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A Local Nonlinear Stability Analysis of Modulated Double Diffusive Stationary Convection in a Couple Stress Liquid

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Abstract

The non-autonomous Ginzburg-Landau equation with time-periodic coefficients is derived for two modulated double-diffusive stationary convection involving couple stress liquid. The heat and mass transports are quantified in terms of Nusselt and Sherwood numbers, which are obtained as functions of the slow time scale. Effects of Prandtl number, Lewis number, solute Rayleigh number and couple stress parameter have been discuused in detail.

Keywords: Rayleigh-Benard convection; Couple stress liquid; Temperature modulation; Gravity modulation; Ginzburg-Landau equation.

Nomenclature

a	horizontal wave number
a_c	critical wave number
b	basic state
c	critical
C	couple stress parameter
D	d/dz
$\mathbf{g}$	gravitational acceleration, $(0,0,-g)$
$\hat{i}$	unit normal vector in x-direction
i	$\sqrt{-1}$
$\hat{j}$	unit normal vector in y-direction
$\hat{k}$	unit normal vector in z-direction
k_c	wave number
l	lower wall
Le	Lewis number
osc	oscillatory
Pr	Prandtl number
p	pressure
$\mathbf{q}$	velocity of the fluid (u,v,w)
Ra_T	Rayleigh number $(Ra_T = \beta_T gd(\Delta T)d^3/\nu\kappa_T)$
Ra_S	concentration Rayleigh number $(Ra_S = \beta_S gd(\Delta S)d^3/\nu\kappa_T)$
β_S	concentration analog of thermal expansion coefficient
st	stationary
t	time
T	temperature
β_T	thermal expansion coefficient
ΔT	temperature difference between the walls
u	upper wall
ρ	density
ε	slow time scale
δ_1	amplitude of temperature modulation
μ	dynamic viscosity
ν	kinematic viscosity
σ	growth rate
ψ	stream function
∇_1^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, horizontal Laplacian
∇^2	$\nabla_1^2+\frac{\partial^2}{\partial z^2}$
0	reference state
$'$	perturbed value
$*$	non-dimensional value

1. Introduction

The theory of couple stress liquids was developed by Stokes (1966) and is now being extensively used as a continuum model in many liquid-based applications involving suspended particles. The onset and heat transport by thethermo gravitational convection in couple stress liquid investigated by Siddheshwar and Pranesh (2004). Subsequently, a number of investigators have reported double diffusive convection in these liquids (See works of Malashetty and co-investigators (2009, 2011, 2006, 2010) and Rani and Raddy (2013). Regulation of thermal or thermohaline convection can be important in situation in which inclean liquids are working media. It is here that the works of Venezian (1969), Rosenblat and Herbert (1970), Rosenblat and Tanaka (1971), Roppo *et al.* (1984), Bhadauria and Bhatia (2002), Siddheswar and Abraham (2003), Siddheshwar and Bhadauria (2012), Siddheshwar *et al.* (2012, 2013), Bhadauria (2003, 2006), Bhadauria and Debnath (2004), Gresho and Sani (1970), Wadih and Roux (1988), Kumar (2012) and Banyal (2013) become important. All the above works have the limited objective of predicting the onset of convection.

A nonlinear study of thermal or thermohaline convection problems can be done in one of the following ways:-

(i) local nonlinear stability analysis- Lorenz-model or Ginzburg-landau model.

(ii) Global nonlinear stability analysis- Lyapunov method.

The Lorenz and Lyapunov aproaches to the thermal problem have been reported by Siddheshwar and Pranesh (2004), and the Lorenz model for modulated thermal or thermohaline system is difficult to solve and the global nonlinear stability analysis cannot be quantify the heat and/ or mass transports. In this paper we consider the mechanisms of external regulation of convection;

Time-periodic boundary temperature (temperature modulation).

Three types of temperature modulation are considered;

(i) the two boundaries are modulated in phase.

(ii) the boundaries are modulated out of phase.

(iii) one of the boundaries are modulated.

The first is an example of symmetric modulation and the second and third are asymmetric modulation. The analysis is made using a Ginzburg- landau amplitude equation that has a time-periodic coefficients. In this study we focus attention only on stationary convection.

2. Mathematical Formulation

Considering the double diffusive convection in couple stress fluid saturated porous layer, confined between two parallel infinite horizontal plates $z=0$ and $z=d$ at a distance apart. The fluid layer is heated from below and cooled from above to maintain a constant gradient temperature $\triangle T$ across the layer. We have taken a cartesian frame of reference in which the origin lies on the lower plate and $z-$axis as vertically upward. The governing euations of motion of an incompressible couple stress fluid in the absence of body couple are given by,

$$\nabla.\vec{q}=0, \tag{1}$$

$$\rho_0[\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}]=-\nabla p-\rho\vec{g}+(\mu-\mu_c\nabla^2)\nabla^2\vec{q}, \tag{2}$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa_T\nabla^2 T, \tag{3}$$

$$\frac{\partial S}{\partial t}+(\vec{q}.\nabla)S=\kappa_S\nabla^2 S, \tag{4}$$

$$\rho=\rho_0[1-\beta_T(T-T_0)+\beta_S(S-S_0)] \tag{5}$$

where $\vec{q}$ is the velocity, ρ_0 is the density at the reference temperature T_0 (temperature of the upper plate), p is the pressure, ρ is the density, $\vec{g}$ is the acceleration due to the gravity, μ is the dynamic coefficient of viscosity, μ_c is the couple stress viscosity, T is the temperature , S is the solute concentration, κ_T is the thermal diffusivity and κ_S is the solute diffusivity of the liquid, β_T is the coefficient of thermal expansion and β_S is the coefficient of solute expansion.

3. Time-periodic boundary temperature

We assume the externally imposed boundary temperatures to oscillate with time, according to the relations used by Venezian (1969),

$$\begin{aligned} T &= T_0+\frac{\triangle T}{2}[1+\varepsilon^2\delta_1\cos(\Omega t)] \quad at\ z=0 \\ &= T_0-\frac{\triangle T}{2}[1-\varepsilon^2\delta_1\cos(\Omega t+\phi)]\ at\ z=1 \end{aligned} \tag{6}$$

where ω is the modulation frequency , ϕ is phase angle . The quantity $\varepsilon^2\delta_1$ is the amplitude of modulation, where ε and δ_1 both are small, resulting the modulation to be of small amplitude.

The basic state is assumed to be quiescent, i.e.,

$$q_b=0,\ \rho=\rho_b(z),\ p=p_b(z),\ T=T_b(z),\ S=S_b(z),$$

which satisfy the following equations,

$$\frac{\partial p_b}{\partial z}=-\rho_b\vec{g} \tag{7}$$

$$\frac{\partial T_b}{\partial z} = \kappa_T \frac{\partial^2 T_b}{\partial^2 z} \tag{8}$$

$$\rho_H = \rho_0[1 - \beta_T(T_H - T_0) + \beta_S(S_b - S_0)] \tag{9}$$

According to the Venezian, we can write the non-dimensionlized basic temperature as,

$$T_b(z,t) = T_0 + 1 - z +^2 \delta_1 F(z,t) \tag{10}$$

where,

$$F(z,t) = Re\left[\{A(\lambda)e^{\lambda z} + A(-\lambda)\exp^{-\lambda z}\}e^{-i\omega t}\right] \tag{11}$$

$$A(\lambda) = \frac{1}{2}\frac{(e^{-i\phi} - e^{-\lambda})}{(e^{\lambda} - e^{-\lambda})}; \qquad \lambda = (1-i)\sqrt{\frac{\omega}{2}}$$

Taking curl on both side of Eq. (2) and introducing stream function $u = -\frac{\partial\psi}{\partial z}$ and $w = -\frac{\partial\psi}{\partial x}$, we get

$$\rho_0\left[\frac{\partial}{\partial t}(\nabla^2\psi) - \frac{(\psi,\nabla^2\psi)}{\partial(x,z)}\right] = \mu\nabla^4\psi - \mu_c\nabla^6\psi - \alpha_T g\frac{\partial T}{\partial x} + \beta_T g\frac{\partial S}{\partial x} \tag{12}$$

Now consider small infinitesimal perturbations to the basic state solution in the form,

$\psi = \psi_b + \psi$, $T = T_b + \Theta$, $\rho = \rho_b + \rho'$, $S = S_b + S'$

Substituting above in Eqs.(1),(2),(3)and (4), we get the following equations,

$$\rho_0\left[\frac{\partial}{\partial t}(\nabla^2\psi) - \frac{(\psi,\nabla^2\psi)}{\partial(x,z)}\right] = \mu\nabla^4\psi - \mu_c\nabla^6\psi - \beta_T g\frac{\partial\Theta}{\partial x} + \beta_S g\frac{\partial S}{\partial x} \tag{13}$$

$$\frac{\partial\Theta}{\partial t} - \frac{\partial(\psi,\Theta)}{\partial(x,z)} = \kappa_T\nabla^2\Theta + \frac{\partial\psi}{\partial x}\frac{\partial T_b}{\partial z} \tag{14}$$

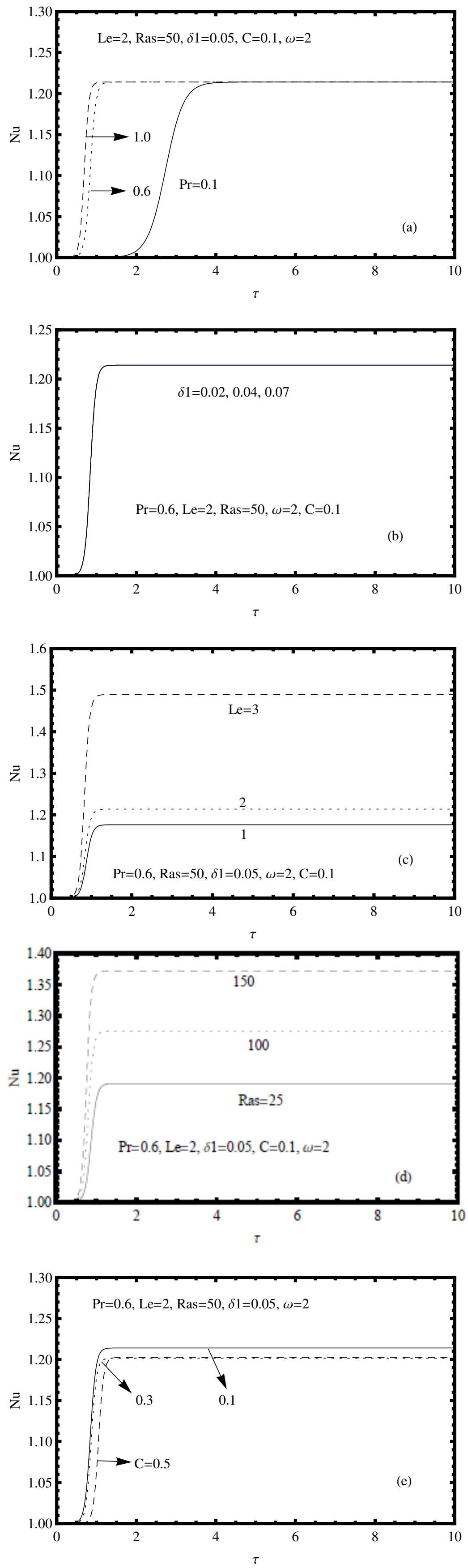
$$\frac{\partial S}{\partial t} - \frac{\partial(\psi,S)}{(x,z)} = -\frac{\partial\psi}{\partial x} + \kappa_S\nabla^2 S \tag{15}$$

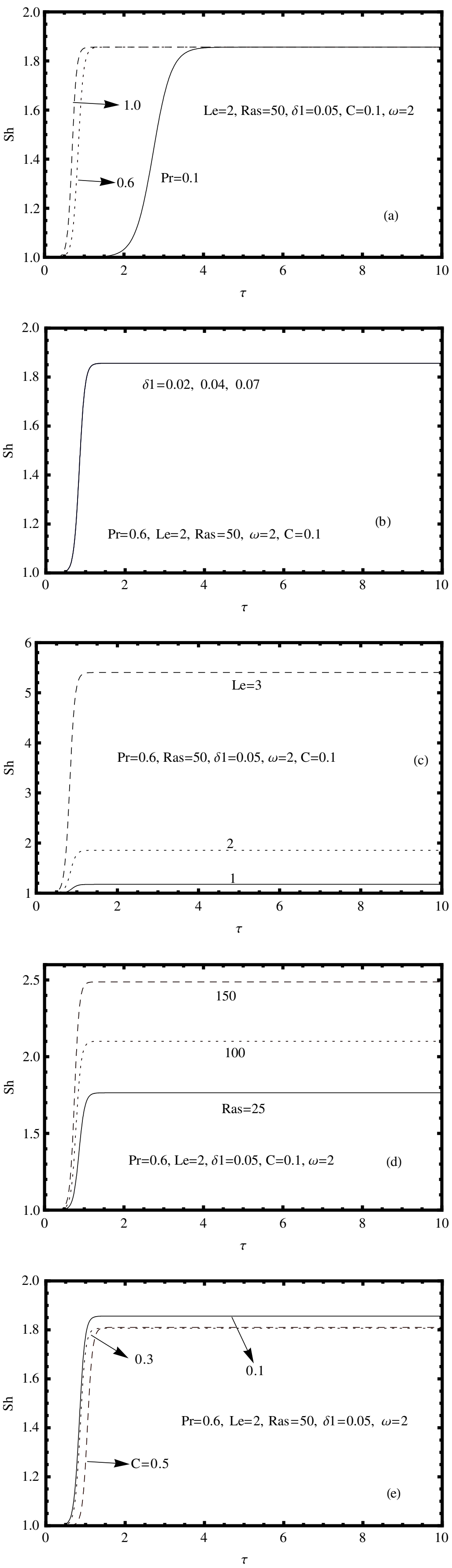
where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

The equations (13)-(15) are rendered dimensionless using the following transformations,

$$\begin{aligned}\psi &= \kappa_T\psi^*, (x,z) = d(x^*,z^*), t = \frac{d^2}{\kappa_T}t^*,\\ \Theta &= (\Delta T)\Theta^*,\\ S &= (\triangle S)S^*,\ T_b = (\triangle T)T_b^*,\ \omega = \frac{\Omega}{\varepsilon^2}\end{aligned}$$





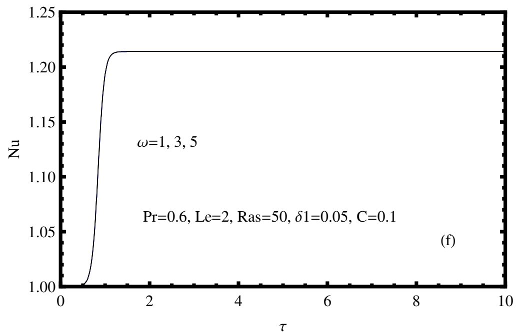


Fig. 1.Variation of Nusselt number Nu with time τ for in phase temperature modulation $(\phi = 0)$ for different values of$(a)Pr, (b)\delta 1, (c)Le, (d)Ra_S, (e)C, (f)\omega$.

The dimensionless equations are written as

$$\frac{1}{Pr}\left[\frac{\partial}{\partial t}(\nabla^2\psi^*) \quad -\frac{\partial(\psi^*,\nabla^2\psi^*)}{\partial(x^*,z^*)}\right] = \nabla^4\psi^* - C\nabla^6\psi^* - Ra_T\frac{\partial\Theta^*}{\partial x^*} + Ra_S\frac{\partial S^*}{\partial x^*} \qquad (16)$$

$$\frac{\partial\Theta^*}{\partial t} = \nabla^2\Theta^* + \frac{\partial\psi^*}{\partial x^*}\frac{\partial T_b^*}{\partial z^*} + \frac{\partial(\psi^*,\Theta^*)}{\partial(x^*,z^*)} \qquad (17)$$

$$\frac{\partial S^*}{\partial t} = -\frac{\partial\psi^*}{\partial x^*} + \frac{1}{Le}\nabla^2 S^* + \frac{\partial(\psi^*,\Theta^*)}{\partial(x^*,z^*)} \qquad (18)$$

where asterisks denote dimensionless values. And Pr=$\frac{\nu}{\kappa_T}$,the Prandtl number, Ra_S=$\frac{\beta_S g\Delta T d^3}{\nu\kappa_T}$, solute Rayleigh number, $Ra_T = \frac{\beta_T g\Delta T d^3}{\nu\kappa_T}$, Rayleigh number, C=$\frac{\mu d^2}{\mu_1}$, the couple stress parameter,Le= $\frac{\kappa_S}{\kappa_T}$, the Lewis number.
The boundary conditions for the perturbed state are given by,

$$\psi = \nabla^2\psi = \Theta = 0 \qquad at\ \ z = 0,1 \qquad (19)$$

The asterisks denote the non-dimensional values. After dropping the asterisks we can write the above equations as,

$$\frac{1}{Pr}\left[\frac{\partial}{\partial t}(\nabla^2\psi) \quad -\frac{\partial(\psi,\nabla\psi)}{\partial(x,z)}\right] = \nabla^4\psi - C\nabla^6\psi - Ra_T\frac{\partial\Theta}{\partial x} + Ra_S\frac{\partial S}{\partial x} \qquad (20)$$

$$\frac{\partial\Theta}{\partial t} = \nabla^2\Theta + \frac{\partial\psi}{\partial x}\frac{\partial T_b}{\partial z} + \frac{\partial(\psi,\Theta)}{\partial(x,z)} \qquad (21)$$

$$\frac{\partial S}{\partial t} = -\frac{\partial\psi}{\partial x} + \frac{1}{Le}\nabla^2 S + \frac{\partial(\psi,\Theta)}{\partial(x,z)} \qquad (22)$$

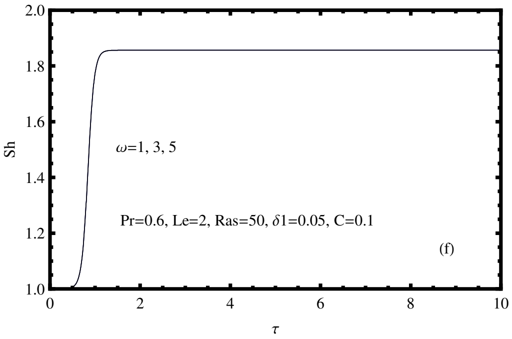


Fig. 2.Variation of Sherwood number *Sh* with time τ for in phase temperature modulation $(\phi = 0)$ for different values of$(a)Pr, (b)\delta 1, (c)Le, (d)Ra_S, (e)C, (f)\omega$.

Now using the value of T_b in Eq. (21) , we have the following equations,

$$\frac{1}{Pr}\left[\frac{\partial}{\partial t}(\nabla^2\psi) - \frac{\partial(\psi,\nabla\psi)}{\partial(x,z)}\right] = \nabla^4\psi - C\nabla^6\psi - Ra_T\frac{\partial\Theta}{\partial x} + Ra_S\frac{\partial S}{\partial x} \quad (23)$$

$$\frac{\partial\Theta}{\partial t} = \nabla^2\Theta + \frac{\partial\psi}{\partial x}\left[-1+\varepsilon^2\delta_1\frac{\partial F}{\partial z}\right] + \frac{\partial(\psi,\Theta)}{\partial(x,z)} \quad (24)$$

$$\frac{\partial S}{\partial t} = -\frac{\partial\psi}{\partial x} + \frac{1}{Le}\nabla^2 S + \frac{\partial(\psi,\Theta)}{\partial(x,z)} \quad (25)$$

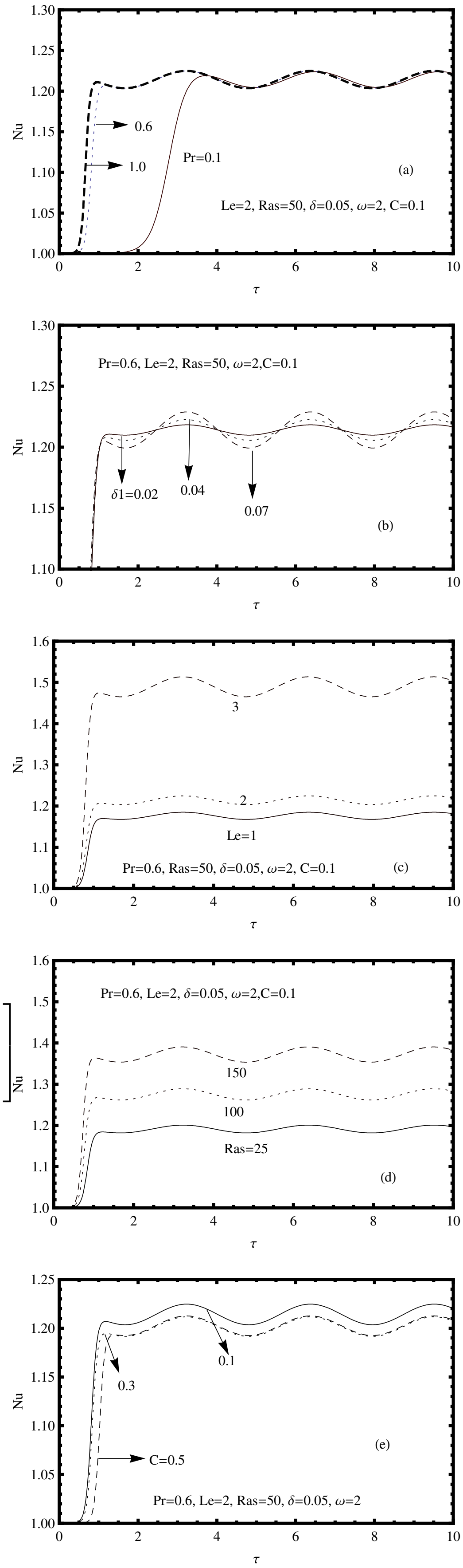
We will use the time variations only at the slow time scale $\tau = \varepsilon^2 t$

$$\begin{bmatrix} \frac{\varepsilon^2}{Pr}\frac{\partial}{\partial\tau}(\nabla^2) - \nabla^4 + C\nabla^6 & Ra_T\frac{\partial}{\partial x} & -Ra_S\frac{\partial}{\partial x} \\ -(-1+\varepsilon^2\delta_1\frac{\partial F}{\partial z})\frac{\partial}{\partial x} & -\varepsilon^2\frac{\partial}{\partial\tau} - \nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix} \times \begin{bmatrix} \psi \\ \Theta \\ S \end{bmatrix} = \begin{bmatrix} \frac{1}{Pr}\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)} \\ \frac{\partial(\psi,\Theta)}{\partial(x,z)} \\ \frac{\partial(\psi,S)}{\partial(x,z)} \end{bmatrix} \quad (26)$$

Now we use following perturbations in equation (3.),

$$\left.\begin{array}{l} Ra_T = Ra_{T_0} + \varepsilon^2 Ra_{T_2} + ... \\ \psi = \varepsilon\psi_1(x,z,t) + \varepsilon^2\psi_2(x,z,t) + \varepsilon^3\psi_3(x,z,t)... \\ \Theta = \varepsilon\Theta_1(x,z,t) + \varepsilon^2\Theta_2(x,z,t) + \varepsilon^3\Theta_3(x,z,t)... \\ S = \varepsilon S_1(x,z,t) + \varepsilon^2 S_2(x,z,t) + \varepsilon^3 S_3(x,z,t)... \end{array}\right\} (27)$$

where classical analysis shows that the first and second order system have the solution of the form: of different orders:



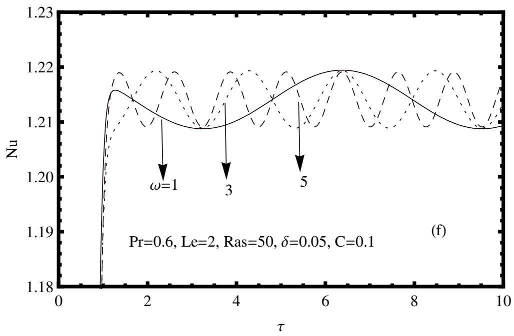


Fig. 3.Variation of Nusselt number Nu with time τ for out-phase temperature modulation ($\phi=\pi$) for different values of$(a)Pr,(b)\delta 1,(c)Le,(d)Ra_S,(e)C,(f)\omega$.

$$\left.\begin{array}{l}\psi_1=A_1(\tau)\sin(k_cx)\sin(\pi z)\\ \Theta_1=B_1(\tau)\cos(k_cx)\sin(\pi z)\\ S_1=C_1(\tau)\cos(k_cx)\sin(\pi z)\\ \psi_2=0\\ \Theta_2=B_2(\tau)\sin(2\pi z)\\ S_2=C_2(\tau)\sin(2\pi z)\end{array}\right\} \tag{28}$$

Substituting Eq.(27) in Eq.(3.)and using Eq. (28) in the resulting equation, we get

$$\left.\begin{array}{l}B_1(\tau)=-\frac{k_c}{\delta^2}A_1(\tau),B_2(\tau)=-\frac{k_c^2}{8\pi\delta^2}[A_1(\tau)]^2,\\ C_1(\tau)=-\frac{k_cLe}{\delta^2}A_1(\tau),C_2(\tau)=-\frac{k_c^2Le}{8\pi\delta^2}[A_1(\tau)]^2,\end{array}\right\} \tag{29}$$

The first order system is an eigen-boundary value problem whose eigenvalues Ra_{T_0} is given by

$$Ra_{T_0}=Ra_SLe+\frac{\delta^6}{k_c^2}+C\frac{\delta^8}{k_c^2} \tag{30}$$

The third order system is given by,

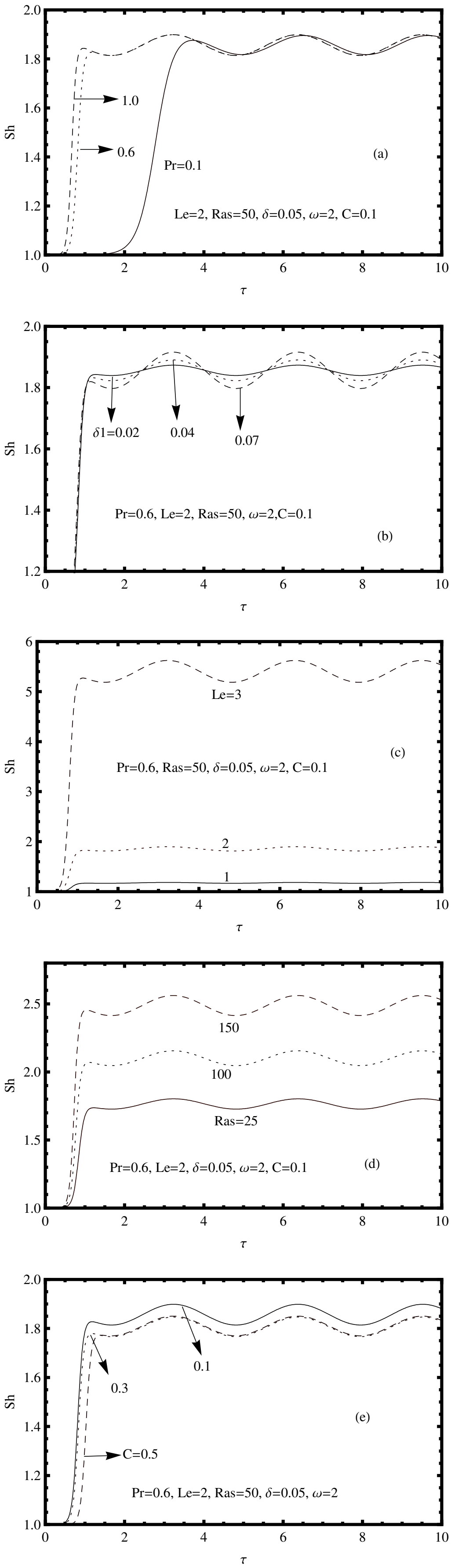
$$\begin{bmatrix}-\nabla^4+C\nabla^6 & Ra_{T_0}\frac{\partial}{\partial x} & -Ra_S\frac{\partial}{\partial x}\\ \frac{\partial}{\partial x} & -\nabla^2 & 0\\ \frac{\partial}{\partial x} & 0 & -\frac{1}{Le}\nabla^2\end{bmatrix}$$

$$\times\begin{bmatrix}\psi_3\\ \Theta_3\\ S_3\end{bmatrix}=\begin{matrix}\Re_{31}\\ \Re_{32}\\ \Re_{33}\end{matrix}, \tag{31}$$

where,

$$\Re_{31}=\Big[\frac{\delta^2}{Pr}\frac{dA_1}{d\tau}-R_2\frac{k_c^2}{\delta^2}A_1\Big]\sin(k_cx)\sin(\pi z) \tag{32}$$

$$\Re_{32}=\Big[\frac{k_c}{\delta^2}\frac{dA_1}{d\tau}+\delta_1\frac{\partial F}{\partial z}k_cA_1-\frac{k_c^3}{4\delta^2}A_1^3\cos(2\pi z)\Big]\times\cos(k_cx)\sin(2\pi z) \tag{33}$$



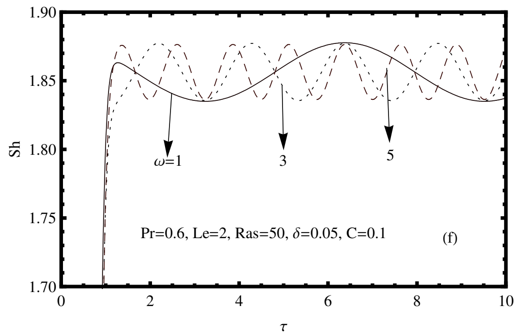


Fig. 4. Variation of Sherwood number Sh with time τ for out-phase temperature modulation $(\phi=\pi)$ for different values of$(a)Pr,(b)\delta 1,(c)Le,(d)Ra_S,(e)C,(f)\omega$.

$$\Re_{33}=\Big[\frac{k_cLe}{\delta^2}\frac{dA_1}{d\tau}\quad-\quad\frac{k_c^3Le^2}{4\delta^2}A_1^3\cos(2\pi z)\Big]\times\quad\cos(k_cx)\sin(2\pi z)\quad(34)$$

The Fredholm alternative condition for the third order solution yields the Ginzburg-Landau equation for the stationary instability with a time-periodic coefficient in the form;

$$\Big[\frac{\delta^2}{Pr}\quad+\quad Ra_{T_0}\frac{k_c^2}{\delta^4}-Ra_SLe^2\frac{k_c^2}{\delta^4}\Big]\frac{dA_1}{d\tau}-f(\tau)A_1(\tau)+\quad\frac{k_c^4}{8\delta^4}\Big[Ra_{T_0}-Le^3Ra_S\Big]A_1^3(\tau)=0\qquad(35)$$

$$f(\tau)=\frac{k_c^2}{\delta^2}[Ra_{T_2}-2Ra_{T_0}\delta_1\mathrm{f}(\tau)]\qquad(36)$$

and

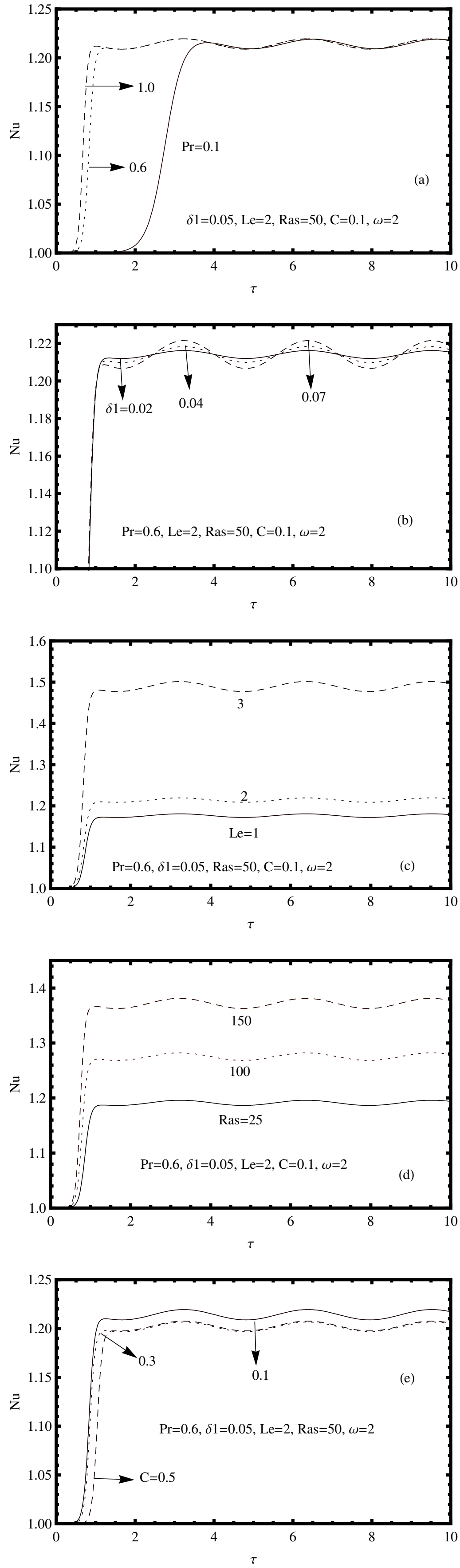
$$\mathrm{f}(\tau)=\int_0^1\Big[\frac{\partial F(z,\tau)}{\partial z}\sin^2(\pi z)\Big]dz,\qquad(37)$$

The solution of Eq.(35), subject to the initial condition $A(0)=a_0$ where a_0 is a chosen initial amplitude of convection, can be solved by using Runge-Kutta method. In our computation we assume $Ra_{T_2}=Ra_{T_0}$ to keep the parameters to minimum.

The horizontally averaged Nusselt number $Nu(\tau)$, and Sherwood number $Sh(\tau)$ for stationary convection is given by;

$$Nu(\tau)=\frac{[\frac{K_c}{2\pi}\int_{x=0}^{\frac{2\pi}{K_c}}(1-z+\Theta_2)_z\,dx]_{z=0}}{[\frac{K_c}{2\pi}\int_{x=0}^{\frac{2\pi}{K_c}}(1-z)_z\,dx]_{z=0}}\qquad(38)$$

$$Sh(\tau)=\frac{[\frac{K_c}{2\pi}\int_{x=0}^{\frac{2\pi}{K_c}}(1-z+S_2)_z\,dx]_{z=0}}{[\frac{K_c}{2\pi}\int_{x=0}^{\frac{2\pi}{K_c}}(1-z)_z\,dx]_{z=0}}\qquad(39)$$



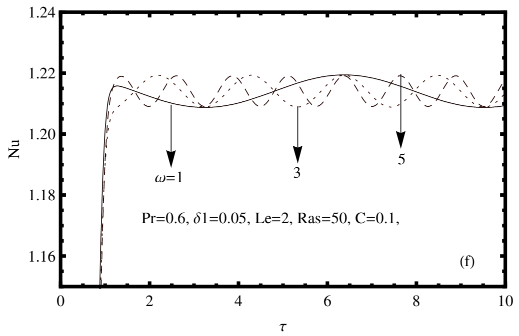


Fig. 5. Variation of Nusselt number Nu with time τ for lower plate modulation $(\phi = -I\infty)$ for different values of$(a)Pr, (b)\delta 1, (c)Le, (d)Ra_S, (e)C, (f)\omega$ for temperature modulation.

Now substituting Eq.(28) in Eq.(38)and Eq.(39)and completing the integration, we get

$$Nu(\tau) = 1 + \frac{k_c^2[A_1(\tau)]^2}{4\delta^2} \tag{40}$$

$$Sh(\tau) = 1 + \frac{k_c^2 Le^2[A_1(\tau)]^2}{4\delta^2} \tag{41}$$

4. RESULT AND DISCUSSIONS

In this paper, effect of temperature and gravity modulation on double diffusive convection in a couple stress liquid has been investigated. The effect of temperature modulation has been discussed in three parts.

1. In-phase modulation $(IPM)(\phi = 0)$
2. Out-phase modulation $(OPM)(\phi = \pi)$
3. Lower boundary modulated only $(LBMO)(\phi = -I\infty)$

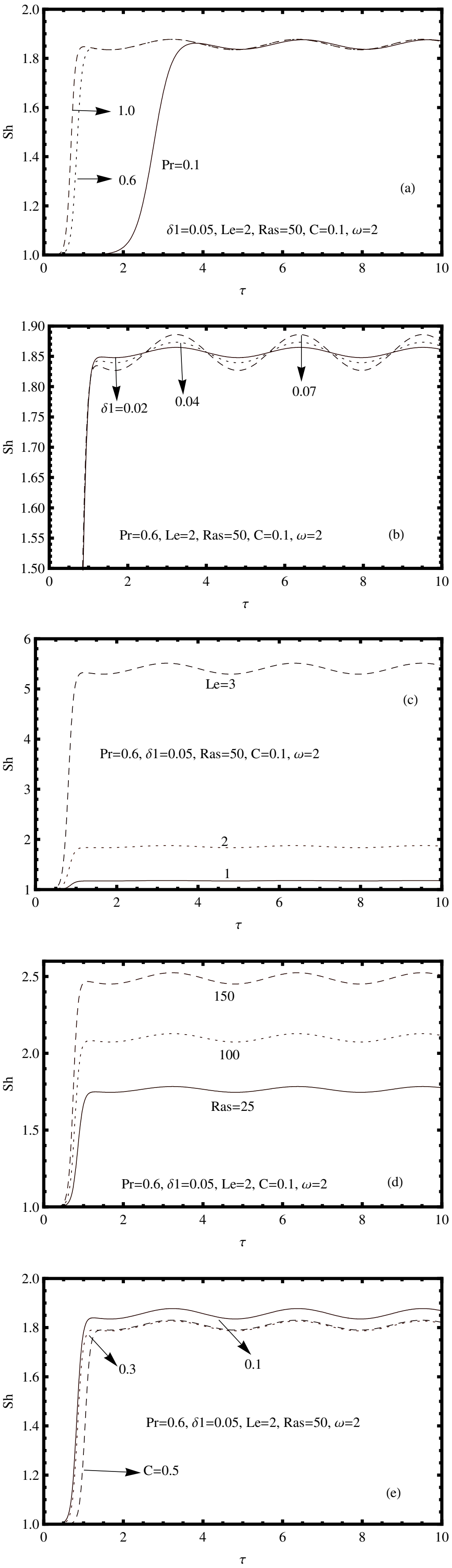
From the figures, it can easily seen that the value of Nusselt number Nu and Sherwood number Sh is found as

$Nu_{IPM} < Nu_{LBMO} < Nu_{OPM}$

$Sh_{IPM} < Sh_{LBMO} < Sh_{OPM}$

The double diffusive convection in a couple stress liquid has been investigated under the influence of time-periodic temperature modulation. A weak non-linear stability analysis has been used to investigate the effect of modulation on the heat and mass transport. Figs.1$(a-f)$ and 2$(a-f)$ are the plots of Nusselt number Nu and Sherwood number Sh with respect to slow time τ respectively for the case of in-phase modulation. We find that for small time τ, Nu and Sh remain constant, then increase on increasing τ and finally become steady on further increasing τ.

From Figs. $1(a, f)$ we observed that the



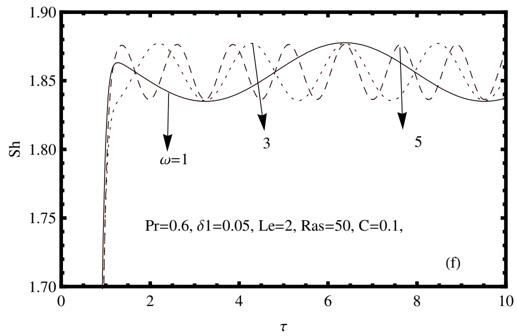


Fig. 6. Variation of Sherwood number Sh with time τ for lower plate modulation $(\phi = -I\infty)$ for different values of$(a)Pr, (b)\delta 1, (c)Le, (d)Ra_S, (e)C, (f)\omega$.

effect of increasing amplitude of modulation δ and the frequency of modulation ω is negligible . From Figs. $1(c,d)$ we examined that on increasing Lewis number Le and solute Rayleigh number Ra_S, the value of Nusselt number is also increases, thus the rate of heat transport and hence advances the convection. From Fig. $1(a)$ we observed that on increasing the Prandtl number Pr , the value of Nusselt number is also increases but when time increases the effect of increasing Pr is negligible. From Fig. $1(e)$ we examined that on increasing the couple stress parameter C, the value of Nusselt number is decreases but when time increases the effect of couple stress parameter is negligible. From the Figs. $2(a-f)$, we obtained the qualitatively similar result as found in Figs. $1(a-f)$.

In Figs. $3(a-f)$ and $4(a-f)$ we depict the variations of Nu and Sh with slow time τ for the case of out-phase modulation. From Fig. $3(a)$, we find that the effect of increasing Pr increases the value of Nu but when time increases the effect of Pr is negligible. From Fig. $3(b)$, we find that the effect of increasing modulation amplitude δ on Nu is to increase the magnitude of Nu, i.e. rate of heat transport increases. From Figs. $3(c-d)$ we observed the effect of increasing Le and Ra_S on Nu is to increase the value of Nu. From Fig. $3(e)$ we examined that on increasing C, the value of Nu is decreases but when time increases the effect of C is negligible. From Fig. $3(f)$ shows that an increase in frequency of modulation ω does't alter the value of Nu but wavelength of oscillation decreases on increasing ω. Figs. $4(a-f)$ are the plots of Sherwood number Sh with the slow time τ and we obtained the qualitatively similar results as found in Figs. $4(a-f)$.

In Figs.$5(a-f)$ and $6(a-f)$ we depict the variations of Nu and Sh with the slow time τ for the case of Lower boundary modulation only. We find the similar results as found in Figs. $3(a-f)$ and $4(a-f)$.

5. Conclusion

The effect of gravity modulation in a couple stress liquid has been investigated. GL equations have been used to solve the problem. On the basis of above discussion we reach on following conclusions:
1. The effect of increasing Pr, Ra_S and Le on Nu and Sh are to increase the rate of heat and mass transfer for each case of temperature modulation.
2. The effect of increasing C on Nu and Sh is to decrease the rate of heat and mass transfer for each case of temperature modulation.

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