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Onset of Darcy-Brinkman Convection in a Maxwell Fluid Saturated Anisotropic Porous Layer

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ABSTRACT

In the present study, the onset of Darcy-Brinkman double diffusive convection in a Maxwell fluid-saturated anisotropic porous layer is studied analytically using stability analysis. The linear stability analysis is based on normal technique. The modified Darcy-Brinkmam Maxwell model is used for the momentum equation. The Rayleigh number for stationary, oscillatory and finite amplitude convection is obtained analytically. The effect of the stress relaxation parameter, solute Rayleigh number, Darcy number, Darcy-Prandtl number, Lewis number, mechanical and thermal anisotropy parameters, and normal porosity parameter on the stationary, oscillatory and finite amplitude convection is shown graphically. The nonlinear theory is based on the truncated representation of the Fourier series method and is used to find the heat and mass transfer. The transient behavior of the Nusselt and Sherwood numbers is obtained by solving the finite amplitude equations using the Runge-Kutta method.

Keywords: Double diffusive convection; Darcy brinkman Maxwell model; Porous layer, Anisotropy; Heat and mass transfer.

NOMENCLATURE

a	Overall horizontal wave number
c	specific heat
Da	Darcy number,
d	height of the porous layer
g	gravitational acceleration
H	rate of heat transport per unit area
J	rate of mass transport per unit area
K	permeability tensor,
Le	Lewis number,
l, m	Horizontal wave numbers
Nu	Nusselt number
p	pressure
$\Pr_D$	Darcy–Prandtl number,
q	velocity vector,
Ra_S	solute Rayleigh number,
Ra_T	thermal Rayleigh number,
S	solute concentration
Sh	Sherwood number
ΔS	salinity difference between the walls
t	time
T	temperature
ΔT	temperature difference between the walls
x, y, z	space coordinates
β_T	thermal expansion coefficient
β_S	solute expansion coefficient
Φ	dimensionless amplitude of concentration perturbation
ϕ	normalized porosity,
γ	ratio of specific heats,
η	thermal anisotropy parameter,
κ_T	thermal diffusivity
κ_S	solute diffusivity
κ	diffusivity
λ	stress relaxation parameter
$\bar{\lambda}$	stress relaxation time
ε	porosity
μ	dynamic viscosity
μ_e	effective viscosity
ν	kinematics viscosity,
Θ	dimensionless amplitude of temperature perturbation
ρ	fluid density
σ	growth rate
Ω	angular velocity of rotation,
ξ	mechanical anisotropy parameter,
ψ	stream function

Other Symbols

∇_h^2 $\quad \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

∇^2 $\quad \nabla_h^2+\frac{\partial^2}{\partial z^2}$

Subscripts and Superscripts

b basic state
c critical
f fluid
h horizontal
m porous medium
0 reference value
s solid
$*$ dimensionless quantity
' perturbed quantity
F finite amplitude
Osc oscillatory state
St stationary state

1. INTRODUCTION

Double diffusive convection in porous medium is prevalent in nature occurring such diverse areas as in polymer processing, chemical separation techniques and transport in biological system. Typical examples of natural porous media include sandstone, wood and human tissue including lungs and blood vessels. Understanding the nature, the behavior and stability characteristics of viscoelastic fluid in porous media is also important in many engineering fields, for example, in oil recovery processes, paper and textile manufacturing, wetting and drying processes and composite manufacturing processes, see for instance, Khuzhayorov *et al.* (2000) and Khan *et al.* (2007).

Double-diffusive convection is referred to buoyancy driven flows induced by combined temperature and concentration gradients. The onset of double diffusive convection in a fluid saturated in porous medium is regarded as a classical problem due to its wide range of applications in many engineering fields such as evaporative cooling of high temperature systems, agricultural product storage, soil sciences, enhanced oil recovery, packed-bed catalytic reactors, and the pollutant transport in underground. A detailed review of the literature concerning double diffusive convection in binary fluid in a porous medium was given by Nield and Bejan (2006), Trevisan and Bejan (1990), and Malashetty and Kollur (2011). Thermal convection in binary fluid driven by the Soret and Dufour effects was investigated by Knobloch (1980) and showed that the equations were identical to the thermosolutal problem except relation between the thermal and solutal Rayleigh numbers.

It is well known that many applications in engineering disciplines as well as in circumstances linked to modern porous media involve high permeability porous layer. For instance, in biomedical hydrodynamic studies, a thin fibrous surface layer coating blood vessels (endothelial surface layer) is found to be a highly permeable, high porosity porous medium is studied by Khaled and Vafai (2003). In such circumstances the use of a non-Darcy model, which takes care of boundary and/or inertia effects is of fundamental and practical interest to obtain accurate results. Further, it is believed that the results of this study are useful in bridging the gap between a non-porous case in which $Da\to\infty$ and a dense porous medium in which $Da\to 0$. A better understanding of the characteristics of the Darcy–Brinkman equation is therefore an important part of more practical problems and thus forms a motivation of the present report.

For the low porosity media, the viscous effects near the boundary are negligible. In such situations Darcy’s laws is a good approximation for the momentum equation. The main advantage of Darcy’s flow model is that it linearizes the momentum equation and thus reduces a significant amount valid for flow through regular structures over the whole spectrum of the porosity. This model is silent about difficulty in solving the governing equations. Further, the classical Darcy model is the flow structure near the bounding surfaces where close packing of the porous material is not possible. Brinkmam model is valid for a sparsely packed porous medium wherein there is more window fluid to flow so that the distortions of velocity give rise to the usual shear force. An analytical and the numerical study of double diffusive convection with parallel flow in a horizontal sparsely packed porous layer under the influence of constant heat and mass flux was performed using a Brinkmam model by Amahmid *et al*. (1999).

As some new technologically significant materials are discovered acting like non-Newtonian fluids therefore mathematicians, physicists and engineers are actively conducting research in rheology.Maxwell fluids can be considered as a special case of a Jeffreys-Oldroyd B fluid, which contain relaxation and retardation time coefficients. Maxwell’s constitutive relation can be recovered from that corresponding to Jeffreys-Oldroyd B fluids by setting the retardation time to be zero. Several fluids such as glycerin, crude oils or some polymeric solutions, behave as Maxwell fluids.

Recently, interest in viscoelastic flows through porous media has grown considerably, due to the demands of such diverse fields as biorheology, geophysics, chemical, and petroleum industries. The works on convective instability thresholds for viscoelastic fluids in porous media (2003-2009) can be found in the literature and have not been given much attention. Recently, Wang and Tan (2008) have made the stability analysis of double diffusive convection of Maxwell fluid in a porous medium.

It is worthwhile to point out that the first viscoelastic rate type model, which is still used widely, due to Maxwell (1866). Maxwell did not develop this model for polymeric liquids, he recognized that such a fluid has a means for storing energy characterizing its viscous nature. Malashetty *et al*. (2009) have studied double diffusive convection in a viscoelastic fluid saturated porous layer using Oldroyd model. More recently, Awad *et al*. (2010) used the Darcy-Brinkman-Maxwell model to study linearstability analysis of a Maxwell fluid with cross-diffusion and double-diffusive convection. They found that the effect of relaxation time is to decrease the critical Darcy-Rayleigh number. Although many works are available on the use of non-Darcy models to study flow and heat transfer in porous media in the recent past, the works on double diffusive convection in an anisotropic sparsely packed porous layer are very sparse and it is in much-to-be desired state.

In the present paper, we intend to analyze of a Darcy-Brinkmam binary Maxwell fluid saturated anisotropic porous layer. Our objective is to study how the onset criterion for oscillatory convection is affected by Maxwell fluid and the other parameters, and also to know their heat and mass transfer in a more general porous medium, in limiting cases, some previously published results can be recovered as the particular cases of our results.

2. MATHEMATICAL FORMATION

We consider an infinite horizontal fluid-saturated anisotropic porous layer confined between the planes $z=0$ and $z=d$, with vertically downward gravity force $\mathbf{g}$ acting on it. A uniform adverse temperature gradient $\Delta T = T_l - T_u$ and a stabilizing concentration gradient $\Delta S = S_l - S_u$ $(T_l > T_u \text{ and } S_l > S_u)$ are imposed at the bottom and top boundaries respectively. The boundaries are impermeable, and we assume that the fluid and solid phases are in local thermal equilibrium. A Cartesian frame of reference is chosen with origin in the lower boundary and z-axis vertically upward. The velocities are assumed to be small so that the advective and Forchheimer inertia effects are ignored. The Boussinesq approximation, which states that the variation in density is negligible everywhere in the conservations except in the buoyancy term, is assumed to hold. The Darcy-Brinkman Maxwell model is employed to describe the flow in the porous media.

The basic state is assumed to be quiescent, and we superpose perturbations on the basic state. The equations for the perturbation are

$$\nabla.\mathbf{q} = 0, \tag{1}$$

$$\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t}+\nabla p-\left(\rho_0[1-\beta_T(T-T_0)+\beta_S(S-S_0)]\right)\mathbf{g}\right) = \left(\mu_e\nabla^2\mathbf{q}-\mu\mathbf{K}\mathbf{q}\right), \tag{2}$$

$$\gamma\frac{\partial T}{\partial t}+(\mathbf{q}.\nabla)T - w\frac{\Delta T}{d} = \kappa_T\,\nabla^2 T, \tag{3}$$

$$\varepsilon\frac{\partial S}{\partial t}+(\mathbf{q}.\nabla)S-\frac{\Delta S}{d} = (\kappa_S\nabla^2 S), \tag{4}$$

where $\mathbf{q} = (u, v, w)$ is the velocity, p is pressure, $\bar{\lambda}$ is the relaxation time, $\mathbf{g} = (0,0,-g)$ is the acceleration due to gravity, μ is the fluid viscosity, ρ is the fluid density, ρ_0 is the reference density, T and S are the temperature and concentration respectively and ε is the porosity of the porous medium, $\mathbf{K} = K_x^{-1}\mathbf{ii}+K_y^{-1}\mathbf{jj}+K_z^{-1}\mathbf{kk}$ is the inverse of the permeability tensor and $\boldsymbol{\kappa}_{\mathbf{T}} = \kappa_{Tx}\,\mathbf{ii}+\kappa_{Ty}\,\mathbf{jj}+\kappa_{Tz}\,\mathbf{kk}$ is the thermal diffusivity tensor. $\gamma = \frac{(\rho c)_m}{(\rho c_p)_f}$, $(\rho c)_m = (1-\varepsilon)(\rho c_p)_f$, c_p is the specific heat of the fluid at constant pressure, c is the specific heat of the solid, the subscripts f,s and m denote fluid, solid and porous medium values respectively, and β_T, β_S, μ_f, μ_e and κ_S are the thermal and solute expansion coefficients, fluid viscosity, effective viscosity, and solute diffusivity, respectively. It is hereby stated that permeability is most strongly anisotropic than solute diffusivity. Therefore, we ignore the solute anisotropy. Notice that, in the case $\bar{\lambda}$ = 0, the model reduces to the Newtonian binary fluid. By operating curl twice on Eq. (2) we eliminate p, and then render the resulting equation and Eqs. (3) and (4) dimensionless using the following transformations

$$(x,y,z)=d\left(x^*,y^*,z^*\right),\ t=\left(\frac{\gamma d^2}{\kappa_{Tz}}\right)t^*,$$
$$(u,v,w)=\frac{\kappa_{Tz}}{d}(u^*,v^*,w^*),\ T=(\Delta T)T^*, \tag{5}$$
$$S=(\Delta S)S^*,$$

to obtain non-dimensional linear equations as (on dropping the asterisks for simplicity),

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{1}{\gamma Pr_D}\frac{\partial}{\partial t}\nabla^2 w - Ra_T\nabla_h^2 T + Ra_S\nabla_h^2 S\right) = \left(-\nabla_h^2-\frac{1}{\xi}\frac{\partial^2}{\partial z^2}+Da\nabla^4\right)w, \tag{6}$$

$$\left(\frac{\partial}{\partial t}-\left(\eta\nabla_h^2+\frac{\partial^2}{\partial z^2}\right)\right)T+\left(\vec{q}.\nabla\right)T-w=0, \tag{7}$$

$$\left(\phi\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)S+\left(\vec{q}.\nabla\right)S-w=0, \tag{8}$$

where $\lambda = \left(\kappa_{Tz}/d^2\right)\bar{\lambda}$ relaxation parameter. The stress relaxation parameter is the ratio of a relaxation time, characterizing the intrinsic fluidity of a material, and the characteristic timescale. The smaller the stress relaxation parameter, the more fluid the material appears. $Ra_T = \beta_T g \Delta T d K_z / \nu\kappa_{Tz}$ the thermal Rayleigh number, $Ra_S = \beta_S g \Delta S d K_z / \nu\kappa_{Tz}$ the solute Rayleigh number, $Da = \mu_e k_z / \mu_f d^2$ the Darcy number, $\Pr_D = \gamma\varepsilon\nu d^2 / K_z \kappa_{Tz}$ the Darcy-Prandtl number. The parameters (namely, $\Pr_D$ and Le) depend on the properties of the fluid. It is worth mentioning here that the Darcy–Prandtl number $\Pr_D$ includes the Prandtl number, Darcy number, porosity, and the specific heat ratio. It depends on the properties of the fluid and on the nature of porous matrix. The Prandtl number affects the stability of the porous system through this combined dimensionless group. $Le = \kappa_{Tz}/\kappa_S$, the ratio between thermal and solutal diffusivities is characterized by the Lewis number. $\xi = k_X / k_Z$ the mechanical anisotropy parameter, $\eta = \kappa_{Tx}/\kappa_{Tz}$ is the thermal anisotropy parameter, $\phi = \varepsilon / \gamma$ normalized porosity. The normalized porosity $\phi$is expressed in terms of the porosity of the porous medium, and the solid to fluid heat capacity ratio. Equation (6) - (8) are solved for stress free, isothermal and isosolutal boundary conditions. Hence the boundary conditions for the perturbation variables are given by

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \quad at \quad z = 0, 1 \,. \tag{9}$$

3. LINEAR STABILITY ANALYSIS

In this section we predict the threshold of both stationary and oscillatory convection using linear theory. The Eigen value problem defined by Eqs. (6) - (8) subject to the boundary conditions (9) is solved using the time-dependent periodic disturbances in a horizontal plane, upon assuming that amplitudes are small enough and can be expressed as

$$\begin{pmatrix} w \\ T \\ S \end{pmatrix} = \begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} \exp\left[i\,(lx + my) + \sigma t \right], \tag{10}$$

where l and m are the wave numbers in the horizontal plane and σ is the growth rate. Infinitesimal perturbations of the rest state may either damp or grow depending on the value of the parameter σ. Substituting Eq. (9) into Eqs. (6) - (8) we obtain

$$(1+\lambda\sigma)\left[\frac{\sigma}{\Pr_D}\left(D^2 - a^2\right)W + a^2 Ra_T \Theta - a^2 Ra_S \Phi\right] = \left[Da\left(D^2 - a^2\right)^2 + a^2 - \frac{1}{\xi}D^2\right]W\,, \tag{11}$$

$$\left[\sigma - \left(D^2 - a^2\eta\right)\right]\Theta - W = 0, \tag{12}$$

$$\left[\phi\sigma - \frac{1}{Le}\left(D^2 - a^2\right)\right]\Phi - W = 0, \tag{13}$$

where $D=d/dz$ and $a^2 = l^2 + m^2$. In case of stress-free boundary conditions, it is possible to solve analytically the system of Eqs. (11) - (13). This is a standard Eigenvalue-Eigen function problem. Here the Rayleigh number is taken as the Eigenvalue and it is expressed as a function of the other parameters which govern the stability of the system.

The corresponding boundary conditions are

$$W = D^2 W = \Theta = \Phi = 0 \quad at \quad Z = 0,1 \tag{14}$$

The solutions of Eqs. (11) - (13) satisfying the boundary conditions (14) are assumed in the from

$$\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} = \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} \sin n\pi z, \left(n = 1, 2, 3........\right). \tag{15}$$

The most unstable mode corresponding to $n=1$ (fundamental mode). Therefore, substituting Eq (15) with $n=1$ into Eqs. (11) - (13), and using the solvability condition we obtain a matrix equation of the form

$$\begin{pmatrix} \frac{\sigma\delta^2}{\Pr_D} + \left(\frac{\delta_1^2 + Da\delta^4}{1+\lambda\sigma}\right) & -a^2 Ra_T & a^2 Ra_S \\ -1 & \sigma + \delta_2^2 & 0 \\ -1 & 0 & \phi\sigma + \frac{1}{Le}\delta^2 \end{pmatrix} \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{16}$$

where $\delta^2 = \pi^2 + a^2$, $\delta_1^2 = \pi^2 \xi^{-1} + a^2$ and $\delta_2^2 = \pi^2 + \eta a^2$.

The conditions of non-trivial solutions of system of homogenous linear equations (16) yields the expression for the thermal Rayleigh number in the form

$$Ra_T = \left[\frac{\sigma\delta^2}{\Pr_D} + \frac{\delta_1^2 + Da\delta^4}{1+\lambda\sigma}\right]\left(\frac{\sigma + \delta_2^2}{a^2}\right) + Ra_S\left(\frac{\sigma + \delta_2^2}{\phi\sigma + Le^{-1}\delta^2}\right), \tag{17}$$

3.1 Marginal State

For validity of the principle of exchange of stabilities (i.e. steady case), we have σ=0 (i.e. $\sigma_r = \sigma_i = 0$) at the margin of stability. Then the Rayleigh number at which marginally stable steady mode exists becomes

$$Ra_T{}^{st} = \left(\frac{\eta a^2+\pi^2}{a^2}\right)\left(Da\left(\pi^2+a^2\right)^2+\left(\frac{\pi^2}{\xi}+a^2\right)\right) + \frac{\left(\eta a^2+\pi^2\right)Le\,Ra_S}{\left(\pi^2+a^2\right)}, \tag{18}$$

The above result is independent of the relaxation time and identical to that of the Newtonian problem. In the absence of $Da \to 0$, i.e., for densely packed porous medium Eq. (18) reduces to

$$Ra_T{}^{st} = \left(\frac{\eta a^2+\pi^2}{a^2}\right)\left(\frac{\pi^2}{\xi}+a^2\right)+\frac{\left(\eta a^2+\pi^2\right)Le\,Ra_S}{\left(\pi^2+a^2\right)}, \tag{19}$$

which is identical with Malashetty and Swamy (2010). Further, for an isotropic porous medium, that is, when $\xi=\eta=1$, Eq. (19) gives

$$Ra_T{}^{st} = \frac{\left(\pi^2+a^2\right)^2}{a^2}+Le\,Ra_S, \tag{20}$$

which is the one obtained by Gaikwad and Bharati (2013).

3.2 Oscillatory State

We now set $\sigma = i\sigma$ in Eq. (17) and clear the complex quantities from the denominator, to obtain

$$Ra_T = \Delta_1 + i\,\sigma_i\,\Delta_2 \tag{21}$$

where

$$\Delta_1 = -\frac{\sigma^2\delta^2}{a^2\,\mathrm{Pr}_D}+\frac{\left(\delta_1^2+Da\,\delta^4\right)\left(\delta_2^2+\lambda\sigma^2\right)}{a^2\left(1+\lambda^2\sigma^2\right)} + Ra_S\left(\frac{\sigma^2\phi+Le^{-1}\delta^2\delta_2^2}{\left(Le^{-1}\delta^2\right)^2+\phi^2\sigma^2}\right), \tag{22}$$

$$\Delta_2 = \frac{\delta_2^2\delta^2}{a^2\,\mathrm{Pr}_D}+\frac{\left(\delta_1^2+Da\,\delta^4\right)\left(1-\lambda\delta_2^2\right)}{a^2\left(1+\lambda^2\sigma^2\right)} + Ra_S\,\frac{\left(Le^{-1}\delta^2-\phi\,\delta_2^2\right)}{\left(Le^{-1}\delta^2\right)^2+\phi^2\sigma^2}, \tag{23}$$

Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (21) it follows that either $\sigma_i = 0$ (steady onset) or $\Delta_2 = 0\ (\sigma_i \neq 0)$ (oscillatory onset).For oscillatory onset $\Delta_2 = 0\ (\sigma_i \neq 0)$ and this gives an expression for frequency of oscillations in the form (on dropping the subscript i)

$$a_0\left(\sigma^2\right)^2 + a_1\left(\sigma^2\right)+a_2 = 0 \tag{24}$$

where the coefficients are given by

$$a_0 = \delta^2\delta_2^2\lambda^2\phi^2,$$

$$a_1 = \delta^2\delta_2^2\phi^2+\delta^2\delta_2^2\lambda^2\left(\delta^2 Le^{-1}\right)^2 - \mathrm{Pr}_D\,\delta_1^2\phi^2\delta_2^2\lambda + \mathrm{Pr}_D\,\delta_1^2\phi^2 + Da\,\mathrm{Pr}_D\,\delta^4\phi^2 - Da\,\mathrm{Pr}_D\,\delta^4\delta_2^2\lambda\phi^2 + a^2 Le^{-1}\,\mathrm{Pr}_D\,Ra_S\,\delta^2\lambda^2 - a^2 Ra_S\,\mathrm{Pr}_D\,\phi\lambda^2\delta_2^2,$$

$$a_2 = \delta^2\delta_2^2\left(\delta^2 Le^{-1}\right)^2+\mathrm{Pr}_D\,\delta_1^2\left(\delta^2 Le^{-1}\right)^2 - \mathrm{Pr}_D\left(\delta^2 Le^{-1}\right)^2\delta_1^2\delta_2^2\lambda + Da\,\mathrm{Pr}_D\,\delta^4\left(\delta^2 Le^{-1}\right)^2 - Da\,\mathrm{Pr}_D\,\delta^4\delta_2^2\lambda\left(\delta^2 Le^{-1}\right)^2 + a^2 Le^{-1}\,\mathrm{Pr}_D\,Ra_S\,\delta^2 - a^2\,\mathrm{Pr}_D\,Ra_S\,\delta_2^2\phi.$$

Now Eq. (23) with $\Delta_2 = 0$ gives,

$$Ra_T{}^{osc} = -\frac{\sigma^2\delta^2}{a^2\,\mathrm{Pr}_D}+\frac{\left(\delta_1^2+Da\,\delta^4\right)\left(\delta_2^2+\lambda\sigma^2\right)}{a^2\left(1+\lambda^2\sigma^2\right)} + Ra_S\,\frac{\left(\sigma^2\phi+Le^{-1}\delta^2\delta_2^2\right)}{\left(Le^{-1}\delta^2\right)^2+\phi^2\sigma^2}, \tag{25}$$

The expression for the oscillatory Rayleigh number given by Eq. (25) in the limit $Da \to 0$ for densely packed porous medium, coincides exactly with the one given by Gaikwad and Bharati (2013) for isotropic case after making necessary rescaling.

4. WEAK NONLINEAR ANALYSIS

In this section, we consider the nonlinear analysis using a truncated representation of the Fourier series considering only two terms. We consider the early stages of nonlinear convection, when the basic structure of the convection is still determined bythe behavior of the linearized solution. In the immediate vicinity of the stability boundary, we could develop a weakly nonlinear analysis in which the amplitudes are no longer small but finite. A weak nonlinear stability analysis is performed using a truncated representation of the Fourier series method.

For simplicity of analysis, we confine ourselves to the two-dimensional rolls, so that all the physical quantities are independent of *y*. We introduce a stream function ψ such that $u=\partial\psi/\partial z,\ w=-\partial\psi/\partial x$ into the Eqs. (3) - (4) to obtain

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\mathrm{Pr}_D}\frac{\partial}{\partial t}\nabla^2\psi + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x}\right] + \left(\frac{\partial^2}{\partial x^2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\psi - Da\nabla^4\psi = 0, \tag{26}$$

$$\frac{\partial T}{\partial t} - \left(\eta\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)T - \frac{\partial\left(\psi,T\right)}{\partial\left(x,z\right)}+\frac{\partial\psi}{\partial x} = 0, \tag{27}$$

$$\left(\phi\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)S-\frac{\partial(\psi,S)}{\partial(x,z)}+\frac{\partial\psi}{\partial x}=0. \tag{28}$$

As mentioned earlier, we set $\xi=\eta=1$, for simplicity. For flows with $Ra_T > Ra_T^{st}$ linear stability analysis is not valid one has to take into account the non-linear effects. The first effect of non-linearity is to distort the temperature and concentration fields through the interaction of ψ, T and also ψ, S. The distortion of these fields will corresponds to a change in the horizontal mean, i.e., a component of the form $\sin(2\pi z)$ will be generated. Thus a minimal Fourier series which describes the finite amplitude free convection is given by

$$\psi = A(t)\sin(ax)\sin(\pi z), \tag{29}$$

$$T = B(t)\cos(ax)\sin(\pi z)+C(t)\sin(2\pi z) \tag{30}$$

$$S = E(t)\cos(ax)\sin(\pi z)+F(t)\sin(2\pi z) \tag{31}$$

where the amplitudes $A(t),\ B(t),\ C(t),\ E(t)\ and\ F(t)$ are to be determined from the dynamics of the system.

Substituting equations (29)-(31) into the coupled nonlinear system of partial differential equations (26)-(28) and equating the coefficients of like terms, we obtain the following non-linear autonomous system of differential equations

$\frac{dX}{dt}=D$ (32) where $X=(A,B,C,E,F)^T$, and $D=(D_1,D_2,D_3,D_4,D_5,D_6)^T$ with

$$D_1 = A,\quad D_2=-\left(aA+\delta_2^2B+\pi aAC\right),$$

$$D_3=-4\pi^2C+\frac{\pi a}{2}AB,$$

$$D_4=-\frac{1}{\phi}\left(aA+\frac{\delta^2}{Le}E+\pi aAF\right),$$

$$D_5=\frac{1}{\phi}\left(\frac{\pi a}{2}AE-\frac{4\pi^2}{Le}F\right),$$

$$D_6=-\frac{Pr_D}{\delta^2\lambda}\left(\begin{array}{l}\frac{\delta^2}{Pr_D}D_1+\left(\delta_1^2+Da\delta^4\right)A\\ +aRa_T\left(B+\lambda D_2\right)-aRa_S\left(E+\lambda D_4\right)\end{array}\right).$$

The non-linear system of autonomous differential equations is not suitable to analytical treatment for the general time-dependent variable and we have to solve it using a numerical method. However, one can make qualitative predictions as discussed below. The system of Eqs. (32) is uniformly bounded in time and possesses many properties of the full problem. Like the original Eqs. (2)-(4), Eqs. (32) must be dissipative. Thus, the volume in the phase space must contract. In order to prove volume contraction, we must show that the flow field has a constant negative divergence. Indeed

$$\frac{\partial}{\partial A}\left(\frac{dA}{dt}\right)+\frac{\partial}{\partial B}\left(\frac{dB}{dt}\right)+\frac{\partial}{\partial C}\left(\frac{dC}{dt}\right)+\frac{\partial}{\partial E}\left(\frac{dE}{dt}\right)+\frac{\partial}{\partial F}\left(\frac{dF}{dt}\right)=-\left(\begin{array}{l}\frac{\Pr_D}{\delta^2\lambda}\left(\delta_1^2+Da\delta^4\right)+\delta_2^2\\ +\frac{\delta^2}{Le}+4\pi^2\left(1+\frac{1}{Le}\right)+\frac{\delta^2}{Le}+\frac{1}{\lambda}\end{array}\right), \tag{33}$$

which is always negative and, therefore, the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase; in particular, they may be attracted to a fixed point, a limit cycle or perhaps, a strange attractor. From Eq. (33) we conclude that if a set of initial points in the phase occupies a region

$$V(t)=V(0)\exp\left\{-\left(\begin{array}{l}\frac{\Pr_D}{\delta^2\lambda}\left(\delta_1^2+Da\delta^4\right)+\delta_2^2+\frac{\delta^2}{Le}+\\ 4\pi^2\left(1+\frac{1}{Le}\right)+\frac{\delta^2}{Le}+\frac{1}{\lambda}\end{array}\right)t\right\}. \tag{34}$$

This expression indicates that the volume decreases exponentially with time. We can also infer that the relaxation parameter and Lewis number tend to enhance contraction.

4.1 Steady Finite Amplitude Motions

From qualitative predictions we look into possibility of an analytical solution. In case of steady motions, Eqs. (26) - (28) can be solved in closed form. The steady-state solutions are useful because they predict that a finite-amplitude solution to the system is possible for subcritical values of the Rayleigh number, and that the minimum values of Ra_T for which a steady solutions is possible lie below the critical values for instability perturbation, setting the left-hand side of Eqs. (32) equal to zero, and writing the amplitudes *B,C,E*, and *F* in terms of *A*, we get

$$A_1x^2+A_2x+A_3=0, \tag{35}$$

where $x=\frac{A^2}{8}$ and

$$A_1=a^4Le^2\left(\delta_1^2+Da\delta^4\right),$$

$$A_2=\left(\delta_1^2+Da\delta^4\right)\left(Le^2\delta_2^2+\delta^2\right)a^2+a^4Le\left(Ra_S-LeRa_T\right),$$

$$A_3=\left(\delta_1^2+Da\delta^4\right)\delta_2^2\delta^2+a^2\left(LeRa_S\delta_2^2-Ra_T\delta^2\right).$$

The required root of Eq. (35) is given by

$$x=\frac{1}{2A_1}\left(-A_2+\left(A_2^2-4A_1A_3\right)^{\frac{1}{2}}\right). \tag{36}$$

When we set the radical in the above equation vanish, we obtain the expression for the finite amplitude Rayleigh number $Ra_T{}^F$, which characterizes the onset of finite amplitude steady motions. The finite- amplitude Rayleigh number can be obtained in the form

$$Ra_T{}^F = \frac{1}{2B_1}\left(-B_2 + \left(B_2{}^2 - 4B_1B_3\right)^{\frac{1}{2}}\right), \tag{37}$$

where

$$B_1 = a^8 Le^2,$$
$$B_2 = -2a^8Le^3Ra_S + 2a^6DaLe^2\delta^6 + 2a^6Le^2\delta^6\delta_1^2 - 2a^6DaLe^4\delta^4\delta_2^2 - 2a^6Le^4\delta_2^2\delta_1^2,$$
$$\begin{aligned} B_3 = {}& a^8Le^2Ra_S{}^2 + 2a^6DaLeRa_S\delta^6 + a^4Da^2\delta^{12} \\ &+ 2a^6LeRa_S\delta^2\delta_1^2 + 2a^4Da\delta^8\delta_1^2 + a^4\delta^4\delta_1^4 \\ &- 2a^6DaLe^3Ra_S\delta^4\delta_2^2 - 2a^4Da^2Le^2\delta^{10}\delta_2^2 \\ &- 2a^6Le^3Ra_S\delta_2^2\delta_1^2 - 4a^4DaLe^2\delta^6\delta_1^2\delta_2^2 \\ &- 2a^4Le^2\delta^2\delta_1^4\delta_2^2 + a^4Da^2Le^4\delta^8\delta_2^4 \\ &+ 2a^4DaLe^4\delta^4\delta_1^2\delta_2^4 + a^4Le^4\delta_1^4\delta_2^4. \end{aligned}$$

The expression for the steady finite-amplitude Rayleigh number given by Eq. (37) is evaluated for critical values and the results are discussed.

4.2 Heat and Mass Transport

In the study of convection in fluids, the quantification of the heat and the mass transport is important. This is because the onset of convection, as the Rayleigh number is increased, is more readily detected by its effect on the heat and mass transport. In the basic state, heat and mass transport is by conduction alone.

If H and $J$are the rate of heat and mass transport per unit area, respectively, then

$$H = -\kappa_{Tz}\left\langle \frac{\partial T_{total}}{\partial z}\right\rangle_{z=0} \quad \text{and}$$
$$J = -\kappa_{Sz}\left\langle \frac{\partial S_{total}}{\partial z}\right\rangle_{z=0}, \tag{38}$$

where the angular brackets correspond to a horizontal average and

$$T_{total} = T_0 - \Delta T\frac{z}{d} + T(x,z,t) \quad \text{and}$$
$$S_{total} = S_0 - \Delta S\ \frac{z}{d} + S\ (x,z,t), \tag{39}$$

substituting Eq. (30) - (31) into Eqs. (39) and using the resultant equation in Eq. (38), we get

$$H = \frac{\kappa_{Tz}\Delta T}{d}(1-2\pi C) \ \text{ and } \ J = \frac{\kappa_{Sz}\Delta S}{d}(1-2\pi F). \tag{40}$$

The Nusselt (Nu) and Sherwood (Sh) numbers are respectively defined by

$$Nu = \frac{H}{\kappa_T\ \Delta T/d} = 1 - 2\pi C, \quad \text{and}$$
$$Sh = \frac{J}{\kappa_S\ \Delta S/d} = 1 - 2\pi F, \tag{41}$$

writing C and F in terms of A, and substituting into Eqs. (41), we obtain

$$Nu = 1 + \frac{2a^2x}{\delta_2^2 + a^2x}, \tag{42}$$

$$Sh = 1 + \frac{2x}{\left(\dfrac{\delta^2}{Le^2a^2} + x\right)}. \tag{43}$$

The second term on the right-hand sides of Eqs. (42) and (43) represent the convective contributions to heat and mass transport, respectively. Further we solved the system of autonomous Eqs. (32) numerically using the Runge–Kutta method, and trace the transient curves for heat and mass transfer for various values of the parameters.

5. RESULT AND DISCUSSION

The onset of double diffusive convection of a Maxwell fluid in a saturated anisotropic Darcy-Brinkman porous layer, which is heated and salted from below, is investigated analytically using both linear and nonlinear theories. The linear theory is used to obtain the criterion for the onset of stationary and oscillatory convection. The weakly nonlinear theory provides the quantification of heat and mass transport.

The neutral stability curves in the $Ra_T^{Osc} - a$ plane for various parameter values are as shown in figures1-5. We fixed the values for the parameters except the varying parameter. From these figures it is clear that the neutral curves are connected in a topological sense. This connection allows the linear stability criteria to be expressed in terms of the critical Rayleigh number Ra_{Tc}, below which the system is stable and unstable above. The stationary critical Rayleigh number is found to be independent of the viscoelastic parameter and therefore concurs with the classical results of double diffusive convection in porous medium for Newtonian fluid (see, e.g Nield and Bejan) (2006). In figure 1 the effect of Darcy number Da, on the onset of the oscillatory convection when all other parameters are fixed is shown. We observe that the minimum of Rayleigh number for oscillatory mode increases with increasing Darcy number Da. The size of Darcy number Da, is related to the importance of viscous effects at the boundaries and reduction in Da decreases this effect, which allows the fluid to move more freely thereby decreasing the oscillatory Rayleigh number.

The neutral stability curves for different values of scaled stress relaxation parameter λ for the oscillatory mode is presented in Fig. 2. We observe that increasing the stress relaxation parameter results in a decrease of the minimum of the oscillatory Rayleigh number up to a particular value of λ. And further increase of λ this trend reverses. Thus, the effect of an increase in the value of the stress relaxation parameter λ is to destabilize the system in oscillatory mode. Furthermore, we find from this figure that the minimum of the Rayleigh number shift toward the smaller values of the wave number with an increase of the stress relaxation parameter.

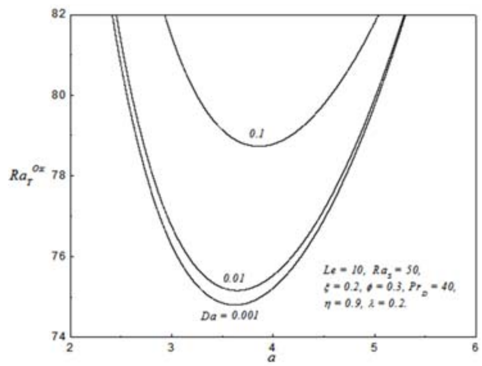

Fig. 1. Oscillatory neutr al stability curves for different values of Darcy number Da.

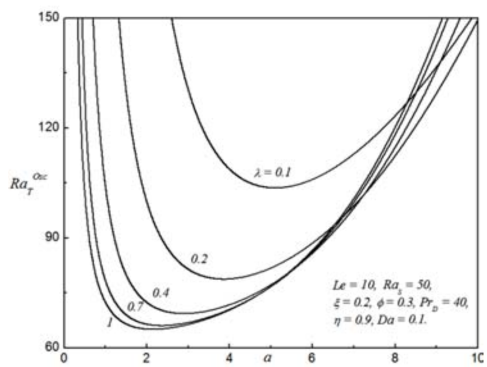

Fig. 2. Oscillatory neutr al stability curves for different values of Stress relaxation parameter λ.

In Fig. 3, the effect of the mechanical anisotropy parameter on the oscillatory Rayleigh number is depicted. We observe that the minimum of the Rayleigh number decreases with increasing mechanical anisotropy parameter, indicating that the mechanical anisotropy parameter advances the onset of oscillatory convection. The effect of mechanical anisotropy can be understood as follows; let us keep the vertical permeability K_z fixed and then an increased horizontal permeability K_x, reduces the critical Rayleigh number. This is due to the fact that increased permeability enhances the fluid mobility in the vertical direction and hence convection sets in early. On the other hand keep horizontal permeability K_x fixed and then increased vertical permeability K_z increases the critical Rayleigh number. Further, we find that the minimum of the Rayleigh number shift towards the smaller values of the wave number with increasing mechanical anisotropy parameter. This indicates that the cell width increases with an increase of mechanical anisotropy parameter.

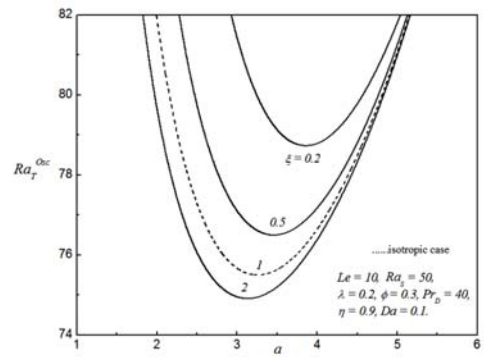

Fig. 3. Oscillatory neutr al stability curves for different values of Mechanical anisotropy parameter ξ.

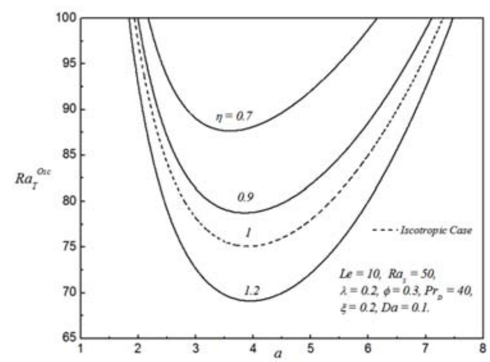

Fig. 4. Oscillatory neutr al stability curves for different values of Thermal anisotropy parameter η.

The effect of thermal anisotropy parameter η with Ra_T^{Osc} for the fixed values of other parameters is displayed in figure 4. It is found that Ra_T^{Osc} for the oscillatory mode decrease. Therefore, the effect of η is to advance the onset of oscillatory convection.

Figure 5 depicts the effect of solute Rayleigh number Ra_S on the neutral stability curves for oscillatory mode. We find that the effect of increasing Ra_S is to increase the value of the Rayleigh number for oscillatory mode. Thus, the solute Rayleigh number Ra_S has a stabilizing effect on the double-diffusive convection in porous medium.

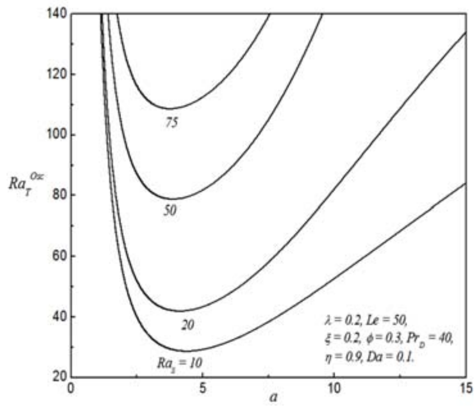

Fig. 5. Oscillatory neutr al stability curves for different values of Solute Rayleigh number Ras.

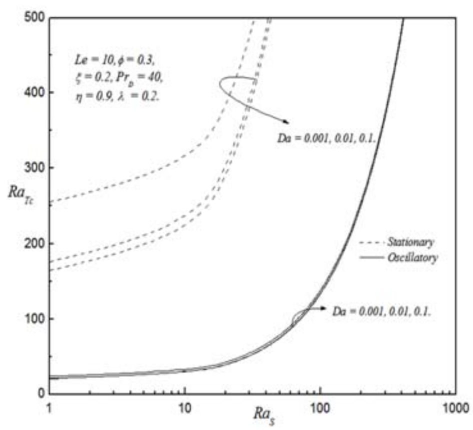

Fig. 6. Variation of the critical Rayleigh number with solute Rayleigh number for different values of the Darcy number Da.

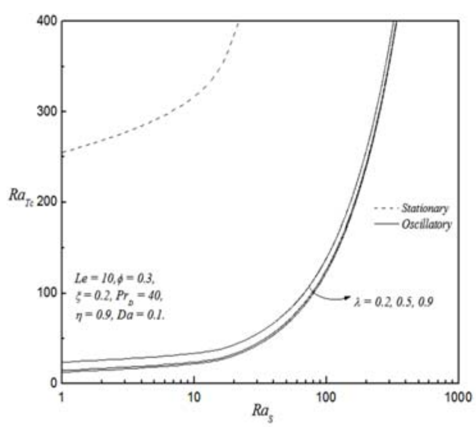

Fig. 7. Variation of the critical Rayleigh number with solute Rayleigh number for different values of stress relxation parameter λ.

The detailed behavior of the critical Rayleigh number for stationary and oscillatory modes with respect to the solute Rayleigh number is analyzed in the $Ra_{Tc} - Ra_S$ plane through figures 6-9. We find that the quantities namely, the critical Rayleigh number for stationary and oscillatory modes is increasing function of the solute Rayleigh number. It is clear that for the parameters chosen for these figures, the oscillatory convection sets in prior to the stationary convection.

Figure 6 displays the variation of Ra_{Tc} with Ra_S for different values of Darcy number Da. It is observed that with an increase of Da the oscillatory and the stationary critical Rayleigh number increase, implying that Da has a stabilizing effect on the system.

Fig. 7 reveals the effect of stress relaxation parameter λ on the critical Rayleigh number for the stationary and oscillatory modes with the solute Rayleigh number. The critical Rayleigh number for the oscillatory mode decreases with an increase of λ, indicating that the effect of the stress relaxation parameter is to destabilize the system in the oscillatory mode.

In Fig. 8, we show the effect of the mechanical anisotropy parameter ξ on the critical Rayleigh number Ra_{Tc} is shown. We find that increasing anisotropy parameter decreases the critical Rayleigh number indicating that the mechanical anisotropy parameter ξ destabilizes the system.

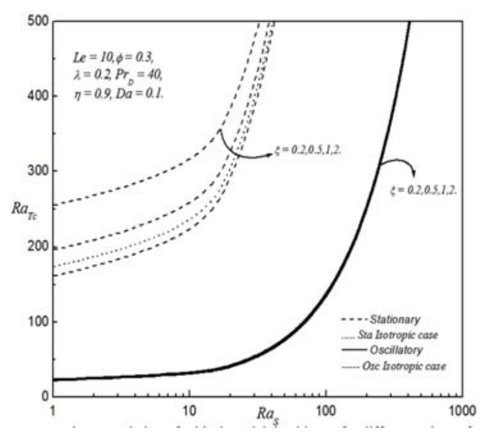

Fig. 8. Variation of the critical Rayleigh number with solute Rayleigh number for different values of mechanical anisotropy parameter ξ

In figure 9 the variation Ra_{Tc} with Ra_S for different values of thermal anisotropy parameter η is presented. It is noticed that an increase of η the critical Rayleigh number decreases for oscillatory mode, whereas it increases for the stationary mode implying that the effect of increasing thermal anisotropy parameter is to advance the onset of oscillatory convection and is to inhibit the onset of stationary convection as compared to the isotropic case.

To know the transient behavior of Nusselt and

Sherwood numbers, the autonomous system of ordinary differential equations is solved numerically using the Runge-Kutta method with suitable initial conditions. Then *Nu* and *Sh* are evaluated as a function of time t . The transient behavior of *Nu* and *Sh* is shown graphically through the figures 10-13. It is observed that both *Nu* and *Sh* start with a conduction state value i.e., 1 at $t=0$ and then oscillate periodically about their steady state value i.e., close to 3 for $t>0$ This periodic variation of *Nu* and *Sh* is very short lived and decays as time progresses. The values of Nusselt and Sherwood then tend toward their steady- state value of 3.

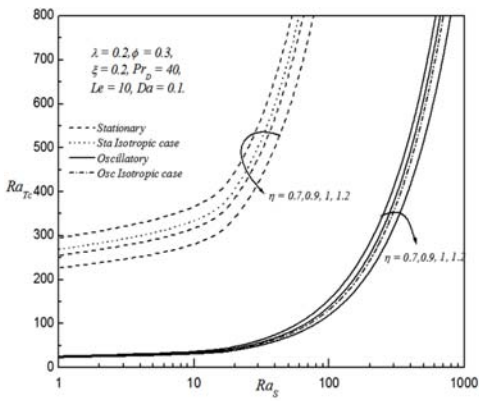

Fig. 9. Variation of the critical Rayleigh number with solute Rayleigh number for different values of thermal anisotropy parameter η.

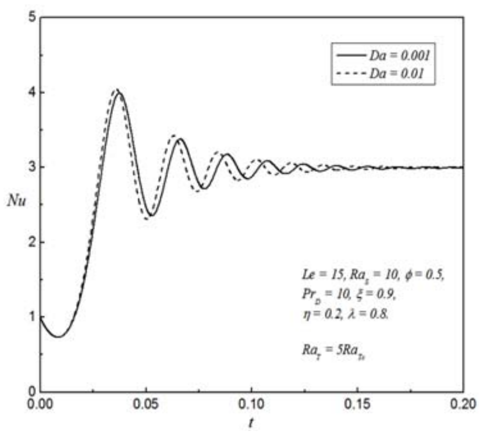

Fig. 10 (a). Variation of the Nusselt number with time for different values of the Darcy number Da.

The effect of the Darcy number is to increase the amplitude of the oscillations of heat and mass flux marginally is shown in figs 10(a) & (b). Figs 11(a) & (b) show the effect of the relaxation parameter λ on heat and mass transfer. We find that an increase in λincreases both *Nu* and *Sh* marginally. Figures 12(a) & (b) show the effect of mechanical anisotropy parameter ξ. We observe that an increase in the value of the mechanical anisotropy parameter suppresses both heat and mass transfer. In the Figs. 13(a) & (b) the effect of thermal anisotropy parameter η is to suppress both *Nu* and *Sh*. From Fig. 14 (a) and (b) we observe that an increase in the value of *Le* Lewis number enhances both heat and mass transfer.

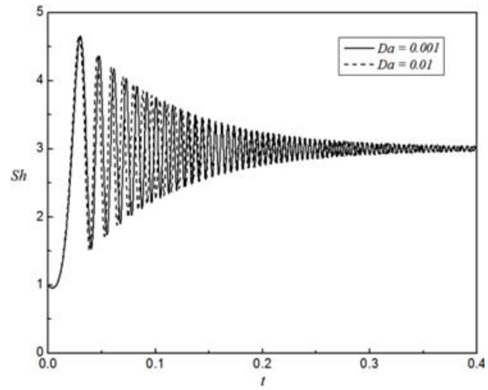

Fig. 10 (b). Variation of the Sherwood number with time for different values of the Darcy number Da.

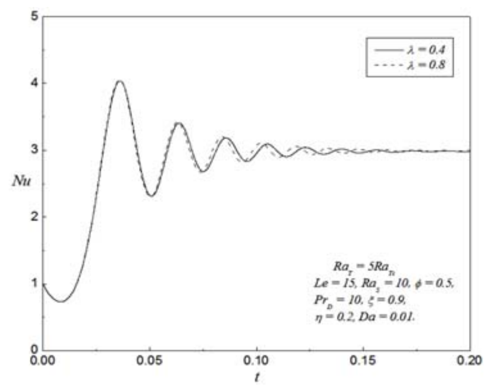

Fig. 11 (a). Variation of the Nusselt number with time for different values of stress relaxation parameter λ.

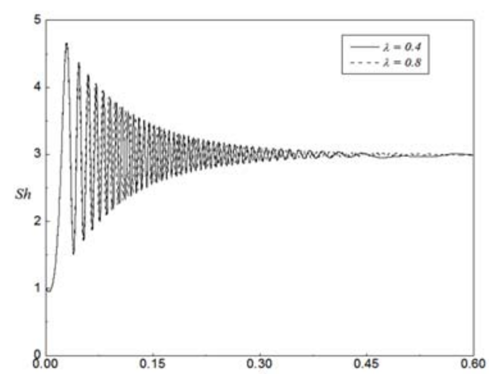

Fig. 11 (b). Variation of the Sherwood number with time for different values of stress relaxation parameter λ.

6. CONCLUSIONS

The onset of double diffusive convection of a Maxwell fluid in a saturated anisotropic Darcy-Brinkman porous layer is studied analytically using both linear and non-linear stability analyses. The linear analysis is based on the usual normal mode technique, while the non-linear analysis is based on truncated representation of the Fourier series. The modified Darcy-Brinkman Maxwell model is used for the momentum equation. The following conclusions are drawn:

1. The oscillatory mode is most favorable for a system with moderate and high values of the Darcy number Da and Solute Rayleigh number Ra_S. However for the large values of stress relaxation parameter λ, mechanical anisotropy parameter ξ and thermal anisotropy parameter is advance the onset of oscillatory convection.

2. The effect of increasing the value of thermal anisotropy parameter η in the presence of critical Rayleigh number Ra_S is to allow the onset of convection to be stationary rather than oscillatory. The value of stress relaxation parameter λ and mechanical anisotropy parameter ξ advance the onset of oscillatory convection. The effect of increasing the value of Darcy number Da in the presence of critical Rayleigh number Ra_S delay the onset of convection, whereas the stress relaxation parameter λ and mechanical anisotropy parameter ξ advances.

3. The effect of Darcy number Da and λ increases both heat and mass transfer. While the ξ mechanical anisotropy parameter suppresses and thermal anisotropy parameter η advances. The transient Nusselt and Sherwood numbers approach the steady state values for large time.

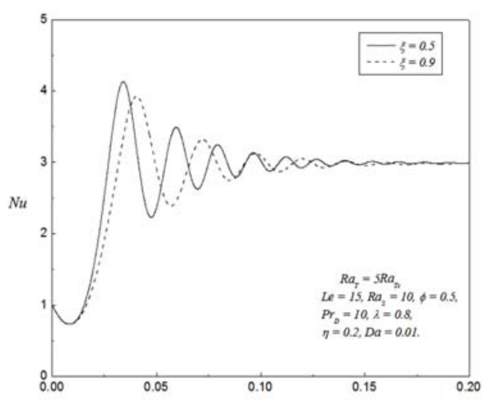

Fig. 12 (a). Variation of the Nusselt number with time for different values of mechanical anisotropy parameter ξ.

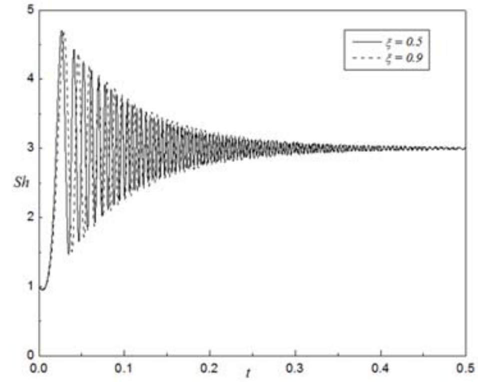

Fig. 12 (b). Variation of the Sherwood number with time for different values of mechanical anisotropy parameter ξ.

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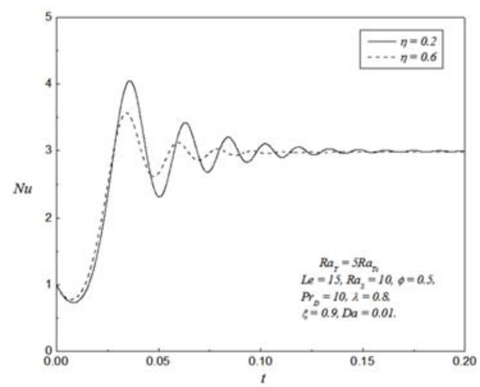

Fig. 13 (a). Variation of the Nusselt number with time for different values of mechanical anisotropy parameter η.

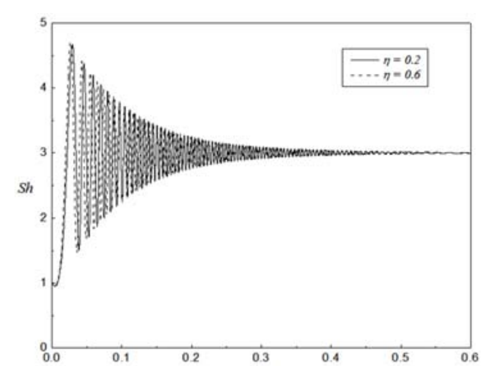

Fig. 13 (b). Variation of the Sherwood number with time for different values of thermal anisotropy parameter η.

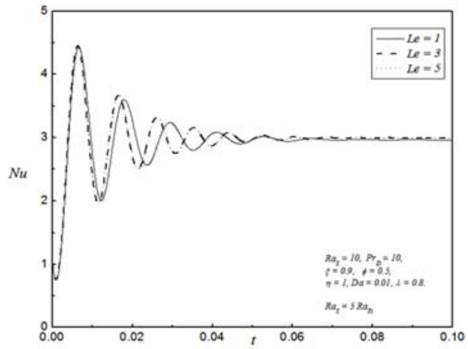

Fig. 14 (a). Variation of Nu with time for different values of Le.

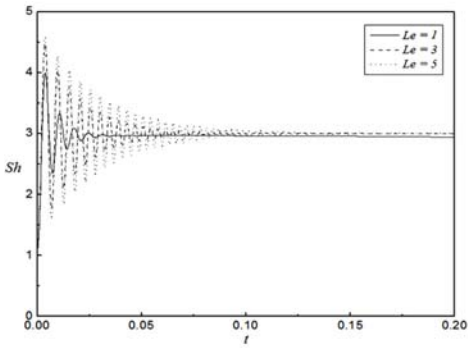

Fig. 14 (b). Variation of Sh with time for different values of Le.

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