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Onset of Convection in Porous Medium Saturated by Viscoelastic Nanofluid: More Realistic Result

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ABSTRACT

The present paper deals with the linear thermal instability analysis of viscoelastic nanofluid saturated porous layer. We consider a set of new boundary conditions for the nanoparticle fraction, which is physically more realistic. The new boundary condition is based on the assumption that the nanoparticle fraction adjusts itself so that the nanoparticle flux is zero on the boundaries. We use Oldroyd-B type viscoelastic fluid that incorporates the effects of Brownian motion and thermophoresis. Expressions for stationary and oscillatory modes of convection have been obtained in terms of the Rayleigh number, which are found to be functions of various parameters. The numerical results have been presented through graphs.

Keywords: Viscoelastic; Nanofluid; Porous media; Nanoparticle flux.

NOMENCLATURE

a	Wave number
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
d	depth of the fluid layer
g	gravitational acceleration
$\mathbf{g}$	gravitational acceleration vector
K	permeability of the porous media
L_e	Lewis number, defined by Eq. (14)
N_A	modified thermophoresis to Brownian-motion diffusivity ratio, defined by Eq. (18)
N_B	modified particle-density increment, defined by Eq. (19)
P	reduced pressure
Pr	Prandtl number
Ra	thermal Rayleigh-darcy number, defined by Eq. (15)
Rm	basic-density Rayleigh number defined by Eq. (16)
Rn	concentration Rayleigh number, defined by Eq. (17)
T	temperature
T_c	temperature at the upper wall
T_h	temperature at the lower wall
t	time
(x,y,z)	Cartesian coordinates
α_m	thermal diffusivity of the porous medium
β	volumetric thermal expansion coefficient
λ_1	relaxation time
λ_2	retardation time
κ_m	effective thermal conductivity of the porous medium
μ	viscosity of the fluid
ε	porosity
ρ_f	fluid density
ρ_p	nanoparticle mass density
$(\rho c)_f$	effective heat capacity of the porous medium
ϕ	nanoparticle volume fraction
ϕ_0	reference value for nanoparticle volume fraction
ω	frequency of oscillation

Other symbols

∇_1^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$
∇^2	$\nabla_1^2+\frac{\partial^2}{\partial z^2}$

Subscripts

b	basic state
c	critical
0	reference value

Superscripts

$'$	perturbed quantity
$*$	dimensionless quantity
Osc	oscillatory
S	stationary

1. INTRODUCTION

The pioneering work of Choi (1995) introduces the term "nanofluids" during his research in Argonne National Laboratory. Nanofluids are colloidal mixture of nanoparticles and a base liquid, its marvelous heat transfer enhancement property now became central part of research and attracts many scientists. The continuous growth in technology demands high class energy efficient devices and power enhancement which requires rapid heat exchangers, where the conventional fluids are not sufficient to improve the rapid heat transfer, therefore we seek a relatively new class of fluid which enhances the heat exchange. It is experimentally verified that nanofluid enhances the heat transfer over the conventional fluid (Eastman *et al*. (2001), Robert *et al*. (2013)). Nanofluid find its application in coolants for advanced nuclear systems, chemical engineering, electronic devices, medical science, storage devices and in solar collectors. Studies related to nanofluid are mainly focused to thermal conductivity, however a satisfactory explanation for the abnormal enhancement in thermal conductivity and viscosity in nanofluid is yet to be found. The attempt of Buongiorno (2006) is found suitable for stability analysis of nanofluid convection which includes the effect of Brownian diffusion and thermophoresis for non-turbulent flow. Rayleigh-Be´nard convection in porous media commonly known as Horton-Rogers-Lapwood convection includes many applications of nanofluid which occur in the porous medium such as electronic cooling system, including food and chemical processes, nuclear reactors, petroleum industry, biomechanics, and geophysical problems. Documented work in this area are well collected and reviewed by Nield and Bejan (2013).

As a growing research in nanofluid convection, several attempts have been made; Nield and Kuznetsov (2009) studied onset of convection in nanofluid saturated porous media, Kuznetsov and Nield (2010a) investigated thermal instability of nanofluid saturated porous layer using Brinkman model, Kuznetsov and Nield (2010b) performed stability analysis for local thermal non-equilibrium convection in porous media saturated with nanofluid, Nield and Kuznetsov (2011) studied the thermal instability of nanofluid convection in porous media considering the effect of vertical throughflow. Recently Hayat *et al*. (2015) studied the mixed convection flow of non-Newtonian nanofluid over a stretching surface including the effect of thermal radiation, heat source/sink and first order chemical reaction by taking Casson fluid model. Author's group, Bhadauria and Agarwal (2011a, b, c), Agarwal and Bhadauria (2011, 2014, 2014a,b,c) and Agarwal *et al*. (2011, 2012) studied thermal stability of nanofluid, considering various physical models and boundary conditions.

Most of the above studies dealt only with Newtonian fluid, however, waxy crude at shallow depth, enhanced oil recovery, paper and textile coating, paint industries are few examples which admit the applications of viscoelastic fluids, therefore the study of viscoelastic fluid is also very important. There are some works related to thermal stability in viscoelastic fluid saturated porous media; Rudraiah *et al*. (1989) studied the stability of a viscoelastic fluid in a densely packed saturated porous layer considering an Oldroyd model. Yoon *et al*.(2003, 2004) made a linear stability analysis to study convection in a viscoelastic fluid saturated porous layer, and obtain the expression of Darcy Rayleigh number for oscillatory case to describe the onset of convection. Bertola and Cafaro (2006) studied theoretically the stability of viscoelastic fluid heated from below. Sheu *et al*. (2008) analysed chaotic convection for viscoelastic fluids, using truncated Galerkin expansion. Choudhury and Das (2014) studied the viscoelastic free convective transient MHD flow over a vertical porous plate through porous media in the presence of radiation and chemical reaction by applying transverse variable suction velocity on the porous plate. Kumar and Bhadauria (2011a) studied thermal instability in a rotating viscoelastic fluid saturated porous layer, and calculate the heat transfer. Also Kumar and Bhadauria (2011b) studied linear and nonlinear double diffusive convection in a viscoelastic fluid saturated porous layer. Further, they (2011c) studied double diffusive convection in a rotating porous layer saturated by a viscoelastic fluid and calculated heat and mass transfer across the fluid layer. However, very few studies are available on convection in a viscoelastic nanofluid saturated porous medium. To the best of authors knowledge only Sheu (2011) have studied thermal instabilty in a porous layer, saturated with viscoelastic nanofluid, using Oldroyd-B type constitutive equation by considering the boundary conditions in which temperature and nanoparticle concentration can be controlled at the boundaries, he suggested that oscillatory instability is possible in both bottom- and top-heavy nanoparticle distributions. It was considered in old boundary conditions that one could control the nanoparticle concentration at the boundaries like in the case of temperature, but in real problem, this is however difficult to control the nanoparticle concentration at the boundaries, further more with the set of new boundary conditions, the concentration Rayleigh number is always positive.

Recently, physically a more realistic model was studied for thermal instability by Nield and Kuznetsov (2014), considering new set of boundary conditions that the normal component of the nanoparticle flux on boundaries is zero. Further, Agarwal (2014) also studied the thermal instability of nanofluid convection in a rotating porous layer considering the new model of Nield and Kuznetsov (2014). Therefore, in this paper, we have made an attempt to study onset of thermal instability in a viscoelastic nanofluid saturated porous medium with the assumption that there is no nanoparticle flux at the boundaries, which is physically a more realistic condition.

2. GOVERNING EQUATIONS

We consider an infinitely extended horizontal porous layer saturated by viscoelastic nanofluid, confined between the planes z = 0 and z = d. We choose Cartesian frame of reference as origin in the lower boundary and the z-axis in vertically upward direction. The gravitational force is acting in vertically downward direction. It is assumed that the fluid and solid phases are in local thermal equilibrium. T_h and T_c are the lower and upper plate temperature respectively with the condition that $T_h > T_c$, T_c is taken as reference temperature. Oldroyd-B model is used to describe the rheological behaviour of the viscoelastic nanofluid. Further, the density variation is considered under Boussinesq approximation. Also for linear theory, it is assumed that the change in temperature in the viscoelastic nanofluid is small as compared to T_c . Then using the approximated buoyancy term, the governing equations under the above considerations are as follows:

$$\nabla.\mathbf{q}_D = 0, \tag{1}$$

$$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}_D}{\partial t} + \nabla p - \{\phi\rho_p + (1-\phi)\rho\,[1-\beta(T-T_c)]\}\mathbf{g}\right) = -\frac{\mu}{K}\left(1+\bar{\lambda}_2\frac{\partial}{\partial t}\right)\mathbf{q}_D, \tag{2}$$

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f\,\mathbf{q}_D.\nabla T = \kappa_m\nabla^2 T + \varepsilon(\rho c)_m\left[D_B\nabla\phi.\nabla T + \left(\frac{D_T}{T_c}\right)\nabla T.\nabla T\right], \tag{3}$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}_D.\nabla\phi = D_B\nabla^2\phi + \left(\frac{D_T}{T_c}\right)\nabla^2 T. \tag{4}$$

We write $\mathbf{q}_D = (u, v, w)$. We assume that the boundaries are held at constant temperature and the nanoparticle flux is zero on the boundaries. Thus the boundary conditions are taken as follows

$$w = 0, T = T_h, D_B\frac{\partial\phi}{\partial z} + \frac{D_T}{T_c}\frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \tag{5}$$

$$w = 0, T = T_c, D_B\frac{\partial\phi}{\partial z} + \frac{D_T}{T_c}\frac{\partial T}{\partial z} = 0 \text{ at } z = d. \tag{6}$$

We introduce dimensionless variable by using the following transformations:

$$\begin{aligned}(x^*, y^*, z^*) &= (x, y, z)/d, \quad t^* = t\alpha_m\sigma d^2,\\ (u^*, v^*, w^*) &= (u, v, w)d/\alpha_m,\\ T^* &= (T-T_c)/(T_h-T_c),\\ p^* &= pK/\mu\alpha_m, \phi^* = (\phi-\phi_0)/\phi_0\end{aligned} \tag{7}$$

Where $\alpha_m = \dfrac{\kappa_m}{(\rho c)_f}$, $\sigma = \dfrac{(\rho c)_m}{(\rho c)_f}$.

The nondimensionlized equations (after dropping the asterisks for simplicity) are:

$$\nabla.\mathbf{q}_D = 0, \tag{8}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial \mathbf{q}_D}{\partial t} + \nabla p - Rm\vec{k} - RaT\vec{k} - Rn\phi\vec{k}\right) = -\frac{\mu}{K}\left(1+\lambda_2\frac{\partial}{\partial t}\right)\mathbf{q}_D, \tag{9}$$

$$\frac{\partial T}{\partial t} + \mathbf{q}_D.\nabla T = \nabla^2 T + \left[\frac{N_B}{Le}\nabla\phi.\nabla T + \frac{N_A N_B}{Le}\nabla T.\nabla T\right], \tag{10}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}_D.\nabla\phi = \frac{1}{Le}\nabla^2\phi + \frac{N_A}{Le}\nabla^2 T, \tag{11}$$

$$w = 0, T = 1, \frac{\partial\phi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \tag{12}$$

$$w = 0, T = 0, \frac{\partial\phi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \text{ at } z = 1. \tag{13}$$

The nondimensional parameters, which appeared in the above equations are defined as follows:

$$Le = \frac{\alpha_m}{D_B} \tag{14}$$

is the Lewis number,

$$Ra = \frac{\rho g\beta K d(T_h - T_c)}{\mu\alpha_m} \tag{15}$$

is the thermal Darcy Rayleigh number,

$$Rm = \frac{[\rho_p\phi_0 + \rho(1-\phi_0)]gKd}{\mu\alpha_m} \tag{16}$$

is the basic density Rayleigh number ,

$$Rn = \frac{(\rho_p - \rho)\phi_0 gKd}{\mu\alpha_m} \tag{17}$$

is the concentration Rayleigh number,

$$N_A = \frac{D_T(T_h - T_c)}{D_B T_c\phi_0} \tag{18}$$

is the modified diffusivity ratio,

$$N_B = \frac{\varepsilon(\rho c)_p}{(\rho c)_f}\phi_0 \tag{19}$$

is the modified particle density increment,

$$\lambda_1 = \frac{\alpha_m}{d^2}\bar{\lambda}_1 \tag{20}$$

is the stress relaxation parameter

$$\lambda_2 = \frac{\alpha_m}{d^2}\bar{\lambda}_2 \tag{21}$$

is the strain retardation parameter.

2.1 Basic Solution

The basic state of the nanofluid is assumed to be quiescent thus, temperature field and nanoparticle volume fraction vary in the z-direction only. This gives the solution of the form

$$u = v = w = 0, \quad T = T_b(z), \quad \phi = \phi_b(z), \tag{22}$$

which satisfy the following equations

$$\frac{d^2T_b}{dz^2} + \frac{N_B}{Le}\frac{d\phi_b}{dz}\frac{dT_b}{dz} + \frac{N_A N_B}{Le}\left(\frac{dT_b}{dz}\right)^2 = 0, \tag{23}$$

$$\frac{d^2\phi_b}{dz^2} + N_A\frac{d^2T_b}{dz^2} = 0. \tag{24}$$

Using the boundary conditions (12–13), Eq. (24) may be integrated to give

$$\frac{d\phi_b}{dz} + N_A\frac{dT_b}{dz} = 0. \tag{25}$$

Using Eq. (25) in Eq. (23), we get

$$\frac{d^2T_b}{dz^2} = 0. \tag{26}$$

The solution of the Eq. (26), subject to the boundary conditions (12–13), is given by

$$T_b = 1 - z, \tag{27}$$

also the Eq. (24) has been solved subjected to the boundary conditions (12–13) using (27), we get

$$\phi_b = \phi_0 + N_A z. \tag{28}$$

2.2 Perturbation State

We apply perturbation to the basic state of the system as

$$q = q', p = p_b + p', T = T_b + T', \phi = \phi_b + \phi'. \tag{29}$$

Substituting the above Eq. (29) in Eqs. (8–13) and neglecting the product of primes to linearize the equations, we get the following set of equations:

$$\nabla . q' = 0, \tag{30}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial q'}{\partial t} + \nabla p' - RaT'\vec{k} - Rn\phi'\vec{k}\right) + \frac{\mu}{K}\left(1+\lambda_2\frac{\partial}{\partial t}\right)q' = 0, \tag{31}$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T + \frac{N_B}{Le}\left(N_A\frac{T'}{dz} - \frac{\partial\phi'}{\partial z}\right) - 2\frac{N_A N_B}{Le}\frac{\partial T'}{\partial z}, \tag{32}$$

$$\frac{1}{\sigma}\frac{\partial\phi'}{\partial t} + \frac{N_A}{\varepsilon}w' = \frac{1}{Le}\nabla^2\phi + \frac{N_A}{Le}\nabla^2 T'. \tag{33}$$

Taking curl twice of the Eq. (31), we get

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2 w) - Ra\nabla_1{}^2 T - Rn\nabla_1{}^2\phi\right) + \left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2 w = 0 \tag{34}$$

$$w' = 0, T' = 0, \quad \frac{\partial\phi'}{\partial z} + N_A\frac{\partial T'}{\partial z} = 0 \ \text{ at } \ z = 0,1. \tag{35}$$

3. LINEAR STABILITY ANALYSIS

We use normal mode technique for linear stability analysis to solve the eigenvalue problem defined by Eqs. (32, 33, 34) subject to the boundary conditions given by Eq. (35). Using time periodic disturbance in horizontal plane, we take normal mode form as:

$$(w', T', \phi') = (W(z), \Theta(z), \Phi(z)) \exp[i(lx + my) + \omega t], \tag{36}$$

where l, m are horizontal wave number in x and y directions respectively, and $\omega = \omega_r + i\omega_i$ is growth rate, which is, in general, a complex quantity. Substitution of the above Eq. (36) in Eqs. (32, 33, 34) gives the following set of equations

$$(1+\lambda_1\omega)\left(\frac{\omega}{Pr}(D^2 - a^2)W + Raa^2\Theta - Rna^2\Phi\right) + (1+\lambda_2\omega)(D^2 - a^2)W = 0, \tag{37}$$

$$W + \left(D^2 - \frac{N_A N_B}{Le}D - a^2 - \omega\right)\Theta - \frac{N_B}{Le}D\Phi = 0, \tag{38}$$

$$\frac{N_A}{\varepsilon}W - \frac{N_A}{Le}(D^2 - a^2)\Theta - \left(\frac{1}{L_e}(D^2 - a^2) - \frac{\omega}{\sigma}\right)\Phi = 0, \tag{39}$$

$$W = 0, \Theta = 0, D\Phi + N_A D\Theta = 0 \ \text{ at } \ z = 0,1 \tag{40}$$

where $D = \frac{d}{dz}$ and $a^2 = l^2 + m^2$. The approximate solution of the above Eqs. (37–39) is obtained by using a Galerkin type weighted residuals method. As trial function (satisfying the boundary conditions), we choose

$$W_p = \Theta_p = \sin(p\pi z), \tag{41}$$

where we have taken

$$W = \sum_{p=1}^{N} A_p W_p, \quad \Theta = \sum_{p=1}^{N} B_p\Theta_p, \quad \Phi = \sum_{p=1}^{N} C_p\Phi_p \tag{42}$$

satisfying the boundary conditions (40). Substitution of above Eqs. (41–42) into Eqs. (37–39) yields a set of $3N$ linear algebraic equations in the unknowns $A_p, B_p, C_p; p = 1, 2, ..., N.$ The orthogonality of the trial function, and vanishing of the determinant of coefficients gives the expression for thermal Rayleigh number as a function of nondimensional parameters. We take trial functions only upto first order i.e corresponding to the value N = 1. We get the expression of thermal Rayleigh number as:

$$Ra=\frac{\delta^2}{a^2}\left\{\frac{\omega}{Pr}+\frac{(1+\omega\lambda_2)}{(1+\omega\lambda_1)}\right\}\left(\delta^2+\omega\right)-N_A Rn\frac{\left\{\delta^2\left(\frac{1}{\varepsilon}+\frac{1}{Le}\right)+\frac{\omega}{\varepsilon}\right\}}{\left(\frac{\delta^2}{Le}+\frac{\omega}{\sigma}\right)} \quad (43)$$

where $\delta^2=\pi^2+a^2$.

For neutral stability state $\omega_r=0$, whereas for $\omega_r<0$ system is always stable and for $\omega_r>0$ system is always unstable.

3.1 Stationary State

The expression of thermal Rayleigh number for the onset of stationary convection at the marginally stable steady state, for which the ex-change of stabilities are valid correspond to the $\omega=0$

(i.e. $\omega_r=0$ and $\omega_i=0$) becomes

$$Ra^S=\frac{\delta^4}{a^2}-N_A Rn\left(1+\frac{Le}{\varepsilon}\right). \quad (44)$$

3.2 Oscillatory State

To obtain the expression of thermal Rayleigh number for oscillatory convection at the marginal state, we substitute $\omega=i\omega_i$ (since the real part of ω for marginal oscillatory state is zero i.e $\omega_r=0$) in Eq. (43) and clear the complex quantity from denominator. After simplification, we get

$$Ra^{Osc}=\Delta_1+i\omega_i\Delta_2 \quad (45)$$

where

$$\Delta_1=\frac{\delta^2}{a^2}\times\left(-\frac{\omega_i^2}{Pr}+\frac{((1+\lambda_1\lambda_2\omega_i^2)\delta^2-(\lambda_2-\lambda_1)\omega_i^2}{(1+\lambda_1{}^2\omega_i^2)}\right), -N_A Rn\frac{\left(\frac{\delta^4}{Le}\left(\frac{1}{\varepsilon}+\frac{1}{Le}\right)-\frac{\omega_i^2}{\varepsilon\sigma}\right)}{\left(\frac{\delta^4}{Le^2}+\frac{\omega_i^2}{\sigma^2}\right)} \quad (46)$$

and

$$\Delta_1=\frac{\delta^2}{a^2}\times\left(-\frac{\delta^2}{Pr}+\frac{((1+\lambda_1\lambda_2\omega_i^2)\delta^2+(\lambda_2-\lambda_1)\delta^2}{(1+\lambda_1{}^2\omega_i^2)}\right)+N_A Rn\frac{\left(\frac{\delta^2}{\sigma}\left(\frac{1}{\varepsilon}+\frac{1}{Le}\right)-\frac{\delta^2}{\varepsilon Le}\right)}{\left(\frac{\delta^4}{Le^2}+\frac{\omega_i^2}{\sigma^2}\right)}. \quad (47)$$

For oscillatory onset of convection, we have $\Delta_2=0$ (since *Ra* is a physical quantity, therefore it must be real, also $\omega_i\neq 0$ for oscillatory convection). This gives a biquadratic equation in ω_i

$$f(\omega_i^2)^2+g(\omega_i^2)+h=0 \quad (48)$$

where

$$f=\frac{\delta^2}{\sigma^2 a^2}\left(\frac{\delta^2}{Pr}\lambda_1{}^2+\lambda_1\lambda_2\right) \quad (49)$$

$$g=\frac{\delta^2}{a^2}\left\{\frac{\delta^2}{\sigma^2 Pr}+\lambda_1{}^2\frac{\delta^2}{Pr}\frac{\delta^4}{Le^2}+\frac{\delta^4\lambda_1\lambda_2}{Le^2}+\frac{1}{\sigma^2}(1+(\lambda_2-\lambda_1)\delta^2)\right\}+N_A Rn\left\{\frac{\delta^2}{\sigma}\left(\frac{1}{\varepsilon}+\frac{1}{Le}\right)-\frac{\delta^2}{\varepsilon Le}\right\}\lambda_1{}^2 \quad (50)$$

$$h=\frac{\delta^2}{a^2}\frac{\delta^4}{Le^2}\left\{\frac{\delta^2}{Pr}+(1+(\lambda_2-\lambda_1)\delta^2)\right\}+N_A Rn\left\{\frac{\delta^2}{\sigma}\left(\frac{1}{\varepsilon}+\frac{1}{Le}\right)-\frac{\delta^2}{\varepsilon Le}\right\} \quad (51)$$

and

$$Ra^{Osc}=\frac{\delta^2}{a^2}\times\left(-\frac{\omega_i{}^2}{Pr}+\frac{((1+\lambda_1\lambda_2\omega^2)\delta^2-(\lambda_2-\lambda_1)\omega_i{}^2}{(1+\lambda_1{}^2\omega_i{}^2)}\right)-N_A Rn\frac{\left(\frac{\delta^4}{Le}\left(\frac{1}{\varepsilon}+\frac{1}{Le}\right)-\frac{\omega_i{}^2}{\varepsilon\sigma}\right)}{\left(\frac{\delta^4}{Le^2}+\frac{\omega_i{}^2}{\sigma^2}\right)}. \quad (52)$$

The possibility of oscillatory convection depends upon the condition that, ω_i must be positive, therefore we seek the set of appropriate values of nondimensional parameters for which oscillatory convection is possible.

4. RESULTS AND DISCUSSION

The rescaled concentration Rayleigh number is defined in terms of particle fraction, so it cannot be negative as considered in the earlier results, therefore we take only positive values of concentration Rayleigh number for our numerical calculations. The expression of thermal Rayleigh number given by Eq. (43) is independent of the modified particle-density increment parameter N_B, this happens due to the orthogonality of the trial functions of first order.

Eq. (44) can be rewritten as

$$Ra^S + N_A Rn\left(1+\frac{Le}{\varepsilon}\right) = \frac{\delta^4}{a^2}. \qquad (53)$$

The minimum value of right hand side with respect to a can be obtained at $a = \pi$, hence the critical value of the right hand side of the Eq. (53) can be given by

$$Ra_c^S + N_A Rn\left(1+\frac{Le}{\varepsilon}\right) = 4\pi^2 \qquad (54)$$

In the absence of nanoparticle, we recover the classical result of Horton-Rogers-Lapwood convection.

In contrast to Newtonian fluid, viscoelastic fluid possesses overstability due to which we get the oscillatory convection. We consider the values of parameters appeared in the expression of thermal Rayleigh number as Rn = 0.1, Le = 200, $Pr = 50$, $N_A = 1$, $\lambda_1 = 1$, $\lambda_2 = 0.5$, $\sigma = 2$, ε=0.9 or otherwise mentioned.

Fig. (5) shows the effect of the Darcy-Prandtl number and is observe from the graph that the Darcy-Prandtl destabilizes the onset of convection for its increasing values. Fig. (6) shows the effect of Lewis number on the onset of convection and is observed from the Lewis number destabilizes the onset of stationary convection while stabilizes the oscillatory convection, for its increasing values. Figs. (7, 8) shows the effect of modified diffusivity ratio on the onset of convection and is observed from the graph that the modified diffusivity ratio destabilizes the onset of stationary convection, while stabilizes the onset of oscillatory convection for its increasing values Fig. (9) shows the effect of heat ratio on the onset of convection and is observed from the graph that heat ratio stabilizes the onset of convection for its increasing values. Figs. (10, 11) shows the effect of porosity on the onset of convection and is observed from the graph that the porosity stabilizes the onset of stationary convection while destabilizes the onset of oscillatory convection for its increasing values.

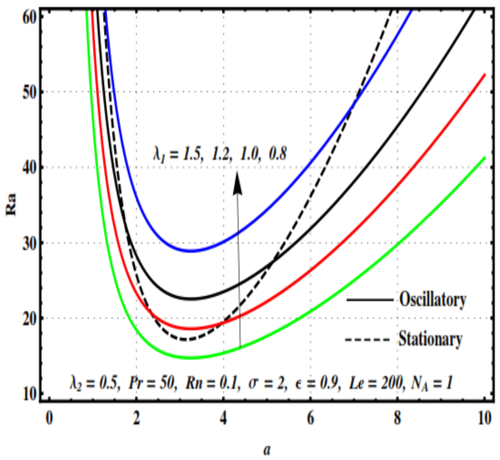

Fig. 1. Neutral stability curves for the different values of λ_1 .

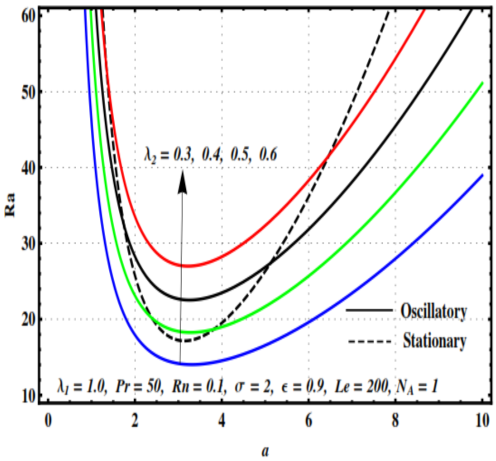

Fig. 2. Neutral stability curves for the different values of λ_2 .

Figs. (1-11) shows the neutral stability curve for different values of parameters. In Fig. (1), we consider the effect of the stress relaxation parameter on the onset of convection, and observe that an increase in the stress relaxation parameter destabilizes the onset of oscillatory convection, as the convection takes place at lower value of the Rayleigh number. Fig. (2) represents the effect of the strain retardation parameter, and from the graph it is clear that the strain retardation parameter stabilizes the onset of oscillatory convection, since the critical value of the Rayleigh number increases on increasing the value of the strain retardation parameter. Figs. (3, 4) shows the effect of the concentration Rayleigh number on the onset of convection and is observed from the graph that the concentration Rayleigh number destabilizes the onset of stationary convection which is similar to the result obtained by Nield and Kuznetsov (2014), while stabilizes the onset of oscillatory convection for its increasing values.

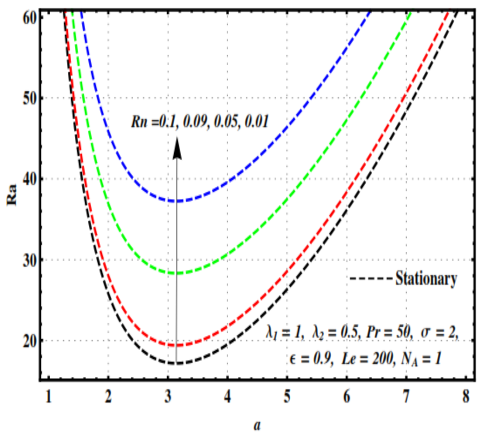

Fig. 3. Neutral stability curves for the different values of *Rn*.

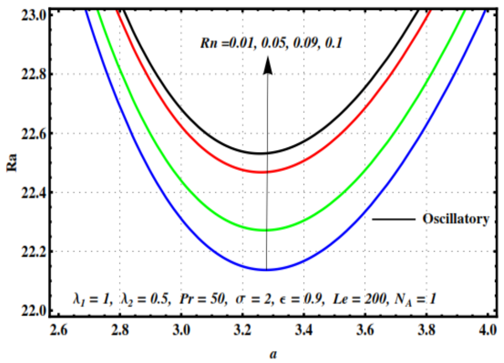

Fig. 4. Neutral stability curves for the different values of *Rn*.

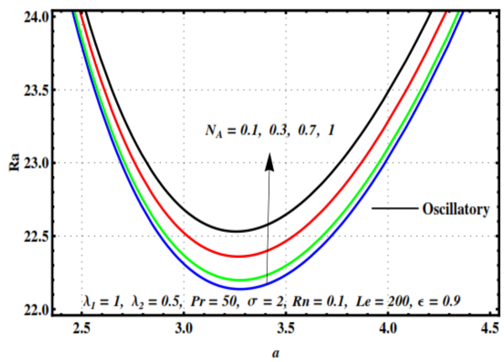

Fig. 8. Neutral stability curves for the different values of N_A.

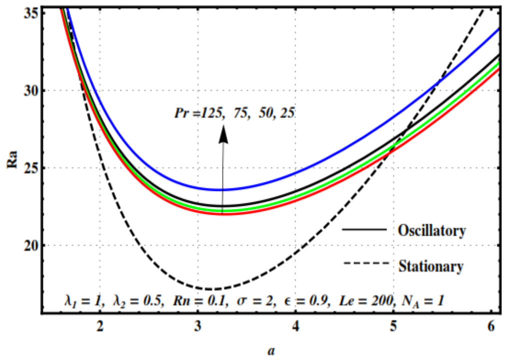

Fig. 5. Neutral stability curves for the different values of *Pr*.

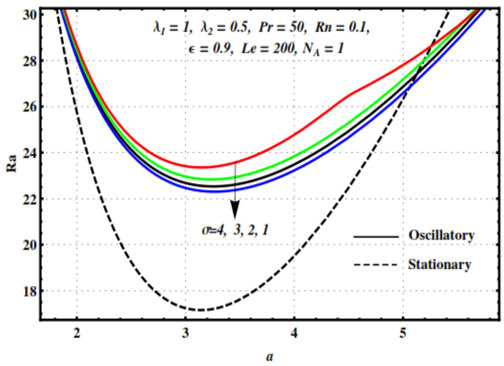

Fig. 9. Neutral stability curves for the different values of σ.

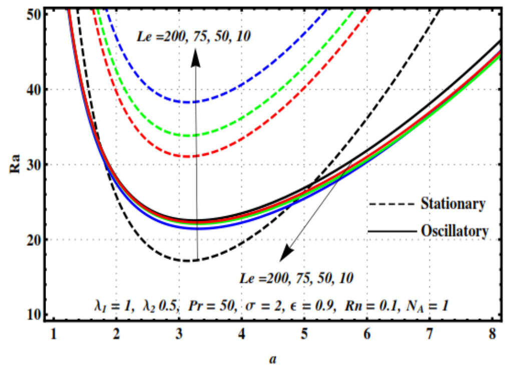

Fig. 6. Neutral stability curves for the different values of *Le*.

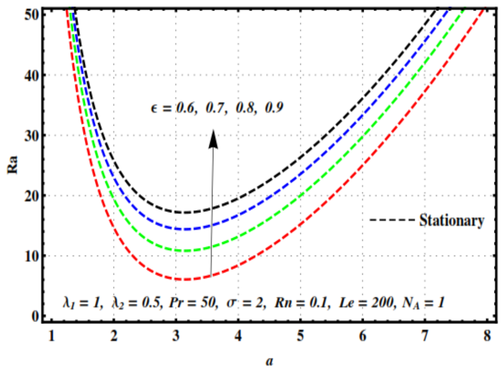

Fig. 10. Neutral stability curves for the different values of ε.

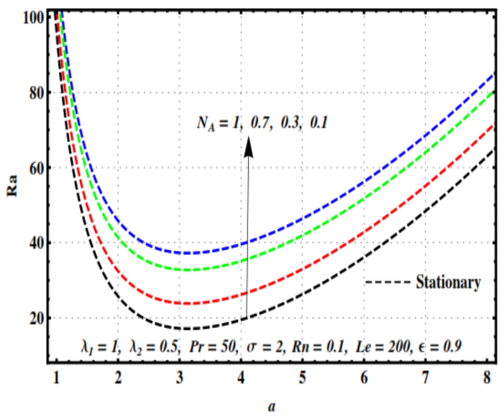

Fig. 7. Neutral stability curves for the different values of N_A.

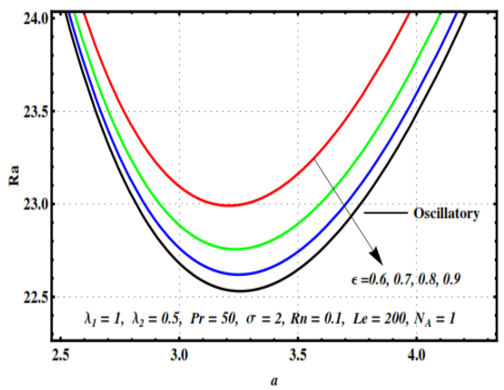

Fig. 11. Neutral stability curves for the different values of ε.

5. CONCLUSIONS

We investigate the onset of convection viscoelastic nanofluid convection in an infinite horizontal porous layer which is heated from with the set of new boundary condition which is physically more realistic. From the expression of *Rn*, it is observed that *Rn* is defined as a typical nanofluid fraction instead of the difference of two fractions so that, *Rn* cannot be negative, the modified diffusion ratio N_A is positive, also it is not necessary to take large values of *Le* as mentioned by Nield and Kuznetsov (2009), moreover, the Eq. (54) can be taken as an upper bound for the value of critical Rayleigh number in case of stationary convection. For the increasing value of various parameters, we found the following results:

1. Relaxation parameter λ_1 : destabilizes the onset of convection.
2. Retardation parameter λ_2 : stabilizes onset of convection.
3. Concentration Rayleigh number *Rn*: destabilizes the onset of stationary convection, stabilizes the onset of oscillatory convection.
4. Modified diffusivity ratio N_A : destabilizes the onset of stationary convection, stabilizes the onset of oscillatory convection.
5. Lewis number *Le*: stabilize the stationary convection, destabilize the oscillatory convection.
6. Darcy-Prandtl number *Pr*: destabilizes the oscillatory convection.
7. Porosity ε: stabilizes the onset of stationary convection, destabilizes the onset of oscillatory convection.
8. Heat ratio σ: stabilizes the onset of convection.

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