



Evaluation Study of Pressure-Strain Correlation Models in Compressible Flow

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ABSTRACT

This paper is devoted to the second-order closure for compressible turbulent flows with special attention paid to modeling the pressure-strain correlation appearing in the Reynolds stress equation. This term appears as the main one responsible for the changes of the turbulence structures that arise from structural compressibility effects. The structure of the gradient Mach number is similar to that of turbulence, therefore this parameter may be appropriate to study the changes in turbulence structures that arise from structural compressibility effects. Thus, the incompressible model (LRR) of the pressure-strain correlation and its corrected form by using the turbulent Mach number, fail to correctly evaluate the compressibility effects at high shear flow. An extension of the widely used incompressible model (LRR) on compressible homogeneous shear flow is the major aim of the present work. From this extension the standard coefficients C_i became a function of the compressibility parameters (the turbulent Mach number and the gradient Mach number). Application of the model on compressible homogeneous shear flow by considering various initial conditions shows reasonable agreement with the DNS results of Sarkar. The ability of the models to predict the equilibrium states for the flow in cases A₁ and A₄ from DNS results of Sarkar is examined, the results appear to be very encouraging. Thus, both parameters M_t and M_g should be used to model significant structural compressibility effects at high-speed shear flow.

Keywords: Compressible; Turbulence; Pressure-strain; Model of turbulence; Shear flow.

NOMENCLATURE

b_{ij}	Reynolds stress anisotropy	U_i	velocity in the direction, x_i
c	speed of sound	ϵ_s	solenoidal part of the dissipation
M_t	turbulent Mach number	δ_{ij}	Kronecker delta
M_g	gradient Mach number	$(\)'$	Reynolds fluctuation
P	pressure	$(\)_{,i}$	spatial gradient
R	specific gas constant	Φ_{ij}	pressure-strain correlation
T	time	γ	specific heat ratio
T	temperature		

1. INTRODUCTION

Compressible turbulence modeling is an essential element for calculations of many problems of practical engineering interest, such as combustion, environment and aerodynamics. The compressibility phenomena have been extensively studied and numerical simulation of compressible turbulent flows using compressible turbulence models have been performed by many authors. Previous studies carried out in the last 30 years have conjectured that compressibility effect

was linked with dilatational dissipation and pressure-dilatation correlation, as it is represented by the models of Zeman (1990), Sarkar (1991) and others. During the last thirty years, many theoretical and experimental works have been developed primarily to understand and predict the behavior of turbulent flows. In fact, the models which developed turbulence gave full satisfaction in simple configurations for the homogeneous flows.

In this context, many studies, Sarkar and Lakshmanan (1991), Bradshaw (1977), H.

Marzougui *et al.* (2005) Carlos A. Gomez (2013), Hechmi *et al.* (2008) of the compressible shear flow which show the changes of the turbulence structures are principally due to the structural compressibility effects which significantly affect the pressure-strain correlation. Thus, the pressure-strain correlation appears as the main factor for the changes in the magnitude of the Reynolds stress anisotropies. The extension of the standard models to compressible flows represents a research topic of great scientific and industrial interest.

A major challenge related to this extension is to take into account the compressibility effects in the classical scheme closure of turbulence.

It is well concluded that the Favre Reynolds stress closure using the standard models of the pressure-strain correlation with the addition of the compressible dissipation and pressure-dilatation correlation models failed to predict high compressible flows (Speziale *et al.* (1995), Sarkar (1995) and Hamba (1999) also performed DNS results of compressible homogeneous shear flow and reached similar conclusions concerning the roles of dilatational terms.

The consequent effects on the pressure-strain correlation may cause significant changes on turbulence structures. According to the DNS results of Sarkar (1995), there is a reduction in the magnitude of the Reynolds shear stress anisotropy and an increase in the magnitude of the normal stress anisotropy. As a consequence, the pressure strain modeling seems to be an important issue in the second order closures for the compressible turbulent flows.

A method of including compressibility effects in the pressure strain correlation is the subject of the present study. The LRR model developed by Launder-Reece and Rodi (1975) has shown a great success in simulating a variety of incompressible complex turbulent flows. Thus, a compressible correction for the LRR model is the major aim of this study. In the present work, the correction concerns essentially the C_i coefficients which became in a compressible situation a function of the turbulent Mach number and the gradient Mach number.

In the present study, we concentrate on evaluating the models of the pressure-strain correlation in the homogeneous shear flow.

2. GOVERNING EQUATIONS

The General equations governing the motion of a compressible fluid are the Navier-Stokes equations. They can be written as follows for mass, momentum and energy conservation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = \frac{\partial \sigma_{ij}}{\partial x_j} \quad (2)$$

$$\frac{\partial(\rho e)}{\partial t} + \frac{\partial(\rho e u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \sigma_{ij} u_i - \frac{\partial}{\partial x_j} (k T_{,j}) \quad (3)$$

Here ρ is the density, u is the velocity, p is the pressure, e is the internal energy, T is the temperature, μ is the viscosity, k is the thermal conductivity and C_v is specific heat at constant volume. Where:

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij} \quad (4)$$

$$e = C_v T \quad (5)$$

$$\tau_{ij} = 2\mu S_{ij} \quad (6)$$

$$S_{ij} = (u_{i,j} + u_{j,i})/2 \quad (7)$$

For an ideal gas, the relation between pressure, density and temperature can be written as follows:

$$p = \rho RT \quad (8)$$

3. BASIC EQUATIONS OF THE FAVRE SECOND-ORDER CLOSURE IN COMPRESSIBLE FLOWS

For compressible homogeneous shear flow, the mean

gradient velocity is given by:

$$\tilde{U}_{i,j} = S\delta_{i1}\delta_{j2} = \begin{cases} S, & \text{if } (i,j) = (1,2) \\ 0, & i \neq 1, j \neq 2 \end{cases} \quad (9)$$

Where S is the constant mean shear rate. These considerations lead to:

$$\tilde{U}_{k,k} = 0, \bar{\rho} = \text{constante} \quad (10)$$

The Favre equations for conservation of mass, momentum and energy are:

$$\frac{\partial}{\partial t} \bar{\rho} + \frac{\partial}{\partial x_i} (\bar{\rho} \tilde{U}_i) = 0 \quad (11)$$

$$\frac{\partial}{\partial t} (\bar{\rho} \tilde{U}_i) + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{U}_i \tilde{U}_j) + \frac{\partial}{\partial x_j} \overline{\rho u_i u_j} = \frac{\partial}{\partial x_j} [\tilde{\tau}_{ij} + \overline{\tau}_{ij} - \bar{p} \delta_{ij}] \quad (12)$$

$$\begin{aligned} \frac{\partial}{\partial t} (\bar{\rho} \overline{C_v T}) + \frac{\partial}{\partial x_i} (\bar{\rho} \overline{C_v T} \tilde{U}_i) = & -\bar{p} \frac{\partial}{\partial x_i} \tilde{U}_i - \\ \bar{p} \frac{\partial}{\partial x_i} \overline{u_i} - \bar{p}' \frac{\partial}{\partial x_i} \overline{u_i} + \overline{\Phi} + \frac{\partial}{\partial x_i} k \frac{\partial}{\partial x_i} T - & \\ \frac{\partial}{\partial x_i} (\bar{\rho} \overline{C_v u_i T}) & \end{aligned} \quad (13)$$

Where :

$$\tilde{\tau}_{ij} = 2\tilde{\mu} \tilde{S}_{ij} - \frac{2}{3} \tilde{\mu} \tilde{U}_{k,k} \delta_{ij} \quad (14)$$

$$\tilde{S}_{ij} = (\tilde{U}_{i,j} + \tilde{U}_{j,i})/2 \quad (15)$$

$$\tau_{ij}'' = 2\tilde{\mu} s_{ij}'' - \frac{2}{3} \tilde{\mu} u_{k,k}'' \delta_{ij} \quad (16)$$

$$\overline{\Phi} = \overline{\tau_{ij} u_{i,j}} \quad (17)$$

Classically, the second-order closure requires a transport equation of the turbulent dissipation rate. The new concept of dissipation in compressible turbulence was proposed by Sarkar *et al.* (1995), Zeman (1991) and Ristorcelli (1997) and can be written as follows

$$\epsilon_{ij} = \frac{2}{3} \epsilon \delta_{ij} \quad (18)$$

Where for homogeneous turbulence $\bar{\rho}\varepsilon_s = \overline{\mu\omega_1\omega_1'}$, ω_1' is the fluctuating vorticity and $\bar{\rho}\varepsilon_c = \frac{4}{3}\mu(u_{1,i}')^2$ represent the solenoidal and compressible parts of ε respectively. Sarkar *et al.* (1990) have mentioned that for moderate Mach numbers, ε_s is insensitive to the compressibility changes. For ε_s , model transport equation, similar to what it was obtained in the incompressible case. Such a model equation is written in (1985), namely:

$$\bar{\rho} \frac{d\varepsilon_s}{dt} = -C_{\varepsilon 1} \bar{\rho} \frac{\varepsilon_s}{k} \widetilde{u_i'' u_j''} \widetilde{U}_{ij} + \overline{p' u_{i,1}'} - C_{\varepsilon 2} \bar{\rho} \frac{\varepsilon_s^2}{k} \quad (19)$$

Where $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are respectively the model constants $C_{\varepsilon 1}=1.44$ and $C_{\varepsilon 2}=1.83$

ε_c is generally taken to be proportional to ε_s through the following equation algebraic equation:

$$\varepsilon_c = f(M_t) \varepsilon_s \quad (20)$$

As it is suggested in model (1991), one can write $f(M_t)=0.5M_t^2$

$f(M_t)$ is a function of the turbulent Mach number The Reynolds stress is solutions of the transport equation:

$$\bar{\rho} \frac{d}{dt} (\widetilde{u_i'' u_j''}) = P_{ij} + \Phi_{ij}^* + \frac{2}{3} \overline{p' d'} \delta_{ij} - \frac{2}{3} \bar{\rho} \varepsilon \delta_{ij} \quad (21)$$

Where P the turbulent kinetic energy production:

$$P_{ij} = -\bar{\rho} \widetilde{u_i'' u_j''} \widetilde{U}_{ij} \quad (22)$$

$$\Phi_{ij}^* = p' \left(\frac{\partial u_i''}{\partial x_j} + \frac{\partial u_j''}{\partial x_i} \right) - \frac{2}{3} \overline{p' d'} \delta_{ij} \quad (23)$$

The turbulent kinetic energy equation is

$$\bar{\rho} \frac{dk}{dt} = -\bar{\rho} \widetilde{u_i'' u_j''} \widetilde{U}_{ij} + \overline{p' d'} - \bar{\rho} \varepsilon \quad (24)$$

The turbulent Mach number is described in (1985) by the transport equation as follows

$$\frac{dM_t}{dt} = M_t \frac{P}{2k} + \frac{M_t}{2\bar{\rho}k} \left[1 + \frac{1}{2} \gamma (\gamma - 1) M_t^2 \right] (\overline{p' d'} - \bar{\rho} \varepsilon) \quad (25)$$

4. MODEL OF TURBULENCE

It is generally accepted that the term of the pressure-dilatation plays an important role in the modeling of turbulent compressible flows and behavior analysis of the turbulence. Indeed, this term which appears in the equation of turbulent kinetic energy seems to have important effects on the evolution of turbulence. Such effect is manifested by a reduction in the turbulent kinetic energy when the turbulent Mach number increases. In this context, the results of the numerical simulation developed by Sarkar *et al.* (1991,1992,1995) and Blaisdell *et al.* (1991) have already shown that the pressure-dilatation correlation is an important indicator for the compressibility.

In this part, we will choose a number of models from literature that are well-known and that express the pressure-dilatation.

Model of Sarkar(1992)

The algebraic form of the model developed by Sarkar *et al.* (1992) and expressing the pressure-dilatation correlation is as follows:

$$\overline{p' d'} = -\alpha_2 P M_t + \alpha_3 \bar{\rho} \varepsilon_s M_t^2 \quad (26)$$

$$\alpha_2 = 0.15$$

$$\alpha_3 = 0.2$$

Model of Zeman(1990)

$$\overline{p' d'} = -\frac{1}{2} \frac{D}{Dt} \left(\frac{\overline{p'^2}}{\gamma \bar{p}} \right) = (\gamma \bar{p})^{-1} \left[\frac{\overline{p'^2} - \overline{p_e^2}}{\tau} + \left(\frac{5-3\gamma}{12} \right) \overline{p'^2} \overline{U_{k,k}} \right] \quad (27)$$

Where τ is the acoustic time scale,

$$\tau = \frac{k}{\varepsilon} M_t \quad (28)$$

$$p_e^2 = 2\bar{\rho}^2 k \gamma R \bar{T} \left(\frac{M_t^2 + M_t^4}{1 + M_t^2 + M_t^4} \right) \quad (29)$$

Model of F. Hamba(1999)

$$\overline{p' d'} = -(1 - C_{pd3} \chi_p) \left[C_{pd1} M_t^2 \frac{D}{Dt} (\bar{\rho} k) + C_{pd2} \gamma \bar{\rho} M_t^2 k \widetilde{U_{i,i}} \right] \quad (30)$$

Where

$$\chi_p = \left(\frac{\overline{p'^2}}{2\bar{\rho}^2 C^2 k} \right) \quad (31)$$

When C_{pdi} , ($i = 1,3$) are numerical constants

Model of El Baz and Launder (1996)

$$\overline{p' d'} = -F \left(P - \frac{2}{3} k U_{k,k} \right), F = 1.5 M_t^2 \quad (32)$$

Model of Ristorcelli (1997)

$$\overline{p' d'} = \frac{C_1 M_t^2}{C_2 + C_1 M_t^2} (R_{ij} \widetilde{U}_{ij} - \varepsilon_s) \quad (33)$$

We note that there are other models using different compressibility parameters as the density variance, for example those developed by Taulbee *et al.* (1991), Hamba (1999), Simone *et al.* (1997), Hanafi (2013), and Bogdanoff *et al.* (1983) .

For compressible homogeneous shear flow, the DNS results of Sarkar (1995) show the change in the magnitude of the components of Reynolds stress anisotropy tensor and the growth rate of turbulent kinetic energy. Thus, the compressibility has a significant effect on the pressure fields and in consequence on its correlations such as the pressure-strain, the pressure-dilatation. The work of Sarkar (1991) recommends to use the gradient Mach number with the turbulent Mach number in order to predict the correct behavior of compressible turbulence. This is the point on which the present work is centered. At first, we have examined some basic assumptions which were used in constructing turbulence models. For compressible homogeneous shear flow, the growth rate of the turbulent kinetic energy is given by the following equation.

$$\begin{aligned} \hat{\Lambda} &= \frac{1}{SK} \frac{dK}{dt} = \frac{P - \varepsilon + \overline{p'd'}}{SK} \\ &= -2b_{12} - \frac{\varepsilon_s}{Sk} + \frac{\overline{p'd'} - \varepsilon_c}{Sk} \end{aligned} \quad (34)$$

Vreman *et al.* (1996) performed DNS results for compressible mixing layers. From which, they deduced that the diagonal rapid part of the pressure-strain correlation are approximately proportional to the growth rate. Pantano and Sarkar (2003) used DNS results to study the compressibility effects in the high-speed turbulent shear layer. They showed that the components of the pressure-strain correlation and the pressure variance normalized by its incompressible counterparts decrease similarly.

$$\frac{\Phi_{ij}}{(\Phi_{ij})^I} \sim \frac{\sqrt{p'^2}}{\sqrt{p'^2}^I} \quad (35)$$

The hypothesis about an approximate proportionality between $\frac{\Phi_{ij}}{(\Phi_{ij})^I}$, $\frac{\sqrt{p'^2}}{\bar{\rho}K} / \left(\frac{\sqrt{p'^2}}{\bar{\rho}K}\right)^I$ and $\hat{\Lambda}/\Lambda_I$ can be supported as in H.Marzougui and al.(2005) and we can write.

$$\frac{\Phi_{ij}}{SK} \frac{1}{(\Phi_{ij})^I} \sim \frac{\hat{\Lambda}}{\Lambda_I} \sim \frac{\frac{\sqrt{p'^2}}{\bar{\rho}K}}{\left(\frac{\sqrt{p'^2}}{\bar{\rho}K}\right)^I} \quad (36)$$

Now, we consider the equation

$$\frac{\Phi_{ij}}{SK} \frac{1}{(\Phi_{ij})^I} = \hat{\Lambda} \quad (37)$$

It comes

$$\frac{\Phi_{ij}}{(\Phi_{ij})^I} = \frac{dK}{dK^I} \frac{dt}{dt} \quad (38)$$

The turbulent kinetic energy K and K^I satisfied the transport equations

$$\frac{dK}{dt} = P - \varepsilon_s - \varepsilon_c + \overline{p'd'} \quad (39)$$

$$\frac{dK^I}{dt} = P^I - \varepsilon_s \quad (40)$$

From the two last equations, the time derivative ratio in RHS of Eq. (38) can be written as

$$\frac{dK}{dK^I} \frac{dt}{dt} = \left[\frac{P - \varepsilon_s - \varepsilon_c + \overline{p'd'}}{P^I - \varepsilon_s} \right] \left(1 - \frac{\varepsilon_c}{\varepsilon_s} \right) \left(1 + \frac{\overline{p'd'}}{P - \varepsilon} \right) \quad (41)$$

The DNS results of Sarkar(1995) show that the relative dissipation ε_s/P is less affected by compressibility and it shows a similar equilibrium value as its incompressible counterpart ε_s/P^I . Thus, the term between the square brackets in (41) can be approximated by 1 and we can obtain

$$\frac{dK}{dK^I} \frac{dt}{dt} = \left(1 - \beta \frac{\varepsilon_c}{\varepsilon_s} \right) \left(1 + \frac{\overline{p'd'}}{P - \varepsilon} \right) \quad (42)$$

$$\beta = \left(\frac{1}{\varepsilon_s} \right) \quad (43)$$

Where, β is a numerical constant can be determined by the equilibrium value of ε_s/P^I . From eqs.(38,42), we have

$$\overline{p'd'} = \left[\frac{\Phi_{ij}^I - 1 + \beta \frac{\varepsilon_c}{\varepsilon_s}}{1 - \beta \frac{\varepsilon_c}{\varepsilon_s}} \right] (P - \varepsilon) \quad (44)$$

Pantano and Sarkar (1991) use two scales to express Φ_{ij}/Φ_{ij}^I by a function of the turbulent Mach number and gradient Mach number.

$$\frac{\Phi_{ij}}{\Phi_{ij}^I} = 1 - f(M_t, M_g) \quad (45)$$

Where $f(M_t, M_g)$ is a function model of turbulent Mach number and gradient Mach number.

$$f(M_t, M_g) = M_t^2 \left(\frac{b - e^{[-(\alpha M_g - \beta)^2]}}{1 + b M_t^2} \right) \quad (46)$$

$$\overline{p'd'} = -f(M_t, M_g)P + \alpha M_t^2 \varepsilon_s \quad (47)$$

$$\alpha=0.9, \beta=0, b=4.2$$

The present work deals with the problem of modeling of the pressure-dilatation, for this reason we choose a number of models from literature that are well-known.

Launder Reece and Rodi(1975)

The models established in compressible turbulent flows are deduced by a simple extension of their incompressible counterparts. This extension concerned only one incompressible model, it is due to Launder, Reece and Rodi (1975), this model is written in the following form:

$$\begin{aligned} \Phi_{ij}^* &= -C_1 \bar{\rho} \varepsilon_s b_{ij} + C_2 \bar{\rho} k \left(\tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{kk} \delta_{ij} \right) + \\ &C_3 \bar{\rho} k \left(b_{ip} \tilde{S}_{jp} + b_{jp} \tilde{S}_{ip} - \frac{2}{3} b_{pq} \tilde{S}_{pq} \delta_{ij} \right) + \\ &C_4 \bar{\rho} k \left(b_{ip} \tilde{\Omega}_{jp} + b_{jp} \tilde{\Omega}_{ip} \right) \end{aligned} \quad (48)$$

Where C_1, C_2, C_3 and C_4 are constants that take on the values of: $C_1=3, C_2=0.8$ et $C_3=1.75$ et $C_4=1.31$

$$\tilde{S}_{ij} = \frac{1}{2} (\tilde{u}_{i,j} + \tilde{u}_{j,i}) \quad (49)$$

$$\tilde{\Omega}_{ij} = \frac{1}{2} (\tilde{u}_{i,j} - \tilde{u}_{j,i}) \quad (50)$$

$$b_{ij} = \frac{1}{2k} \widetilde{u_i u_j} - \frac{1}{3} \delta_{ij} \quad (51)$$

Model of Adumitroaie et al. (1999)

Adumitroaie *et al.* (1999) assumed that incompressible modeling approach of the pressure-strain correlation can be used to develop turbulent models taking into account compressibility effects. Considering a none zero divergence for the velocity fluctuation called the compressibility continuity constraint and using different models for the pressure dilatation which is proportional to the trace of the pressure strain, their model for the linear part

of this term is written as:

$$\begin{aligned} \Phi_{ij}^* = & -C_1 \bar{\rho} \varepsilon_s b_{ij} + \bar{\rho} k \left(\frac{4}{5} + \frac{2}{5} d_1 \right) \left(\tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{11} \delta_{ij} \right) + \\ & 2\bar{\rho} k (1 - C_3 + 2d_2) \left[b_{ik} \tilde{S}_{jk} + b_{jk} \tilde{S}_{ik} - \right. \\ & \left. \frac{2}{3} b_{ml} \tilde{S}_{ml} \delta_{ij} \right] - (1 - C_4 - 2d_2) \bar{\rho} k \left(b_{ik} \tilde{\Omega}_{jk} + \right. \\ & \left. b_{jk} \tilde{\Omega}_{ik} - \frac{4}{3} d_2 \tilde{S}_{kk} b_{ij} \right) \end{aligned} \quad (52)$$

Where: $C_3 = C_3^I + 0.3M_t$;

$C_4 = C_4^I - 0.3M_t$

Present Model

$$\overline{p'd'} = -f(M_t, M_g)P + \alpha M_t^2 \varepsilon_s \quad (53)$$

Where ; $C_3 = C_3^I + M_t^2 \left(\frac{b - e^{-(\alpha M_g - \beta)^2}}{1 + bM_t^2} \right)$

$C_4 = C_4^I - M_t^2 \left(\frac{b - e^{-(\alpha M_g - \beta)^2}}{1 + bM_t^2} \right)$

In Eqs. (52) and (53), the coefficients C_3^I and C_4^I are those of the incompressible LRR model

$C_3^I = 1.75$

$C_4^I = 1.31$

5. RESULTS AND DISCUSSION

The transport Eqs. (19), (21), (24) and (25) on which the second order closure for compressible homogeneous shear flow is based, are solved using the fourth-order accurate Runge-Kutta numerical scheme.

The ability of the proposed model to predict the anisotropy of compressible homogeneous turbulent shear flow will be now considered. The model predictions will be compared with DNS results conducted by Sarkar (1995) for cases A₁ and A₄. These cases correspond to different initial conditions for which the initial values of the gradient Mach number Mg change by changing the initial values of Sk/ε and taking Mt constant as it is listed in Table1.

Figures (1, 2), (3, 4) and (5, 6) show the non-dimensional time (St) variation of the Reynolds stress anisotropies b₁₁, b₂₂, and b₁₂. From these figures, it is clear that at low-Mg values, Figs. (1, 3 and 5), the Adumitroaie *et al.* (1999) model is in agreement with DNS data but at high-Mg values Figs. (2, 4, 6), the proposed model appears to be able to predict correctly the significant decrease in the normalized turbulent production term $-2b_{12}$ and the increase in the streamwise term b₁₁, as well as the transverse b₂₂ Reynolds stresses anisotropies with increasing the gradient Mach number. Results obtained with Adumitroaie *et al.*'s model (1999) disagree with DNS data, especially at high-Mg values in case A₄, this model is still unable to predict the changes in the magnitude of the Reynolds stress anisotropy when the compressibility is higher.

Table 1 Initial condition for the DNS results of Sakar (1995) in compressible homogeneous shear flow

Case	Mt	Mg	Sk/ε
A ₁	0.4	0.22	1.6
A ₂	0.4	0.44	3.6
A ₃	0.4	0.66	5.4
A ₄	0.4	1.32	10.8

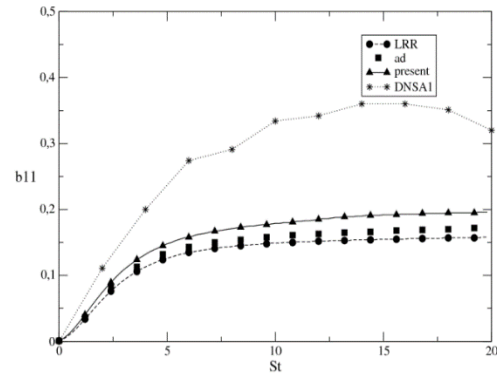


Fig. 1. Time evolution of the Reynolds-stress anisotropy b₁₁ in case A₁.

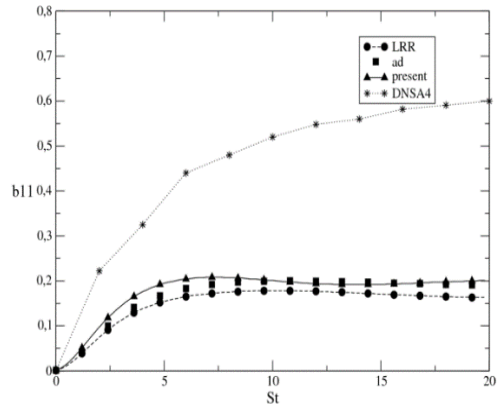


Fig. 2. Time evolution of the Reynolds-stress anisotropy b₁₁ in case A₄.

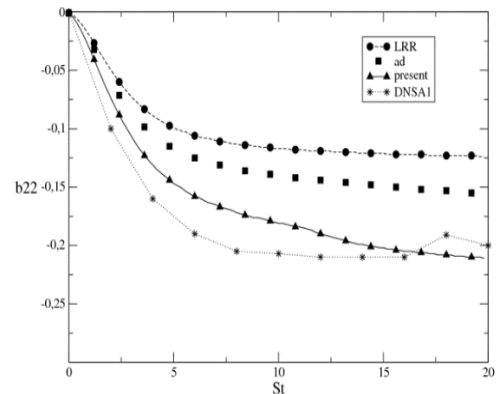


Fig. 3. Time evolution of the Reynolds-stress b₂₂ in case A₁.

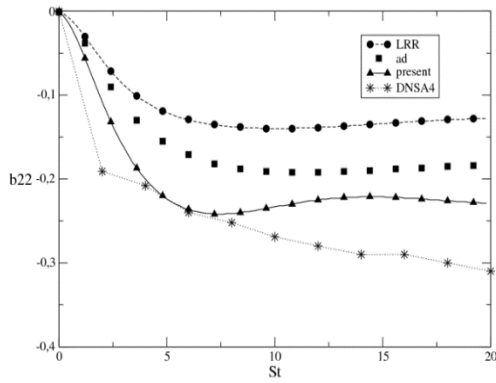


Fig. 4. Time evolution of the Reynolds-stress b_{22} in case A4.

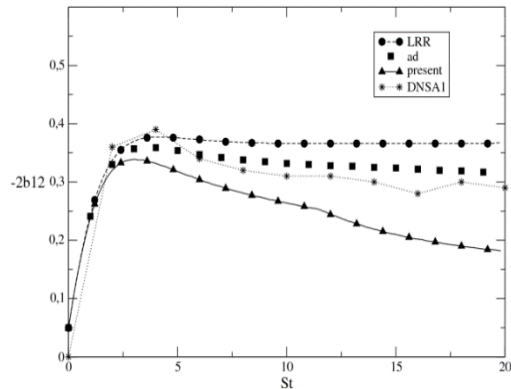


Fig. 5. Time evolution of the Reynolds-stress anisotropy $-2b_{12}$ in case A1.

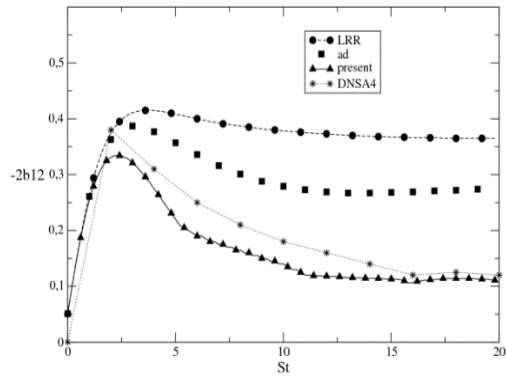


Fig. 6. Time evolution of the Reynolds-stress anisotropy $-2b_{12}$ in case A4.

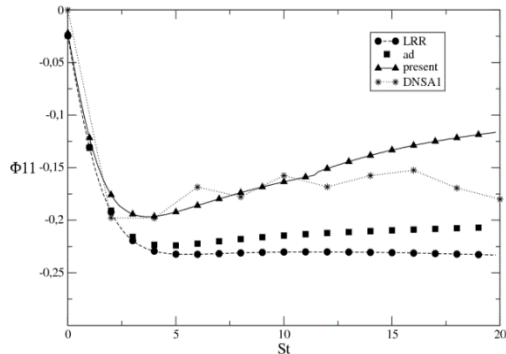


Fig. 7. Time evolution of the pressure-strain correlation Φ_{11} in case A1.

From all the figures, the incompressible Laufer, Reece and Rodi model (1975) is unable to predict the dramatic changes in the magnitude of the Reynolds stress anisotropy that arise from compressibility, while the present Model provides an acceptable performance in reproducing the DNS results in cases A4. This model explains the importance of the evolving of M_g with the commonly used the parameter M_t in modeling the high compressible turbulent flow.

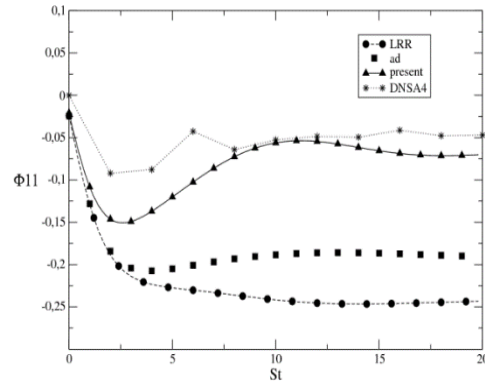


Fig. 8. Time evolution of the pressure-strain correlation Φ_{11} in case A4.

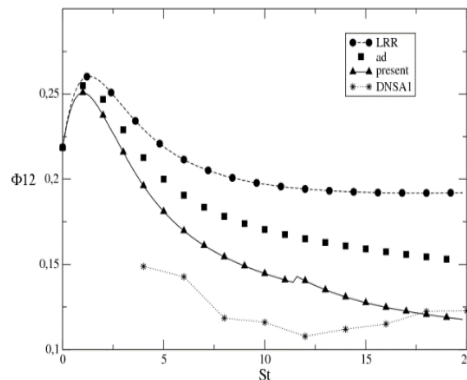


Fig. 9. Time evolution of the pressure-strain correlation Φ_{12} in case A1.

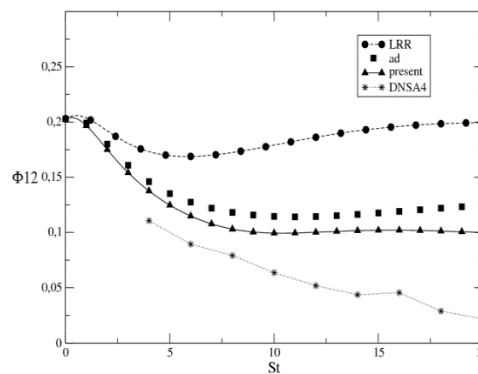


Fig. 10. Time evolution of the pressure-strain correlation Φ_{12} in case A4.

Figs. ((7, 8); (9, 10) and (11, 12)) present the behavior of the pressure-strain correlation. As can

be seen in these figures, the present model yields acceptable results that are in good qualitative agreement with the DNS data, especially at high gradient Mach number (case A4).

Figs. (13, 14) present the behavior of the normalized dissipation ϵ_s/Sk , ($\epsilon_s/Sk = -2b_{12} \epsilon_s P$) for cases A1 and A4. It can be seen that there is a decrease in ϵ_s/Sk when M_g increases, since the compressibility effects cause the significant reduction in the Reynolds turbulent shear stress b_{12} from numerical simulation cases A1 and A4 of the previous DNS results. It is clear that the proposed model is in accordance with the DNS results.

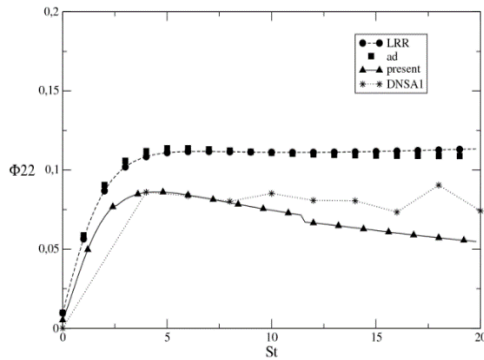


Fig. 11. Time evolution of the pressure-strain correlation Φ_{22} in case A1.

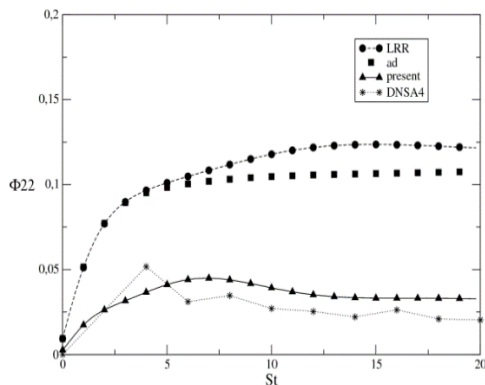


Fig. 12. Time evolution of the pressure-strain correlation Φ_{22} in case A4.

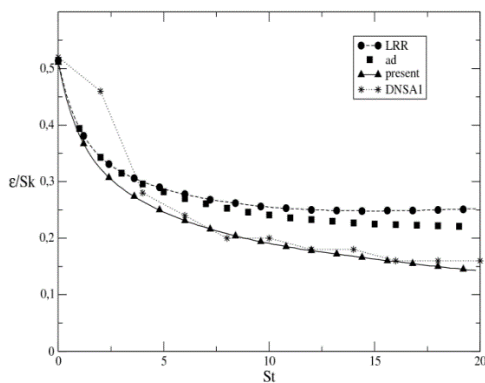


Fig. 13. Time evolution of the turbulent dissipation rate: ϵ_s/Sk in cases A1.

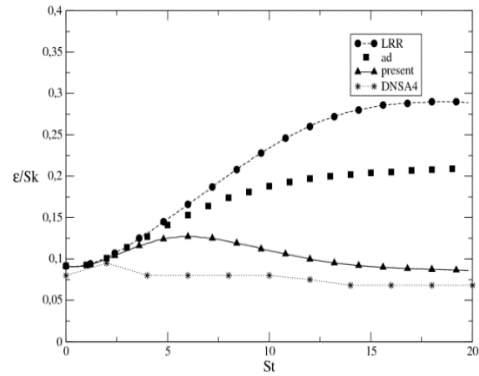


Fig. 14. Time evolution of the turbulent dissipation rate: ϵ_s/Sk in cases A4.

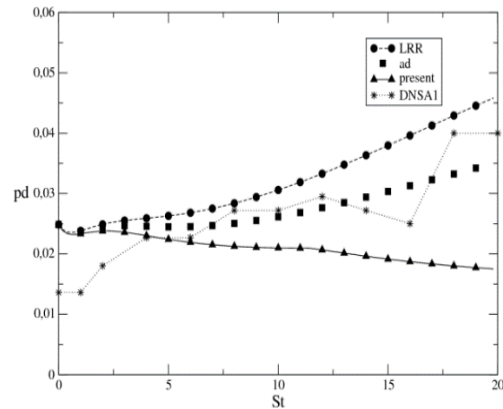


Fig. 15. Time evolution of the pressure-dilatation correlation $pd = \frac{(-p'd'/\rho + \epsilon_c)}{Sk}$ in case A1.

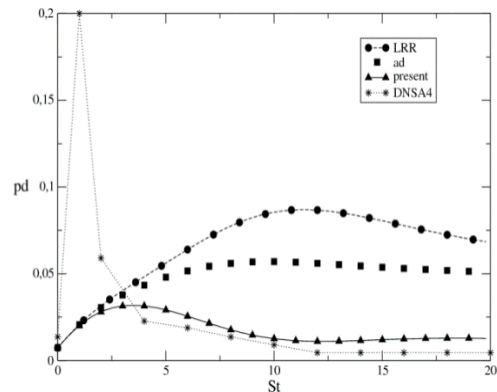


Fig. 16. Time evolution of the pressure-dilatation correlation $pd = \frac{(-p'd'/\rho + \epsilon_c)}{Sk}$ in case A4.

Figs. (15, 16) present the behavior of the dilatational terms in cases A1 and A4. It will be shown that these terms are much smaller to explain the compressibility effect on the turbulence. Using Eq. (34). One can notice that the compressibility effect of decreased growth rate of turbulent kinetic energy is due to a decrease of the normalized production term. It will be shown from cases A1, A2, A3 and A4 that the asymptotic values of turbulent parameters are highly dependent on the initial conditions when M_g is changed. This shows that the

gradient Mach number is an important parameter that describes the level of stabilizing effect of compressibility.

Our model is actually designed to study cases of high- compressibility and to the limit moderate compressibility. Whereas, the case A1 correspond to a low-compressibility, it is clear that even the incompressible models can describe the evolution of the pressure-dilatation term such as is shown in Fig. 15.

Generally the modeling of the pressure-strain correlation is calibrated for a large time relatively to equilibrium conditions, therefore, all the models cannot predict accurately these terms for enough small time.

6. CONCLUSION

In this study, the compressible models are used to describe the evolution of the turbulence and the performances of these models which are compared to the results of numerical simulation of Sarkar (1995). The standard model for the pressure-strain correlation of L. R. R. yields poor predictions for compressible homogeneous shear flow. It is clear that this model is unable to predict the effect of compressibility, while the predictions of the compressible models using the turbulent Mach number yield encouraging result. The present model is an extension of the LRR model involving the gradient Mach number M_g with the commonly used M_t appears to be able to predict accurately the structural compressibility effects. Therefore, the gradient Mach number M_g is concluded to be an important parameter in addition to M_t in the modeling of the pressure-strain correlation for high compressible homogeneous turbulence.

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REFERENCES

- Adumitroaie, V., J. R. Ristorcelli and D. B. Taulbee (1999). Progress in Favre Reynolds stress closures for compressible flows. *Phys. Fluids A* 9, 2696-2719.
- Bagdanoff, D. W. (1983). Compressibility effect in turbulent shear layers. *AIAA J* 21, 926-927.
- Blaisdell, G. A., N. M. Mansour and W. C. Reynolds (1991). Numerical simulations of compressible homogeneous turbulence. *Report TF-50*, Stanford University, Stanford, California.
- Bradshaw, P. (1977). Compressibility effects on turbulence. *Ann. Rev. Fluid Mech.* 9, 33-54.
- El Baz, A. M. and B. E. Launder (1993). Second moment modeling of compressible mixing layers. In W. Rodi and F. Martelli (Eds), *Engineering Turbulence Modeling and Experiments* 2, 63-72.
- Gomez, C. A. and S. S. Girimaji (2013). Toward second-moment closure modelling of compressible shear flows. *Journal of F. Mechanics* 733, 325-369
- Hamba, F. (1999). Effects of pressure fluctuations on turbulence growth compressible homogeneous shear flow. *Phys. Fluids* A11, 1623-1625.
- Hanafi, A., H. Khelifi and T. Lili (2013). The effect of compressibility on turbulent shear flow: direct numerical simulation study. *Journal of Fluid Engineering ASME* 135, 101201-5.
- Hechmi, K. and L. Taieb (2011). An extension of the SSG model on compressible turbulent flow. *Journal of Applied Fluid Mechanics* 4, 101-111.
- Huang, S. and S. Fu (2008). Modeling of pressure-strain correlation in compressible turbulent flow. *Acta Mech. Sin* 24, 37-43.
- Launder, B. E., G. J. Reece and W. Rodi (1975). Progress in the development of a Reynolds stress turbulence closure. *J. Fluid Mech.* 68, 537-566.
- Marzougui, H., H. Khelifi and T. Lili (2005). Extension of the Launder, Reece and Rodi on compressible homogeneous shear flow. *The European Physical Journal B* 45, 147-154.
- Pantano, C. and S. Sarkar (2003). A study of compressibility effects in the high-speed turbulent shear layer using direct simulation. *J. Fluid Mech.* 451, 329-371.
- Ristorcelli, J. R. (1977). A pseudo-sound constitutive relationship for the dilatational covariance in compressible turbulence. *Journal of Fluid Mech.* 347, 37-70.
- Sarkar, S. (1992). The pressure dilatation correlation in compressible flows. *Physics of Fluids A* (4), 2674-2682.
- Sarkar, S. (1995). The stabilizing effects of compressibility in turbulent shear flows. *Journal of Fluid Mech.* 282, 163-186.
- Sarkar, S. and B. Lakshmanan (1991). Application of a Reynolds stress turbulence model to the compressible shear layer. *AIAA J.* 29, 743-749.
- Sarkar, S., G. Erlebacher and M. Y. Hussaini (1991). Direct simulation of compressible turbulence in a shear flow. *Journal of Theoretical and Computational Fluid Dynamics* 2, 291-305.
- Sarkar, S., G. Erlebacher, M. Y. Hussaini and H. O. Kreiss (1991). The analysis and modeling of dilatational terms in compressible turbulence. *J. Fluid Mech.* 227, 473-493.
- Simone, A., G. N. Coleman and C. Cambon

- (1997). The effect of compressibility on turbulent shear flow: a rapid distortion theory and direct numerical simulation study. *J. Fluid Mech.* 330,307.
- Speziale, C. G., R. Abid and N. N. Mansour (1995). Evaluation of Reynolds stress turbulence closures in compressible homogeneous shear flow. *J. Appl. Math. Phys.* 46, 5717-5736.
- Taulbee, D. and J. V. Osdol (1991). Modeling turbulent compressible flows: The massfluctuating velocity and squared density. *AIAA paper* 91-0524.
- Vreman, A. W., N. D. Sandham and K. H. Luo (1996). Compressible mixing layer growth rate and turbulence characteristics. *J. Fluid Mech.* 320, 320-325.
- Zeman, O. (1990). Dilatational dissipation, the concept and application in modeling compressible mixing layers. *Phys. Fluids A2*, 178-188.
- Zeman, O. (1991). On the decay of compressible isotropic turbulence. *Phys. Fluids A3*, 951.