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Design of Centralized Fractional order PI Controller for Two Interacting Conical Frustum Tank Level Process

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ABSTRACT

The mathematical model for two interacting conical frustum tank level (TICFTL) process is proposed. The control of TICFTL process is difficult due to the nonlinearity, interaction effect between input flow and output level of tanks. Hence, the nonlinear process is linearised around proper operating points. Then, the decentralized PI, centralized PI, centralized FOPI controllers are designed and then the control parameters are tuned using genetic algorithm. The closed loop controller performances are simulated and compared in terms of settling time, rise time and integral error criteria. It is found that centralized PI controller has better servo and regulatory response than the decentralized PI, centralized FOPI controller. The liquid inflow rates are optimally manipulated by the centralized PI controller to track the set point variation and to reject the disturbance effectively than centralized FOPI, decentralized PI controller.

Keywords: Two interacting conical frustum tank level process; Flow control; Level control; Centralized PI; FOPI; Decentralized PI controller.

NOMENCLATURE

a_1, a_2, a_{12}	cross sectional area of pipe
dt	time step
g	gravity
$Gp(s)$	process transfer function
h_1, h_2	liquid level in tank 1,2
H_1, H_2	maximum height of tank 1,2
IV_1, V_2	input voltage to pump1, 2
Kpp_1	pump gain of pump 1
Kpp_2	pump gain of pump 2
R_1, R_2	top Radius of conical frustum tank
r_{in1}, r_{in2}	bottom radius of tank1,2
β_1	valve co-efficient of tank1 outlet
β_{12}	valve co-efficient of interaction pipe
β_2	valve co-efficient of tank2 outlet
$\gamma Gc(s)$	controller transfer function
λ	order of FOPI controller

1. INTRODUCTION

Controlling of liquid level and flow in process tanks are the challenging control problem in the process industries. Generally, the liquids are pumped and stored in the tanks for processing; again it is pumped to other tanks for other operations. The conical tanks are widely used in liquid treatment industry, concrete industry and hydrometallurgical industries (Ravi *et al*. 2014). The TICFTL process is a typical two input two output (TITO) process which exhibits nonlinear characteristics and dynamic coupling effect between inputs and outputs. The control of TITO process requires dedicated multiloop or multivariable control system. Commonly, the process industries employ multiloop PID controller because of its simple structure, robustness and failure tolerance (Astrom *et al*. 2002). The multiloop PID controllers produce better control performance for the system with modest interaction. But it fails to provide desirable control performance for the system with severe interaction effect between inputs and outputs. Such a highly coupled multivariable system requires a decoupler based centralized control or multivariable centralized controller scheme for compensating the interaction disturbances effectively. These two types of centralized control (full matrix control) strategies are employed with PI/PID controller for enhancing the servo tracking and regulatory response of controller performance.

Vijay kumar *et al*. (2012) developed the centralized PI controller for TITO process. Yuling Shen *et al.* (2014**)** Xiaoli Luan *et al*. (2014) have proposed full matrix controller based on the effective open loop transfer function model. Juan Garrido *et al*. (2012) have developed multivariable centralized control design using inverted decoupling method. Wieder chang *et al*. (2007) has developed multivariable PID controller for TITO process using genetic algorithm with modified cross over formula. In this approach, the IAE is considered as the objective function and then the centralized PID controller 12 tunable parameters are tuned using GA. The effectiveness of modified GA based PID controller is compared with BLT method based PID controller and traditional GA based PID controller. Ravi V R *et al*. (2014) have developed multiloop PI controller with decoupler for two interacting conical tank level process. The multiloop PI controller tuned using stability boundary equation method and genetic algorithm.

The past two decades fractional calculus has been gained popularity in science and engineering application, especially in control theory. Podlubny *et al*. (1999) introduced generalization of PID controller called FOPID controller with additional degree of freedom to enhance the robustness of closed loop system. In the process control industry, more than 90% of the controller loops are controlled by PI/PID controller. However, many authors have demonstrated the advancement of FOPID controller in closed loop control system. The additional non-integer order integration and differentiation operator makes FOPID more robust than integer order PID control. The additional two parameters in FOPID controller introduced complexity in the controller tuning. Various analytical based methods are reported in the literature. However, the optimization based tuning methods have gained popularity in fractional order control design.

The global optimization such as genetic algorithm is widely used to solve complex engineering problems. The GA has been utilized in many control engineering problem to obtain optimal tuning parameters to provide desirable control performance.

Puneet mishra *et al*. (2015) designed fractional order fuzzy PID control for binary distillation process (TITO process) where the FOPID controller parameter tuned using GA with weighted sum of integral sum of absolute error as an objective function. Morteza moradi (2014) proposed multivariable FOPID control for TITO process where the 20 FOPID controller tuning parameters are tuned using modified GA with weighted sum of IAE as an objective function. It was claimed that multivariable FOPID controller provides better performance than H_∞ synthesis based FOPID controller. Conception A monje *et al*. (2004) tuned FOPI controller using iteration based optimization with robust specifications. The phase margin, gain margin and robustness to plant gain variation are considered as the constraints of the optimization and then FOPI controller are tuned to meet these three specifications. pan I and das S (2013) has designed multiloop FOPID controller for bench mark TITO process using particle swarm optimization algorithm. In which, the multiloop FOPID controller parameters obtained by optimizing the time multiplied squared error of both loops. Priya c *et al*. (2014) has designed multiloop PI control and fractional order PI control for spherical-conical interconnected tank systems.

The decoupled control scheme and multivariable centralized controller are designed with linear PI/PID controller, which are failed to produce reasonable performance for the nonlinear system. Hence, the adaptive PI/PID control has been designed for nonlinear MIMO systems by combining family of linear PI/PID controller with gain scheduling scheme. Ananda natrajan *et al*. (2006) has designed PI controller for single conical tank process, where the multimodel based gain scheduler and neural network based gain scheduler are utilized for controlling the entire operating regimes of conical tank process. Many adaptive mechanisms have been presented for nonlinear process, in which family of linear controller with gain scheduler is extensively used as adaptive mechanism. The conventional gain schedule may change the PID controller parameter abrupt across the boundaries of the operating regions. But the fuzzy logic based gain scheduling method utilizing bumpless transfer function to adjust the controller parameter smoothly (Balametee TP *et al*. 2000).

Zhao Z-Y *et al*. (1993) have demonstrated the fuzzy gain scheduler based PID control for process control application. It has been claimed that fuzzy logic based gain scheduler provides satisfactory control performance for nonlinear system (Dhanalakshmi Vinodha R *et al*. (2013)). An adaptive fuzzy based gain scheduler method is developed to provide pre-specified control objectives for nonlinear system. Vijayalakshmi *et al*. (2014) used multimodal based gain scheduler to control the level of liquid in the single conical tank process, where the family of linear PID controller parameters is found for three operating regimes and then weighted scheduler is designed to adjust the controller parameter based on the operating regimes. Nithya *et al*. (2008) has developed fuzzy logic controller for nonlinear spherical tank level process. In that, fuzzy logic rule base is tuned using genetic algorithm. It has been claimed that fuzzy logic control is superior than the conventional PI control. Blanchett *et al*. (2000) has designed improved fuzzy gain scheduler to enhance the PID control performance for nonlinear process. The comparative results of fuzzy logic gain scheduler PID with model predictive control is demonstrated. Kamala *et al*. (2012) has demonstrated fuzzy gain scheduler based PID controller for nonlinear MIMO process. Almeida otacilio da *et al*. (2002) has developed fuzzy rules for MIMO system based on the information of human expertise about the gain

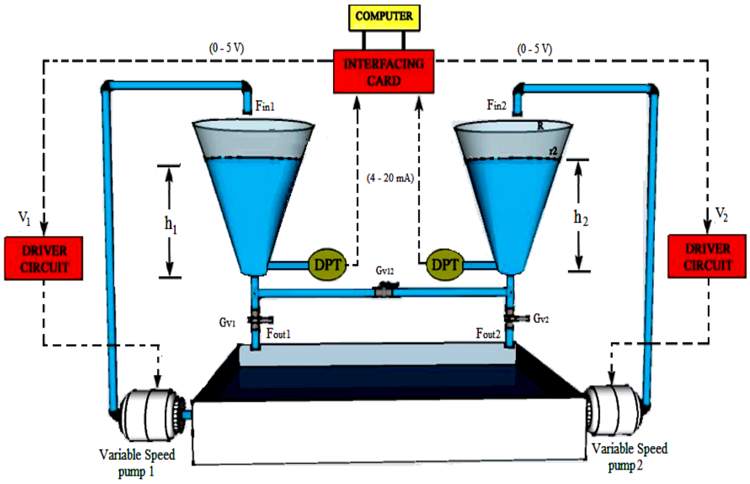

Fig. 1. Schematic diagram of Two Interacting Conical Frustum tank level (TICFTL) process.

and phase margin of the specific multivariable closed loop system.

The objective of this work is to develop a mathematical model for proposed TICFTL process and then to design an optimal closed loop control system to control the liquid level of tank 1, tank 2. In this paper, genetic algorithm is used to found the Decentralized PI, centralized PI, FOPI controller parameters for minimum value of integral error indices. The controller parameters are tuned to overcome the interaction effect between loops and to improve the servo, regulatory performance of closed loop system.

This paper organized as follows. The mathematical model for proposed TICFTL process is briefly explained in section 2. The decentralized PI controller design procedure is given in section 3. The centralized PI, FOPI control scheme and genetic algorithm based tuning procedures are detailed in section 4. The simulation results of controller are analyzed in section 5. Finally the conclusion of proposed work is highlighted in section 6.

2. TWO INTERACTING FRUSTUM CONICAL TANK PROCESS DESCRIPTION

The proposed system consists of two interacting conical frustum tanks connected by interacting pipe. The heights of tanks are 50cm and top and bottom radius of conical tanks are 40cm and 14cm. The gate values Gv_1, Gv_2 and interaction valve Gv_{12} are partially opened and kept fixed. The interaction effect of process can be changed by the hand value Gv_{12}. The two tanks getting inflow of liquid from variable speed pumps. The manipulated inputs of system are the voltage applied to the pumps. The range of input voltage is 0 to 5V, which is directly proportional to rate of change of inflow. The differential pressure transmitter used for measuring the level in terms of milliamps. The main aim is to control the liquid level in the tanks by manipulating the applied input voltages to motor pumps1 and 2.

2.1 Mathematical Modeling of TICFTL Process

The mathematical model of TICFTL process is derived from the mass balance equation. The single conical frustum tank system shown in Fig. 2.

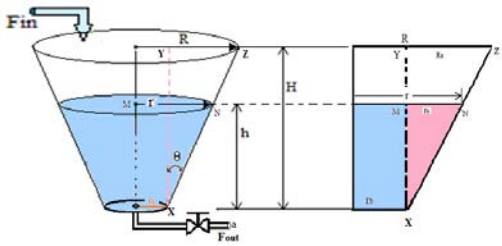

Fig. 2. Volume of liquid in the frustum tank.

The mathematical model for single frustum conical tank process is derived using the conservation of mass and Bernoulli's principle as follows,

Rate of accumulation = Rate of inflow – Rate of Outflow

$$\frac{dVol}{dt} = Fin - Fout \tag{1}$$

where Vol is a volume of liquid in the cone frustum tank. The volume of liquid change due its varying surface area of the tank. The Volume of cone frustum tank Vol is

$$Vol = \frac{\pi}{3}\left(r_{in}^2 + r^2 + r_{in}r\right) \tag{2}$$

where r_{in} bottom radius of tank and r is the top radius of liquid. The varying top radius of liquid level is found using trigonometric law.

$$\tan\theta = \frac{NM}{XN} = \frac{YZ}{XY} \tag{3}$$

where θ is the angle of frustum conical slope.

Table 1 Operating point conditions

Region	Operating point Fin1 Vs h1		Operating point Fin2 Vs h2	
	Fin1	h1	Fin2	h2
1	(0-1.5) V	0-4.67	(0-1.5)V	0-4.67
2	(1.5-3)V	4.42-11.62	(1.5-3)V	4.42-11.62
3	(3-5)V	11.62-27.26	(3 – 5)V	11.62-27.26

Table 2 The transfer function matrix for regime (Tank 1/Regime 1, 2, 3 ; Tank 2/Regime 1, 2)

	Tank 2	
Tank 1	Regime 1	Regime 2
1	$\begin{bmatrix} \frac{0.157s+0.03073}{s^2+0.393s+0.02109} & \frac{0.02085}{s^2+0.393s+0.02109} \\ \frac{0.02085}{s^2+0.393s+0.02109} & \frac{0.1573s+0.0309}{s^2+0.393s+0.02109} \end{bmatrix}$	$\begin{bmatrix} \frac{0.1417s+0.005834}{s^2+0.0904s+0.001561} & \frac{0.002961}{s^2+0.0904s+0.001561} \\ \frac{0.002961}{s^2+0.0904s+0.001561} & \frac{0.1323s+0.006514}{s^2+0.0904s+0.001561} \end{bmatrix}$
2	$\begin{bmatrix} \frac{0.1323s+0.00651}{s^2+0.0904s+0.00156} & \frac{0.00296}{s^2+0.0904s+0.00156} \\ \frac{0.002961}{s^2+0.0904s+0.00156} & \frac{0.1417s+0.005834}{s^2+0.0904s+0.00156} \end{bmatrix}$	$\begin{bmatrix} \frac{0.117s+0.0131}{s^2+0.2237s+0.002914} & \frac{0.01146}{s^2+0.2237s+0.002914} \\ \frac{0.01146}{s^2+0.2237s+0.002914} & \frac{0.117s+0.0131}{s^2+0.2237s+0.002914} \end{bmatrix}$
3	$\begin{bmatrix} \frac{0.095s+0.00242}{s^2+0.0414s+0.000338} & \frac{0.0009032}{s^2+0.0414s+0.000338} \\ \frac{0.0009032}{s^2+0.0414s+0.000338} & \frac{0.1227s+0.001966}{s^2+0.0414s+0.000338} \end{bmatrix}$	$\begin{bmatrix} \frac{0.0811s+0.0015}{s^2+0.03497s+0.000193} & \frac{0.00089}{s^2+0.03497s+0.000193} \\ \frac{0.00089}{s^2+0.03497s+0.000193} & \frac{0.0886s+0.00145}{s^2+0.03497s+0.000193} \end{bmatrix}$

NM=r_s, is an incremental radius of liquid level due to slope surface. $r_s = \frac{R_s}{H}h = \frac{(R-r_{in})}{H}h$.

The top radius of liquid level, r = r_{in}+ r_s;

$$r = r_{in} + \frac{(R-r_{in})}{H}h \quad (4)$$

After substituting 'r' value in Eq. 2, the volume of liquid in cone frustum becomes,

$$Vol = \frac{\pi}{3}\left[3r_{in}^2 h + 3r_{in}\left(\frac{R-r_{in}}{H}\right)h^2 + \left(\frac{R-r_{in}}{H}\right)^2 h^3\right] \quad (5)$$

After substituting Eq. 5 in Eq. 1,

$$\frac{dh}{dt} = \frac{F_{in} - \beta a\sqrt{2gh}}{\frac{\pi}{3}\left[3r_{in}^2 + 6r_{in}\left(\frac{R-r_{in}}{H}\right)h + 3\left(\frac{R-r_{in}}{H}\right)^2 h^2\right]} \quad (6)$$

Where $F_{out} = \beta a\sqrt{2gh}$, 'a' is a cross sectional area of outlet pipe and β is the ratio of gate valve opening (β varies from 0 to 1). When the valve is fully closed, β is 0, when the valve is fully open β is 1. V is input voltage, Kpp is the pump gain.

Similarly, the mathematical model for TICFTL process is developed,

$$\frac{dh_1}{dt} = \frac{k_{pp1}V_1 - \beta_1 a_1\sqrt{2gh_1} - sign(h_1-h_2)\beta_{12}a_{12}\sqrt{2g\left|h_1-h_2\right|}}{\frac{\pi}{3}\left[3r_{in1}^2 + 6r_{in1}\left(\frac{R_1-r_{in1}}{H_1}\right)h_1 + 3\left(\frac{R_1-r_{in1}}{H_1}\right)^2 h_1^2\right]} \quad (7)$$

$$\frac{dh_2}{dt} = \frac{k_{pp2}V_2 + sign(h_1-h_2)\beta_{12}a_{12}\sqrt{2g\left|h_1-h_2\right|} - \beta_2 a_2\sqrt{2gh_2}}{\frac{\pi}{3}\left[3r_{in2}^2 + 6r_{in2}\left(\frac{R_2-r_{in2}}{H_2}\right)h_2 + 3\left(\frac{R_2-r_{in2}}{H_2}\right)^2 h_2^2\right]} \quad (8)$$

The flow rates are function of applied input voltage.

The tanks are identical tanks, so the bottom radius r_{in1}=r_{in2}=14cm, top radius R_1=R_2=20cm. The H_1, H_2 are the height of frustum conical tank (H_1, H_2 = 50 cm). The h_1,h_2 are the liquid level of tank 1,2. The valve coefficient also same for both tanks (i.e,β_1 = β_2 = 0.33) and interaction pipe valve coefficient β_{12}=0.2.

The TICFTL process exhibits nonlinear characteristic, hence the operating regimes are found using piecewise linearization method for controller design. The operating points obtained from input-output characteristic and tabulated in the Table.3. The k_{pp1}, k_{pp2} are the pump 1, 2 gains (25

Table 3 The transfer function matrix for (Tank 1/Regime 1, 2, 3; Tank 2 / Regime 3)

	Tank 2
Tank1	Regime 3
Regime 1	$\begin{bmatrix} \frac{0.1227s+0.001966}{s^2+0.0414s+0.0003378} & \frac{0.0009032}{s^2+0.0414s+0.0003378} \\ \frac{0.0009032}{s^2+0.0414s+0.0003378} & \frac{0.095s+0.00242}{s^2+0.0414s+0.0003378} \end{bmatrix}$
Regime 2	$\begin{bmatrix} \frac{0.0886s+0.001457}{s^2+0.03497s+0.0001935} & \frac{0.000894}{s^2+0.03497s+0.0001935} \\ \frac{0.000894}{s^2+0.03497s+0.0001935} & \frac{0.08116s+0.001503}{s^2+0.03497s+0.0001935} \end{bmatrix}$
Regime 3	$\begin{bmatrix} \frac{0.0121s+0.002686}{s^2+0.0422s+0.000444} & \frac{0.00163}{s^2+0.0422s+0.000444} \\ \frac{0.00163}{s^2+0.0422s+0.000444} & \frac{0.015s+0.002573}{s^2+0.0422s+0.000444} \end{bmatrix}$

cm^3/v.sec). V1, V_2 are the voltage applied to the pumps (0V - 5V).

The state space model and transfer function model is obtained around the operating points using Jacobian linearization.
State equation is,

$$\dot{X} = AX+BU \tag{9}$$

where X is the states of the process [h_1,h_2] and U is the input vector of process [V_1, V_2]. The A, B matrixes are the state matrix and input matrix of the state space model.

$$\begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial h_1} & \frac{\partial f_1}{\partial h_2} \\ \frac{\partial f_2}{\partial h_1} & \frac{\partial f_2}{\partial h_2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial V_1} & \frac{\partial f_1}{\partial V_2} \\ \frac{\partial f_2}{\partial V_1} & \frac{\partial f_2}{\partial V_2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{10}$$

where f_1 is the function dh_1/dt, f_2 is the fucntion dh_2/dt. The output equation of state space model is given below,

$$Y=C\,X + D\,U \tag{11}$$

where Y is the output vector [h_1,h_2] , C is the output matrix, D is the feedforward input matrix.

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{12}$$

Where Y_1, Y_2 are the outputs of the TICFT process. The state space model is converted into transfer function model which is tabulated for 3 X 3operating condtions.

3. DECENTRALIZED PI CONTROLLER

The interaction between input and output is analyzed using Relative gain array. Then, the decentralized PI controller is designed for diagonal element using ziegler Nichols method and then the diagonal controllers are detuned.

$$Gc(s)=\begin{bmatrix} kc_{11}\left(1+\frac{1}{\tau_{11}s}\right) & 0 \\ 0 & kc_{22}\left(1+\frac{1}{\tau_{22}s}\right) \end{bmatrix} \tag{13}$$

$diag[kc_{ii}] = kc_{ii}/F$; $diag[\tau_{ii}] = \tau_{ii}F$;
where i=1,2. The recommended value of detuning parameter 'F' is between 2 to 5.

4. CENTRALIZED PI, FRACTIONAL ORDER PI CONTROLLER

4.1. Centralized PI Controller

The centralized PI Controller is designed for TICFTL process using the gain array of the plant. The gain matrix of operating region is used to find the centralized PI controller parameter (Davison EJ 1976).

$$Kc= \delta_i\left[G(s=0)\right]^{-1} \tag{14}$$

$$K_i= \varepsilon_i\left[G(s=0)\right]^{-1} \tag{15}$$

The controller tuning parameters δ_i , ε_i are called as rough tuning parameters. This parameter adjusted to change the controller gain of centralized PI controller.

4.2. Centralized Fractional Order PI Controller

The fractional order integrator is included in the conventional FOPI controller to improve the performance of PI controller.

$$Gc(s)=\begin{bmatrix} \left(kp_{11}+\frac{k_{i11}}{s^{\lambda}}\right) & kc_{11}\left(1+\frac{1}{\tau_{11}s^{\lambda}}\right) \\ kc_{11}\left(1+\frac{1}{\tau_{11}s^{\lambda}}\right) & kc_{22}\left(1+\frac{1}{\tau_{22}s}\right) \end{bmatrix} \quad (16)$$

Consider the transfer function model and gain matrix of operating point 33 (Tank1/Regime 3, Tank2 /Regime 3),

$$G_p(s)=\begin{bmatrix} \frac{0.0121s+0.002686}{s^2+0.0422s+0.000444} & \frac{0.00163}{s^2+0.0422s+0.000444} \\ \frac{0.00163}{s^2+0.0422s+0.000444} & \frac{0.015s+0.002573}{s^2+0.0422s+0.000444} \end{bmatrix} \quad (17)$$

The gain array at steady state is,

$$[G_p(0)]_{33}=\begin{bmatrix} 6.05 & 3.26 \\ 3.26 & 5.78 \end{bmatrix} \quad (18)$$

The inverse of gain matrix,

$$[G_p(0)]^{-1}=\begin{bmatrix} 0.2378 & -0.134 \\ -0.134 & 0.2489 \end{bmatrix} \quad (19)$$

The complex centralized problem is simplified and control scheme is given below,

$$G_c(s)=\begin{bmatrix} 0.2378*\delta_f+\frac{0.2378*\varepsilon_f}{s^{\lambda}} & -0.134*\delta_f-\frac{0.134*\varepsilon_f}{s^{\lambda}} \\ -0.134*\delta_f-\frac{0.134*\varepsilon_f}{s^{\lambda}} & 0.2489*\delta_f+\frac{0.2489*\varepsilon_f}{s^{\lambda}} \end{bmatrix} \quad (20)$$

The parameters (δ_f, ε_f) are the tuning parameters of Centralized FOPI controller. The tuning parameter δ_f change the proportional gain of centralized FOPI controller and parameter ε_f adjust the integral gain of centralized FOPI controller. The order of integrator is ‘λ’. The centralized FOPI controller parameters are obtained optimally to achieve minimum integral time absolute error (ITAE).

4.3. Genetic Algorithm Based Controller Tuning

Genetic algorithm is most widely used optimization algorithm to solve nonlinear multimodal problems. Many researchers have used genetic algorithm (GA) for tuning the controller with integral error criteria as an objective function. In this paper, the centralized fractional order FOPI controller is tuned using matlab genetic algorithm tool box (Houck et.al 1995).

Steps involved in Genetic optimization algorithm

Step 1: Initialize the objective function

$$J=w_1\int_0^{\infty} t\left|h_{1sp}(t)-h_1(t)\right| \text{dt}+w_2\int_0^{\infty} t\left|h_{2sp}(t)-h_2(t)\right| \text{dt} \quad (21)$$

Minimize J, subject to

$$\delta_f^{\min}\le\delta_f\le\delta_f^{\max};\ \varepsilon_f^{\min}\le\varepsilon_f\le\varepsilon_f^{\max};$$

$$\lambda^{\min}\le\lambda\le\lambda^{\max}$$

Where ‘h_{1sp}’, ‘h_{2sp}’ are the reference set point of 1st loop and 2nd loop. The ‘$h_1(t)$’ ‘$h_2(t)$’ are the liquid level of the 1st tank and 2nd tank. The w_1, w_2 are the weigtage of sub objective function.($W_1 = W_2 = 0.5$)

Step 2: The population size is fixed as 20, number of iteration is fixed as 100 and other optimization parameters such as mutation, cross over parameters are selected as 0.8 and constraint dependent.

Step 3: The fitness values objective functions is obtained based on rank based scaling function.

Step 4: At each iteration the control vectors are updated to achieve minimum value of objective function J.

Step 5: The selection function is fixed as stochastic uniform function and the reproduction function such as elite count and cross over fraction is set as 0.05, 0.8. The initial guess for optimization problem for three cases such as decentralized PI, centralized PI and centralized FOPI is given below, case i: For decentralized PI controller, the controller parameters for each diagonal transfer function model is obtained using Z-N tuning formula and then, detuning factor F_1, F_2 fixed in between the range of 2-5.

Case ii; For the centralized controller, the initial guess of tuning parameters δ_i fixed between 0.1 to 50 and ε_i fixed between 0.01 to 10.

Case iii: For the centralized FOPI controller, the parameters are fixed as $\delta_f\in[0.1,50]$, $\varepsilon_f\in[0.01,10]$, $\lambda\in[0.1,1.2]$.

Step 6: Run the optimization until the minimum objective function is obtained. The stopping criteria for the optimization are maximum iteration or minimum value convergence.

The controller parameters for objective functions J are obtained using GA and the obtained parameters are tabulated in Table. 4, 5. The block diagram of GA based decentralized/centralized controller tuning for TITO system is shown in Fig.3.

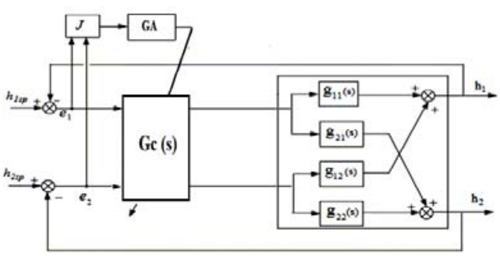

Fig. 3. Tuning of centralised PI/FOPI control parameter tuning using GA.

4.2 Fuzzy Based Gain Scheduling Method

The centralized PI controller designed for 3 X 3 operating points. The controller around one

Table 4 Optimal GA based controller parameter for 3-3 operating point region.

Operating point (3-3)	$\begin{bmatrix} \frac{0.0121s+0.002686}{s^2+0.042s+0.00044} & \frac{0.00163}{s^2+0.042s+0.00044} \\ \frac{0.00163}{s^2+0.042s+0.00044} & \frac{0.015s+0.002573}{s^2+0.042s+0.00044} \end{bmatrix}$
Decentralized PI Controller	$\begin{bmatrix} 0.721+\frac{0.0066}{s} & 0 \\ 0 & 0.814+\frac{0.0075}{s} \end{bmatrix}$
Centralized PI Controller	$\begin{bmatrix} 0.2433+\frac{0.0046}{s} & -0.1541-\frac{0.0029}{s} \\ -0.1541-\frac{0.0029}{s} & 0.254+\frac{0.0048}{s} \end{bmatrix}$
Centralized FOPI controller	$\begin{bmatrix} 0.3439+\frac{0.004}{s^{1.051}} & -0.2179-\frac{0.0026}{s^{1.051}} \\ -0.2179-\frac{0.0026}{s^{1.051}} & 0.359+\frac{0.0042}{s^{1.051}} \end{bmatrix}$

operating region cannot produce satisfactory performance for complete process because of change in process gain and time constant. In order to accommodate nonlinearity, the gain scheduler is used in controller to vary the controller parameters according to different operating condition. The PI controller for each operating regions are combined using fuzzy gain scheduling method. The trapezoidal membership function is chosen and fuzzy rules are developed based on the operating regions. The tank levels are the input for fuzzy gain scheduler and controller parameters such that proportional gain (Kp_{11}, K_{p12}, K_{p21}, Kp_{22}), integral gain (Ki_{11}, K_{i12}, K_{i21}, Ki_{22}) are the outputs of the fuzzy gain scheduler. The memberships functions and rule viewer of fuzzy gain scheduler is shown in figs. 4, 5.

Table 5 Optimal GA based decentralized PI controller parameter for minimum ITAE

	Decentralized PI			
Regime				
	Kp1	Ki1	Kp2	Ki2
1-1	3.19	0.48	3.24	0.482
1-2	1.88	0.29	1.94	0.412
1-3	2.1	0.23	2.6	0.31
2-1	1.94	0.412	1.88	0.29
2-2	1.42	0.092	1.53	0.104
2-3	0.621	0.0136	0.54	0.0113
3-1	2.6	0.31	2.1	0.23
3-2	0.54	0.0113	0.621	0.0136
3-3	0.721	0.0066	0.814	0.0075

Table 6 Optimal GA based controller parameter for minimum ITAE

	Centralized PI		Centralized FOPI		
Regime	δ_i	ε_i	δ_f	ε_f	λ
1-1	20.76	0.85	20.51	0.84	0.991
1-2	15.76	0.44	16.51	0.56	0.979
1-3	9.76	0.34	10.51	0.26	0.982
2-1	15.76	0.44	16.51	0.56	0.979
2-2	2.76	0.248	2.91	0.024	1.04
2-3	1.76	0.14	1.97	0.02	1.03
3-1	9.76	0.34	10.51	0.26	0.982
3-2	1.76	0.14	1.97	0.02	1.03
3-3	0.906	0.017	1.281	0.015	1.051

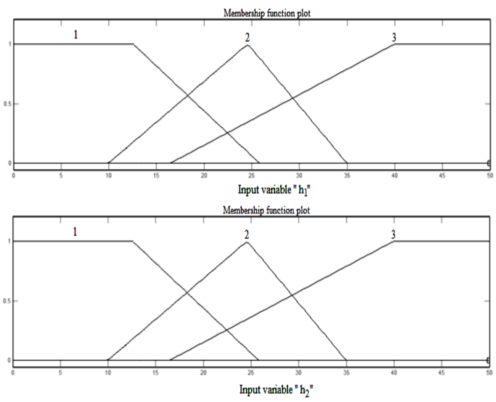

Fig. 4. Membership function of fuzzy gain scheduler.

Fig. 5. Fuzzy Rule viewer of fuzzy gain scheduler.

The membership function and fuzzy rules are selected based on the operating condition. The adaptive centralized PI/FOPI controller is shown in Fig. 6.

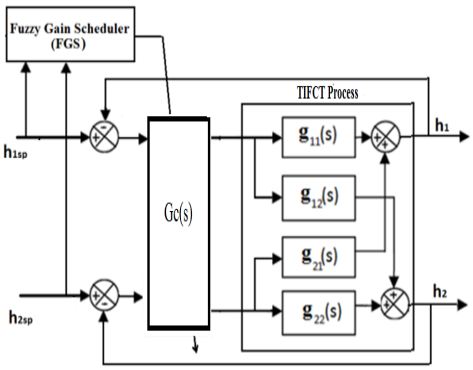

Fig. 6. Adaptive Centralized PI/ FO-PI controller.

5. Simulation Results and Analysis

In the proposed work, the TICFTL process model is developed based on the mass balance equation. The open loop data was generated and the operating regimes are selected from the input output characteristics. The linear state space model is developed and converted into transfer function model. The decentralized PI, centralized PI, FOPI controller parameters are tuned using GA. The multivariable controller design is treated as multi objective optimization problem. The local linear controllers are combined using fuzzy gain scheduling method.

The servo and regulatory response of adaptive decentralized PI, centralized PI, centralized FOPI controllers responses are compared and shown in Figs. 7,8,9,10. The setpoint changes are introduced to tank 1, 2 at 1000 sec. The first tank level set point is fixed as 20 cm from 0 sec to 2000 sec and then the set point is changed at 2000sec from 20 cm to 15cm. The tank 2 level set point changes applied at each 1000 sec. The step change in set point changes 0-15 cm , 15-25 cm , 25-20 cm are applied at 0,1000,2000 sec.

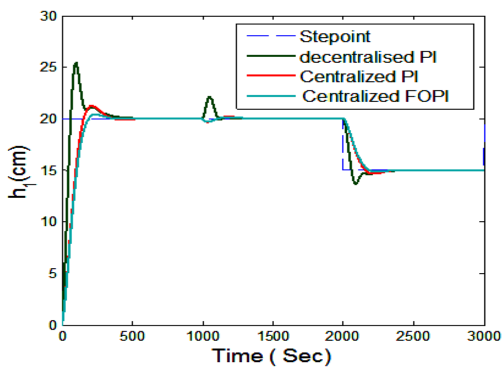

Fig. 7. Servo response of controller for TICFTL process (loop1).

The servo tracking response of proposed centralized PI controller has better servo tracking and disturbance rejection response. The performance indexes are found and tabulated for comparison. It clearly shows that centralized PI controller has better servo tracking than decentralized PI, centralized FOPI controller.

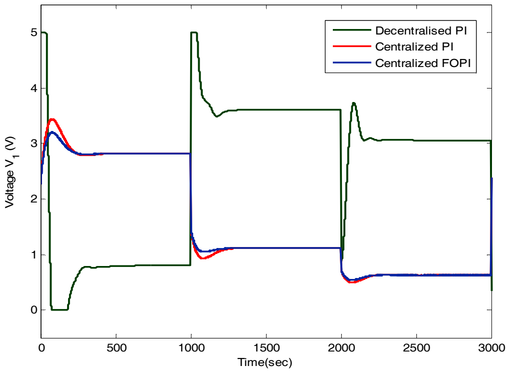

Fig. 8. Controller output voltage (manipulated input V_1).

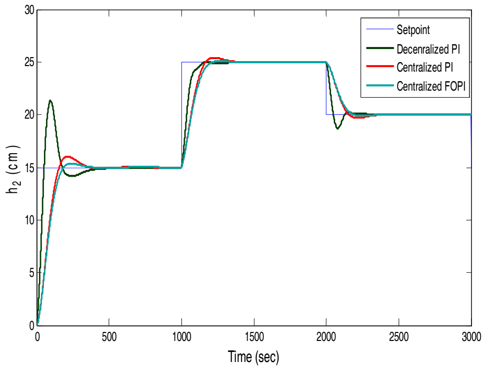

Fig. 9. Servo response of controller for TICFTL process (loop 2).

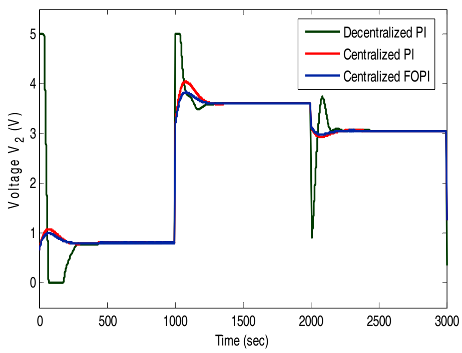

Fig. 10. Controller output voltage (manipulated input V_2).

The FOPI controller has three tuning parameters ($\delta_f, \varepsilon_f, \lambda$), the order of FOPI controller (λ) provides additional flexibility in controller tuning. It is believed that the fractional order integrator improves the performance of FOPI than PI controller. But the non integer integrator order results in the offset in the closed loop response which increases the settling time and offset. The FOPI controller provides better response when the order of integrator is equals to 1. It can be easily inferred that the centralized PI controller has better performance than decentralized PI, centralized FOPI controller.

The proposed controller validated with servo tracking and regulatory responses. The centralized FOPI also produce reasonable controller response with faster settling time, minimum overshoot, but the integral error such as IAE, ITAE is larger for centralized FOPI controller. Because of the non-integer order integrator provides offset in the closed loop control performance.

Table 7 Comparison of decentralized PI, centralized PI, centralized FOPI controller

	IAE		ITAE	
	Loop1	Loop2	Loop1	Loop2
Decentralized PI	381.2	215.4	968.82	863.4
Centralized PI	309.7	174.13	905.5	813.72
Centralized FOPI	322.6	186.8	929.3	895.05

6. CONCLUSION

The two interacting frustum conical tank interacting process is proposed and mathematical model was developed using mass balance equation. The interaction effect between input and output is analyzed for controller design. The genetic algorithm based centralized FOPI controller is presented and controller parameters are optimally tuned using genetic algorithm. The centralized FOPI controllers are compared with decentralized PI, centralized PI controller in terms of settling time, ISE, IAE, ITAE. From the simulation studies it is infer that the GA based centralized PI controller provides control response with faster settling time with minimum integral error criteria than centralized FOPI controller. It is concluded that, the fractional order PID may produce better response than the integer order PID controller, but FOPI controller does not improve the performance of closed loop control system than PI controller.

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