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Blasius Problem with Generalized Surface Slip Velocity

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ABSTRACT

This paper considers the classical problem of the steady boundary layer flow past a semi-infinite flat plate first considered by Blasius in 1908 with generalized surface slip velocity. Numerical solutions are obtained by solving the nonlinear similarity equation using the bvp4c function from MATLAB for several values of the slip parameters.

Keywords: Blasius problem; Generalized slip velocity; Numerical results.

NOMENCLATURE

a, b	constants
C_f	skin friction coefficient
Re_x	is the local Reynolds number
u, v	dimensional velocity components along the axes x, y
u_t	tangential sheet velocity
u_w	velocity at $y = 0$
U	constant velocity of the external flow
x, y	dimensionless Cartesian coordinates
α	dimensionless velocity slip parameter
β	dimensionless critical shear rate
μ	dynamic viscosity
ν	kinematic viscosity
τ_w	skin friction
ψ	dimensionless stream function

1. INTRODUCTION

The usual velocity and thermal boundary conditions that are applied on a body's surface in the study of forced or convective flows in a viscous fluid are either a prescribed wall velocity or prescribed temperature or surface heat flux. The idea of a wall slip (or partial slip) condition was suggested originally by Beavers and Joseph (1967) and has been applied recently in several contexts. Such flows have been frequently analyzed under a large variety of both geometrical and physical conditions, as described for example in the paper by Aziz (2009) for the classical problem of hydrodynamic boundary layer over a flat plate in a uniform stream of fluid and by Magyari (2009) for the classic problem of thermal boundary layer over a flat plate. Wang (2006), Mukhopadhyay and Andersson (2009), Fang *et al*. (2010), etc., have examined the effect of the slip velocity on the flow and heat transfer past a moving surface or over stretching/shrinking sheets. Wang and Ng (2011) have considered the effects of wall slip flow due to a stretching cylinder, while Crane and McVeigh (2011, 2010) have treated the accelerated flow, or nonlinear shear flow, over cylinders with wall slip conditions.

Fang and Lee (2005), Kumaran and Pop (2011) and Cai (2015) generalized the problem of Blasius (1908) by adding a slip boundary condition where the velocity on the wall is based on the Maxwell's approximation. However, here we consider the classical boundary layer flow past a semi-infinite flat plate with a different generalized surface slip velocity as suggested by Thompson and Troian (1997). The physical influence of the controlling governing parameters are presented and discussed in detail. To the best author's knowledge, the present problem has not been studied before by other researchers.

2. BASIC EQUATIONS

Consider the boundary layer flow of a viscous and incompressible fluid past a semi-infinite flat plate with generalized slip velocity. We assume that the plane surface is located at $y = 0$, where y is the coordinate measured in the direction normal to the plate and x is the coordinate measured along the surface in the upward direction, respectively. It is assumed that the constant velocity of the external (inviscid) flow is U. The basic boundary layer equations can be written in Cartesian coordinates x and y as (see Bejan, 2013)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial x}=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Following Thompson and Troian (1997), we assume that the generalized slip velocity condition is given by

$$u_t(x)=\alpha^*(1-\beta^*\tau_w)^{-1/2}\,\tau_w \tag{3}$$

Where τ_w is the skin friction or shear stress along the surface of the plate, and is given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{4}$$

Here u and v are the velocity components along x and y axes, ν is the kinematic viscosity, μ is the dynamic viscosity, u_t is the tangential sheet velocity, α^* corresponds to Navier's constant slip length and β^* is the reciprocal of some critical shear rate. Thus, we assume that the boundary conditions of Eqs. (1) and (2) are

$$\begin{aligned} &v=0,\quad \text{at } y=0\\ &u=u_t(x)=\\ &\quad=\alpha^*(x)\left[1-\beta^*(x)\frac{\partial u}{\partial y}\right]^{-1/2}\frac{\partial u}{\partial y}\ \text{at } y=0\\ &u=U\quad \text{as}\quad y\to\infty \end{aligned} \tag{5}$$

3. SOLUTION

In order to solve Eqs. (1) and (2) along with the boundary conditions (5), we introduce the following similarity variables

$$\psi=(2U\nu x)^{1/2}f(\eta),\qquad \eta=(U/2x\nu)^{1/2}\,y \tag{6}$$

where ψ is the stream function with $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. Substituting (6) into Eq. (2) it results in the following ordinary (similarity) equation

$$f'''+f\,f''=0 \tag{7}$$

subject to the boundary conditions

$$\begin{aligned} &f(0)=0,\\ &f'(0)=\alpha(x)\left[1-\beta(x)f''(0)\right]^{-1/2}f''(0)\\ &f'(\eta)\to 1\quad \text{as}\quad \eta\to\infty \end{aligned} \tag{8}$$

where primes denote differentiation with respect to η. The two parameters $\alpha(x)$ and $\beta(x)$ appearing in Eq. (8) denote the dimensionless velocity slip parameter and the dimensionless critical shear rate, which are defined as

$$\begin{aligned} &\alpha=\sqrt{\frac{U}{2\nu}}\,\alpha^*(x)x^{-1/2},\\ &\beta=U\sqrt{\frac{U}{2\nu}}\,\beta^*(x)x^{-1/2} \end{aligned} \tag{9}$$

As it was suggested by Aziz (2009), for Eq. (7) to have a similarity solution, the quantities $\alpha(x)$ and $\beta(x)$ must be constants and not functions of the variable x as in Eq. (8). This condition can be met if $\alpha^*(x)$ and $\beta^*(x)$ are proportional to $x^{1/2}$. We therefore assume

$$\alpha^*(x)=a\,x^{1/2},\quad \beta^*(x)=b\,x^{1/2} \tag{10}$$

where a and b are constants. With the introduction of (10) into (9), we have

$$\alpha=\sqrt{\frac{U}{2\nu}}\,a,\quad \beta=U\sqrt{\frac{U}{2\nu}}\,b \tag{11}$$

We mention that with α and β defined by (11), the solution of Eq. (7) yields the similarity solutions. However, with α and β defined by (9), the solutions generated are the local similarity solutions. We notice that for $\alpha=0$ the equation (7) along with the boundary condition (8) reduces to the well-known Blasius (1908) problem. Further, for $\beta=0$ the problem reduces to one with classical slip wall velocity (see Martin and Boyd, 2001).

The quantity of physical interest in this problem is the skin friction coefficient C_f, which is defined as

$$C_f=\frac{\tau_w}{\rho U^2} \tag{12}$$

where τ_w is given by (4). Using (4) and (12), we get

$$\mathrm{Re}_x^{1/2}\,C_f=\frac{1}{\sqrt{2}}f''(0) \tag{13}$$

where $\mathrm{Re}_x=U\,x/\nu$ is the local Reynolds number.

4. RESULTS AND DISCUSSION

Equation (7) along with the boundary conditions (8) has been solved numerically using function bv4c from MATLAB for several values of the governing parameters, such as: $\alpha=1$ and β =0 ,1 and 2, $\alpha=5$ and β =0, 1, 2.5 and 4, $\alpha=10$ and $\beta=0,3,5$ and 7, and $\alpha=20$ and $\beta=0$, 5,10 and 20 (see Shampine *et al.*, 2010, 2003). The values of $f'(0)$ and $f''(0)$ are given in Table 1. We notice from this table that the value of $f''(0)$ obtained for $\alpha = 0$ is in very good agreement with that of the classical Blasius problem (see Leal, 2007). Thus we are confident that the present results are correct. It is also worth mentioning that the patterns of the dimensionless velocity are similar for $\beta = 0$ with those obtained by Martin and Boyd (2001). Unfortunately, in the above paper numerical results are not reported.

Table 1 shows that the dimensionless wall slip velocity $f'(0)$ increases, while the skin friction coefficient on the plate $f''(0)$ decreases with the increase values of the both parameters α and β. However, for large values of the parameter α, the skin friction coefficient $f''(0)$ goes to zero and the slip velocity on the plate $f'(0)$ goes to one. This is agreement with the proposed mathematical model. Further, Figs. 1 and 2 show that the boundary layer thickness decreases with the increase of the parameter α when $\beta = 0$ (Fig. 1) and also decreases when $\beta \neq 0$. It is also evident from these figures that the far field boundary condition (8) that $f'(\eta) \to 1$ as $\eta \to \infty$ is approached asymptotically. Thus, it supports the numerical results obtained for the boundary-value problem (7) and (8).

Table 1 Values related to the dimensionless wall slip velocity $f'(0)$ and skin friction coefficient $f''(0)$.

α	β	$f'(0)$	$f''(0)$
0			0.469600 (0.469)
1	0	0.382443	0.382443
	0.5	0.410545	0.370565
	1	0.443431	0.355884
	2	0.523434	0.316821
5	0	0.787646	0.157529
	1	0.801546	0.147974
	2.5	0.821595	0.133998
	4	0.840233	0.120805
10	0	0.884796	0.088480
	3	0.898218	0.078531
	5	0.906426	0.072401
	7	0.913984	0.066723
20	0	0.939938	0.046997
	5	0.946373	0.042051
	10	0.952134	0.037605
	20	0.961638	0.030232

5. CONCLUSION

We have studied the steady boundary layer flow of a viscous and incompressible fluid past a semi-infinite flat plate, with generalized surface slip velocity. The bvp4c MATLAB program (collocation method) is used to obtain the numerical solutions. The numerical results are verified with the earlier study from the open literature, and have been found to be in very good agreement. We mention again that such a generalization of the Blasius problem has not been reported before, so that the reported results are new and original.

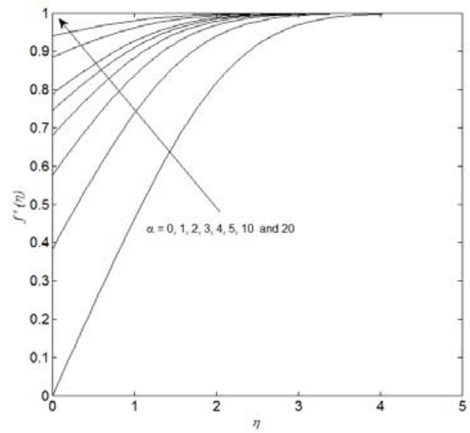

Fig. 1. Variation of dimensionless velocity velocity profiles for $\beta = 0$ and $\alpha = 0, 1, 2, 3, 4, 5$ and 20.

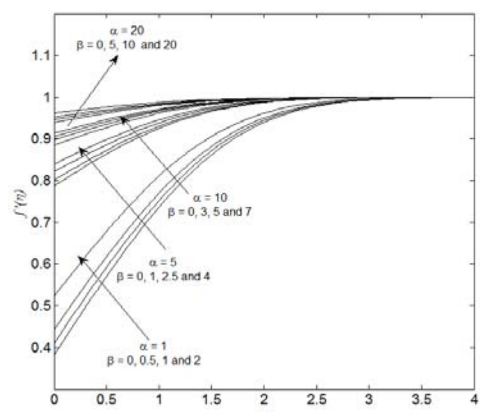

Fig. 2. **Variation of dimensionless profiles for $\alpha = 1, 5, 10$ and 20 and for different values of β.**

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