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Effect of Cubic Temperature Profiles on Ferro Convection in a Brinkman Porous Medium

C. E. Nanjundappa[1], I. S. Shivakumara[2] and R. Arunkumar [3†]

[1] *Department of Mathematics, Dr. Ambedkar Institute of Technology, Bangalore -560 056, India*
[2] *UGC-Centre for Advanced Studies in Fluid Mechanics, Department of Mathematics, Bangalore University, Bangalore – 560 001, India*
[3] *Department of Mathematics, Sai Vidya Institute of Technology, Bangalore- 560 064, India*

†*Corresponding Author Email: rakrrce@gmail.com*

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ABSTRACT

The effect of cubic temperature profiles on the onset ferroconvection in a Brinkman porous medium in presence of a uniform vertical magnetic field is studied. The lower and upper boundaries are taken to be rigid-isothermal and ferromagnetic. The Rayleigh-Ritz method with Chebyshev polynomials of the second kind as trial functions is employed to extract the critical stability parameters numerically. The results indicate that the stability of ferroconvection is significantly affected by cubic temperature profiles and the mechanism for suppressing or augmenting the same is discussed in detail. It is observed that the effect of Darcy number Da, magnetic number M_1 and nonlinearity of the fluid magnetization parameter M_3 is to hasten, while an increase in the ratio of viscosity parameter λ and Biot number Bi is to delay the onset of ferroconvection in a Brinkman porous medium. Further, increase in Bi, M_1, M_3 and decrease in λ, Da is to decrease the size of the convection cells.

Keywords: Ferrofluid; Cubic temperature profiles; Ferro convection in Brinkman porous medium; Rayleigh-Ritz technique.

NOMENCLATURE

a	overall horizontal wave number
$\vec{B}$	magnetic induction field
Bi	Biot number
$C_{V,H}$	specific heat at constant volume and magnetic field
$\vec{H}$	magnetic field intensity
Da	Darcy number
h_t	heat transfer coefficient
k	permeability of the porous medium
K	pyromagnetic coefficient
$\vec{M}$	magnetization
M_0	constant mean value of magnetization
M_1	magnetic number
M_2	magnetic parameter
M_3	nonlinearity of magnetization parameter
p	pressure
$\vec{q}$	velocity vector
R_m	magnetic Rayleigh number
R_t	thermal Rayleigh number
T	temperature
T_b	basic temperature
$\overline{T}$	average temperature
W	amplitude of vertical perturbed velocity
α_t	thermal expansion coefficient
χ	magnetic susceptibility
ε	porosity of the porous medium
∇	del operator
∇^2	Laplacian operator
∇_1^2	horizontal Laplacian operator
κ	thermal diffusivity
λ	ratio of viscosities

μ	dynamic viscosity
μ_0	free space magnetic permeability of vacuum
φ	perturbed magnetic potential
Φ	amplitude of perturbed magnetic potential
ρ	density
ρ_0	reference density at T_0
Θ	amplitude of perturbed temperature
ω	growth rate

1. INTRODUCTION

Ferrofluids are stable colloidal suspensions of magnetic nano-particles suspended in a carrier liquid with low electrical conductivity. In the absence of an external magnetic field the magnetic moments of the particles are randomly orientated and there is no net macroscopic magnetization. In an external magnetic field, however, the magnetic moments of particles easily orient and a large (induced) magnetization prevails. There are two additional features in ferrofluids not found in ordinary fluids, the Kelvin force and the body couple (Rosensweig 1985). In addition, in an external magnetic field, a ferrofluid exhibits additional rheological properties such as a field-dependent viscosity, special adhesion properties, and a non-Newtonian behavior, among others (Odenbach 2003). The theory of thermal convective instability in a ferrofluid layer began with Finlayson (1970) and extensively continued over the years (Stiles and Kagan 1990, Ganguly *et al*. 2004, Nanjundappa *et al*. 2008, Shivakumara *et al*. 2012, Nanjundappa *et al*. 2015).

Thermal convection of ferrofluids saturating a layer of porous medium has also attracted considerable attention in the literature owing to its importance in controlled emplacement of liquids or treatment of chemicals and emplacement of geophysically imageable liquids into particular zones for subsequent imaging etc. Rosensweig *et al*. (1978) have studied experimentally the penetration of ferrofluids in the Hele-Shaw cell. The stability of the magnetic fluid penetration through a porous medium in high uniform magnetic field oblique to the interface is studied by Zhan and Rosensweig (1980). The thermal convection of ferrofluid saturating a porous medium in the presence of a vertical magnetic field is studied by Vaidyanathan *et al*. (1991) by employing the Brinkman equation. Qin and Chadam (1995) have carried out the non-linear stability analysis of ferroconvection in a porous layer by including the inertial effects to accommodate high velocity. The experimental results of the behavior of ferrofluids in porous media consisting of sands and sediments are presented in detail by Borglin *et al*. (2000). Sunil and Mahajan (2008) have used generalized energy method to study nonlinear convection in a magnetized ferrofluid saturated porous layer heated uniformly from below for the stress-free boundaries case. Shivakumara *et al*. (2009) have investigated theoretically the onset of convection in a layer of ferrofluid saturated porous medium for various types of velocity and temperature boundary conditions. Sunil *et al*. (2011) have investigated the effect of rotation in a magnetized ferrofluid with internal angular momentum, heated and soluted from below subject to transverse uniform magnetic field. Nanjundappa *et al*. (2012) have explored a model for penetrative ferroconvection via internal heat generation in a ferrofluid saturated porous layer using the Brinkman-Lapwood extended Darcy equation with fluid viscosity different from effective viscosity to describe the flow of porous medium. Nanjundappa *et al*. (2013) have investigated the effect of penetrative ferroconvection via internal heat generation in a ferrofluid anisotropic porous layer theoretically using a Brinkman extended-Darcy equation with fluid viscosity different from effective viscosity. Nanjundappa *et al*. (2014) have studied effect of cubic temperature profiles and MFD viscosity on Benard-Marangoni ferroconvection with convective surface boundary conditions. Recently, Ram *et al*. (2014) have studied the effect of viscous dissipation and variable viscosity on rotationally symmetric ferrofluid flow in porous medium subjected to applied vertical magnetic field.

The objective of the present paper is to make clear the effects of cubic temperature profiles on the onset of ferroconvection in a Brinkman porous medium in the presence of a uniform vertical magnetic field. In investigating the problem, the lower and upper boundaries are taken to be rigid-isothermal and ferromagnetic. The study helps in understanding control of ferroconvection by cubic temperature profiles in a Brinkman porous medium, which is useful in many heat transfer related problems. The resulting eigenvalue problem is solved numerically by employing the Rayleigh Ritz method with Chebyshev polynomials of the second kind as trial functions.

2. MATHEMATICAL FORMULATION

The system considered is an initially quiescent magnetic fluid saturated horizontal porous layer of characteristic thickness d in the presence of an applied magnetic field H_0 in the vertical direction. The physical configuration is as shown in Fig. 1. The horizontal extension of the porous layer is sufficiently large so that edge effects may be neglected. A Cartesian co-ordinate system (x, y, z) is used with the origin at the bottom of the porous layer and z -axis is directed vertically upward. Gravity acts in the negative z -direction, $\vec{g} = -g\,\hat{k}$, where $\hat{k}$ is the unit vector in the z -direction.

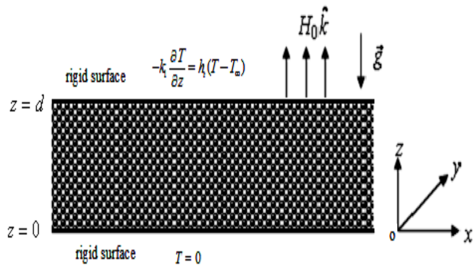

Fig. 1. Physical Configuration.

The basic governing equations for the flow of an incompressible ferrofluid are:

Continuity Equation:

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

Momentum Equation:

$$\rho_0 \left[\frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2}(\vec{q}.\nabla)\vec{q}\right] = -\nabla \mathrm{p}$$
$$+\rho_0\left[1-\alpha_t\,(T - T_0)\right]\vec{g} + \mu_0(\vec{M}\cdot\nabla)\vec{H} \tag{2}$$
$$+\mu(\nabla^2\vec{q}) - \frac{\mu}{k}\vec{q}$$

Energy/Temperature Equation:

$$\varepsilon\left[\rho_0 C_{V,H} - \mu_0 \vec{H}\cdot\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt}$$
$$+(1-\varepsilon)(\rho_0 C)_S\frac{\partial T}{\partial t} + \mu_0 T\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\cdot\frac{D\vec{H}}{Dt} \tag{3}$$
$$= k_t \nabla^2 T$$

Maxwell's Equations:

$$\nabla\cdot\vec{B} = 0,\ \nabla\times\vec{H} = 0 \text{ or } \vec{H} = \nabla\varphi \tag{4 a,b}$$

$$\vec{B} = \mu_0(\vec{M} + \vec{H}) \tag{5}$$

$$\vec{M} = \frac{\vec{H}}{H}M(H, T) \tag{6}$$

Linearized Equation of Magnetic state:

$$M = M_0 + \chi(H - H_0) - K(T - \overline{T}) \tag{7}$$

where, the quantities have their predefined meaning as in Nomenclature.

The basic state is assumed to be quiescent and it is given by

$$\vec{q}_b = 0,\ p = p_b(z),\ -\frac{dT_b}{dz} = f(z),$$

$$\vec{H}_b(z) = \left[H_0 - \frac{K\beta z}{1+\chi}\right]\hat{k}$$
$$\vec{M}_b(z) = \left[M_0 + \frac{K\beta z}{1+\chi}\right]\hat{k} \tag{8}$$

To study the stability of the system, we perturb all the variables in the form

$$[q,\ p, T,\ \vec{H}, \vec{M}] = [\vec{q}',\ p_b(z)$$
$$+ p', T_b(z) + T',\ \vec{H}_b(z) + \vec{H}', \vec{M}_b(z) + \vec{M}'] \tag{9}$$

where, $\vec{q}'$, p', T', $\vec{H}'$ and $\vec{M}'$ are perturbed variables and are assumed to be small.

Substituting Eq. (9) into momentum Eq. (2), linearizing, eliminating the pressure term by operating curl twice, the z -component of the resulting equation is:

$$\left(\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t} + \frac{\mu}{k} - \mu\nabla^2\right)\nabla^2 w = \rho_0\alpha_t g\nabla_1^2 T$$
$$-\mu_0 K f(z)\frac{\partial}{\partial z}(\nabla_1^2\varphi) + \frac{\mu_0 K^2}{1+\chi}f(z)\nabla_1^2 T \tag{10}$$

As before, substituting Eq. (9) into energy Eq. (3), linearizing, we obtain (after neglecting primes)

$$(\rho_0 C)_1\frac{\partial T}{\partial t} - \mu_0 K T_0\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial z}\right) = \left((\rho_0 C)_2 - \frac{\mu_0 K^2 T_0}{1+\chi}\right)w f(z)$$
$$+ k_t\nabla^2 T \tag{11}$$

where,
$(\rho_0 C)_1 = \varepsilon\,\rho_0 C_{V,H} + \varepsilon\,\mu_0 H_0 K + (1-\varepsilon)(\rho_0 C)_s$
and $(\rho_0 C)_2 = \varepsilon\,\rho_0 C_{V,H} + \varepsilon\,\mu_0 H_0 K$.

Equations (4a, b), after substituting Eq. (9), may be written as (after dropping the primes)

$$\left(1 + \frac{M_0}{H_0}\right)\nabla_1^2\varphi + (1+\chi)\frac{\partial^2\varphi}{\partial z^2} - K\frac{\partial T}{\partial z} = 0\,. \tag{12}$$

Since there are no physical mechanisms to introduce oscillatory motions, the principle of exchange of stability is assumed to be valid and hence the normal mode expansion of the dependent variables are taken in the form

$$\{\mathrm{w, T, \varphi}\} = \{\mathrm{W}(z), \Theta(z), \Phi(z)\}\,\mathrm{e}^{\mathrm{i}(L\,\mathrm{x} + \mathrm{m\,y})} \tag{13}$$

where, ℓ and m are wave numbers in the x and y directions, respectively.

Substituting Eq. (13) into Eqs. (10)-(12) and non-dimensionalizing the variables by setting

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), \quad f(z)^* = \frac{1}{\beta}f(z),$$
$$W^* = \frac{d}{\nu A}W,\ \Theta^* = \frac{\kappa}{\beta\nu d}\Theta,\ \Phi^* = \frac{(1+\chi)\,\kappa}{K\,\beta\,\nu\,d^2}\Phi \tag{14}$$

where, $\nu = \eta_0/\rho_0$ is the kinematic viscosity,

$\kappa = k_t/(\rho_0 C)_2$ is the thermal diffusivity and $A = (\rho_0 C)_1/(\rho_0 C)_2$ is the ratio of heat capacities, we obtain (after ignoring the asterisks)

$$\left[\lambda(D^2-a^2)-Da^{-1}\right](D^2-a^2)W = a^2 R_t\,\Theta - a^2 R_m f(z)\left[D\Phi-\Theta\right] \tag{15}$$

$$(D^2-a^2)\,\Theta = -(1-M_2)\,W f(z) \tag{16}$$

$$(D^2-a^2 M_3)\Phi - D\Theta = 0. \tag{17}$$

In the above equations, D is the differential operator, a the overall horizontal wave number, R_t the thermal Rayleigh number, R_m the magnetic Rayleigh number, Da^{-1} the inverse Darcy number, λ the non-dimensional viscosity ratio parameter, M_3 the measure of nonlinearity of fluid magnetization parameter. The typical value of M_2 for magnetic fluids with different carrier liquids turns out to be of the order of 10^{-6} and hence its effect is neglected as compared to unity. The non-dimensional basic temperature gradient $f(z)$ is given by

$$f(z) = a_1^* + 2a_2^*(z-1) + 3a_3^*(z-1)^2. \tag{18}$$

Three types of basic temperature gradients are considered for discussion as mentioned below.

Reference steady-state temperature gradients	$f(z)$	a_1^*	a_2^*	a_3^*
Linear	1	1	0	0
Cubic 1	$3(z-1)^2$	0	0	1
Cubic 2	$0.66+1.02(z-1)^2$	0.66	0	0.34

Equations (15)-(17) are solved using the following boundary conditions:

$$W = DW = \Theta = \Phi = 0 \quad \text{at } z = 0$$

$$W = DW = D\Theta + Bi\,\Theta = \Phi = 0 \quad \text{at } z = 1 \tag{19a, b}$$

where, Bi is the Biot number. The case $Bi = 0$ and $Bi \to \infty$ respectively correspond to constant heat flux and isothermal conditions at the upper boundary.

3. METHOD OF SOLUTION

Equations (15)-(17) together with the corresponding boundary conditions constitute an eigenvalue problem with thermal Rayleigh number R_t as an eigenvalue. The resulting eigenvalue problem is solved numerically using the Rayleigh Ritz method. In this method, the test (weighted) functions are the same as the base (trial) functions. Accordingly, W, Θ and Φ are written as

$$W = \sum_{i=1}^{n} A_i\, W_i(z),\quad \Theta(z) = \sum_{i=1}^{n} C_i\,\Theta_i(z),\quad \Phi(z) = \sum_{i=1}^{n} D_i\,\Phi_i(z) \tag{20}$$

where, the trial functions $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ will be generally chosen in such a way that they satisfy the respective boundary conditions, and A_i, C_i and D_i are constants.

We select the trial functions as

$$W_i = (z^4-2z^3+z^2)T_{i-1}^*,\quad \Theta_i = z(1-z/2)T_{i-1}^*,\quad \Phi_i = (z^2-z)T_{i-1}^*. \tag{21}$$

where, T_i^* s are the Chebyshev polynomials of the second kind. Substituting Eq.(20) into Eqs.(15)-(17), multiplying the resulting momentum Eq. (15) by $W_j(z)$, energy Eq. (16) by $\Theta_j(z)$ and magnetic potential Eq. (17) by $\Phi_j(z)$, performing the integration by parts with respect to z between $z=0$ and $z=1$ and using the boundary conditions (19a, b), we obtain a system of linear homogeneous algebraic equations in A_i, C_i and D_i. A nontrivial solution to the system requires the characteristic determinant of the coefficient matrix must vanish and this leads to a relation in the form

$$f(R_t, \lambda, Da^{-1}, Bi, M_1, M_3, a_1^*, a_2^*, a_3^*, a) = 0 \tag{22}$$

The critical values of R_{tc} are found as a function of wave number a for various values of physical parameters.

4 RESULTS AND DISCUSSIONS

The linear stability analysis is carried out to investigate the effect of different forms of basic temperature profiles on the onset of ferroconvection in a ferrofluid Brinkmann porous layer. The bounding surfaces of the ferrofluid layer are considered to be rigid ferromagnetic and the resulting eigenvalue problem is solved numerically by employing the Galerkin technique. The results presented here are for $i = j = 6$ the order at which the convergence is achieved, in general.

Figure 2(*a*) represents the variation of critical thermal Rayleigh number R_{tc} as a function of Da^{-1} for various values of ratio of viscosity parameter λ. It is observed that an increase in λ is to delay the onset of ferroconvection. This is because increase in the value of λ is related to increase in viscous effect which has the tendency to retard the fluid flow and hence higher heating is required for the onset of ferroconvection.

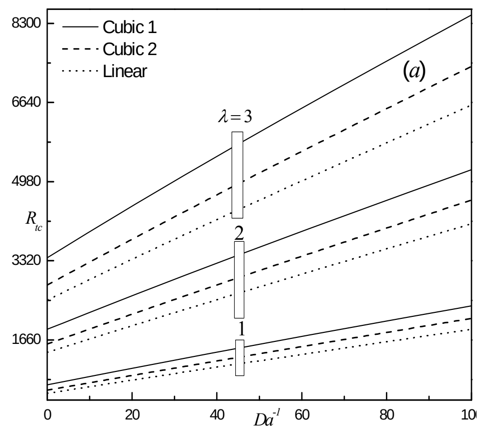

Fig. 2(*a*). Variation of R_{tc} as a function of Da^{-1} for different values of λ when $M_1 = 2$, $M_3 = 1$ and $Bi = 2$.

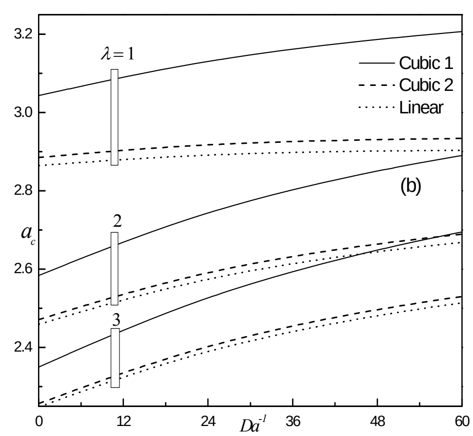

Fig. 2(b). Variation of a_c as a function of Da^{-1} for different values of λ when $M_1 = 2$, $M_3 = 1$ and $Bi = 2$.

In other words, higher value of λ is more effective in suppression of ferroconvection in a ferrofluid saturated porous medium. It is seen that R_{tc} increases with increasing Da^{-1} and hence its effect is to delay the onset of ferroconvection. For a fixed thickness of the porous layer, increase in Da^{-1} leads to decrease in the permeability of the porous medium which in turn retards the flow of ferrofluid. Therefore, higher heating and hence higher value of thermal Rayleigh number is required for the onset of ferroconvection in a porous medium. Moreover, $(R_{tc})_{\text{linear}} < (R_{tc})_{\text{cubic 2}} < (R_{tc})_{\text{cubic1}}$ suggesting cubic 1 basic temperature profile is more stabilizing than cubic 2 temperature profile and the linear temperature profile is the least stable. Thus, it is possible to control ferroconvection in a Brinkmann porous medium effectively by the choice of different forms of basic temperature profiles. The variation in critical wave number a_c as a function of Da^{-1} is elucidated in Fig. 2(b) for different forms of basic temperature profile with different values of λ. It may be noted that the critical wave number a_c increases with increasing Da^{-1}. Moreover, an increase in the value of λ is to decrease a_c and hence its effect is to reduce the size of convection cells and also it is observed that $(a_c)_{\text{linear}} < (a_c)_{\text{cubic 2}} < (a_c)_{\text{cubic1}}$.

Figure 3(*a*), shows the plot of R_{tc} as the function of Da^{-1} for different values of Bi and fixed values of λ, M_1 and M_3 with three different forms of basic temperature profiles. From the figure it is evident that an increase in the value of Bi is to increase R_{tc} and thus its effect is to delay the onset of ferroconvection in a porous medium. This may be attributed to the fact that with increasing Bi, the thermal disturbances can easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the top surface and hence higher heating is required to make the system unstable. Fig. 3(b) represents the variation of a_c as a function of Da^{-1} for different values of Bi and we note that as Da^{-1} increases the critical wave number and hence its effect is to contract the size of convection cells.

Figure 4(*a*) reveals that critical thermal Rayleigh number R_{tc} as a function of Da^{-1} for different values of M_1 with different forms of basic temperature profile. Physically, increase in M_1 leads to either increase in destabilizing magnetic force or decrease in stabilizing viscous force on the system and hence it has a destabilizing effect on the system. A closer inspection of the figure further reveals that the magnetic force is to reinforce together with buoyancy force and to hasten the onset of ferroconvection when compared to their effect in isolation. Besides, it may be noted that the difference in the critical thermal Rayleigh numbers among different values of M_1 diminishes as the value of M_1 increases. The variation in a_c as a function of Da^{-1} is elucidated in Fig. 4(b) for different forms of basic temperature profile with different values of M_1. It may be noted that the critical wave number a_c increases with increasing Da^{-1}. Moreover, an increase in the value of magnetic parameter M_1 is to increase the value of critical wave number a_c and thus its effect is to increase the dimension of convection cells.

Figure 5 shows the locus of critical thermal Rayleigh number R_{tc} and magnetic Rayleigh number R_{mc} for different values of non-linearity of fluid magnetization, denoted through the parameter M_3, on the onset of ferroconvection in a Brinkman porous medium. In the figure, the regions above and below the curves, correspond respectively to unstable and stable ones.

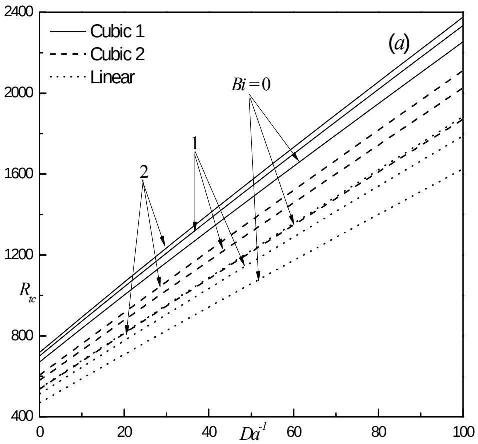

Fig. 3(a). Variation of R_{tc} as a function of Da^{-1} for different values of Bi when $M_1 = 2$, $\lambda = 1$, and $M_3 = 1$.

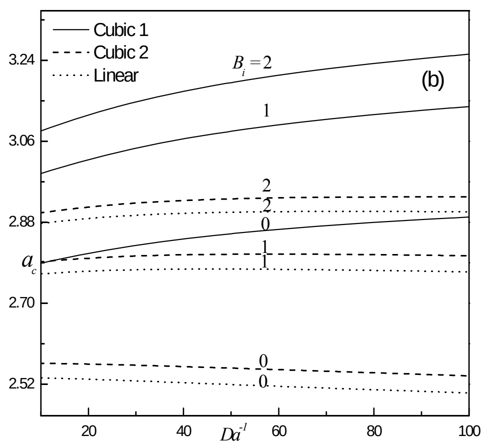

Fig. 3(b). Variation of a_c as a function of Da^{-1} for different values of Bi when $\lambda = 1$, $M_1 = 2$ and $M_3 = 1$.

It is observed that there is a strong coupling between R_{tc} and R_{mc} such that an increase in the one decreases the other. Thus, when the buoyancy force is predominant, the magnetic force becomes negligible and vice-versa. The stability curves are slightly convex and in the absence of buoyancy forces ($R_{tc} = 0$), the instability sets in at higher values of R_{mc} indicating the system is more stable when the magnetic forces alone are present. Fig. 5 demonstrated that an increase in M_3 is to decrease R_{tc} and R_{mc} and thus it has a destabilizing effect on the system. This may be due to the fact that the application of magnetic field makes the ferrofluid to acquire larger magnetization which in turn interacts with the imposed magnetic field and releases more energy to drive the flow faster. Hence, the system becomes unstable with a smaller temperature gradient as the value of M_3 increases. Alternatively, a higher value of M_3 would arise either due to a larger pyromagnetic coefficient or larger temperature gradient. Both these factors are conducive for generating a larger gradient in the Kelvin body force field, possibly promoting the instability. The variation of a_c as a function of Da^{-1} is shown in Fig. 6 for different values of M_3. From the figure, we note that increasing M_3 and Da^{-1} is to increase a_c and hence to decrease the dimension of convection cells.

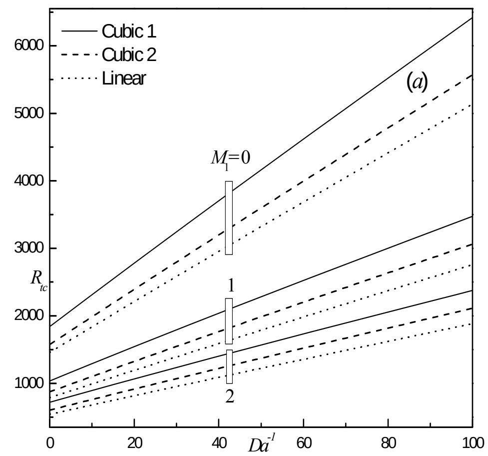

Fig. 4(a). Variation of R_{tc} as a function of Da^{-1} for different values of M_1 when $\lambda = 1$, $M_3 = 1$ and $Bi = 2$.

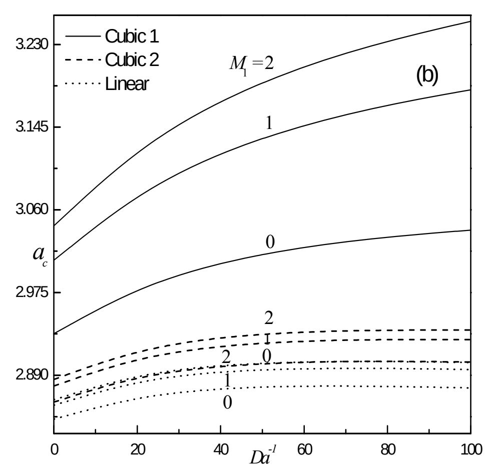

Fig. 4(b). Variation of a_c as a function of Da^{-1} for different values of M_1 when $\lambda = 1$, $M_3 = 1$ and $Bi = 2$.

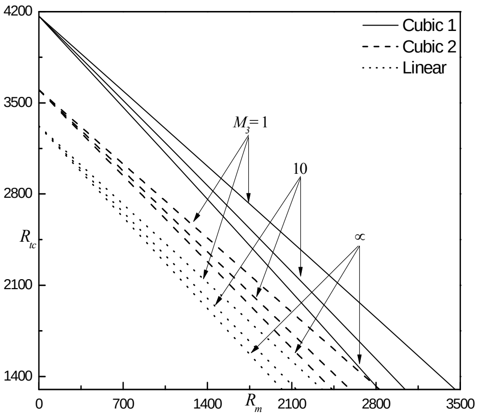

Fig. 5. Locus of R_{tc} versus R_m for different values of M_3 when $\lambda = 1,\ Bi = 2$ and $Da^{-1} = 50$.

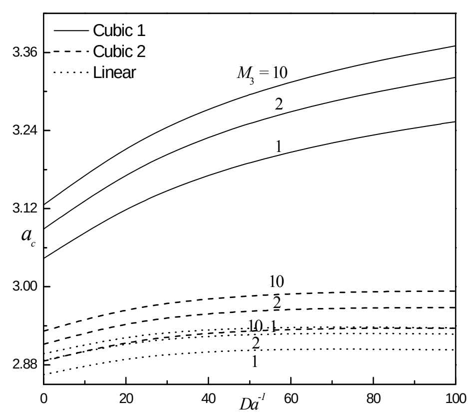

Fig. 6. Variation of a_c as a function of Da^{-1} for different values of M_3 when $\lambda = 1,\ M_1 = 2$ and $Bi = 2$.

5. CONCLUSIONS

The linear stability theory is used to investigate the effect of different forms of basic temperature profile on onset of ferroconvection in a Brinkman porous medium. The lower and upper boundaries is taken to be rigid-ferromagnetic and insulated to temperature perturbations. The Galerkin technique is used to find the eigenvalues as this technique is found to be more convenient to tackle different forms of basic temperature profiles.

From the foregoing study, it is observed that

(i) The cubic 1 basic temperature profile delays, while linear profile hastens the onset of ferroconvection. That is

$(R_{tc})_{\text{linear}} < (R_{tc})_{\text{cubic}\,2} < (R_{tc})_{\text{cubic}\,1}$.

(ii) The critical thermal Rayleigh number R_{tc} increases with an increase in the value of ratio of viscosity parameter λ, Biot number Bi and thus their effect is to delay the onset of ferroconvection.

(iii) The effect of increase in the value of Darcy number Da, magnetic number M_1 and non-linearity of the fluid magnetization parameter M_3 is to reinforce together and to hasten the onset of ferroconvection.

(iv) The magnetic and buoyancy forces are complementary with each other and the system is more stabilizing when the magnetic forces alone are present.

(v) The effect of increasing Bi, Da^{-1}, M_1 and M_3 as well as decrease in λ is to increase the critical wave number a_c and hence their effect is to narrow the convection cells.

(vi) The critical wave numbers a_c for cubic 1 basic temperature profile are higher than those of cubic 2 basic temperature profile and linear temperature profiles. That is,

$(a_c)_{\text{linear}} < (a_c)_{\text{cubic}\,2} < (a_c)_{\text{cubic}\,1}$.

(vii) It is possible to either augment or suppress ferroconvection in a porous medium by tuning the physical parameters of the system.

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