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Effects of Horizontal Magnetic Field and Rotation on Thermal Instability of a Couple-Stress Fluid through a Porous Medium: a Brinkman Model

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ABSTRACT

A linear stability analysis is carried out to discuss the effects of horizontal magnetic field and horizontal rotation on thermal instability problem of a couple-stress fluid through a Brinkman porous medium. After employing normal mode method on the dimensionless linearized perturbation equations, it is noted that for the stationary state, Taylor number T_A promotes stabilization, whereas medium porosity $\in$ hastens the onset of convection. The medium permeability P, magnetic field Q, couple-stress Υ and Darcy-Brinkman parameter D_A play dual role in determining the stability/instability of the system under certain restrictions. Also, the sufficient conditions responsible for the non-existence of overstability are gained and the principle of exchange of stabilities holds good for a magneto-rotary system.

Keywords: Couple-stress fluid; Magnetic field; Rotation; Brinkman porous medium.

NOMENCLATURE

c_s	heat capacity of solid material
c_v	specific heat of the fluid at constant volume
t	time co-ordinate,
d	depth of fluid layer
D	differentiation Operator
D_{A_1}	Modified Darcy-Brinkman number
$\mathbf{H}$	horizontal magnetic field having components
$\mathbf{h}$	perturbation in magnetic field strength
k_1	darcy-Brinkman medium permeability
k_T	coefficient of heat conduction
k_x	wave number in x direction
k_y	wave number in y direction
k	Resultant wave number
$\mathbf{K}$	vertical component of magnetic field
n	frequency of the harmonic disturbance
p	pressure
P_l	dimensionless medium permeability
p_1	thermal Prandtl number
p_2	magnetic Prandtl number
$\mathbf{q}$	velocity of fluid having components
Q_1	Modified Chandrasekhar's number
R_1	Modified Darcy-Brinkman thermal Rayleigh number,
T_0	reference temperature
T	temperature
T_{A_1}	Modified Taylor's number
$\mathbf{w}$	vertical fluid velocity,
$\mathbf{w}^*$	complex conjugate of after applying normal mode method
$\mathbf{W}$	vertical component of fluid velocity after applying normal mode method
$\mathbf{X_i}$	gravitational acceleration vector
$\mathbf{X}$	vertical component of current density after applying normal mode method
$\mathbf{Z}$	vertical component of vorticity after applying normal mode method
∇p	pressure gradient term
$\in$	darcy-Brinkman medium porosity
ρ_0	density of fluid
ρ_s	density of solid material
μ	Fluid viscosity
μ'	couple-stress fluid viscosity

μ_{ef}	effective viscosity
μ_e	magnetic permeability
∂	curl Operator
α	co-efficient of thermal expansion
β	adverse temperature gradient
η	electrical resistivity
Θ	temperature component after applying normal mode method
δp	perturbation in fluid pressure
$\delta\rho$	perturbation in fluid density
υ	kinematic viscosity
υ'	kinematic viscoelasticity
κ	thermal diffusivity
ζ	z-component of vorticity
ξ	z-component of current density
$\mathbf{\Omega}$	horizontal rotational vector
∇^2	3-dimensional Laplacian operator
Υ_1	modified couple-stress parameter
σ	growth rate of harmonic disturbance after applying normal mode method,
θ	perturbation in temperature T
λ_i	vertical unit vector,

1. INTRODUCTION

A basic and rigorous overview about various fascinating and widespread engineering applications of fluid mechanical phenomena has been provided in detail by Bansal (2004) and Gupta and Gupta (2013). The problem of the onset of thermal instability of an incompressible Newtonian fluid has been studied widely by Chandrasekhar (1981) and Drazin and Reid (1981).Stokes (1966) has proposed and formulated the theory of couple-stress fluids which has become the objective of the scientific and technical research because of its vital importance to understand the mechanism and functioning of lubrication process in synovial joints. Magneto-hydrodynamics (MHD) theory of electrically conducting fluids in the presence of magnetic field has several scientific and practical applications in astrophysics, geophysics, space sciences etc. Magnetic field is also used in several clinical areas such as neurology and orthopaedics for probing and curing the internal organs of the body in several diseases like tumours detection, heart and brain diseases, stroke damage etc. Thermo-convective phenomenon in a rotating system is of practical significance and finds its applications in several scientific and industrial areas such as in rotating machinery, crystal growth, food processing industry, centrifugal casting of metals and thermal power plant. In an electric power plant, electricity is generated by the rotation of turbine blades. Kumar *et al*. (2014a) analyzed theoretically thermal instability problem of a compressible ferromagnetic fluid under the effects of rotation and heat source strength through a porous medium. Thermal instability problem of an electrically conducting couple-stress fluid heated from below through a porous medium in the presence of a uniform magnetic field has been investigated by Sharma and Thakur (2000). Sharma and Sharma (2004) have considered the effect of suspended particles on couple-stress fluid heated from below in the presence of vertical rotation and vertical magnetic field and noted that the effect of rotation is to stabilize the system, whereas the suspended particles have destabilizing effects. Thermosolutal convective problem for a couple-stress fluid through a porous medium under the influences of vertical magnetic field and vertical rotation has been studied by Kumar (2012) and observed that rotation has a stabilizing effect, whereas magnetic field and couple-stress have both stabilizing and destabilizing effects on the system.

The flow of fluid through a porous medium is of vital importance in several sectors such as in solidification, biological studies, chemical processing industry, geophysical fluid dynamics, petroleum industry, filtering equipment, recovery of crude oil from earth's interior etc. A comprehensive and detailed investigation of thermo-convective problem through various porous mediums such as Darcy model, Brinkman model and Forchheimer model has been given in the famous book by Nield and Bejan (2006). It has been found, both theoretically and experimentally, that Darcy's equation provides unsatisfactory results of the hydrodynamic conditions, particularly near the boundaries of a porous medium (Beavers *et al*. 1970). It is also believed that for a flow of high porosity Brinkman model is more superior over the usual Darcy model and also makes the system thermally more stable than the Darcy model. The physical properties of comets, meteorites and interplanetary dust strongly motivate to study the impact of porosity in astrophysical situations (McDonnel 1978). The global stability problem for thermal convection in a couple-stress fluid through a porous medium using thermal non-equilibrium model has been carried out by Sunil *et al*. (2013) and concluded that both couple-stress parameter and Darcy-Brinkman number expands the region of stability, whereas medium porosity contracts the stability region. Kumar *et al*. (2013, 2014b, 2015) have investigated theoretically the thermal instability problem of an Oldroydian and couple-stress fluid by considering the effects of various parameters such as vertical rotation, vertical magnetic field, suspended particles and variable gravity considering Darcy or Brinkman porous medium.

The intention is to study the impact of horizontal magnetic field and horizontal rotation for the present theoretical investigation and also to find out any similarities or differences between the present findings with those the previous findings (Kumar 2012, Kumar *et al*. 2014b, Sharma and Sharma 2004).

2. GOVERNING EQUATIONS

Here, we consider an infinite horizontal layer of an incompressible couple-stress fluid bounded by two horizontal boundaries separated at a distance *d* apart through a Brinkman porous medium. The fluid layer is subjected to a uniform horizontal magnetic field $\mathbf{H}=(H,0,0)$ and a uniform horizontal rotation $\mathbf{\Omega}=(\Omega,0,0)$. A uniform temperature gradient $\beta=|dT/dz|$ is maintained across the layer by underside heating.

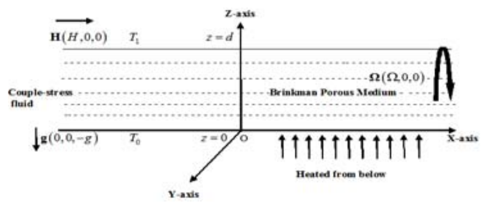

Fig. a. Geometrical sketch of the physical problem.

The governing equations of conservation of linear momentum and conservation of mass for a couple-stress fluid saturating a Brinkman (1947a, b) porous medium and subjected to Boussinesq approximation (1903) are presented as

$$\frac{\rho_0}{\in}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\in}(\mathbf{q}\nabla)\mathbf{q}\right]=\left[\begin{array}{l}-\nabla p+\rho_0\mathbf{X_i}-\frac{1}{k_1}\left(\mu-\mu'\nabla^2\right)\mathbf{q}+\frac{2\rho_0}{\in}\\ \left(\mathbf{q}\times\mathbf{\Omega}\right)+\left(\frac{\tilde{\mu}_{ef}}{\in}\right)\nabla^2\mathbf{q}+\frac{\mu_e}{4\pi}(\nabla\times\mathbf{H})\times\mathbf{H}\end{array}\right] \tag{1}$$

$$\nabla.\mathbf{q}=0 \tag{2}$$

The equation for temperature balance is presented as

$$\left[\in\rho_0 c_v+\rho_s c_s(1-\in)\right]\frac{\partial T}{\partial t}+\rho_0 c_v(\mathbf{q}.\nabla)T=k_T\nabla^2 T \tag{3}$$

The Maxwell's equations (1866) of electromagnetism are presented as

$$\in\frac{\partial \mathbf{H}}{\partial t}=\nabla\times(\mathbf{q}\times\mathbf{H})+\in\eta\nabla^2\mathbf{H} \tag{4}$$

and $\nabla.\mathbf{H}=0$ (5)

The density equation of state is given by

$$\rho=\rho_0\left[1+\alpha\left(T_0-T\right)\right] \tag{6}$$

3. PERTURBATION TECHNIQUE AND NORMAL MODE METHOD

To determine the stability or instability of the basic state of the system, infinitesimal perturbations are introduced in various physical quantities. Let $\mathbf{q}(u,v,w),\theta,\delta p,\delta\rho,\mathbf{h}(h_x,h_y,h_z)$ denote, respectively, the perturbations in $\mathbf{q}(0,0,0),T,p,\rho,\mathbf{H}$.

The density variation $\delta\rho$ due to perturbation in temperature (θ) is given by

$$\delta\rho=-\alpha\theta\rho_0 \tag{7}$$

By ignoring the nonlinear terms and assuming the perturbation quantities to be very small, the governing linearized perturbation equations (after eliminating the pressure gradient term) are defined as

$$\frac{1}{\in}\left\{\frac{\partial}{\partial t}\left(\nabla^2\mathbf{w}\right)\right\}=\left[\begin{array}{l}\mathbf{g}\alpha\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\theta-\frac{2\Omega}{\in}\left(\frac{\partial\zeta}{\partial x}\right)+\\ \left(\frac{\tilde{\mu}_{ef}}{\rho_0\in}\right)\nabla^4\mathbf{w}+\frac{\mu_e\mathbf{H}}{4\pi\rho_0}\left\{\frac{\partial}{\partial x}\left(\nabla^2\mathbf{h_z}\right)\right\}\\ -\frac{1}{k_1}\left(\upsilon-\upsilon'\nabla^2\right)\nabla^2\mathbf{w}\end{array}\right] \tag{8}$$

$$\frac{1}{\in}\left(\frac{\partial\zeta}{\partial t}\right)=\left[\begin{array}{l}\frac{\mu_e\mathbf{H}}{4\pi\rho_0}\left(\frac{\partial\xi}{\partial x}\right)-\frac{1}{k_1}\left(\upsilon-\upsilon'\nabla^2\right)\zeta\\ +\frac{2\Omega}{\in}\left(\frac{\partial\mathbf{w}}{\partial x}\right)+\left(\frac{\tilde{\mu}_{ef}}{\rho_0\in}\right)\left(\nabla^2\zeta\right)\end{array}\right] \tag{9}$$

$$\in\left(\frac{\partial\mathbf{h_z}}{\partial t}\right)=\mathbf{H}\left(\frac{\partial\mathbf{w}}{\partial x}\right)+\in\eta\left(\nabla^2\mathbf{h_z}\right) \tag{10}$$

$$\in\left(\frac{\partial\xi}{\partial t}\right)=\mathbf{H}\left(\frac{\partial\zeta}{\partial x}\right)+\in\eta\left(\nabla^2\xi\right) \tag{11}$$

$$E\left(\frac{\partial\theta}{\partial t}\right)=\beta\mathbf{w}+\kappa\nabla^2\theta \tag{12}$$

where,

$\upsilon=\frac{\mu}{\rho_0}$, $\upsilon'=\frac{\mu'}{\rho_0}$, $\kappa=\frac{k_T}{\rho_0 c_v}$, $\zeta=\left(\frac{\partial\mathbf{v}}{\partial x}-\frac{\partial\mathbf{u}}{\partial y}\right)$,

$\xi=\left(\frac{\partial\mathbf{h_y}}{\partial x}-\frac{\partial\mathbf{h_x}}{\partial y}\right)$,

$\nabla^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$,

$\lambda_i=(0,0,1)$ and $E=\in+(1-\in)\left(\frac{\rho_s c_s}{\rho_0 c_v}\right)$.

Now, using normal mode method by decomposing the disturbances in the following form

$$\left[\mathbf{w},\theta,\zeta,\mathbf{h_z},\xi\right]=\left[\mathbf{W}(\mathbf{z}),\Theta(z),\mathbf{Z}(\mathbf{z}),\mathbf{K}(\mathbf{z}),\mathbf{X}(\mathbf{z})\right]\exp\left(ik_x x+ik_y y+nt\right) \tag{13}$$

and also making the equations (8)-(12) dimensionless by introducing the scaling of the form

$z^*=\left(\frac{z}{d}\right)$, $k=\left(\frac{a}{d}\right)$, $\sigma=\frac{nd^2}{\upsilon}$,

$P_l=\frac{k_1}{d^2}$, $p_1=\frac{\upsilon}{\kappa}$, $p_2=\frac{\upsilon}{\eta}$, $\Upsilon=\frac{\upsilon'}{\upsilon d^2}$

and $D_A=\left(\frac{\tilde{\mu}_{ef}P_l}{\mu}\right)$, We obtain the following dimensionless equations (after dropping the asterisk for convenience) as

$$\left[\frac{\sigma}{\in}+\frac{1}{P_l}\left\{1-\Upsilon\left(D^2-a^2\right)\right\}-\frac{D_A}{P_l\in}\left(D^2-a^2\right)\right]\left(D^2-a^2\right)$$
$$\mathbf{W}(\mathbf{z})+\frac{g\alpha a^2d^2}{\upsilon}\Theta(z)+\frac{2\mathbf{\Omega}ik_xd^4}{\in\upsilon}\mathbf{Z}(\mathbf{z})-\frac{\mu_e\mathbf{H}ik_xd^2}{4\pi\rho_0\upsilon}$$
$$\left(D^2-a^2\right)\mathbf{K}(\mathbf{z})=0 \quad (14)$$

$$\left[\frac{\sigma}{\in}+\frac{1}{P_l}\left\{1-\Upsilon\left(D^2-a^2\right)\right\}-\frac{D_A}{P_l\in}\left(D^2-a^2\right)\right]\mathbf{Z}(\mathbf{z})$$
$$=\frac{2\mathbf{\Omega}ik_xd^2}{\in\upsilon}\mathbf{W}(\mathbf{z})+\frac{\mu_e\mathbf{H}ik_xd^2}{4\pi\rho_0\upsilon}\mathbf{X}(\mathbf{z}) \quad (15)$$

$$\left[p_2\sigma-\left(D^2-a^2\right)\right]\in\mathbf{K}(\mathbf{z})=\left(\frac{\mathbf{H}ik_xd^2}{\eta}\right)\mathbf{W}(\mathbf{z}) \quad (16)$$

$$\left[p_2\sigma-\left(D^2-a^2\right)\right]\in\mathbf{X}(\mathbf{z})=\left(\frac{\mathbf{H}ik_xd^2}{\eta}\right)\mathbf{Z}(\mathbf{z}) \quad (17)$$

$$\left[\left(D^2-a^2\right)-p_1E\sigma\right]\Theta(z)=-\left(\frac{\beta d^2}{\kappa}\right)\mathbf{W}(\mathbf{z}) \quad (18)$$

where, k_x and k_y are horizontal wave numbers, $k^2=\left(k_x{}^2+k_y{}^2\right)$ is a dimensionless resultant wave number and *n* is the growth rate (the stability parameter) of harmonic disturbance.

The appropriate boundary conditions (for free boundaries and non-conducting medium) are defined as

$$\left.\begin{array}{l} W=D^2W=\Theta=DZ=X=0,\text{ at } z=0 \text{ and } 1\\ \text{and } h_x, h_y, h_z \text{ are continuous at } z=0 \text{ and } 1.\end{array}\right\} \quad (19)$$

Eliminating $\Theta(z)$, $\mathbf{X}(\mathbf{z})$, $\mathbf{Z}(\mathbf{z})$ and $\mathbf{K}(\mathbf{z})$ from the equations (14) - (18) and taking a suitable solution for *W* of the form

$$W=W_0\sin l\pi z;\quad l=(1,2,3,4........) \quad (20)$$

the dispersion relation is obtained as

$$\left[A_1A_2A_3\left\{A_1A_3\in+Q_1x\cos^2\theta\right\}\right](1+x)+\left(\frac{T_{A_1}}{\in}\right)$$
$$\left(A_2A_3{}^2x\cos^2\theta\right)+\left(\frac{Q_1}{\in}\right)\left\{A_1A_3\in+Q_1x\cos^2\theta\right\}$$
$$\left\{A_2x\cos^2\theta(1+x)\right\}-R_1xA_3\left[A_1A_3\in+Q_1x\cos^2\theta\right]=0 \quad (21)$$

where, in obtaining equation (21), following modified parameters are assumed as

$$R_1=\frac{R}{l^4\pi^4},\ x=\frac{a^2}{l^2\pi^2},\ i\sigma_1=\frac{\sigma}{l^2\pi^2},\ P_l=\frac{P}{l^2\pi^2},$$
$$k^2=\frac{l^2\pi^2x}{d^2},Q_1=\frac{Q}{l^2\pi^2},\ \Upsilon=\frac{\Upsilon_1}{l^2\pi^2},T_{A_1}=\frac{T_A}{l^4\pi^4},$$
$$D_A=\frac{D_{A_1}}{l^2\pi^2},\qquad k_x=k\cos\theta,\qquad R=\frac{g\alpha\beta d^4}{\upsilon\kappa}$$

the Darcy-Brinkman thermal Rayleigh number, $T_A=\frac{4\mathbf{\Omega}^2d^4}{\upsilon^2}$ the Taylor number, $Q=\frac{\mu_e\mathbf{H}^2d^2}{4\pi\rho_0\upsilon\eta}$ the Chandrasekhar's number,

$$A_1=\left[\frac{i\sigma_1}{\in}+\frac{1}{P}\left\{1+\Upsilon_1(1+x)\right\}+\frac{D_{A_1}}{P\in}(1+x)\right],$$
$$A_2=\left[(1+x)+ip_1E\sigma_1\right]\text{ and }A_3=\left[(1+x)+ip_2\sigma_1\right].$$

Equation (21) is the required dispersion relation accounting the effects of horizontal magnetic field, horizontal rotation, medium permeability and medium porosity on thermal instability of a couple-stress fluid saturating a Brinkman porous medium.

4. CASE OF STATIONARY STATE

For the case of stationary convection, the marginal state will be defined by $\sigma=0$ i.e. when the growth rate vanishes. Substituting $\sigma=0$ in equation (21), an expression for the case of stationary instability is obtained as

$$R_1=\frac{1}{x}\left[\begin{array}{l}\frac{(1+x)^2}{P}\left\{1+\left(\Upsilon_1+\frac{D_{A_1}}{\in}\right)(1+x)\right\}+\frac{Q_1x(1+x)\cos^2\theta}{\in}\\ +\frac{T_{A_1}x(1+x)^2\cos^2\theta}{\in\left[\frac{\in(1+x)}{P}\left\{1+\left(\Upsilon_1+\frac{D_{A_1}}{\in}\right)(1+x)\right\}+Q_1x\cos^2\theta\right]}\end{array}\right] \quad (22)$$

The effect of various embedded parameters on thermal instability problem can be examined with the help of the following derivatives $\frac{dR_1}{dQ_1},\frac{dR_1}{dT_{A_1}},\frac{dR_1}{dP},\frac{dR_1}{d\in},\frac{dR_1}{d\Upsilon_1}$ and $\frac{dR_1}{dD_{A_1}}$, analytically.

On differentiating equation (22) w.r.t. different parameter yields

$$\frac{dR_1}{dQ_1}=\frac{(1+x)\cos^2\theta}{\in}\left[1-\frac{T_{A_1}P^2x(1+x)\cos^2\theta}{\left[\in(1+x)G+Q_1Px\cos^2\theta\right]^2}\right] \quad (23)$$

$$\frac{dR_1}{dT_{A_1}}=\left[\frac{(1+x)^2P\cos^2\theta}{\in\left[\in(1+x)G+Q_1Px\cos^2\theta\right]}\right] \quad (24)$$

$$\frac{dR_1}{dP}=\frac{(1+x)^2G}{x}\left[-\frac{1}{P^2}+\frac{T_{A_1}x(1+x)G\cos^2\theta}{\left[\in(1+x)G+Q_1Px\cos^2\theta\right]^2}\right] \tag{25}$$

$$\frac{dR_1}{d\in}=-\frac{1}{x}\left[\begin{array}{c}\frac{D_{A_1}(1+x)^3}{P\in^2}+\frac{Q_1x(1+x)\cos^2\theta}{\in^2}+\\ \frac{T_{A_1}Px(1+x)^2\cos^2\theta}{\in\left[\in(1+x)G+Q_1Px\cos^2\theta\right]}\\ \left\{\frac{(1+x)\{1+\Upsilon(1+x)\}}{\left[\in(1+x)G+Q_1Px\cos^2\theta\right]}+\frac{1}{\in}\right\}\end{array}\right] \tag{26}$$

$$\frac{dR_1}{d\Upsilon_1}=\frac{(1+x)^3}{Px}\left[1-\frac{T_{A_1}P^2x(1+x)\cos^2\theta}{\left[\in(1+x)G+Q_1Px\cos^2\theta\right]^2}\right] \tag{27}$$

$$\frac{dR_1}{dD_{A_1}}=\frac{(1+x)^3}{\in Px}\left[1-\frac{T_{A_1}P^2x(1+x)\cos^2\theta}{\left[\in(1+x)G+Q_1Px\cos^2\theta\right]^2}\right] \tag{28}$$

where, $G=\left\{1+\left(\Upsilon_1+\frac{D_{A_1}}{\in}\right)(1+x)\right\}$.

From the derivative equations (23) – (28), the stabilizing effect of Taylor number and destabilizing effect of medium porosity is confirmed. The magnetic field, couple-stress and Darcy-Brinkman parameter have stabilizing (or destabilizing) effect and the medium permeability has a destabilizing (or stabilizing) on thermal instability if $\left[\in(1+x)G+Q_1Px\cos^2\theta\right]^2>(\text{or}<)T_{A_1}P^2x(1+x)\cos^2\theta$ respectively. In the absence of rotation $(i.e.\,T_{A_1}=0)$, magnetic field, couple-stress and Darcy-Brinkman parameter always delay the onset of thermal convection, whereas medium permeability assures the destabilizing effect on the system.

5. PRINCIPLE OF EXCHANGE OF STABILITIES ANDOSCILLATORY MODES

Now, the conditions for which principle of exchange of stabilities is satisfied and the possibility of oscillatory modes for the couple-stress fluid under the effects of horizontal rotation and horizontal magnetic field through a Brinkman porous medium are determined.

For this, multiplying equation (14) by W^*, integrating it over the range of z and using equations (15)-(18) leads to

$$\left\{\left(\frac{\sigma}{\in}+\frac{1}{P_l}\right)I_1+\frac{(\in\Upsilon+D_A)}{P_l\in}I_2\right\}-\frac{\mu_e\in}{4\pi\rho_0p_2}\left(p_2\sigma^*I_3+I_4\right)$$
$$-\frac{g\alpha a^2}{\beta p_1}\left(I_5+p_1E\sigma^*I_6\right)+\frac{\mu_e d^2\in}{4\pi\rho_0p_2}(p_2\sigma I_7+I_8)-d^2$$
$$\left\{\left(\frac{\sigma^*}{\in}+\frac{1}{P_l}\right)I_9+\frac{(\in\Upsilon+D_A)}{P_l\in}I_{10}\right\}=0 \tag{29}$$

where, $I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz$,

$I_2=\int_0^1\left(|D^2W|^2+a^4|W|^2+2a^2|DW|^2\right)dz$,

$I_3=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz$,

$I_4=\int_0^1\left(|D^2K|^2+a^4|K|^2+2a^2|DK|^2\right)dz$,

$I_5=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz$, $I_6=\int_0^1\left(|\Theta|^2\right)dz$,

$I_7=\int_0^1\left(|X|^2\right)dz$, $I_8=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz$,

$I_9=\int_0^1\left(|Z|^2\right)dz$, $I_{10}=\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz$

The integrals I_1-I_{10} are positive definite. Putting $\sigma=\sigma_r+i\sigma_i$ in equation (29) and equating the real and imaginary parts gives

$$\sigma_r\left\{\frac{I_1}{\in}-\frac{g\alpha a^2EI_6}{\beta}-\frac{\mu_e\in I_3}{4\pi\rho_0}-d^2\left(\frac{I_9}{\in}\right)+\frac{\mu_e\in d^2I_7}{4\pi\rho_0}\right\}$$
$$+\left\{\begin{array}{l}\frac{I_1}{P_l}+\frac{(\in\Upsilon+D_A)}{P_l\in}\left(I_2+d^2I_{10}\right)-\frac{g\alpha a^2I_5}{\beta p_1}\\ -\frac{\mu_e\in I_4}{4\pi\rho_0p_2}-d^2\left(\frac{I_9}{P_l}\right)+\frac{\mu_e\in d^2I_8}{4\pi\rho_0p_2}\end{array}\right\}=0 \tag{30}$$

and

$$\sigma_i\left\{\frac{I_1}{\in}+\frac{g\alpha a^2EI_6}{\beta}+\frac{\mu_e\in I_3}{4\pi\rho_0}+d^2\left(\frac{I_9}{\in}\right)+\frac{\mu_e\in d^2I_7}{4\pi\rho_0}\right\}=0 \tag{31}$$

Equation (30) implies that either $\sigma_r>0$ or $\sigma_r<0$ which implies that the system may be unstable or stable. Hence the modes may be oscillatory or non-oscillatory, respectively. It is obvious from equation (31) that the quantity inside the bracket is positive. Thus, $\sigma_i=0$ which shows that the oscillatory modes are not allowed in the system and the principle of exchange of stabilities is satisfied.

6. CASE OF OVERSTABILITY

Here, the possibility of whether instability may occur as overstability has been examined. Equating the real and imaginary parts of equation (21) yields

$$\begin{bmatrix} \left\{G^{*2}-\dfrac{\sigma_1^2}{\in^2}\right\}\left\{(1+x)^3-\sigma_1^2 p_2(1+x)(p_2+2p_1E)\right\} \\ -\left(\dfrac{2\sigma_1^2G^*}{\in}\right)\left[\left\{(1+x)^2-\sigma_1^2p_2^2\right\}p_1E+2p_2(1+x)^2\right] \end{bmatrix}$$

$$\in(1+x)+2Q_1x\cos^2\theta(1+x)\begin{bmatrix} G^*\left\{(1+x)^2-\sigma_1^2p_1p_2E\right\} \\ -\dfrac{\sigma_1^2(1+x)(p_1E+p_2)}{\in} \end{bmatrix}$$

$$-R_1x\in\left[G^*\left\{(1+x)^2-\sigma_1^2p_2^2\right\}-\frac{2\sigma_1^2p_2(1+x)}{\in}\right]-R_1x^2$$

$$Q_1\cos^2\theta(1+x)+\frac{Q_1^2x^2\cos^4\theta(1+x)}{\in}+\frac{T_{A_1}x\cos^2\theta(1+x)}{\in}$$

$$\left[\left\{(1+x)^2-\sigma_1^2p_2^2\right\}-2\sigma_1^2p_1p_2E\right]=0 \tag{32}$$

and

$$\begin{bmatrix} \left(\dfrac{2\sigma_1G^*}{\in}\right)\left\{(1+x)^3-\sigma_1^2p_2(1+x)(p_2+2p_1E)\right\}+\sigma_1 \\ \left\{G^{*2}-\dfrac{\sigma_1^2}{\in^2}\right\}\left[\left\{(1+x)^2-\sigma_1^2p_2^2\right\}p_1E+2p_2(1+x)^2\right] \end{bmatrix}\in$$

$$(1+x)+2Q_1x\cos^2\theta\sigma_1(1+x)\begin{bmatrix} G^*(1+x)(p_1E+p_2) \\ +\dfrac{\left\{(1+x)^2-\sigma_1^2p_1p_2E\right\}}{\in} \end{bmatrix}-R_1x$$

$$\sigma_1\in\left[2G^*p_2(1+x)+\frac{\left\{(1+x)^2-\sigma_1^2p_2^2\right\}}{\in}\right]-R_1x^2Q_1\cos^2\theta\sigma_1p_2$$

$$+\frac{T_{A_1}x\cos^2\theta\sigma_1}{\in}\left[2p_2(1+x)^2+p_1E\left\{(1+x)^2-\sigma_1^2p_2^2\right\}\right]=0 \tag{33}$$

Eliminating R_1 from equations (32) and (33) and assuming $\sigma_1^2=y$, a three degree polynomial in *y* is obtained as

$$a_0y^3+a_1y^2+a_2y+a_3=0 \tag{34}$$

where,

$$a_0=\left[\frac{(1+x)^2p_2^4}{\in}+2p_1p_2^3E(1+x)\left\{p_2G^*+\frac{(1+x)}{\in}\right\}\right] \tag{35}$$

and

$$a_3=\begin{bmatrix} G^{*2}\in(1+x)^6+G^{*3}\in^2p_1E(1+x)^5+(1+x)^5x \\ \cos^2\theta\left\{2Q_1G^*-\dfrac{T_{A_1}}{\in}\right\}+Q_1x\cos^2\theta G^{*2}\in(1+x)^4 \\ (3p_1E-p_2)+2Q_1^2x^2\cos^4\theta\left\{p_1E(1+x)-p_2\right\} \\ G^*(1+x)^2+\dfrac{Q_1^2x^2\cos^4\theta(1+x)}{\in}\begin{Bmatrix} 2(1+x)^3- \\ Q_1x\cos^2\theta p_2 \end{Bmatrix} \\ +\dfrac{Q_1x^2\cos^4\theta(1+x)^3}{\in}\left\{T_{A_1}(p_1E+p_2)-Q_1\right\}+T_{A_1} \\ x\cos^2\theta p_1EG^*(1+x)^4 \end{bmatrix} \tag{36}$$

where, $G^*=\left(\dfrac{G}{P}\right)=\dfrac{1}{P}\left\{1+\left(\Upsilon_1+\dfrac{D_{A_1}}{\in}\right)(1+x)\right\}.$

The coefficients a_1 and a_2 involving large number of terms are not included as they don't play any role in determining the overstability of the system.

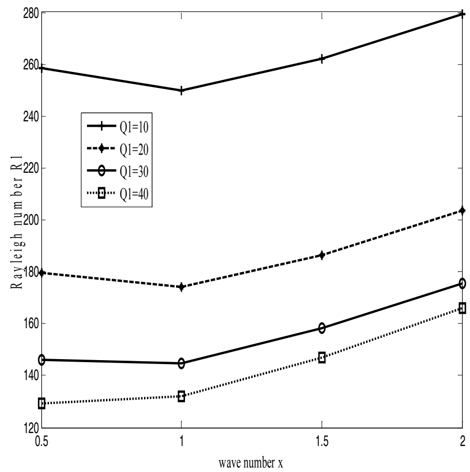

Fig. 1. Variations of Rayleigh number R_1 with wave number x for various values of magnetic field $Q_1=(10,20,30,40,)$ and fixed values of $\in=1, P=5, D_{A_1}=2, \Upsilon_1=2, T_{A_1}=1000, \theta=45°$.

Since σ_1 must be real for overstability to occur, therefore all the three roots of *y* should be positive.

From equation (34), the product of roots $=\left(-\dfrac{a_3}{a_0}\right)$ i.e. negative and this has to be positive.

Since a_0 is always positive as obvious from equation (35) and a_3 will be positive if

$$2\in Q_1G^*>T_A,\ 3p_1E>p_2,\ p_1E(1+x)>p_2,$$
$$2(1+x)^3>Q_1p_2x\cos^2\theta, T_{A_1}(p_1E+p_2)>Q_1. \tag{37}$$

The inequalities (37) are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

The effects of various embedded parameters (medium permeability, medium porosity, magnetic field, rotation, couple-stress, Darcy-Brinkman) on thermo-convective problem and also the variations in Rayleigh number under these physical parameters are depicted graphically in the Figs. 1 - 6.

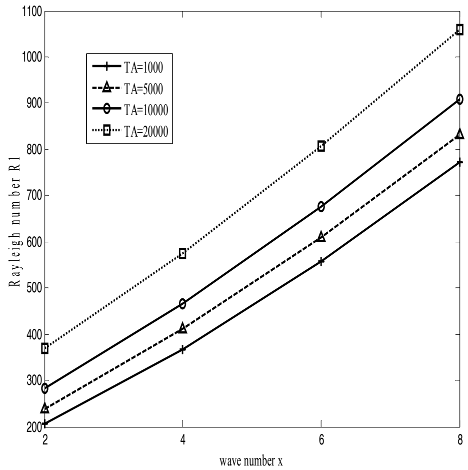

Fig. 2. Variations of Rayleigh number R_1 with wave number X for various values of rotation parameter T_{A_1} $(1000, 5000, 10000, 20000)$ and fixed values of $\in = 2, P = 3, D_{A_1} = 10, \Upsilon_1 = 5, Q_1 = 200, \theta = 45°$.

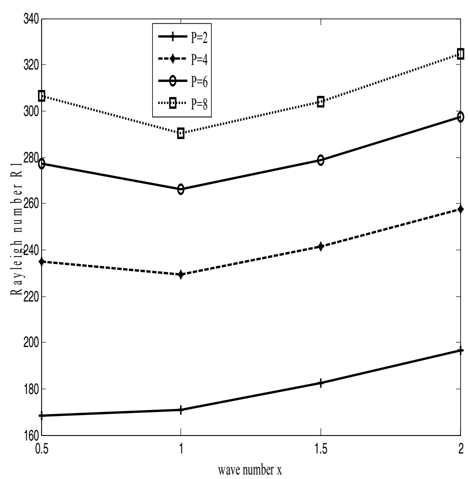

Fig. 3. Variations of Rayleigh number R_1 with wave number X for various values of permeability $P = (2,4,6,8)$ **and fixed values of** $\in = 1, Q_1 = 10, D_{A_1} = 2, \Upsilon_1 = 2, T_{A_1} = 1000, \theta = 45°$.

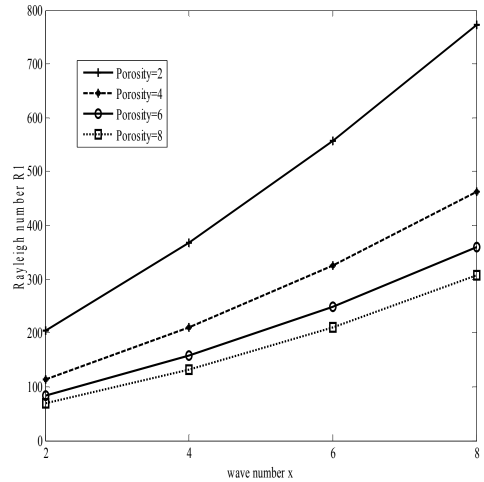

Fig. 4. Variations of Rayleigh number R_1 with wave number X for various values of Porosity $\in = (2,4,6,8)$ **and fixed values of** $P = 3, Q_1 = 200, D_{A_1} = 10, \Upsilon_1 = 5, T_{A_1} = 1000, \theta = 45°$.

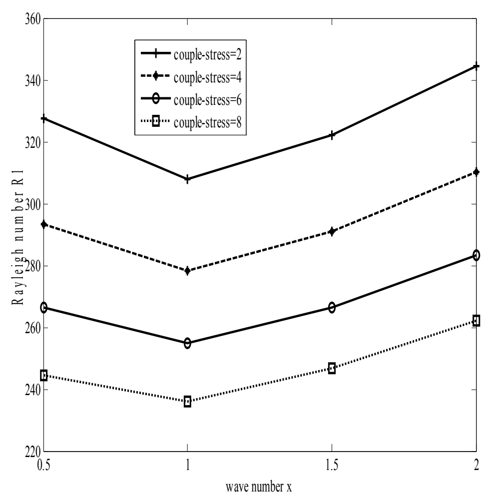

Fig. 5. Variations of Rayleigh number R_1 with wave number x for various values of couple-stress parameter $\Upsilon_1 (2,4,6,8)$ **and fixed values of** $\in = 1, P = 10, D_{A_1} = 2, T_{A_1} = 1000, Q_1 = 10, \theta = 45°$.

7. CONCLUSIONS

In the present note, the effects of various embedded parameters have been analyzed theoretically on thermal convection problem in a couple-stress fluid through a Brinkman porous medium using normal mode method. The following results are drawn while investigating the problem:

(a). For the case of stationary convection, it is concluded that

- the rotational parameter rules out the possibility of the onset of convection, whereas medium porosity accelerates the onset of thermal convection.

- the medium permeability, magnetic field, couple-stress and Darcy-Brinkman parameter have both stabilizing and destabilizing effects in the presence of rotation, whereas for a non-rotating system, magnetic field, couple-stress and Darcy-Brinkman number have stabilizing effects and medium permeability has a destabilizing effect.

(b). The principle of exchange of stabilities (PES) holds good in the presence of both horizontal rotation and magnetic field. Also, the sufficient conditions for the non-existence of overstability are obtained.

(d). The practical relevance and importance of the present finding is that the couple-stress parameter in a Darcy-Brinkman model have a stabilizing impact on the system in the absence of rotational effects. In addition, it has also been observed that for the horizontal magnetic field and horizontal rotation, the result for PES is quite different with those of previous findings (Kumar 2012, Sharma and Sharma 2004) for vertical magnetic field and vertical rotation.

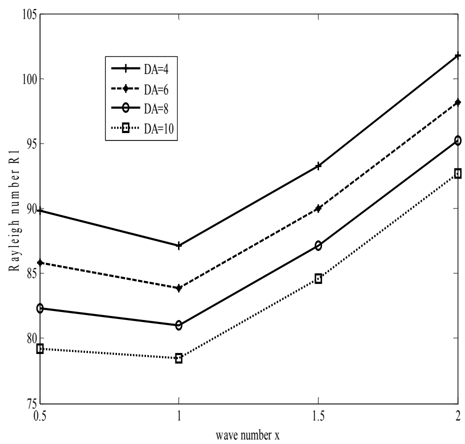

Fig. 6. Variations of Rayleigh number R_1 with wave number x for various values of Brinkman parameter D_{A_1} $(4,6,8,10)$ and fixed values of

$\in = 2, P = 10, \Upsilon_1 = 2, T_{A_1} = 1000, Q_1 = 20, \theta = 45^\circ$.

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