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Effects of Soret and Non-Uniform Heat Source on MHD Non-Darcian Convective Flow over a Stretching Sheet in a Dissipative Micropolar Fluid with Radiation

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ABSTRACT

This study presents a numerical analysis on the effects of Soret, variable thermal conductivity, viscous-Ohmic dissipation, non-uniform heat sources, on steady two-dimensional hydromagnetic mixed convective heat and mass transfer flow of a micropolar fluid over a stretching sheet embedded in a non-Darcian porous medium with thermal radiation and chemical reaction. The governing differential equations are transformed into a set of non-linear coupled ordinary differential equations which are then solved numerically by using the fifth-order Runge-Kutta-Fehlberg method with shooting technique. Numerical solutions are obtained for the velocity, angular velocity, temperature and concentration profiles for various parametric values, and then results are presented graphically as well as skin-friction coefficient, and also local Nusselt number and local Sherwood number for different physical parameters are shown graphically and in tabular form. A critical analysis with earlier published papers was done, and the results were found to be in accordance with each other.

Keywords: Soret number; Thermal radiation; Non-Darcian number; Non-uniform heat source/sink; MHD; Micropoalr fluid; Stretching sheet.

1. INTRODUCTION

Heat and mass transfer past over a stretching sheet is an excellent study in industrial applications such as glass fiber production, hot rolling and wire drawing, the aerodynamic extrusion of plastic sheets, glass blowing, metal spinning and drawing plastic films under different heating processes. The quality of the final product depends on the rate of heat transfer at the stretching surface. A comprehensive study on boundary layer flow caused by the stretching of an elastic flat sheet was studied by McCormack and Crane (1973). Crane (1970) investigated the flow caused by a stretching plate. Many authors such as Gupta and Gupta (1977); Chen and Char (1988); Dutta *et al*. (1985) extended the study of Crane (1970) by including the effects of heat and mass transfer under various situations.

Micropolar fluids may present the non-Newtonian fluid models which can be used to analyze the behavior of exotic lubricants colloidal suspensions or polymeric fluid and liquid crystals. Ariman *et al*. (1973, 1974); Lukaszewicz (1999) and Eringen (1966, 1972, 1972) have provided a comprehensive discussion of micropolar fluid subject and application of micropolar fluid mechanics. Heat and mass transfer effects on MHD flow of a micropolar fluid on a circular cylinder was studied by Mansour *et al*. (2000). El-Hakiem (2000) proposed the dissipation effects on magnetohydrodynamic free convective flow past over a non-isothermal surface in a micropolar fluid. El-Hakiem *et al*. (1999) explained the Joule heating effects on MHD free convective flow of a micropolar fluid. Mass transfer effect on MHD free convective flow in a micropolar fluid in the presence of constant suction was investigated by El-Amin (2001). Kim (2001) derived the unsteady MHD free convection flow of micropolar fluid past a vertically moving porous plate in a porous medium.

In all the previous studies, the effect of radiation on the flow and heat transfer has not been considered. The effect of thermal radiation on MHD flow and heat transfer problem has become very important industrially. At high temperature, thermal radiation can significantly affect the heat transfer and the temperature distribution of a micropolar fluid through different mediums. Abo-Eldohad and Ghonaim (2005) studied the thermal radiation effects on heat transfer of a micropolar fluid through a porous medium. Rahman and Sultana (2008) proposed the thermal radiation effects on a steady convective flow of a micropolar fluid past a vertical porous flat plate with variable heat flux in a porous medium. Recently Srinivas *et al*. (2015) have studied the non- Darcian flow of a micropolar fluid over a porous stretching sheet in the presence of radiation and chemical reaction. The effects of radiation on a micropolar fluid were also studied by researchers such as Mahmoud (2007); Chamkha *et al*. (2011); Ishak (2010); Ibrahim *et al*. (2008).

The study of heat source/sink effects on heat transfer is very important in view of several physical problems. Afore-mentioned studies include only the effect of a uniform heat source/sink (i.e., temperature dependent heat source/sink) on heat transfer. Pal and Chatterjee (2010, 2015), Subhas Abel and Mahesha (2008), Rahman *et al*. (2009), Bataller (2007), have included the effect of a non-uniform heat source, but confined to the case of viscous fluids only. Bhukta *et al*. (2015) investigated dissipation effect on MHD mixed convection flow over a stretching sheet through a porous medium with a non-uniform heat source/sink.

Due to presence of multi-component species in a system, chemical reaction is bound to occur. It is well known that, in convective heat and mass transfer process, diffusion rates can alter tremendously by chemical reaction. It is observed that, when diffusion is much faster than chemical reaction, then only chemical factors influence the chemical reaction rate. Otherwise, the diffusion and kinetics interact to produce very different effects. Our investigation is particularly for cases in which diffusion and chemical reaction occur roughly at the same speed. A large amount of research work has been reported in the study of heat and mass transfer with chemical reactions due to its considerable importance in the chemical and hydrometallurgical industries. A notable contribution in chemical reactions phenomena was made by Astarita (Astarita 1967). The effects of chemical reaction on non-Newtonian / Newtonian fluid were studied by researchers such as Raptis and Perdikis (2006); El-Amin *et al*. (2008); Patil and Kulakarni (2008); Mohammed Ibrahim *et al*. (2015); Mansour *et al*. (2008); Rahman and Al-Lawatia (2010); Das (2011).

So far, in the above studies, mass diffusion due to temperature gradient, called thermal-diffusion or Soret number, has not been taken into account. The importance of Soret number effect in different convective flows has been explained by many researchers such as Jha and Singh (1990) and Kafoussias (1992), Alam and Sattar (1999), Alam *et al*. (2005). But none of the above mentioned research works discusses the combined effect of Soret, thermal conductivity and non-uniform heat source/sink.

Motivated by the above literature and application, the present paper explores heat and mass transfer by mixed convection from a vertical flat plate embedded in an electrically conducting micropolar fluid saturated porous medium, using the Darcy-Brinkmann Forchheimer Boussinesq model in the presence of a uniform magnetic field, anon-uniform heat source/sink, thermal radiation Ohmic dissipation with variable thermal conductivity and Soret number. The nonlinearity of the basic equations and associated mathematical difficulties have led us to use numerical method. Thus the transformed dimensionless governing equations are solved numerically by using the fifth order Runge-Kutta-Fehlberg method (RKF45) along with shooting technique. It is hoped that the results obtained from the present study will provide useful information for different industrial applications. To the best of our knowledge such a study has not so far appeared in scientific literature.

2. FORMULATION OF THE PROBLEM

We consider a two-dimensional steady mixed convection flow of an incompressible, electrically conducting micropolar fluid towards a surface coinciding with the plane $y = 0$ and the flow region $y>0$.The$x$-axis is taken in the direction along the motion of the sheet, and y – axis is taken perpendicular to it. The flow is generated by the action of two equal and opposite forces along the x-axis and the sheet is stretched in such a way that the velocity at any instant is proportional to the distance from the origin$(x = 0)$. Further, the flow field is exposed to the influence of an external transverse magnetic field of strength $\vec{B} = (0, B_0, 0)$. Frictional heating due to viscous dissipation and Ohmic heating due to application of magnetic field and Soret effects are considered in the present model. The diagrammatic of the problem is displayed in Fig .A. In the light of the above assumptions, the boundary layer equations for mass, momentum, angular momentum, energy and concentration can be written as follows:

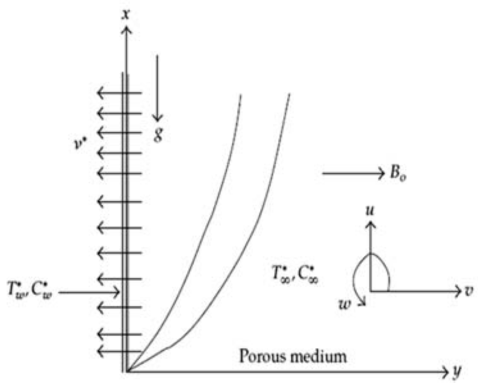

Fig. A. Schematic of the problem.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\left(\nu+\frac{k_1}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k_1}{\rho}\frac{\partial w}{\partial y}-\frac{\nu\varphi}{k}u-\frac{C_b}{\sqrt{k}}\varphi u^2$$

$$-\frac{\sigma B_0^2}{\rho}u+g\beta(T-T_\infty)+g\beta^*(C-C_\infty) \qquad (2)$$

$$\rho j\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=\gamma\frac{\partial^2 w}{\partial y^2}-k_1\left(2w+\frac{\partial u}{\partial y}\right) \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+$$

$$+\frac{\sigma B_0^2}{\rho c_p}u^2+\frac{q'''}{\rho c_p}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (4)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}-$$

$$-kr'(C-C_\infty) \qquad (5)$$

The appropriate physical boundary conditions for the problem under study are given by

$$u=u_w=bx,\ v=0,\ w=-m_0\frac{\partial u}{\partial y},$$

$$T=T_w=T_\infty+A_0\left(\frac{x}{l}\right)^2,\ C=C_w=C_\infty+A_1\left(\frac{x}{l}\right)^2$$

at y = 0

$$u\to 0,\ w\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty \qquad (6)$$

where u and v are the velocity components along the *x* and *y* directions, μ is the dynamic viscosity, ρ is the density of the fluid, ν is the kinematic viscosity, *T* is the temperature of the fluid, *C* is the concentration of the fluid, C_b is the form of the drag coefficient which is independent of viscosity and other properties of the fluid, but depends on the geometry of the medium, *k* is permeability of the porous medium, σ is the electrical conductivity of the fluid, φ is the porosity of the porous medium, β is the coefficient of the thermal expansion, β^* is the coefficient of concentration expansion, c_p is the specific at constant pressure, *w* is the components of microrotation or angular velocity whose rotation is in the direction of the *xy* – plane and j,γ and k_1 are the microinertia per unit mass, spin gradient viscosity and vortex viscosity, respectively. The spin gradient viscosity γ, which defines the relationship between the coefficient of viscosity and microinertia*j* is as follows (Kim and Kim (2007), $\gamma=\mu\left(1+K/2\right)j$, where $K=k_1/\mu$ (>0) is the material parameter. Here all the material constants γ,μ,K and j are non-negative and we take $j=\nu/b$ as a reference length. T_m is the mean fluid temperature, k_T is the thermal diffusion ratio, *l* is the characteristic length, T_w is the wall temperature of the fluid, T_∞ is the temperature of the fluid far away from the sheet, C_w is the wall concentration of the solute and C_∞ is the concentration of the solute far away from the sheet, kr' is the chemical reaction rate constant, D_m is the mass diffusivity, m_0 is a constant such that $0\le m_0\le 1$, and A_0, A_1 are constants.

The non-uniform heat generation/absorption q''' has taken as

$$q'''=\frac{\kappa u_w}{x\nu}\left(Q_0(T_w-T_\infty)e^{-\eta}+Q_1(T-T_\infty)\right),$$

where Q_0 and Q_1 are the coefficients of space and temperature- dependent heat source/sink, respectively. The case $Q_0>0$ and $Q_1>0$ corresponds to internal heat generation while $Q_0<0$ and $Q_1<0$ corresponds to internal heat absorption

The thermal conductivity κ is assumed to vary linearly with temperature and it is of the form

$$\kappa=\kappa_\infty\left(1+\varepsilon\theta(\eta)\right)$$

where $\theta(\eta)=\left.(T-T_\infty)\right/(T_w-T_\infty)$ and $\varepsilon=\left.(\kappa_w-\kappa_\infty)\right/\kappa_\infty$, which depends on the nature of the fluid and it is small parameter. In general, $\varepsilon>0$ for air and liquids such as water, while $\varepsilon<0$ for fluids such as lubrication oils. Following Rosseland approximation (Hsiao (2007)) the radiative heat flux q_r is modeled as,

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. T^4 can be expressed by using Taylor's series as

$$T^4\cong 4T_\infty^3 T-3T_\infty^4$$

Thus, we have from Eq. (4) as

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\left(\kappa+\frac{16T_\infty^3\sigma^*}{3k^*}\right)\frac{\partial T}{\partial y}\right)+$$

$$+\frac{\sigma B_0^2}{\rho c_p}u^2+\frac{q'''}{\rho c_p}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (7)$$

To render dimensionless solutions and facilitate numerical analysis, we define the following dimensionless variables:

$$\eta=\sqrt{\frac{b}{\nu}}y,\qquad w=bx\sqrt{\frac{b}{\nu}}g(\eta),\ u=bxf'(\eta),$$

$$v = -\sqrt{b\nu} f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},$$

$$T - T_\infty = A_0\left(\frac{x}{l}\right)^2 \theta(\eta),\ T_w - T_\infty = A_0\left(\frac{x}{l}\right)^2,$$

$$C - C_\infty = A_1\left(\frac{x}{l}\right)^2 \phi(\eta),\ C_w - C_\infty = A_1\left(\frac{x}{l}\right)^2. \tag{8}$$

where f is the dimensionless stream function, g is the dimensionless microrotation function, η is the similarity variable, l is the characteristic length,

Making use of Eq. (8), the governing Eqs. (2), (3), (7) and (5) with boundary conditions (6) are transformed into

$$(1+K)f''' + ff'' - f'^2 - Da^{-1}f' - cf'^2 + $$

$$+ Kg' - Mf' + Gr\theta + Gm\phi = 0 \tag{9}$$

$$\left(1+\frac{K}{2}\right)g'' - K\left(2g + f''\right) - f'g + fg' = 0 \tag{10}$$

$$(1+R+\varepsilon\theta)\theta'' + \Pr(f\theta' - 2f'\theta) + \varepsilon\theta'^2 + \Pr MEcf'^2 +$$

$$+ \Pr Ecf''^2 + (1+\varepsilon\theta)(Q_0 f' + Q_1\theta) = 0 \tag{11}$$

$$\phi'' + Sc\left(\phi' f - 2\phi f'\right) + ScSo\theta'' - Sc\gamma\phi = 0 \tag{12}$$

The appropriate boundary condition (6) now becomes

$$f(\eta) = 0,\ f'(\eta) = 1,\ g(\eta) = -m_0 f''(\eta),\quad \theta(\eta) = 1,$$
$$\phi(\eta) = 1\ at\ \eta = 0 \tag{13}$$

$$f'(\eta) \to 0,\ g(\eta) \to 0,\ \theta(\eta) \to 0,\ \phi(\eta) \to 0\ as$$
$$\eta \to \infty \tag{14}$$

where $Da^{-1} = \frac{\varphi\nu}{kb}$ is inverse Darcy number, $M = \frac{\sigma}{b\rho}B_0^2$ is the magnetic field parameter, $Gr = \frac{g\beta(T - T_\infty)}{b^2 l}$ is the Grashof number, $Gm = \frac{g\beta^*(C - C_\infty)}{b^2 l}$ is modified Grashof number, $K = \frac{k_1}{\mu}$ is material parameter, $\Pr = \frac{\mu c_p}{\kappa_\infty}$ is Prandtl number, $Ec = \frac{b^2 l^2}{A_0 c_p}$ is Eckert number, $R = \frac{16 T_\infty^3 \sigma^*}{3k^*\kappa_\infty}$ is thermal radiation parameter, $Sc = \frac{\nu}{D_m}$ is Schmidt number, $\gamma = \frac{kr'x}{b}$ is the chemical reaction parameter, $So = \frac{k_T(T_w - T_\infty)}{T_m(C_w - C_\infty)}$ is Soret number.

The most important physical quantities for the problem are skin-friction coefficient (C_f), local Nusselt number (Nu_x) and Sherwood number (Sh_x) which are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2 / 2},\ Nu_x = \frac{xq_w}{\kappa(T_w - T_\infty)},$$

$$Sh_x = \frac{xm_w}{D_m(C_w - C_\infty)} \tag{15}$$

The skin-friction on the flat plate τ_w, rate of heat transfer q_w and rate of mass transfer m_w are given by

$$\tau_w = \left([\mu + k_1]\frac{\partial u}{\partial y}\right)_{y=0},\ q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$m_w = -D_m\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

Thus we from Eq. (15) as

$$C_f \operatorname{Re}_x^{1/2} = (1+K)f''(0),\ Nu_x = -\sqrt{\operatorname{Re}_x}\,\theta'(0),$$
$$Sh_x = -\sqrt{\operatorname{Re}_x}\,\phi'(0)$$
(16)

where $\operatorname{Re}_x = \frac{u_w x}{\nu}$ is the local Reynolds number.

3. METHOD OF SOLUTION

In this study, an efficient numerical scheme Runge-KuttaFehlberg method (Faires and Burden2012) has been employed to investigate the flow model defined by Eqs. (9)-(12) with the boundary conditions Eqs. (13) and (14) for different values of controlling parameters. The method of solution is described in (Faires and Burden 2012) and therefore we have not given any details here. The effects of the emerging parameters on the dimensionless velocity, angular velocity, temperature, concentration, skin friction coefficient, local Nusselt and local Sherwood numbers are investigated. The step size and convergence criteria were chosen to be 0.001 and 10^{-6} respectively. The asymptotic boundary conditions in Eq. (14) were approximated by using a value of 10 for η_{max} as follows:

$$\eta_{max} = 10,\ f'(10) = \theta(10) = \phi(10) = 0 \tag{17}$$

This ensures that all numerical solutions approached the asymptotic values correctly.

4. RESULTS AND DISCUSSIONS

In the present paper our main focus is to investigate the effects of different physical parameters with the help of some important graphs. To validate the present solution, comparison has been made with the previously published data from literature for skin friction in table 1, local Nusselt number in

Table 1 Comparison of $\left(\sqrt{\mathrm{Re}_x}Cf\right)$ for different values of K and other all parameters are zero.

K	Qasim *et al.* (2013)	Present Results
0	-1.000000	-1.000008
1	-1.367872	-1.367996
2	-1.621225	-1.621575
3	-	-1.827382
4	-2.004133	-2.005420
5	-	-2.1648230

Table 2 Comparison of $(\theta'(0))$ for different values of Pr and other all parameters are zero

Pr	Grubka and Bobba (1985)	Chen and Char (1988)	Ishak *et al.* (2008)	Pal *and* Chatterjee (2015)	Present Results
1	1.3333	1.33334	1.3333	1.333333	1.33333334
3	2.5097	2.50997	2.5097	2.509725	2.50972157
10	4.7969	4.79686	4.7969	4.796873	4.79687059

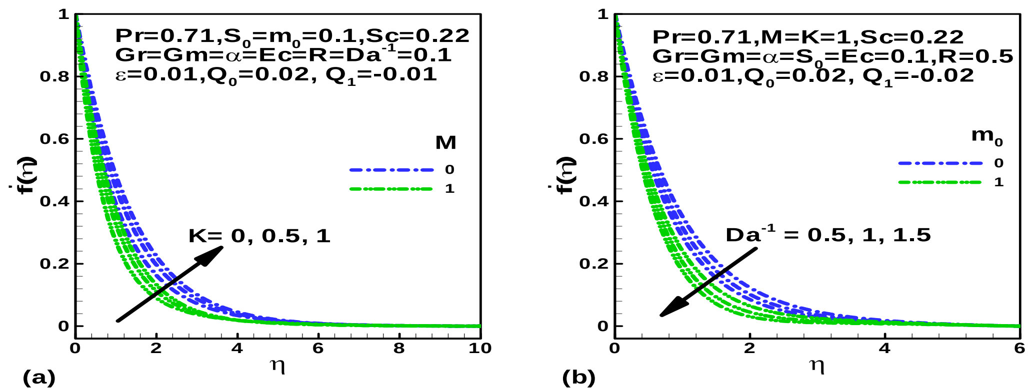

Fig. 1. Effects of M, K, m_0 and Da^{-1} on velocity $f'(\eta)$.

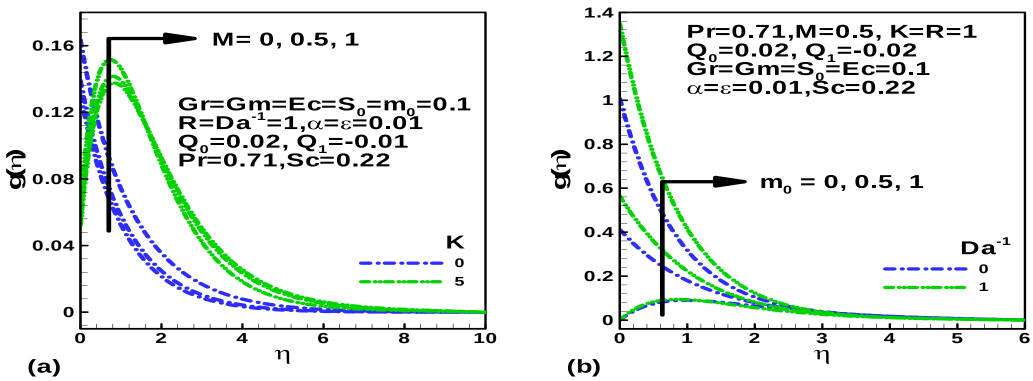

Fig. 2. Effects of M, K, m_0 and Da^{-1} on angular velocity $g(\eta)$.

Table 2, and they are found to be in a favorable agreement.

Fig. 1(a) shows the graph of velocity profile for various values of material parameter and magnetic field parameter. It is noticed from this figure that velocity profile decreases with increase in the value of magnetic field parameter, whereas the reverse effect is observed for increase in the material parameter. Fig. 1(b) depicts the graph of velocity profile for various values of inverse Darcy number and. It is found from this figure that velocity profile for both decrease and increase of the parameters Da^{-1} and m_0.

Figs. 2(a) - 2(b) represent the graph of angular velocity profile $g(\eta)$ with magnetic field parameter, material parameter, constant m_0 and inverse Darcy number, we understand by analyzing the graphs that the effect of M, K, m_0 and Da^{-1} is to increase the angular velocity.

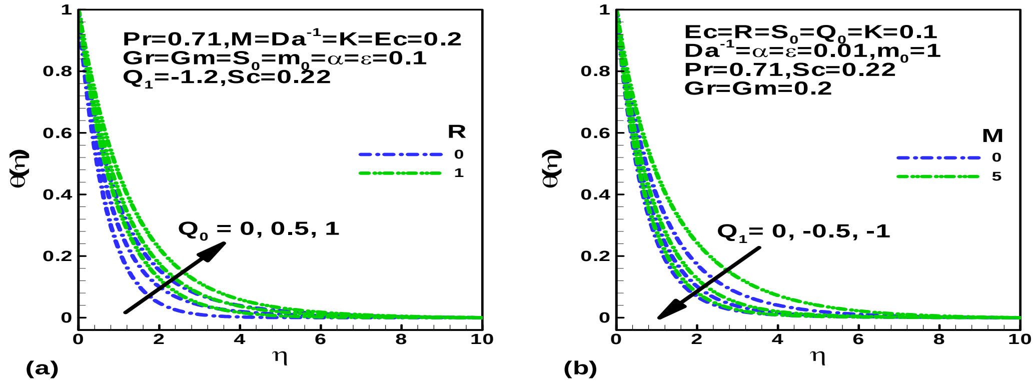

Fig. 3. Effects of R, Q_0, Q_1 and M on temperature $\theta(\eta)$.

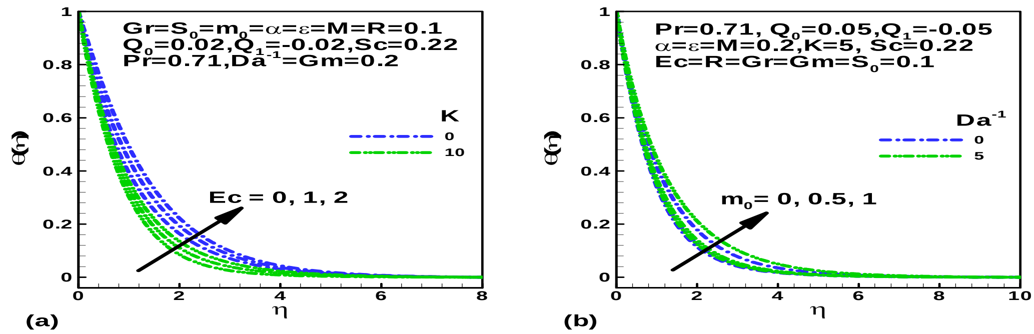

Fig. 4. Effects of K, Ec, m_0 and Da^{-1} on temperature $\theta(\eta)$.

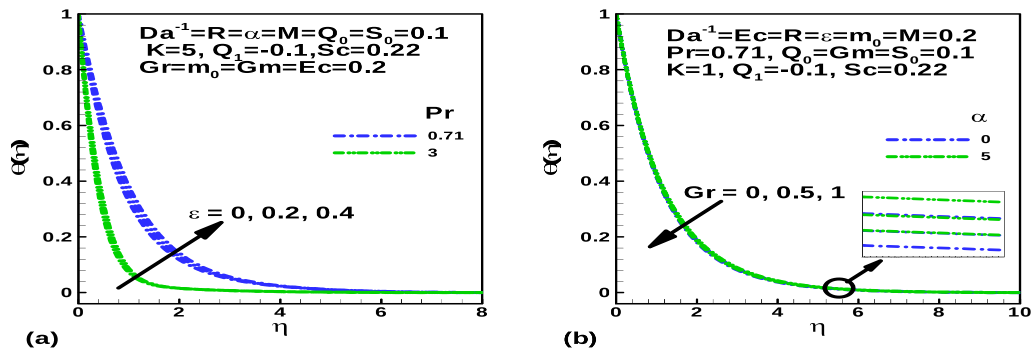

Fig. 5. Effects of K, Ec, m_0 and Da^{-1} on temperature $\theta(\eta)$.

In Figs. 3(a) and 3(b), we depict the effects of non-uniform heat source parameter Q_0, thermal radiation parameter *R*, temperature dependent heat source parameter Q_1 and magnetic field parameter *M* on the dimensionless temperature profile $\theta(\eta)$ for the other fixed parameters. It is found that, with the increasing values of Q_0, *R* and *M*, the thermal boundary layer thickness increases. The dimensionless temperature decreases with an increase in the Q_1.

In Figs. 4(a) and 4(b), the effects of Eckert number *Ec*, material parameter *K*, constant m_0 and inverse Darcy number Da^{-1} on the dimensionless temperature $\theta(\eta)$ are displayed respectively. With an increase in *Ec*, m_0 and Da^{-1}, the dimensionless temperature increases, while the dimensionless temperature decreases with increase in *K*.

The dimensionless temperature profiles $\theta(\eta)$ for various physical parameters porous ε, Prandtl number *Pr*, Granshof number *Gr* and local inertia-coefficient α are displayed in Figs. 5(a) and 5(b).Temperature profiles increase with increase in the value of the ε and α while, temperature decreases with increase in the value of *Pr* and *Gr*.

The influence of Schmidt number *Sc*, Soret number *So*, modified Grashof number *Gm* and magnetic field parameter *M* on the dimensionless concentration are presented in Figs. 6(a) and 6(b). With an increase in *Sc* and *Gm*, the dimensionless concentration decreases, while the dimensionless concentration increases with increase of *So* and *M*.

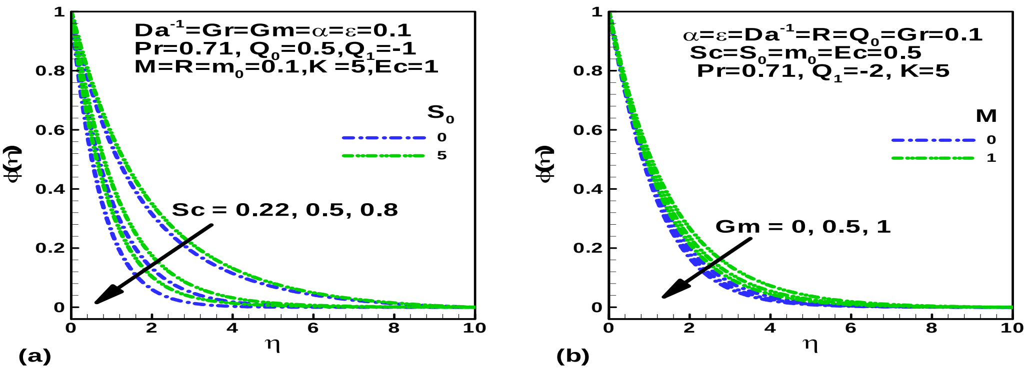

Fig. 6. Effects of Sc, S_0, Gm **and** M **on concentration** $\phi(\eta)$.

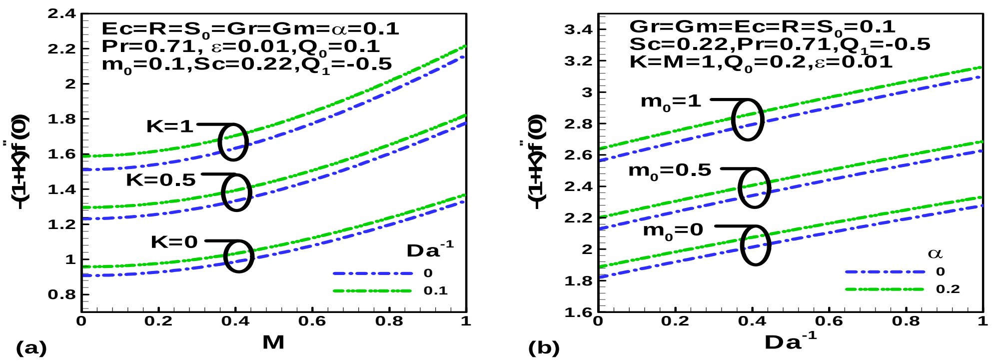

Fig. 7. Variation of skin friction coefficient with M, Da^{-1}, K, m_0, **and** α .

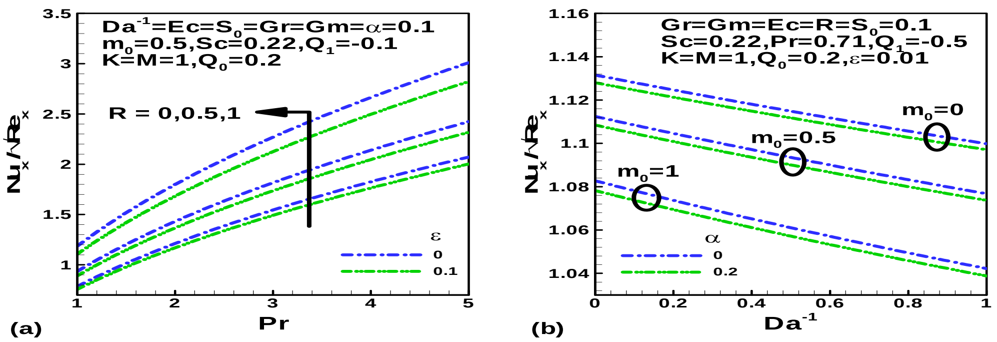

Fig. 8. Variation of heat transfer rate with $\Pr$, ε, R, Da^{-1}, m_0 **and** α .

Fig. 7(a) depicts the nature of the skin-friction factor with material parameter *K*, inverse Darcy number Da^{-1}, and magnetic field parameter *M*. It is noted that the skin-friction coefficient monotonically increases with an increase in *K* and *M*. Fig. 7(b) shows the nature of skin-friction coefficient with m_0, α and Da^{-1}. It is observed that skin-friction coefficient monotonically increases with an increase in m_0 and Da^{-1}.

The variation of the Nusselt number (dimensionless heat transfer rate at the surface) is presented for different parameters in Figs 8(a) and 8(b). Fig. 8(a) presents the behavior of the heat transfer rate against Prandtl number *Pr* with different values of thermal radiation parameter *R* and porous parameter ε. It is noticed that with the increasing of *R*, the rate of heat transfer increase with *Pr*.

Fig. 8(b) exhibits the variation of heat transfer rates with inverse Darcy number Da^{-1} for different values of m_0 and local inertia-coefficient α . It is observed that an increase in m_0 and α decreases the heat transfer rate at the surface.

The variation of the Sherwood number (dimensionless mass transfer rate at the surface) is displayed for various parameters in graphs 9(a) and 9(b). Fig. 9(a) shows the behavior of the mass transfer rate against inverse Darcy number Da^{-1} with various values of Schmidt number *Sc* and Soret number *So*. The mass transfer rate increases with an increase in the Schmidt number *Sc* and inverse Darcy number Da^{-1}.

Fig. 9(b) explores the variation of mass transfer rates with magnetic field parameter *M* for different

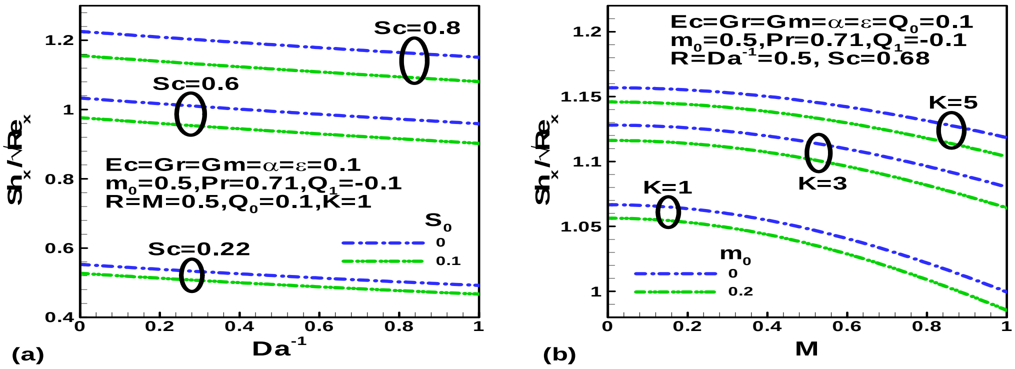

Fig. 9. Variation of mass transfer rate with Da^{-1}, Sc, S_0, M, m_0 **and** K.

Table 3 Variation of $f''(0), -\theta'(0)$ **and** $-\phi'(0)$ **with different values of parameters when** $\Pr=1$**,** $\alpha = Da^{-1} = Gm = Gr = Q_o = 0.1,$ **and** $Q_1 = -0.1$

K	M	m_0	R	ε	Ec	S_0	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	0.2	0.2	0.3	0.1	0.1	0.1	-0.9761	1.0670	0.4990
0.5	0.2	0.2	0.3	0.1	0.1	0.1	-0.8280	1.0979	0.5212
1	0.2	0.2	0.3	0.1	0.1	0.1	-0.7282	1.1191	0.5378
2	0.5	0.2	0.3	0.1	0.1	0.1	-0.6568	1.1263	0.5478
0.5	1	0.2	0.3	0.1	0.1	0.1	-0.8288	1.0621	0.5058
0.5	2	0.2	0.3	0.1	0.1	0.1	-1.3176	0.8795	0.4098
0.5	5	0	0.3	0.1	0.1	0.1	-2.8826	0.3705	0.2712
0.5	0.2	0.4	0.3	0.1	0.1	0.1	-3.1698	0.3287	0.2445
0.5	0.2	0.7	0.3	0.1	0.1	0.1	-3.4231	0.2824	0.2202
0.5	0.2	1	0.5	0.1	0.1	0.1	-3.7168	0.2059	0.1914
0.5	0.2	0.2	1	0.1	0.1	0.1	-3.7166	0.1871	0.1921
0.5	0.2	0.2	2	0.1	0.1	0.1	-3.7165	0.1645	0.1928
0.5	0.2	0.2	3	0.2	0.1	0.1	-3.7164	0.1529	0.1934
0.5	0.2	0.2	0.3	0.3	0.1	0.1	-3.7164	0.1544	0.1934
0.5	0.2	0.2	0.3	0.4	0.1	0.1	-3.7164	0.1559	0.1934
0.5	0.2	0.2	0.3	0.5	0.4	0.1	-3.7160	0.1789	0.2009
0.5	0.2	0.2	0.3	0.1	0.7	0.1	-3.7156	0.5149	0.2084
0.5	0.2	0.2	0.3	0.1	1	0.1	-3.7152	0.8509	0.2159
0.5	0.2	0.2	0.3	0.1	2	0.5	-3.7141	1.9701	0.4217
0.5	0.2	0.2	0.3	0.1	0.1	0.8	-3.7142	1.9702	0.5573
0.5	0.2	0.2	0.3	0.1	0.1	1	-3.7143	1.9703	0.6476
0.5	0.2	0.2	0.3	0.1	0.1	2	-3.7148	1.9704	1.0994

values of material parameter K and m_0. It is noted that an increases in K, increase the mass transfer rate at the surface. Table 3 displays the effects of the skin- friction coefficient, Nusselt number, and Sherwood number for various values of pertinent parameters. It is seen from the table that the effect of increasing values of K, R, ε and Ec is to increase skin-friction coefficient, whereas increasing M, m_0 and So decreases skin-friction coefficient. Further, from table 3 one can note that the Nusselt number increases with an increase in the values of K, ε, Ec, and *So*, while it decreases with the increase of m_0, *M*, and *R*. In addition, the value of Sherwood number increases as *K, R,* ε, *Ec,* and *So* increases. But Sherwood number decreases on increasing the values of *M* and m_0.

5. CONCLUSIONS

The paper deals with the numerical analysis of combined heat and mass transfer effects on steady MHD non-Darcian convective flow past over a stretching surface in a micropolar fluid in the presence of Soret, viscous dissipation and thermal radiation. The resulting partial differential equations were transformed to a set of ordinary differential equations and then these equations are solved numerically, using the Runge-Kutta-Fehlberg method along with shooting techniques. The following conclusions are drawn from the numerical results as follows:

(a) Increase in magnetic field parameter tends to decrease in velocity while increase in temperature and concentration profiles.
(b) Temperature increases with increase in the value of the Eckert number and inverse Darcy number.
(c) Angular velocity increases with increase in the value of m_0, magnetic field parameter M, material parameter K, inverse Darcy number Da^{-1}.
(d) Velocity profiles and inverse Darcy number are inversely proportional to each other
(e) Angular velocity and thermal Grashof number are directly proportional to each other.
(f) The rate of mass transfer increases with Soret number and the rate of mass transfer decreases with increase in Schmidt number.

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