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# Empirical Modeling of Flow Characteristics in Suddenly Expanding Channels

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## ABSTRACT

Different flow characteristics namely sequent depth ratio, relative height of jump, relative energy loss, efficiency, relative length of jump and relative length of roller in suddenly expanding channel against inflow Froude number varying between 2 to 9 at different expansion ratios $B_1/B_2$ (0.4, 0.5, 0.6 and 0.8) as third variable are experimentally studied. Physical explanations of the variation of these characteristics with Froude number are discussed based on the results from experiments. Empirical models are proposed for all the six characteristics for rectangular and suddenly expanding channels using Buckingham π-method which gives quite satisfactory results when compared with other researchers result. Effectiveness of baffle blocks and sills (with different configurations) were also discussed in dissipating maximum energy. Due weightage has been given to Froude and Reynold's number in present study as reported literature as well. As a result, baffle block and sills caused a significant improvement in sequent depth ratio by an amount of 30%, reduction in relative length of jump ratio and relative length of the roller by an amount of 38% and 37% respectively. Hence, energy dissipation increases due to appurtenances.

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## 1. INTRODUCTION

In order to design the energy dissipating structure on the downstream side of dams, the flow characteristics can be analyzed such that no scouring and erosion of the bed would take place. The hydraulic jump formation is an alternative in dissipating energy downstream of spillways for incoming stream. There can be a horizontal rectangular channel or suddenly expanding channel downstream of the spillway of a rigid or nonrigid dam. In these channels mainly two problems are faced by hydraulic engineers; determination of height of jump (sequent depth) and estimation of energy loss of jump (efficiency).

As for as simple rectangular basin is concerned, the solution for the hydraulic jump or flow characteristics is simple and therefore can be determined easily, moreover analytical and empirical solutions for rectangular basin are also available. But for the analysis of suddenly expanding rectangular channel either few studies or solution to limited channel conditions are available, and therefore there analysis is quite cumbersome. Also, it seems that this channel has attracted less attention of the researchers compared to rectangular channels. Abruptly widening channels have a considerable impact on the development of symmetric flows downstream of the channel in addition to altering the hydraulic jump characteristics.

Few analytical and experimental studies have been made by Jan and Chang (2009), Agarwal (2001), Ranga Raju (1993) and Hager (1985) but the review of literature shows that the analysis for the experimental studies is devoted mainly to study the variation of sequent depth ratio and relative energy loss against incoming Froude number and results are not entirely consistent.

Good work has been reported by Ranga Raju et al. (1980), Pagliara and Chiavaccini (2006) on using baffle/sills for energy loss in hydraulic jump characteristics. It has been demonstrated by Negm et al. (2000) and Negm (2000) that the expansion ratio and approach Froude number determine the sequent depth ratio. Daneshfaraz et al. (2021a, b, c) modeled the stepped spillway flow numerically using Fluent. Results clearly indicate significant increase in the energy dissipation. They also came to the conclusion that the finite element approach produces less satisfactory outcomes than the finite volume method. Numerical and experimental modeling on energy dissipation has been done by Ghaderi and Abbasi (2019), Daneshfaraz et al. (2019a, b), Zhou et al. (2020), Ghaderi et al. (2020) and in special cases of jump in expanding spillways.

**NOMENCLATURE**

| | | | |
|---|---|---|---|
| $B$ | bottom width of rectangular channel | $R_{e1}$ | Reynold's number |
| $B_1$ | approach channel width | $S$ | spacing between baffle and channel |
| $B_2$ | expanded channel width | $V_1$ | velocity at section 1 |
| $B_1/B_2$ | expansion ratio | $V_2$ | velocity at section 2 |
| $E_1$ | energy per unit weight at section 1 (before the jump) | $W_{ss}$ | width of sill |
| $E_2$ | energy per unit weight at section 2 (after the jump) | $W_b$ | width of baffle |
| $E_L$ | energy loss per unit weight ($E_1 – E_2$) | $x$ | distance of the toe from expansion section |
| $F_{r1}$ | approach Froude number | $x_{bi}$ | position of baffle from inlet gate |
| $h_b$ | height of baffle | $x_b$ | position of baffle from expanded part |
| $h_j$ | height of jump | $x_s$ | position of sill from expanded part |
| $h_s$ | height of sill | $Y_1$ | pre-jump depth |
| $L_{bb}$ | length of baffle at base | $Y_2$ | post-jump depth |
| $L_{bt}$ | length of baffle at top | $Y_2/Y_1$ | sequent depth ratio |
| $L_r$ | length of roller | $h_j/Y_1$ | relative height of jump |
| $L_{ss}$ | length of sill | $E_L/E_1$ | relative energy loss |
| $h_j$ | height of jump | $E_2/E_1$ | efficiency of jump |
| $h_s$ | height of sill | $L_j/Y_1$ | relative length of jump |
| $L_{bb}$ | length of baffle at base | $L_r/Y_1$ | relative length of roller |
| $L_{bt}$ | length of baffle at top | $\rho$ | density of water |
| $L_j$ | length of roller | $\upsilon$ | kinematic viscosity |
| $n$ | number of baffle blocks | $\varepsilon$ | surface roughness |
| $Q$ | discharge | $\mu$ | dynamic viscosity of water |

Impact of the sill's height and its location on the characteristics of the jump in a suddenly expanding channel was investigated by Zare and Doering (2011) and Bai et. al (2022). Making geometric parameter dimensionless, they proved that depth of the jump decreases. Bremen (1990) had studied the hydraulic jump characteristics with appurtenances and proposed empirical relations for relative length of jump and roller.

## 2. THEORY

The first significant theoretical and experimental work on the hydraulic jump was carried out by Bidone (1819) and Bélanger (1849). Based on the momentum principle, they proposed a theoretical solution to the question of the ratio of sequent depths. Hager (1985) may have reported the first experimental data on dimensionless free surfaces. Authors found that the length and sequence depth of a classical hydraulic jump increase with increasing Froude number.

Hydraulic jumps in a stilling basin depend on the level of tailwater. When tailwater levels are low, sudden expansion results in the formation of hydraulic jumps Daneshfaraz et. al (2019a, b). According to the studies, hydraulic jumps in a sudden expansion channel are classified as repelled hydraulic jumps, spatial hydraulic jumps and transitional hydraulic jumps (Bremen, 1990; Daneshfaraz et al., 2019a, b).

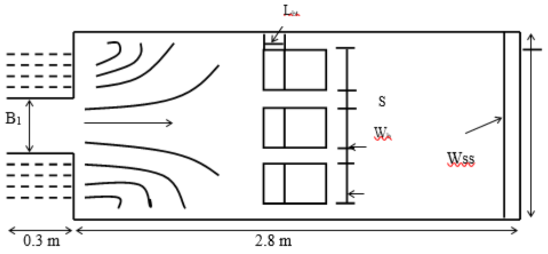
As per Hager (1985) and Bremen (1990), effectiveness of the stilling basin can be improved by designing symmetrical suddenly expanding channel using appurtenances. Herbrand (1973) explored channel with smooth bed and applied the energy condition (ignoring the impact of disturbance, air entrainment, wall grinding and the tension power on the extension walls) and proposed an experimental connection for sequent profundity proportion. Numerical study of symmetric spatial hydraulic jumps is carried out by Jesudhas et al. (2019) and showed that the rollers are formed and influences the characteristics strongly near the wall than in its centre.

Zare and Doering (2011) given due weightage to sill height and its location. Flow characteristics in suddenly expanding channels get modified by adding solid sills. It controls the flow and scour patterns.

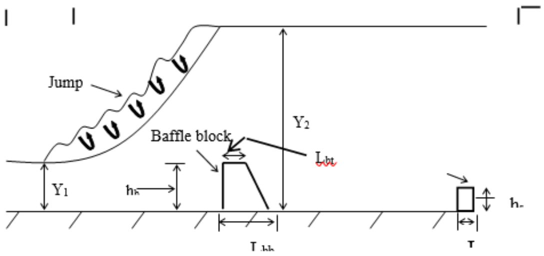
The analytical results for sequent depth ratio and relative energy loss in a suddenly expanding channel can be obtained using equations given by Ranga Raju (1993) and Agarwal (2001). These equations have been solved graphically by Agarwal (2001). This monograph gives quite an accurate estimate for $Y_2/Y_1$, $E_L/E_1$ and $E_2/E_1$ for various values of $B_1/B_2$ for known values of $F_{r1}$. The empirical relation given by Herbrand's (1973) holds good for $3 < F_{r1} < 9$ and gives satisfactory estimation of $Y_2$.

Bremen and Hager (1993) concentrated on momentary pressure driven bounce in which the toe is found upstream from the extension area. In their examination, they considered abruptly extending rectangular channel with flat bed. They created observational condition for the sequent profundity proportion in view of trials and a worked on hypothesis. As indicated by Bremen and Hager (1993), spatial jump happens when its toe is situated at the extension area.

Khosravinia et al. (2018), Kumar and Lodhi (2016), and Alhamid (2004) studied experimentally the jump characteristics for different expansion ratios on smooth bed. The outcomes showed that jump have less sequent depth proportion and higher efficiency contrasted with




(a) Plan




(b) Elevation

**Fig. 1 (a, b) shows plan and elevation of suddenly expanding channel**

the rectangular channel. When an enlargement is employed in the channel; effectiveness of the jump as an energy dissipater is affected by the position of the toe. Sometimes tail water level also influences the height and position of jump formation (Herbrand, 1973; Bremen & Hager, 1993; Sadeghfam et al., 2017; Torkamanzad et al., 2019; Bai et al. 2023). Stabilizing non-symmetric flow is carried out by maintaining expansion ratio between the flow regions (Graber et al., 2001). Inflow Froude number, the expansion factor, the aspect ratio and the level of submergence are important factors to determine whether an asymmetrical flow occurs in a submerged hydrodynamic jump.

As soon as a channel is suddenly extended with baffles and sills due to design restrictions; it now becomes useful to control hydraulic jumps when the tail water depth has not been sufficient to produce good jumps and more likely it also reduces the basin length. Experimental and dimensionless approach has been made in the present study to find solutions to such limitations. Empirical model produced in the study for different flow characteristics can be used directly to the field; specially for determining relative length of jump and roller where locating exactly the position from the toe is difficult.

## 3. Experimental Work and Data Acquisition

All the experiments for mentioned 6 different flow characteristics namely sequent depth ratio, relative height of jump, relative energy loss, efficiency, relative length of jump and length of roller ($Y_2/Y_1$, $h_j/Y_1$, $E_L/E_1$, $E_2/E_1$, $L_j/Y_1$ and $L_r/Y_1$) for different channel types and conditions for Froude number $F_{r1}$ varying between 2 to 9 were conducted in rectangular and suddenly expanding channel (for the expansion ratios $B_1/B_2$ = 0.4, 0.5, 0.6 and 0.8) with and without appurtenances. The upstream face of inlet regulating gate is covered by stilling basin of length 2 m to prevent side wave reflection and surface undulation so that the stabilised flow is available at the inlet of main channel. The waves still remaining get stabilised during their travelling in rest of the length of stilling basin before feeding to main channel. The experiment was performed in 2.1 m long channel with 0.445 m width. Pointer gauges are used for measuring the depth of flow both across the width and along the length of the channel at different points from the inlet gate, whereas discharge is measured by volumetric method. In order to obtain accurate and reliable experimental data, the main design considerations have been to achieve minimum water losses, symmetric flow, depth at three points across the main channel and discharge at the downstream end and their accurate measurement. Figure 1 shows schematic plot of plan and elevation of channel.

Details of measured and varied parameters under different channel types are summarized in Table 1. Dimensions of appurtenances varied under different channel types are provided in Table 2; whereas range of values of flow characteristics calculated are shown in Table 3.

**Table 1 Measured and Varied Parameters**

| Channel Types | Measured Parameters | Varied Parameters |
|---|---|---|
| Rectangular channel | $Y_1$, $Y_2$, Q, $L_j$, $L_r$ | $F_{r1}$ |
| Suddenly expanding channel | $Y_1$, $Y_2$, Q, $L_j$, $L_r$ | $F_{r1}$ |
| Suddenly expanding channel with baffle blocks & sill | $Y_1$, $Y_2$, Q, $L_j$, $L_r$ | $F_{r1}$, $B_1/B_2$, $h_b$, $W_b$, $n_b$, $x_b$ |

**Table 2 Details of the dimensions (in meters) of appurtenances used under different**

| Channel with Appurtenances | $L_{bt}$ | $h_b$ | $W_b$ | $L_{bb}$ | $n_b$ | $h_s$ | $W_{ss}$ | $L_{ss}$ |
|---|---|---|---|---|---|---|---|---|
| Suddenly expanding channel with 2 baffle blocks & sill for $B_1/B_2$ = 0.4, 0.5, 0.6 and 0.8 | 0.01 | 0.04 | 0.025 | 0.04 | 2 | 0.02 | 0.3 | 0.02 |
| Suddenly expanding channel with 1 baffle block & sill for $B_1/B_2$ = 0.4, 0.5, 0.6 and 0.8 | 0.01 | 0.05 | 0.03 | 0.04 | 1 | 0.02 | 0.3 | 0.02 |

**Table 3 Range of different flow characteristics calculated ($F_{r1}$=2 to 9)**

| Channel Types | | $Y_2/Y_1$ | $h_j/Y_1$ | $E_L/E_1$ | $L_r/Y_1$ | $E_2/E_1$ | $L_j/Y_1$ |
|---|---|---|---|---|---|---|---|
| Rectangular Channel | | 4 - 11 | 3 - 10 | 0.24 - 0.73 | 8 - 55 | 0.27 - 0.78 | 30 - 62 |
| Suddenly expanding channel with baffle & sill | $B_1/B_2$ = 0.4 | 2 - 11 | 2 - 10 | 0.15 - 0.80 | 3 - 30 | 0.22 - 0.82 | 12 - 55 |
| | $B_1/B_2$ = 0.5 | 1.5 - 9 | 0.8 - 7 | 0.21 - 0.8 | 2 - 32 | 0.20 - 0.70 | 7 - 46 |
| | $B_1/B_2$ = 0.6 | 2.3 - 12 | 2 - 11 | 0.10 - 0.75 | 2 - 45 | 0.26 - 0.90 | 11 - 65 |
| | $B_1/B_2$ = 0.8 | 3 - 16 | 3 - 14 | 0.01 - 0.7 | 2 - 64 | 0.21 - 0.99 | 20 - 79 |

## 4. Development Of Empirical Models: Testing And Validation

Literature describes number of empirical models for flow characteristics. As per the study of Noor and Bushra (2002) and Ohtsu et al. (2003), it is demonstrated that Reynold's number play an important role in influencing flow behavior. Good work has been reported by Ranga Raju et al. (1980), Pagliara and Chiavaccini (2006) on using appurtenances for energy loss and efficiency respectively. Rajaratnam (1964) and Tyagi et al. (1978) studied the effect of drag on baffles and sills during hydraulic jumping. Unny (1961) proposed semi-empirical formula in terms of expansion ratio, approach Froude number and the shape factor. The important variables affecting flow characteristics are $Y_1$, $Y_2$, $V_1$, $V_2$, $L_r$, $L_j$, $E_L$, υ g, ρ and ε which can be explored as mentioned in eqn (1). Using Buckinham's π-theorem approach and treating $Y_1$, μ and g as repeating variables, the following dimensionless groups are obtained:

$$f\left(Y_1, Y_2, V_1, V_2, L_r, L_j, E_L, \mu, g, \rho, \varepsilon\right) = 0 \quad (1)$$

For rectangular channels, the dimensionless groups can be expressed as:

$$f\left(\frac{Y_2}{Y_1},\frac{h_j}{Y_1},\frac{E_L}{E_1},\frac{E_2}{E_1},\frac{L_j}{Y_1},\frac{L_r}{Y_1},\frac{V_1^2}{gY_1},\frac{\rho V_1 Y_1}{\mu},\frac{\varepsilon}{Y_1}\right)=0 \quad (2)$$

All the flow characteristics namely $Y_2/Y_1$, $h_j/Y_1$, $E_L/E_1$, $E_2/E_1$, $L_j/Y_1$, $L_r/Y_1$ in rectangular channel are found to be a function of Froude number and Reynold's number. For example, the sequent depth ratio can be written as a function:

$$\frac{Y_2}{Y_1}=f\left(\frac{V_1^2}{gY_1},\frac{\rho V_1 Y_1}{\mu}\right) \quad (3)$$

In terms of kinematic viscosity 'υ', it can be further expressed as

$$\frac{Y_2}{Y_1}=f\left(\frac{V_1^2}{gY_1},\frac{V_1 Y_1}{\nu}\right) \quad (4)$$

The remaining flow characteristics can be expressed similar to Eq. (4).

For suddenly expanding channel, the dimensionless groups are expressed as:

$$f\left(\frac{Y_2}{Y_1},\frac{h_j}{Y_1},\frac{E_L}{E_1},\frac{E_2}{E_1},\frac{L_j}{Y_1},\frac{L_r}{Y_1},\frac{B_1}{B_2},\frac{V_1^2}{gY_1},\frac{\rho V_1 Y_1}{\mu},\frac{\varepsilon}{Y_1}\right)=0 \quad (5)$$

The expression for suddenly expanding channels can be expressed (for sequent depth ratio) as:

$$\frac{Y_2}{Y_1}=f\left(\frac{B_1}{B_2},\frac{V_1^2}{gY_1},\frac{\rho V_1 Y_1}{\mu}\right) \quad (6)$$

For suddenly expanding channel (with baffle blocks and sill), the dimensionless groups can be expressed as:

$$f\left(\frac{Y_2}{Y_1},\frac{h_j}{Y_1},\frac{E_L}{E_1},\frac{E_2}{E_1},\frac{L_j}{Y_1},\frac{L_r}{Y_1},\frac{\varepsilon}{Y_1},\frac{B_1}{B_2},\frac{V_1^2}{gY_1},\frac{\rho V_1 Y_1}{\mu},\frac{x_b}{x_s},\frac{h_b}{nW_b},\frac{h_s}{W_s},\frac{L_{bb}}{L_{ss}},\frac{L_{bt}}{S}\right)=0 \quad (7)$$

Flow characteristics are found to be a relation of expansion ratio, Froude number, Reynold's number and position & dimension of baffle blocks/sill. Therefore, sequent depth ratio can be expressed as:

$$\frac{Y_2}{Y_1}=f\left(\frac{B_1}{B_2},\frac{V_1^2}{gY_1},\frac{\rho V_1 Y_1}{\mu},\frac{x_b}{x_s},\frac{h_b}{nW_b},\frac{h_s}{W_s},\frac{L_{bb}}{L_{ss}},\frac{L_{bt}}{S}\right) \quad (8)$$

Similarly, the remaining flow characteristics can be expressed. It is to be noted that the effect of surface roughness are neglected and not be considered in developing there groups due to experimental limitations.

The regression models has been developed for rectangular and suddenly expanding channels for all 6 flow characteristics and provided in Table 4. A best fit model with $R^2$ value are shown in Fig. 2 (a to f) for rectangular as well as suddenly expanding channels. The developed model equations with $R^2$ values for two channel conditions i.e. $B_1/B_2$ = 0.4 and 0.5 are plotted in Fig. 3 (a to n). Similarly, the developed model equations with $R^2$ values for two channel conditions i.e. $B_1/B_2$ = 0.4 and 0.5 using two baffles and sill for each flow characteristics are shown in Fig. 4 (a to l).

## 5. RESULTS AND DISCUSSION

### 5.1 Flow Characteristics for Different Channels

The comparison of all flow characteristics are presented in Fig. 5 without appurtenances using experimental data. Figure 5 (a) shows linear variations of sequent depth ratio ($Y_2/Y_1$) against the approach Froude number ($F_{r1}$) for all types of channel. The linear variation with $R^2$ values of 1 and 0.99 are obtained for suddenly expanding channel with $B_1/B_2$ ratio of 0.4, 0.5, 0.6, and

0.8 for rectangular channels. $R^2$ values in each case show less scattering of data from the best fit line. The linear variation of $Y_2/Y_1$ with $F_{r1}$ were also reported in the studies of Ranga Raju (1993), Reinauer and Hager (1995) and Ohtsu et al. (1995,1996,1997). It is clear from this figure that the sequent depth ratio increases with the increment in $B_1/B_2$ ratio in suddenly expanding channel.

Figure 5 (b) shows a non-linear variation of relative height of jump ($h_j/Y_1$) against approach Froude number ($F_{r1}$). It is clear that the relative jump height increases as the Froude number increases. $R^2$ values are close to 0.94 indicates better fitting of the data points. It is clear from figure that the relative height of jump is comparatively higher in case of suddenly expanding channel with $B_1/B_2$ ratio of 0.8. Bremen (1990), Chow (1959) and Ranga Raju (1993) have also shown the similar variations from their experimental and analytical studies. Bakhmeteff and Matzke (1936) have emphasized that the relative height of jump depends on expansion ratio and shown good agreement at high Froude numbers.

**Table 4. Developed Empirical models for all the three channel cases**

| **Rectangular Channel (Eqn. 9 – 14)** | **Suddenly Expanding Channel (Eqn. 15 – 20)** | **Suddenly expanding channel with appurtenances (Eqn. 22 – 27)** |
|---|---|---|
| $\frac{Y_2}{Y_1}=10371\left(\frac{F_{r1}^2}{R_{e1}}\right)+3.1015$ | $\frac{Y_2}{Y_1}=201.24\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^2}{R_{e1}^{0.5}}\right)+1.53$ | $\frac{Y_2}{Y_1}=2\times10^6\left[\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}}{R_{e1}}\right)^{0.5}\left(\frac{x_b}{x_s}\right)\left(\frac{h_b}{nW_b}\right)\left(\frac{h_s}{W_s}\right)\left(\frac{L_{bb}}{L_{ss}}\right)\left(\frac{L_{bt}}{S}\right)\right]-1.4949$ |
| $\frac{h_j}{Y_1}=10371\left(\frac{F_{r1}^2}{R_{e1}}\right)+2.1015$ | $\frac{h_j}{Y_1}=201.24\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^2}{R_{e1}^{0.5}}\right)+0.53$ | $\frac{h_j}{Y_1}=2\times10^6\left[\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}}{R_{e1}}\right)^{0.5}\left(\frac{x_b}{x_s}\right)\left(\frac{h_b}{nW_b}\right)\left(\frac{h_s}{W_s}\right)\left(\frac{L_{bb}}{L_{ss}}\right)\left(\frac{L_{bt}}{S}\right)\right]-2.4949$ |
| $\frac{E_L}{E_1}=8.8814\left(\frac{F_{r1}^{0.05}}{R_{e1}^{0.001}}\right)-8.9832$ | $\frac{E_L}{E_1}=3.114\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^{0.3}}{R_{e1}^{0.05}}\right)-0.3583$ | $\frac{E_L}{E_1}=601531\left[\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^{0.2}}{R_{e1}^{0.5}}\right)\left(\frac{x_b}{x_s}\right)\left(\frac{h_b}{nW_b}\right)\left(\frac{h_s}{W_s}\right)\left(\frac{L_{bb}}{L_{ss}}\right)\left(\frac{L_{bt}}{S}\right)\right]-0.5503$ |
| $\frac{E_2}{E_1}=-8.8814\left(\frac{F_{r1}^{0.05}}{R_{e1}^{0.001}}\right)+9.9832$ | $\frac{E_2}{E_1}=-3.114\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^{0.3}}{R_{e1}^{0.05}}\right)+1.3583$ | $\frac{E_2}{E_1}=-601531\left[\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^{0.2}}{R_{e1}^{0.5}}\right)\left(\frac{x_b}{x_s}\right)\left(\frac{h_b}{nW_b}\right)\left(\frac{h_s}{W_s}\right)\left(\frac{L_{bb}}{L_{ss}}\right)\left(\frac{L_{bt}}{S}\right)\right]+1.5503$ |
| $\frac{L_j}{Y_1}=465409\left(\frac{F_{r1}^{1.2}}{R_{e1}}\right)+8.0555$ | $\frac{L_j}{Y_1}=312.81\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^2}{R_{e1}^{0.5}}\right)+3.6454$ | $\frac{L_j}{Y_1}=4\times10^6\left[\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^{0.2}}{R_{e1}^{0.5}}\right)\left(\frac{x_b}{x_s}\right)\left(\frac{h_b}{nW_b}\right)\left(\frac{h_s}{W_s}\right)\left(\frac{L_{bb}}{L_{ss}}\right)\left(\frac{L_{bt}}{S}\right)\right]-43.485$ |
| $\frac{L_r}{Y_1}=170\left(\frac{F_{r1}^2}{R_{e1}^{0.5}}\right)+2.1631$ | $\frac{L_r}{Y_1}=15.191\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^2}{R_{e1}^{0.2}}\right)-0.5501$ | $\frac{L_r}{Y_1}=1.5\times10^6\left[\left(\frac{B_1}{B_2}\right)\left(\frac{F_{r1}^{1.2}}{R_{e1}^{0.5}}\right)\left(\frac{x_b}{x_s}\right)\left(\frac{h_b}{nW_b}\right)\left(\frac{h_s}{W_s}\right)\left(\frac{L_{bb}}{L_{ss}}\right)\left(\frac{L_{bt}}{S}\right)\right]-2.5508$ |

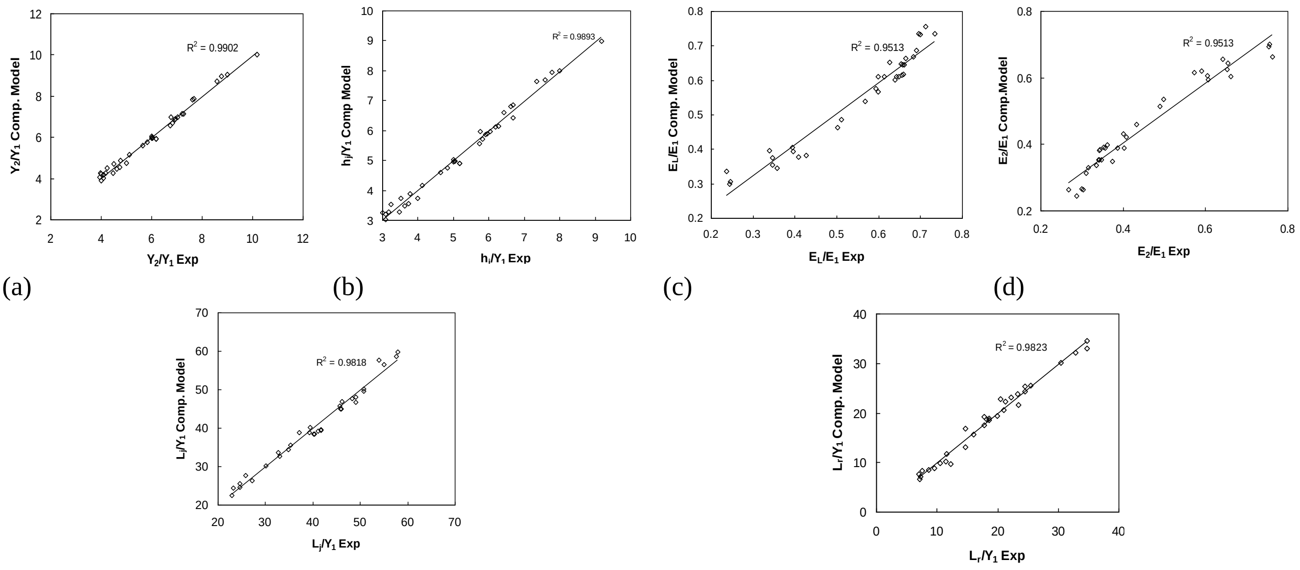
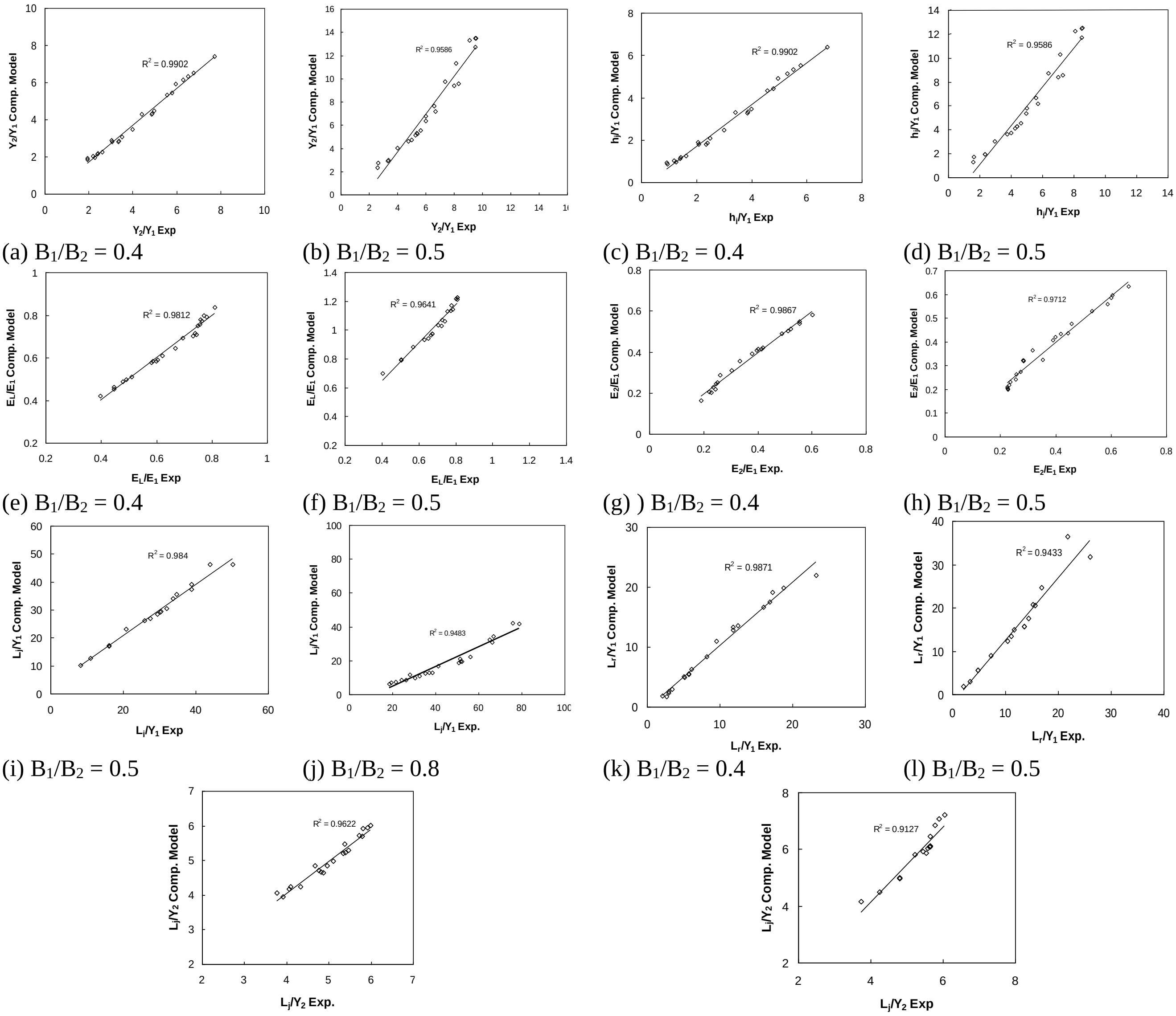



**Fig. 2 (a to f) Linear fit of the empirical models (Eqs. 9 – 14) for flow characteristics in rectangular channel**




(a) $B_1/B_2 = 0.4$ (b) $B_1/B_2 = 0.5$ (c) $B_1/B_2 = 0.4$ (d) $B_1/B_2 = 0.5$

(e) $B_1/B_2 = 0.4$ (f) $B_1/B_2 = 0.5$ (g) ) $B_1/B_2 = 0.4$ (h) $B_1/B_2 = 0.5$

(i) $B_1/B_2 = 0.5$ (j) $B_1/B_2 = 0.8$ (k) $B_1/B_2 = 0.4$ (l) $B_1/B_2 = 0.5$

(m) $B_1/B_2 = 0.4$ (n) $B_1/B_2 = 0.5$

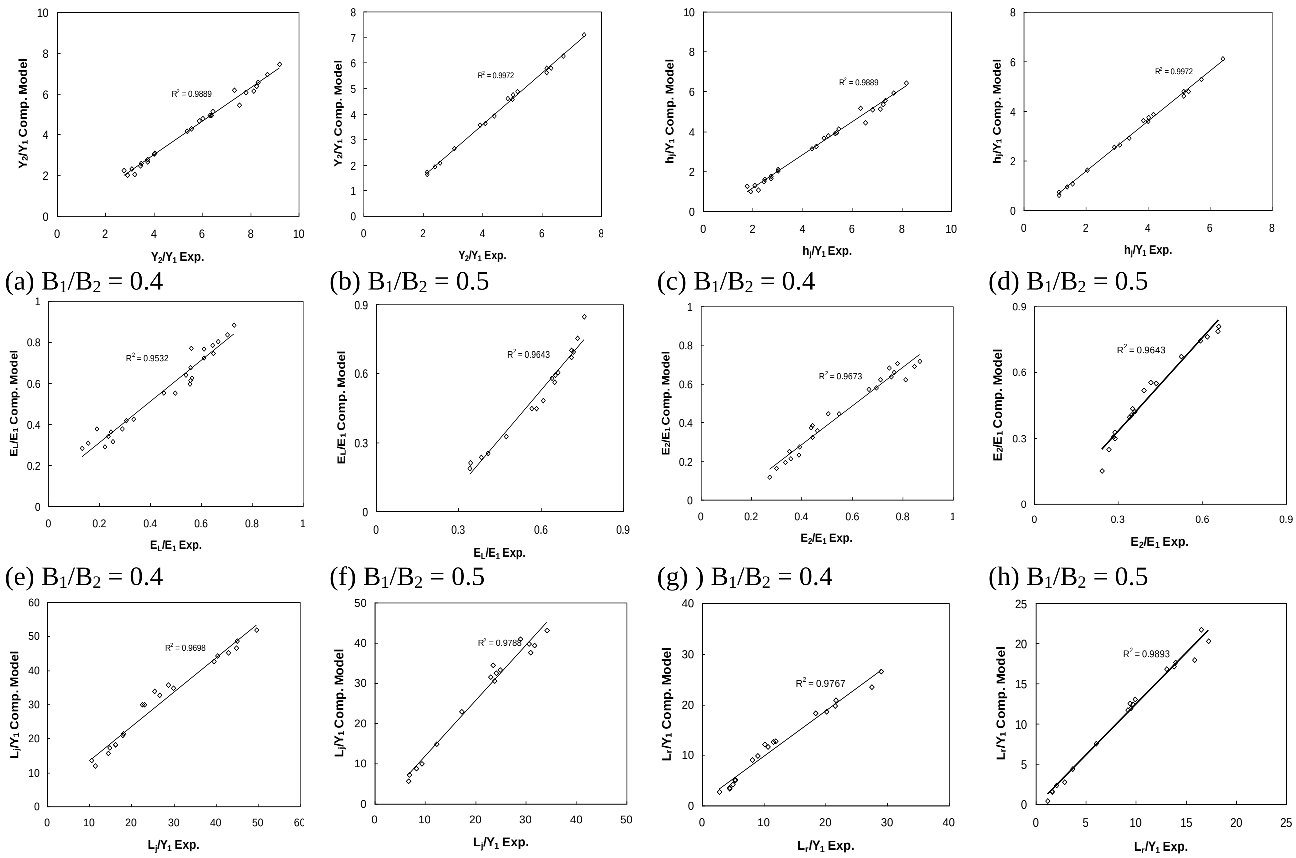
**Fig. 3 (a to n) Linear fit of the empirical models (Eqs. 15 to 20) for flow characteristics in suddenly expanding channel with $B_1/B_2$ = 0.4 and 0.5.**

Figure 5 (c) shows non linear increasing trend of relative energy loss ($E_L/E_1$) with approach Froude number ($F_{r1}$). A logarithmic fitting of the experimental data with $R^2$ values of 0.97, 0.97, 0.98 and 0.98 in suddenly expanding channels with $B_1/B_2$ ratios 0.4, 0.5, 0.6 and 0.8 respectively shows good agreement of the experimental data. It is revealed that low value of lateral expansion of rectangular channel dissipate more energy than higher $B_1/B_2$ ratio.

Figure 5 (d) shows a decreasing pattern of variation of efficiency of jump ($E_2/E_1$) with Froude number ($F_{r1}$). In this figure, the value of $R^2$ is near to 0.97 for all types of channel ratios. Similar observations were noted by Jamil and Khan (2008), Bremen (1990) and Chow (1959) for such channels.

Figure 5 (e) shows non linear increasing trend of relative length of jump ($L_j/Y_1$) against Froude number ($F_{r1}$) when it varies between 2 to 9 for rectangular and suddenly expanding channels. Similar trend of variation is also reported by the other researchers (Afzal & Bushra, 2002; Omid et al., 2008). From this figure, it can be seen that a maximum relative length of jump is observed in suddenly expanding channel with $B_1/B_2$ = 0.8 at higher Froude number 9, whereas $L_j/Y_1$ is lower in other cases. $R^2$ value of 0.99 for suddenly expanding channel shows good fitting of experimental data, whereas $R^2$ = 0.91 for rectangular channel shows somewhat scattered fitting of data points. The reason may be attributed to inaccuracy in measurement of the length of the jump as it was difficult to judge the exact position of the starting and end of jump precisely (Chow, 1959; Bai et al., 2021).

Figure 5 (f) shows a non linear exponential increase of relative length of roller ($L_r/Y_1$) against the Froude number ($F_{r1}$) for all types of channel considered. Relative length of roller is more for rectangular channel between $F_{r1}$ = 2 to 9. $R^2$ value is approximately same for $B_1/B_2$ = 0.4 and 0.5. However, it is 0.65 for suddenly expanding




(a) $B_1/B_2 = 0.4$ (b) $B_1/B_2 = 0.5$ (c) $B_1/B_2 = 0.4$ (d) $B_1/B_2 = 0.5$

(e) $B_1/B_2 = 0.4$ (f) $B_1/B_2 = 0.5$ (g) ) $B_1/B_2 = 0.4$ (h) $B_1/B_2 = 0.5$

(i) $B_1/B_2 = 0.4$ (j) $B_1/B_2 = 0.5$ (k) $B_1/B_2 = 0.4$ (l) $B_1/B_2 = 0.5$

**Fig. 4 (a to 1) Linear fit of the empirical models [Eqs. (22 to 27)] for flow characteristics in suddenly expanding channel with 2 baffle blocks and sill for $B_1/B_2$ = 0.4 and 0.5.**

channels with $B_1/B_2$ = 0.8; which may be due to inaccuracy in deciding the position of the toe and length of jump. Similar results are also reported by other researchers (Peterka, 1958; Ranga Raju et al., 1980; Bretz, 1987; Achour, 2000).

### 5.2 Flow Characteristics for Channels with 2 Baffle & Sill

This section describes the comparison of all flow characteristics using 2 baffles and sill. Figure 6 (a) shows a linear variation of sequent depth ratio ($Y_2/Y_1$) against the approach Froude number ($F_{r1}$) for expanding channel having 2 baffles and sill. The $R^2$ value of 0.99 for $B_1/B_2$ = 0.4, 0.5 and 0.6 show good fitting of the experimental data, however scattering of data points are observed for $B_1/B_2$ = 0.8, due to formation of surface roller and asymmetric jumps. The linear variation of $Y_2/Y_1$ with $F_{r1}$ are well highlighted in the literature (Reinauer & Hager, 1995, Ranga Raju, 1993; Ohtsu et al., 1995, 1996, 1997).

Figure 6 (b) shows a non linear variation of relative height of jump against Froude number. Figure shows that relative height of jump ($h_j/Y_1$) increases with increase in approach Froude number. From this figure, a significant increment in $h_j/Y_1$ with higher expansion ratio with appurtenances is noticed. It indicates the effectiveness of the dimension of baffles and sill used for the flow conditions. Bremen (1990) and Chow (1959) have also shown the similar variations of $L_j/Y_1$ from their experimental and analytical studies. Also, the results obtained are found to be in good agreement with the observations of Bakhmeteff and Matzke (1936) at higher Froude numbers. Deviation of the data points may be attributed to the formation of surface rollers.

Figure 6 (c) shows an increasing trend of change of relative energy loss ($E_L/E_1$) with approach Froude number ($F_{r1}$) for suddenly expanding channel with appurtenances. A logarithmic fitting of experimental data with $R^2$ value of 0.99 for suddenly expanding channels with $B_1/B_2$ = 0.4, 0.5, 0.6 shows good fitting of experimental data. More relative energy loss is observed with $B_1/B_2$ ratio of 0.5 than that for other expansion ratios, which shows the effectiveness of dimension of baffle blocks and sill used.

Figure 6 (d) shows decreasing trend of variation of efficiency ($E_2/E_1$) with Froude number ($F_{r1}$) varying between 2 to 9 for suddenly expanding channel with appurtenances. The higher value of $R^2$ of 0.99, 0.98, 0.96 for $B_1/B_2$ = 0.5, 0.4, 0.6 shows good fitting of the experimental data. A maximum efficiency is observed in suddenly expanding channel with $B_1/B_2$ ratio of 0.8, which shows the suitability of dimension of baffle blocks and sill for higher expansion ratios. A similar trend on these channels with appurtenances was also observed by Jamil and Khan (2008) and Bremen (1990).

Figure 6 (e) shows a non linear variation of relative length of jump ($L_j/Y_1$) against Froude number ($F_{r1}$) varying between 2 to 9. Figure shows, the relative

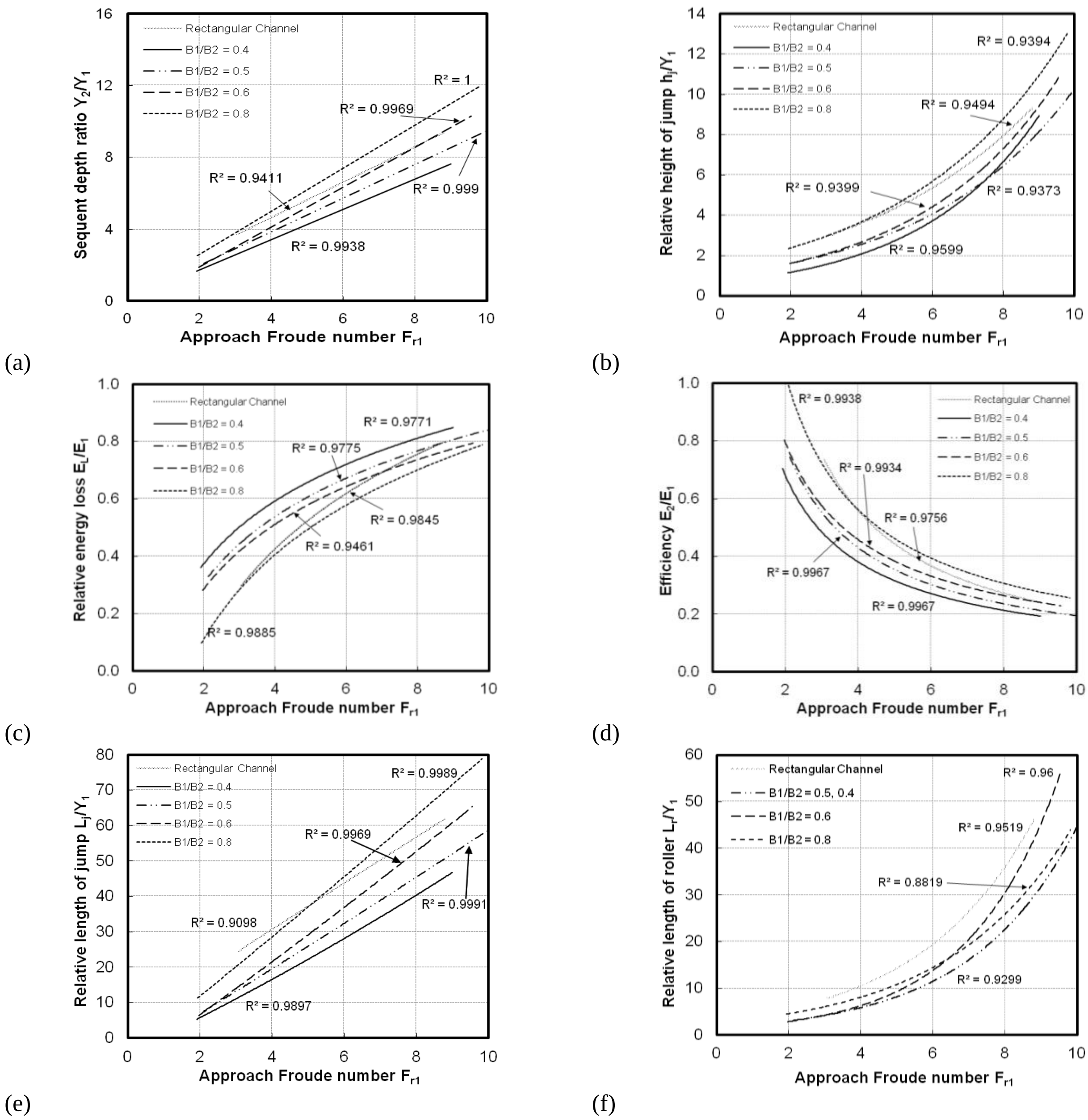



**Fig. 5 (a)-(f) Variation of Flow Characteristics against approach Froude number**

length of jump is minimum in case of $B_1/B_2$ = 0.5, whereas it is higher for other ratios. Higher $R^2$ values shows good fitting of experimental data except $B_1/B_2$ = 0.8, which can be attributed to the inaccuracy in measurement of jump length as it is difficult to judge precisely the position of toe and end of jump (Chow, 1959). Relative length of jump is observed maximum at approach Froude number ranging between 5 to 9 for all expansion ratios. Similar trend of variation of $L_j/Y_1$ is reported by Afzal and Bushra (2002), Hager (1989), Esmaeeli (2005) and Omid et al. (2008) for their studies.

Figure 6 (f) shows a non linear exponential variation of relative length of roller ($L_r/Y_1$) against Froude number ($F_{r1}$) for suddenly expanding channels. From this figure, it is seen that the relative length of roller is higher for suddenly expanding channel for $B_1/B_2$ ratio of 0.8 at $F_{r1}$ between 6 and 7. $R^2$ values for the curves drawn show a relatively poor fitting of data except for $B_1/B_2$ = 0.8, which may be attributed to inaccurate judgment of toe position and end point of the jump.

### 5.3 Flow Characteristics for Channels with 1 Baffle & Sill

In this section, flow characteristics for suddenly expanding channel with 1 baffle & sill as appurtenances are made. The comparative plots for these characteristics are shown below. Figure 7 (a) predicts linear variation of sequent depth ratio ($Y_2/Y_1$) with the approach Froude number ($F_{r1}$) for suddenly expanding channel with 1 baffle and sill. It is clear from figure that sequent depth ratio increases with increase in $B_1/B_2$ ratio in case of suddenly expanding channel for Froude number ranging between 6 to 9. The $R^2$ value of 0.99 for $B_1/B_2$ = 0.4, 0.5, 0.6 and 0.8 shows satisfactory fitting of the experimental data, however, a small scattering of data is observed due to formation of surface roller and asymmetric flow. Linear variation of $Y_2/Y_1$ with $F_{r1}$ were well reported in

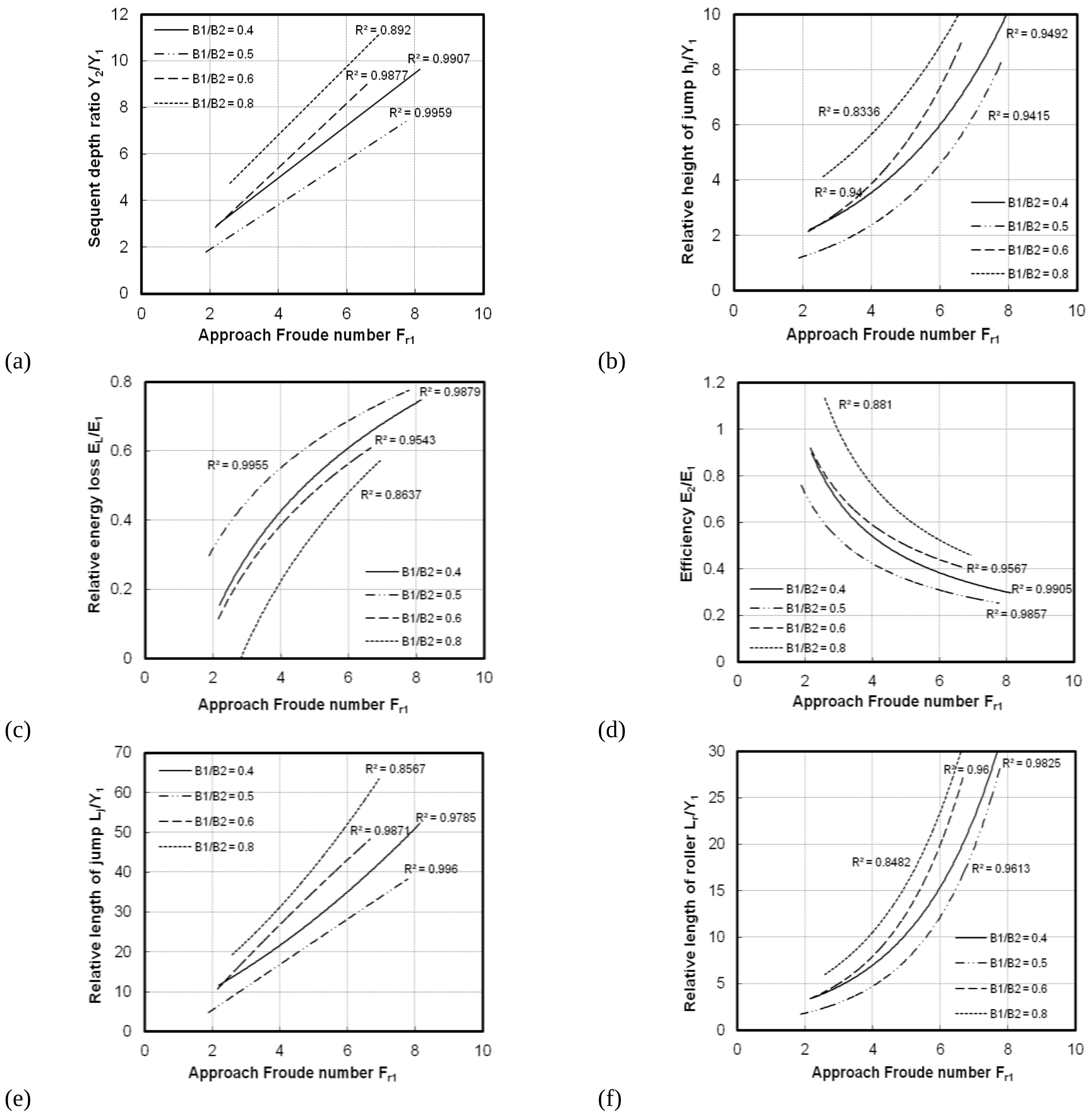


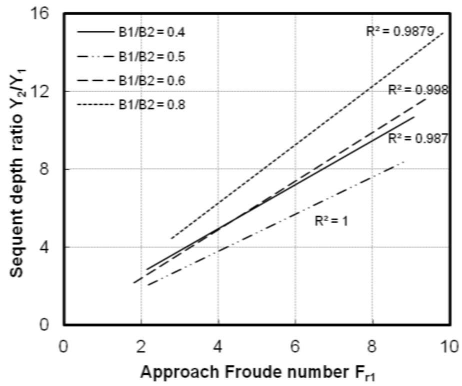

**Fig. 6 (a)-(f) Variation of Flow Characteristics against approach Froude number**

the literature by Ohtsu et al. (1995, 1996, 1997), Ranga Raju (1993) and Reinauer and Hager (1995). From this figure, it also seen that increase in sequent depth ratio with Froude number is in order of suddenly expanding channel with $B_1/B_2$ ratios 0.4 > 0.5 > 0.6 > 0.8 (with 1 baffle block).

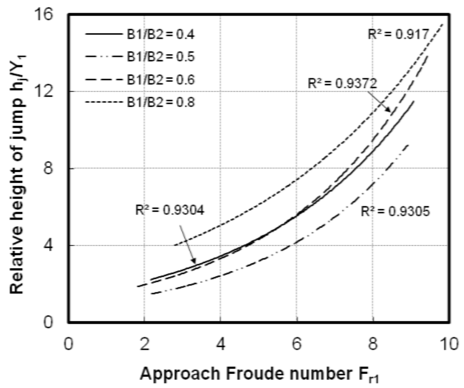
Figure 7 (b) shows a non linear variation of relative height of jump against Froude number varied between 2 to 9 for suddenly expanding channel with 1 baffle and sill. This figure shows that relative height of jump ($h_j/Y_1$) increases with increase in Froude number ($F_{r1}$). From the figure, the relative height of jump is seen higher for suddenly expanding channels with $B_1/B_2$ ratio of 0.8 than that for other expansion ratios, which shows the effectiveness of the dimension of baffles and sill used in the present study. Bremen (1990) and Chow (1959) have also reported similar observations from their experimental as well as analytical results.

Figure 7 (c) shows an increasing trend of relative energy loss ($E_L/E_1$) with approach Froude number ($F_{r1}$) for suddenly expanding channels with appurtenances as described earlier. Logarithmic fitting of experimental data with $R^2$ value of 0.99 for $B_1/B_2$ ratios of 0.5, 0.6 with single baffle shows good fitting of the experimental data, however it is 0.93 for $B_1/B_2$ = 0.8 showing scattering of some data points. The relative energy loss is observed higher for relatively lower expansion ratios in suddenly expanding channels and the maximum relative energy loss is observed for $B_1/B_2$ = 0.5 than other expansion ratios.

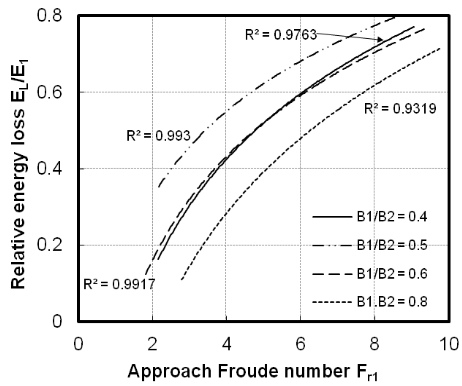
Figure 7 (d) shows a non linear decreasing trend of variation of efficiency of jump ($E_2/E_1$) with Froude number ($F_{r1}$) varied from 2 to 9 for suddenly expanding channel with appurtenances. In this figure, higher values of $R^2$ for all the cases shows good fitting of the experimental data. However, the maximum efficiency is observed in suddenly expanding channel with $B_1/B_2$ ratio



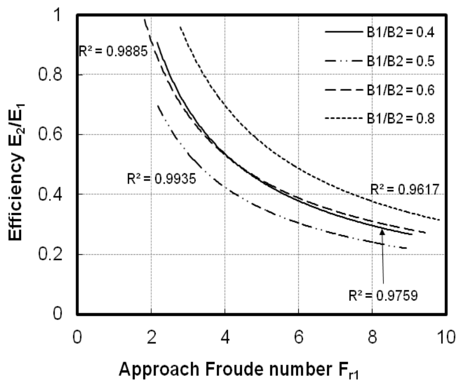

(a)



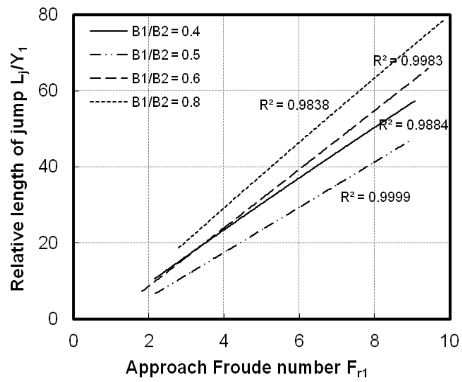

(b)



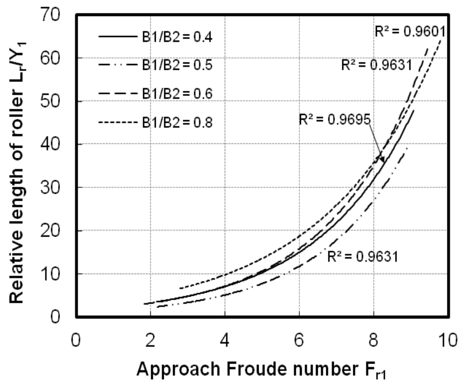

(c)




(d)




(e)




(f)

**Fig. 7 (a)-(f) Variation of Flow Characteristics against approach Froude number**

of 0.8 at $F_{r1}$ = 3, which shows that dimension of baffle blocks and sill used are sufficient to increase the efficiency of energy dissipation. Similar trend on these channels with appurtenances have also been reported by Jamil and Khan (2008) and Bremen (1990) for trapezoidal and suddenly expanding channels.

Figure 7 (e) shows variation of relative length of jump ($L_j/Y_1$) against Froude number ($F_{r1}$) varied between 2 to 9 for suddenly expanding channel with appurtenances. Figure shows relative length of jump is minimum for $B_1/B_2$ = 0.5, whereas it is higher for suddenly expanding channel with $B_1/B_2$ = 0.8. $R^2$ values of 0.98, 0.99, 0.99, 0.99 and 0.94 for suddenly expanding channel having $B_1/B_2$ = 0.8, 0.6, 0.4, 0.5 shows good fitting of the experimental data. Similar results are also reported by other researchers (Hager, 1989; Afzal & Bushra 2002; Esmaeeli 2005; Omid et al., 2008).

Figure 7 (f) shows a non linear exponential increase of relative length of roller ($L_r/Y_1$) against the Froude number ($F_{r1}$) for suddenly expanding channel. It is observed that relative length of roller is higher for $B_1/B_2$ ratio of 0.8 in suddenly expanding channel and lower for $B_1/B_2$ = 0.5. Some scatter in the data points is observed, which may be due to errors in evaluating the current positions of the start and end of the jump.

From the above comparative studies made for suddenly expanding channel with different expansion ratios and use of baffle and sill, it is observed that all the flow characteristics shows a definite relationship with approach Froude numbers.

## 6. CONCLUSION

A comparative study of all the flow characteristics without appurtenances with 2 baffles & sill and with single baffle & sill in suddenly expanding channel are presented.

It is found that due to applications of appurtenances, the sequent depth ratio, energy loss and length of jump/roller are getting modified; designed baffle and sills are found suitable for increasing jump height and overall efficiency. Dimensions of baffles and sill used were observed adequate for achieving better jump pattern and higher energy loss. Sequent depth ratio increases by an amount of 30% and significant reduction in relative length of jump & roller by an amount of more than 35% is noticed. It indicates the efficacy of appurtenances dimensions.

Since flow follows a definite pattern, hence it is suggested to use appurtenances to modify the flow characteristics for higher energy dissipation. Further, a detailed comparison of all the characteristics is well reported for each characteristic for channel types and conditions. Empirical models developed for different characteristics shows, there exists the correlation between the flow characteristics, Froude number and Reynold's number as given in table 4.

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## CONFLICT OF INTEREST

I have no conflicts to disclose.

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