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Newtonian and Joule Heating Effects in Two-Dimensional Flow of Williamson Fluid

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ABSTRACT

In this article, we have studied the combined effects of Newtonian and Joule heating in two-dimensional flow of Williamson fluid over the stretching surface. Mathematical analysis is presented in the presence of viscous dissipation. The governing partial differential equations are reduced into the ordinary differential equations by appropriate transformations. Both series and numerical solutions are constructed. Graphical results for the velocity and temperature fields are displayed and discussed for various sundry parameters. Numerical values of local skin friction coefficient and the local Nusselt number are tabulated and analyzed.

Keywords: Heat transfer; Joule heating; Williamson fluid; Newtonian heating.

NOMENCLATURE

A_1	first Rivlin Erickson tensor
B_0	uniform magnetic field
c	dimensional constant
C_{fx}	local skin friction coefficient
c_p	specific heat
$(C_i, i = 1-5)$	constants
Ec	Eckert number
h_s	heat transfer coefficient
I	identity tensor
k	thermal conductivity
L_1, L_2	Linear operators
M	the Hartman number
Nu_x	local Nusselt number
p	pressure
$\Pr$	Prandtl number
q	embedded parameter
q_w	surface heat flux
Re_x	local Reynolds number
$\left(\mathbf{R}_m^f, \mathbf{R}_m^\theta\right)$	m^th order nonlinear operators
T	temperature
T_∞	ambient fluid temperature
U_w	stretching surface velocity
(u, v)	velocity components
We	Weissenberg number
(x, y)	spatial co-ordinates
ν	kinematics viscosity
f	dimensionless stream function
(f_0, θ_0)	initial approximations of velocity and temperature
(f_m^*, θ_m^*)	particular solutions of velocity and temperature fields
μ_∞	dynamic viscosity
μ_0	shear rate viscosity
ρ	specific heat
Γ	time rate constant
$\dot{\gamma}_{ij}$	components of shear stress
γ	conjugate parameter
θ	dimensionless temperature
Π	second invariant strain tensor
η	transformed coordinate
τ	shear stress
τ_{xy}	wall shear stress
$(\hbar_f, \hbar_\theta)$	non-zero auxiliary parameters
σ	electrical conductivity

1. INTRODUCTION

Boundary layer flows over a stretching surface have great importance in industrial and engineering processes. Such types of flows occur in glass fiber and paper production, extrusion processes, electronic chips, crystal growing etc. (Makinde (2011), Hayat *et al*. (2012) and Turkyilmazoglu and Pop (2013)). On the other hand many researchers are involved to investigate the boundary layer flows of non-Newtonian fluids. This is due to the fact that the rate of heat transfer in non-Newtonian fluid is quite different from those of a Newtonian fluid. Thus several studies dealing with the flow and heat transfer in non-Newtonian fluids exist (Hayat *et al*. (2012), Baoku *et al*. (2013), Rashidi *et al*. (2012), Hayat *et al* 2012), Shateyi *et al*. (2010), Bhattacharyya *et al*. (2011) and Mukhopadhyay (2013)). It is known that the non-Newtonian fluids in view of their diverse characteristics are already described by many constitutive equations. Williamson fluid is one of the subclasses of non-Newtonian fluids which has not been given due attention. The Cauchy stress tensor in such fluid is

$$\boldsymbol{\tau} = -p\mathbf{I} + \left[\mu_\infty + \left(\mu_0 - \mu_\infty\right)\left(1 - \Gamma\dot{\gamma}\right)^{-1}\right]A_1, \qquad (1)$$

where p is the pressure, μ_0 is the zero shear rate viscosity, μ_∞ is the infinite shear rate viscosity, Γ is the time constant and $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i\sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{\Pi}{2}}. \qquad (2)$$

where Π is the second invariant strain tensor. We consider the constitutive equation (1) for the case when $\mu_\infty = 0$ and $\Gamma\dot{\gamma} < 1$. The component of extra stress tensor therefore becomes

$$\boldsymbol{\tau} = \mu_0\left(1 + \Gamma\dot{\gamma}\right)A_1. \qquad (3)$$

where the first Rivlin -Erickson tensor is

$$A_1 = \left(\text{grad}V\right) + \left(\text{grad}V\right)^*$$

in which $(*)$ denotes the matrix transpose. Quite recently, Dapra and Scarpi (2007) analyzed the perturbation solution for pulsatile flow of a non-Newtonian Williamson fluid in a rock fracture. Noreen *et al*. (2012) investigated the peristaltic flow of a Williamson fluid in an inclined asymmetric channel. Interaction of heat transfer in peristaltic pumping of Williamson fluid in a channel has been studied by Vasudev *et al*. (2010). Nadeem and Akram (2010) have discussed peristaltic flow of Williamson model in an asymmetric channel. Nadeem and Akber (2012) studied the effects of mixed convection heat and mass transfer on peristaltic flow of Williamson fluid in a vertical annulus.

Newtonian heating is the heat transfer rate for which a finite heat capacity is proportional to the local surface temperature from the bounding surface. It is usually known as the conjugate convective flow. Salleh *et al*. (2010) considered the steady mixed convection boundary layer flow about a solid surface generated by Newtonian heating in which the heat transfer from the surface is proportional to the local surface temperature. They solved the problem numerically by using an implicit finite difference scheme known as the Keller-box method. Hayat *et al*. (2012) studied the boundary layer flow and heat transfer in a second grade fluid over a stretching sheet in the presence of Newtonian heating. They noted that temperature profiles and heat transfer rate significantly increase by increasing the conjugate parameter for Newtonian heating. Magnetohydrodynamic three-dimensional flow of couple stress fluid in the presence of Newtonian heating was addressed by Ramzan *et al*. (2013). Niu *et al*. (2010) analyzed the stability of thermal convection of an Oldroyd-B fluid in a porous medium with Newtonian heating.

It is found that the Newtonian heating effects in two-dimensional flow of Williamson fluid over a stretching surface are not reported yet in the literature. Therefore the object of present communication is to study this problem. The series solutions of velocity and temperature are developed by homotopy analysis method HAM (Liao (2012), Hayat *et al*. (2012), Farooq *et al*. (2014), Hayat *et al*. (2014), Rashidi *et al*. (2013), Abbasbandy *et al*. (2013), Turkyilmazoglu (2012), Shafiq *et al*. (2013), Rashidi *et al*. (2014), Motsa *et al*. (2012), Ellahi *et al*. (2012), Hayat *et al*. (2014), Hayat *et al*. (2015), Sheikholeslami *et al*. (2014) and Hayat *et al*. (2015)). The numerical solutions are obtained by MATLAB. The effects of various parameters on the velocity and temperature profiles are discussed through graphs. The skin friction coefficient and local Nusselt number are computed and examined.

2. MATHEMATICAL FORMULATION

We consider the two-dimensional boundary layer flow of an incompressible Williamson fluid. The flow is induced due to the stretching sheet with linear velocity. Constant magnetic field is applied perpendicular to the plane of stretching surface i-e along $y-$ axis. There is no external electric field and thus polarization effects are neglected. Induced magnetic field is ignored subject to the assumption of small magnetic Reynolds number. Heat transfer analysis is carried out in the presence of Newtonian heating. The viscous dissipation and Joule heating effects are present. The governing two-dimensional boundary layer flow equations for the flow under consideration are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (4)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + 2\nu\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u, \qquad (5)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_0}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\nu\Gamma}{c_p}\left(\frac{\partial u}{\partial y}\right)^3+\frac{\sigma B_0^2}{\rho c_p}u^2, \tag{6}$$

with the boundary conditions

$$u(x,0)=U(x)=cx,\ v(x,0)=0,\ \frac{\partial T(x,0)}{\partial y}=-h_s T,$$
$$u\to 0,\quad T\to T_\infty \quad \text{when } y\to\infty, \tag{7}$$

where u and v are the velocity components along the $x-$ and $y-$ directions respectively, ρ the fluid density, σ the electrical conductivity of the fluid, ν the kinematic viscosity, c the stretching rate, K the thermal conductivity, T the temperature of the fluid, c_p the specific heat, h_s the heat transfer parameter and T_∞ the ambient temperature.

Using

$$u(x,y)=cxf'(\eta),\ v(x,y)=-2\sqrt{c\nu}\, f(\eta),$$
$$\theta=\frac{T-T_\infty}{T_\infty},\ \eta=\sqrt{\frac{c}{\nu}}y, \tag{8}$$

the incompressibility condition is automatically satisfied while the other equations and boundary conditions give

$$f'''-(f'')^2+2ff''+2Wef''f'''-M^2f'=0,$$

$$f'(0)=1,\ f(0)=0,\ f'(\infty)=0, \tag{9}$$

$$\theta''+2\Pr f\ \theta'+\Pr Ecf''^2+We\Pr Ecf''^3$$
$$+M^2\Pr Ecf'^2=0,$$

$$\theta'(0)=-\gamma(1+\theta(0)),\ \theta(\infty)=0, \tag{10}$$

where prime denotes differentiation with respect to η, $\Pr$ the Prandtl number, We the local Weissenberg number, M is the Hartman number, Ec is the Eckert number and γ is the conjugate parameter for Newtonian heating. These quantities are defined as

$$M^2=\frac{\sigma B_0^2}{\rho c},\ \nu=\frac{\mu_0}{\rho},\ We=\Gamma U\sqrt{\frac{c}{\nu}},$$
$$\Pr=\frac{\mu_0 c_p}{K},\ Ec=\frac{(cx)^2}{c_pT_\infty},\ \gamma=h_s\sqrt{\frac{\nu}{a}}. \tag{11}$$

The skin friction coefficient C_{fx} and the local Nusselt number Nu_x are given by the following expressions

$$C_f=\frac{\tau_{xy}}{\rho(cx)^2},\quad Nu_x=\frac{xq_w}{K(T-T_\infty)}, \tag{12}$$

in which the wall skin friction τ_{xy} and the wall heat flux q_w are

$$\tau_{xy}=\left[\mu_0\frac{\partial u}{\partial y}+\Gamma\mu_0\left(\frac{\partial u}{\partial y}\right)^2\right]_{y=0},$$
$$q_w=-K\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{13}$$

Equations (12) and (13) through dimensionless variables yield

$$(\mathrm{Re}_x)^{1/2}C_f=\{1+We\,f''(0)\}f''(0),$$
$$(\mathrm{Re}_x)^{-1/2}Nu_x=\gamma\left(1+\frac{1}{\theta(0)}\right), \tag{14}$$

where $\mathrm{Re}_x^{1/2}=\sqrt{\frac{cx^2}{\nu}}$ is the local Reynolds number.

3. METHODS OF SOLUTION

3.1 Homotopy Analytic Solution

The velocity and temperature for homotopy solutions can be expressed in the set of base functions

$$\left\{\eta^k\exp(-n\eta)\,\middle|\,k\ge 0, n\ge 0\right\}, \tag{15}$$

with

$$f_m(\eta)=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}a_{m,n}^k\eta^k\exp(-n\eta), \tag{16}$$

$$\theta_m(\eta)=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}b_{m,n}^k\eta^k\exp(-n\eta), \tag{17}$$

where $a_{m,n}^k$ and $b_{m,n}^k$ are the constants. We have chosen the following initial guess $f_0(\eta)$ and $\theta_0(\eta)$ and the auxiliary linear operators L_1 and L_2 from the rule of solution expression and the boundary conditions

$$f_0(\eta)=1-\exp(-\eta),\ \theta_0(\eta)=\frac{\gamma\exp(-\eta)}{1-\gamma}, \tag{18}$$

$$L_1[f(\eta)]=\frac{d^3f}{d\eta^3}-\frac{df}{d\eta},\ L_2[\theta(\eta)]=\frac{d^2\theta}{d\eta^2}-\theta. \tag{19}$$

The operators have the following properties

$$L_1[C_1+C_2\exp(\eta)+C_3\exp(-\eta)]=0, \tag{20}$$

$$L_2[C_4\exp(\eta)+C_5\exp(-\eta)]=0. \tag{21}$$

where $C_i\ (i=1-5)$ are the constants.

3.2 Numerical Solution

Numerical solution is accomplished with MATLAB built-in-function bvp4c. bvp4c is constructed to solve a boundary value problems (BVPs). The

MATLAB built-in-function bvp4c is a higher order finite difference method which implements 3-stage Lobatto IIIa formula. The results obtained with bvp4c are highly accurate. The only challenging part while using bvp4c is to suggest a suitable initial guess for the ODEs.

4. CONVERGENCE ANALYSIS

The convergence of series solutions and the approximation rate depend upon auxiliary parameters $\hbar_f$ and $\hbar_\theta$. The appropriate values of auxiliary parameters $\hbar_f$ and $\hbar_\theta$ are useful to adjust and control the convergence of the obtained solutions. Therefore Fig. 1 includes the $\hbar-$ curves for velocity and temperature fields at 14th order of approximation. It is noticed that the suitable ranges of $\hbar_f$ and $\hbar_\theta$ are $-1.7 \le \hbar_f < -0.5$ and $-1.6 \le \hbar_\theta < -0.3$.

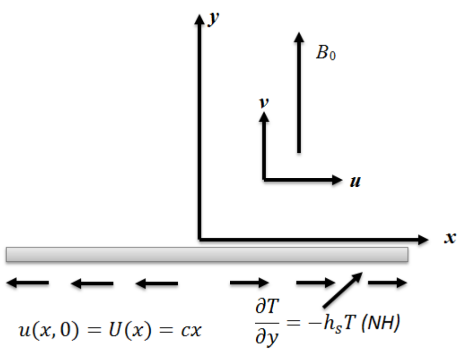

Fig. 1. Flow sketch.

Table 1 Convergence of homotopy solutions when $We = 0.2$, $M = 0.2$, $\gamma = 0.2$, $\Pr = 1$, and $Ec = 0.7$.

Order of approximation	$-f''(0)$	$-\theta'(0)$
1	1.1830	0.28635
5	1.4149	0.35999
15	1.4784	0.37516
20	1.4829	0.37550
29	1.4853	0.37559
30	1.4854	0.37559
50	1.4854	0.37559

5. RESULTS AND DISCUSSION

The aim of this section is to examine the effects of Weissenberg number We, Hartman number M, Prandtl number $\Pr$, Eckert number Ec and conjugate parameter of Newtonian heating γ on the velocity and temperature fields. Figs. 2-8 analyze the variations of such parameters. The variation of skin friction coefficient $\mathrm{Re}_x^{1/2} C_{fx}$ and the local Nusselt number $\mathrm{Re}_x^{-1/2} Nu_x$ for different parameters are also computed in the Tables 2 and 3. Fig. 2 illustrates the influence of Weissenberg number We on the velocity f'. Clearly f' and the associated momentum boundary layer thickness decrease when We increases. The influence of M on the velocity profile f' is observed from Fig. 3. It has been noticed that the magnetic field retards the flow.

Table 2 Numerical values of skin friction coefficients $\mathrm{Re}_x^{1/2} C_{fx}$ for different values of physical parameters.

M	We	$-\mathrm{Re}_x^{1/2} C_x$	
		HAM	Numerical
0.1	0.2	1.044	1.044
0.2		1.052	1.052
0.3		1.067	1.068
0.4		1.088	1.088
0.1	0.0	1.178	1.178
	0.1	1.119	1.119
	0.2	1.043	1.043
	0.3	1.001	1.004

Table 3 Numerical values of Nusselt number $\mathrm{Re}_x^{-1/2} Nu_x$ for different values of physical parameters.

M	We	Pr	Ec	$\mathrm{Re}_x^{-1/2} Nu_x$	
				HAM	Numerical
0.0	0.2	1	0.7	0.4296	0.4295
0.1				0.4279	0.4279
0.2				0.4228	0.4229
0.3				0.4149	0.4150
0.1	0.0	1	0.7	0.4298	0.4298
	0.1			0.4292	0.4293
	0.2			0.4279	0.4279
	0.3			0.4242	0.4242
0.1	0.2	1.0	0.7	0.4279	0.4279
		1.1		0.4354	0.4354
		1.2		0.4406	0.4405
		1.3		0.4454	0.4453
0.1	0.2	1	0.5	0.4779	0.4780
			0.6	0.4505	0.4506
			0.7	0.4279	0.4279
			0.8	0.4090	0.4091

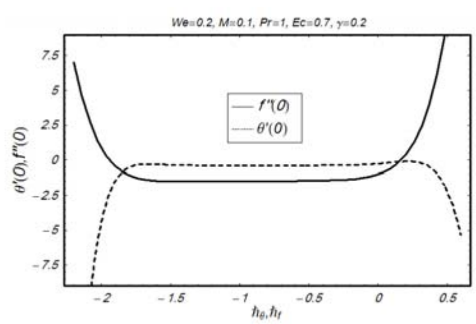

Fig. 2. $\hbar-$ curves for the functions f and θ at 14th-order of approximation.

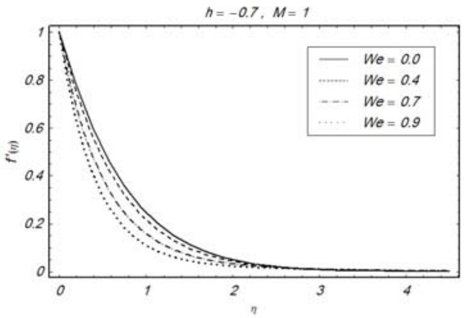

Fig. 3. The influence of *We* on *f′*.

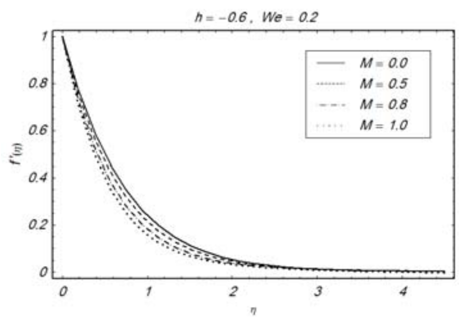

Fig. 4. Influence of M on *f′*.

Figs. 4-8 show the effects of M , We , Pr , Ec and γ on the dimensionless temperature $\theta(\eta)$. The influence of M on the temperature is observed in Fig. 4. The temperature and thermal boundary layer thickness are increasing function of M . Lorentz force is a resistive force which opposes the fluid motion. As a result heat is produced and thus the thermal boundary layer thickness increases. Magnetic field can control the flow and heat transfer characteristics. Fig. 5 clearly indicates that an increase in the Weissenberg number We leads to a decrease in the temperature profile and thermal boundary layer thickness. Fig. 6 portrays the effects of Pr on $\theta(\eta)$ versus η. It is observed that when we increase the value of Prandtl number Pr the dimensionless temperature $\theta(\eta)$ increases and its related boundary layer thickness is reduced. Physically the Prandtl number is the ratio of momentum to thermal diffusivity. Larger values of Pr has higher momentum diffusivity while smaller thermal diffusivity. This higher momentum diffusivity and smaller thermal diffusivity corresponds to thinning of thermal boundary layer thickness. Figs. 7 and 8 describe the effects of Ec and γ on $\theta(\eta)$ respectively. Both Ec and γ increase the temperature profile $\theta(\eta)$. The Newtonian heating parameter γ depends on the heat transfer coefficient h_s . Increasing in γ leads to an increase in h_s that corresponds to the higher temperature. The numerical values of skin friction coefficient $\mathrm{Re}_x^{1/2} C_{fx}$ and the local Nusselt number $\mathrm{Re}_x^{-1/2} Nu_x$ for different values of M , We , Pr and Ec are computed in the Tables 2 and 3. From Table 2 it is clearly seen that the numerical and analytical solutions are in a very good agreement. The magnitude of skin friction coefficient increases for larger values of M whereas it decreases for We . Table 3 shows that the local Nusselt number increases for larger values of Pr while it has opposite behavior for M , We and Ec . It is also clear from this table that both numerical and analytical solutions are in a very good agreement.

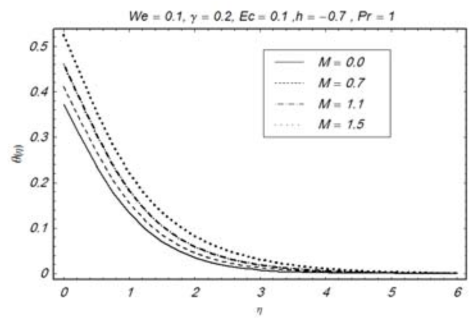

Fig. 5. Influence of M on θ.

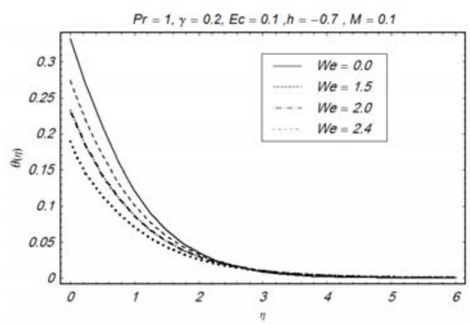

Fig. 6. Influence of We on θ.

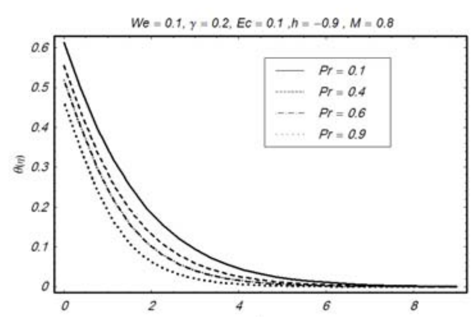

Fig. 7. Influence of Pr on θ.

6. FINAL REMARKS

This attempt examined the influence of Newtonian heating in flow of Williamson fluid over a stretching surface with viscous dissipation and Joule heating. The analytic and numerical solutions have been computed by HAM and the built in solver bvp4c of the software MATLAB respectively. The key points of present study are listed below.

- Table 1 ensures that the convergence of the functions f and $\theta(\eta)$ are obtained at only 24 th order of approximation.
- Behaviors of We and M on the velocity and boundary layer thickness are similar.
- Influence of Pr is to decrease the temperature field $\theta(\eta)$ while temperature increases for higher values of Eckert number Ec .
- Skin friction coefficient increases for larger values of M
- Behaviors of M and We on the temperature $\theta(\eta)$ are opposite.
- Analytical results are in an excellent agreement with the numerical solutions for all values of the physical parameters.

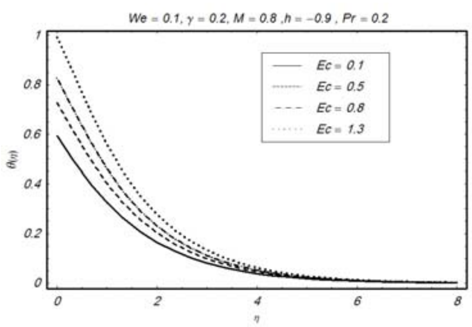

Fig. 8. Influence of Ec on θ.

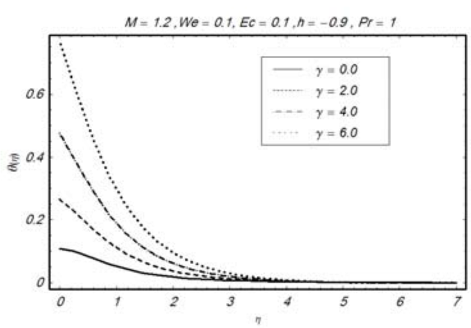

Fig. 8. Influence of γ on θ.

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