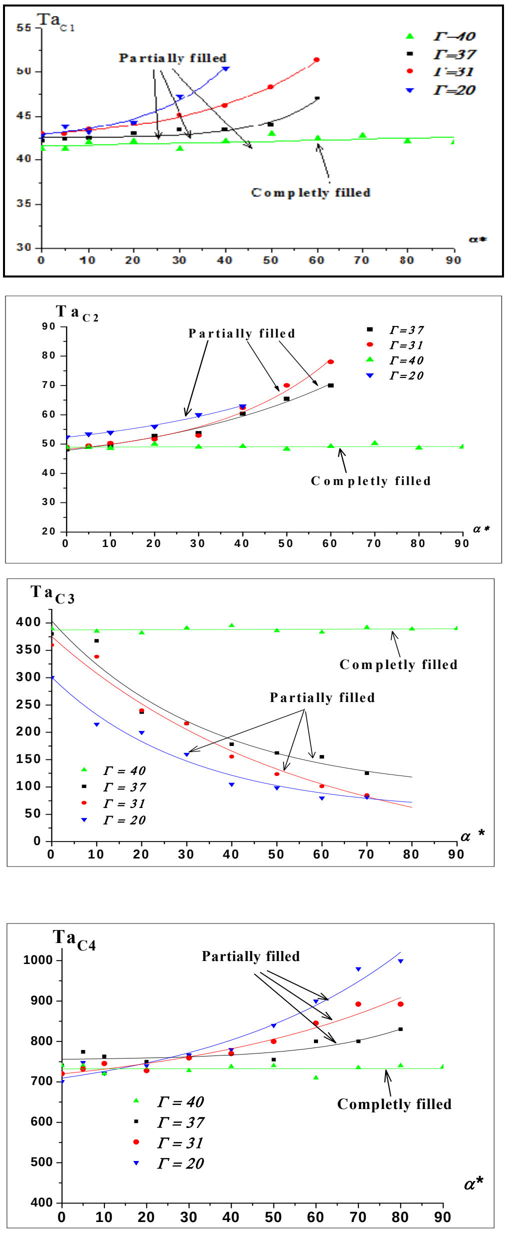


Fig. 9. Evolution of critical Taylor number Ta_{c1}, Ta_{c2}, Ta_{c3} and Ta_{c4} versus α.

When the system is completely filled $\Gamma = \Gamma_{max}$, the inclination angle has no effect on the flow.

Table 3 Critical angle α^*_c versus Γ associated to non-occurrence of TVF and WVF

Modes \ Γ	37	31	20
TVF	70°	60°	40°
WVF	70°	60°	45°

However, the inclination angle has a significant influence in a system partially filled, $\Gamma<\Gamma_{max}$ giving place to various significant modifications of the movement. In addition, we have revealed the effect of relaminarization of the flow for a given critical angle α_c value of the system as Γ is fixed. Furthermore, the determination of the critical angle limits of the different flow regimes occurrence versus filling ration is useful for wide applications such as tribology and journal bearing.

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