Investigating Rising Bubbles in Air-nanofluid Two-phase Flow: A Vertical Channel Simulation Approach

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ABSTRACT

The study analyzes the unique behavior of two-phase flows when incorporating nanofluids containing aluminum trioxide (Al_2O_3) and copper (Cu) nanoparticles in a vertical channel. The main goal is to investigate the behavior of air-nanofluid mixtures in this setting, with potential implications for industrial and exploration applications. Research in this area could provide valuable insights into the dynamics of these flows and their impact on heat transfer, fluid dynamics, and material science. This study includes an analysis of upwelling dynamics, the effect of fluid characteristics on bubble growth, and the system's thermal efficiency. Using numerical and quantitative visualization techniques, we seek to understand the behavior of these particles at the interface between the liquid and gas phases by integrating Al_2O_3 and Cu nanoparticles into the VOF approach. Because of their superior thermal conductivity, copper nanoparticles have a higher volumetric density and provide more efficient heat transfer, leading to quick and efficient thermal dissipation. Smaller nanoparticles offer an increased surface area-to-volume ratio, which improves heat transfer capabilities and ensures uniform heat dissipation throughout the material. Consequently, copper nanoparticles emerge as the preferred choice for applications necessitating high thermal transfer and optimal performance. These results significantly impact the design of more efficient heat exchangers and optimize recovery techniques by elucidating the interactions between these nanoparticles and the surrounding fluids. Furthermore, the selection of smaller copper nanoparticles further amplifies thermal transfer, maximizing performance across diverse applications.

1. INTRODUCTION

Diphasic flows, which refer to the simultaneous presence of two distinct phases, are prevalent in both everyday situations and a wide range of industrial and natural systems. Examples of diphasic flows in everyday life include gaseous emissions and aerosols, while in industrial contexts, they play a crucial role in processes such as fluidized beds (Chen et al., 2023; Leung et al., 2023; Liu et al., 2023; Soria-Verdugo et al., 2023), petroleum extraction (Hassan et al., 2010; Pang et al., 2023; Mei et al., 2023), and various manufacturing applications. Additionally, diphasic flows are integral to natural systems, evident in phenomena like blood circulation and meteorological processes. The study and understanding of diphasic flows hold significant importance due to their prevalence across various applications, ranging from environmental considerations to optimizing industrial processes and comprehending fundamental natural phenomena. Additionally, two-phase flow is a crucial aspect of CO_2 refrigeration technology, influencing heat transfer and phase change processes. It impacts system pressures, temperatures, and refrigerant behavior. Optimizing two-phase flow is essential for efficient and sustainable refrigeration systems, ensuring proper functioning and safety (Dai et al., 2024a, b, c). The objective of this introduction is to explore the complex interconnections and consequences linked to

NOMENCLATURE			
Cp	specific heat	*Greek symbols*	
d	diametre of bubble	φ	fraction of nanoparticle in fluid
F	force of surface tension	μ	dynamic viscosity
g	gravity acceleration	ρ	density
H	channel height	*Super and subscript*	
k	thermel conductivity	c	cold
L	channel length	f	fluid
P	static pressure	h	hot
q	heat flux	l	liquid
T	temperature	nf	nanofluid
t	time	np	nanoparticle
u	velocity	s	solid
V	volume	v	vapor

biphasic flows in various scientific and engineering disciplines, establishing the foundation for further research.

These flows entail the simultaneous movement of two distinct phases, gas-liquid or liquid-liquid mixtures. This scenario is common in numerous industrial processes, including oil and gas production, chemical engineering, and thermal management systems. Moreover, an interface between immiscible fluids characterizes each process, akin to our specific situation. 'Free surface' frequently describes the boundary between a liquid and a gas.

The modeling and description of moving interfaces can take various approaches (Kothe, 1998; Shyy et al., 2001). Surface representation techniques depict interfaces through distinctive marker points approximated using piecewise polynomials. This approach enables precise computation of interface curvature, a crucial factor for incorporating surface tension forces. However, it encounters limitations when simulating the merging and separation of interface surfaces, as particles may exhibit varying distances from one another, resulting in reduced resolution.

Multiple methodologies for handling surface representations are available, including front tracking, and level-set (Brackbill et al., 1992; Ambrosio & Soner, 1996; Sethian, 1999; Osher et al., 2004; Ningegowda & Premachandran, 2014), lattice Boltzmann methods (Watanabe & Ebihara, 2003), and volume methods (Gopala & Van Wachem, 2008; Ma et al., 2012; Merabtene et al., 2023; Vaishnavi et al., 2023). Front tracking employs massless marker particles to trace the interface, while level-set approaches define the interface using a distance function where it is set to zero. Volume methods employ markers to delineate fluids on both sides of the boundary. However, the precise position of the boundary isn't explicitly known, necessitating specific techniques for reconstructing well-defined interfaces. Marker particle techniques offer accuracy and robustness but are computationally intensive due to the substantial number of particles required, especially in three-dimensional simulations. Meanwhile, the volume of fluid method (VOF) utilizes a scalar field to distinguish between distinct fluids. This approach is precious for simulating free surface flows and multiphase flows with different interfaces.

The VOF method, first proposed by Hirt and Nichols (1981), is a significant approach in fluid dynamics for multiphase flow simulations. It incorporates averaged flow equations and a phase indicator function, denoted as α, which assigns a numerical value to discrete points within the computational domain, representing the concentration of a particular component at each station. This method allows for highly accurate predictions of fluid behavior and determines the interface topology. However, the VOF method is often underestimated for surface tension force calculations. On the other hand, the phase-field method convects the fluid interface, ensuring correct total energy reduction. Both methods offer advantages and limitations, allowing users to select the most appropriate method for their specific application.

Multiphysics solvers model the intricate interactions among multiple physical processes, including fluid dynamics, thermal conduction, and structural mechanics. These tools are designed to tackle complex problems encompassing various physical phenomena. Among the prominent solvers, PARISDO stands out for its well-established reputation in achieving high precision and computational efficiency when seeking optimal solutions. Its capacity to handle large-scale optimization challenges is further bolstered by an array of advanced functionalities, including robust error handling, adaptive mesh refinement, and parallel processing capabilities. Consequently, we are empowered to confidently engage in demanding simulations, ensuring the reliable generation of accurate results.

Recent developments in manufacturing technology have created opportunities for the fabrication of particles at the nanoscale level, thereby giving rise to a distinct class of fluids termed as nanofluids (Al-Rashed et al., 2017; Rahimi et al., 2018; Menni et al., 2019, 2020; Boursas et al., 2021; Ghachem et al., 2021; Mahammedi et al., 2021; Maouedj et al., 2021; Hammid et al., 2024). These nanofluids are characterized by the presence of two phases, comprising a liquid medium and ultra-fine metallic particles typically measuring in the nanometer range, spanning from 1 to 100 nanometers. Nanofluids have garnered considerable attention due to their unique properties and prospective utility across various domains, including electronics cooling, solar energy systems, and biomedical applications.

The base fluids employed in nanofluid formulations encompass a spectrum of substances, including water, oils, etc. The nanoparticles themselves are fabricated from chemically stable materials such as metals, metal nitrides, metal carbides, oxides, oxide ceramics, and diverse carbon allotropes. These nanoparticles serve the primary function of elevating thermal conductivity within heat transfer fluids. Integrating nanometer-scale particles mitigates issues related to erosion and sedimentation, while concurrently enhancing the thermal performance of these fluids in contexts such as thermal conductivity and liquid-to-vapor heat exchange.

The augmentation of heat transfer within nanofluids, driven primarily by the Brownian motion of nanoparticles, has led to increased particle collision frequencies and subsequently enhanced convective heat transfer. This characteristic renders nanofluids a compelling candidate for improving the efficacy of heat exchange processes within industrial applications. The inception of heat transfer nanofluids can be traced back to Choi's pioneering work at Argonne National Laboratory in 1995. Empirical investigations have unequivocally demonstrated that nanofluids manifest notably improved heat transfer characteristics when contrasted with their base fluid counterparts. Distinctive attributes of nanofluids encompass heightened thermal conductivities, non-linear temperature dependence concerning effective thermal conductivity, modulation in heat transfer behavior within single-phase flow configurations, alterations in nucleate pool boiling heat transfer, and elevated critical heat fluxes in pool boiling scenarios. The gas-liquid interface, influenced by nanoparticles, significantly impacts heat transmission and bubble dynamics in nanoparticle liquids. Understanding and anticipating these impacts is crucial for applications such as environmental cleanup, biomedical engineering, and nanofluid-based heat transfer systems. Accounting for the effects of nanoparticles in simulations and predictions can improve process design and optimization.

A comprehensive understanding and investigation of two-phase flows are paramount in optimizing industrial processes, enhancing operational efficiency, and assuring safety protocols. Moreover, delving into the complexities of two-phase flows has yielded substantial advancements across various scientific domains, including heat transfer, fluid dynamics, and the modeling of multiphase flow phenomena. The ability to comprehend and analyze these flows is thus an indispensable prerequisite for addressing the manifold requirements of industrial applications, including system design, dimensioning, and mitigating safety concerns. A thorough exploration of the influence of surface tension on the behavior of bubble rebound phenomenon necessitates comprehensive analysis (Yan et al., 2019; Mundhra et al., 2023). Many researchers have investigated the alteration of bubble dynamics near solid walls, particularly at varying surface tension levels (Han et al., 2010; Xue et al., 2019). Furthermore, the ramifications of bubble size on the associated recoil motion have been scrutinized, thereby contributing to an enhanced comprehension of this intricate phenomenon (Crha et al., 2023). In conjunction with computational studies (Battistella et al., 2020; Liao et al., 2022;

Seropian et al., 2023), methodically planned experimental inquiries have been executed to provide complementary insights (Mei et al., 2023). In addition to these investigations, the dynamics surrounding spiraling and dissociation in nucleate boiling, as well as the effect of the current load scale requirements on the growth of bubbles, have been subjected to meticulous examination (Zhao et al., 2021; Agarwal et al., 2022; Liao et al., 2022). This multifaceted research endeavor facilitates a comprehensive exploration of the underlying mechanisms governing two-phase flow phenomena, yielding critical insights relevant to various scientific and industrial applications.

This research explores the intricate dynamics of nanofluid-air flow within a vertical channel, driven by a broader interest in understanding two-phase flow phenomena. Such flows, characterized by the simultaneous movement of two distinct phases, are highly relevant in various contexts, including everyday life and industrial applications like fluidized beds, petroleum extraction, and natural systems. The interplay of immiscible fluids and free surfaces in these scenarios is pivotal for numerous industrial processes.

This study investigates nanofluid-air flow in a vertical channel to contribute to understanding these phenomena. It seeks to enhance comprehension of the upwelling dynamics and the influence of fluid properties on bubble growth. Additionally, the research aims to evaluate the thermal efficiency of this system, with potential implications for heat exchangers and thermal management systems. By examining the complex interplay between nanofluids, heat transfer, and bubble dynamics, employing advanced numerical and quantitative visualization techniques, the study sheds light on the transformative potential of nanomaterials in altering fluid properties.

The findings highlight the interconnected relationship between heat and mass transfer phenomena, influenced by distinct nanomaterials, offering valuable insights into enhancing thermal and mass transfer processes in engineering applications. This research fills a crucial gap in understanding the effects of nanoparticle-infused nanofluids on two-phase flow, providing new perspectives in the field.

2. PROBLEM STATEMENT

The model shown in Fig. 1 provides a complete representation of a vertical channel facilitating two-phase, two-dimensional flows involving an air-nanofluid combination. This nanofluid, which is the subject of the study, comprises water as the base fluid.

A comparison between heat flow in the base fluid alone and with nanoparticles is conducted to analyze fluid system behavior and assess the impact of nanoparticle addition on heat transport. The study evaluates the effects of Al_2O_3 and Cu nanoparticles on liquid phase transition and heat flow while also considering the significance of nanoparticle ratios and their influence on heat transmission. This dual approach yields valuable insights into fluid system behavior and

Table 1 An overview of the thermophysical attributes associated with Al₂O₃, Cu, and H₂O.

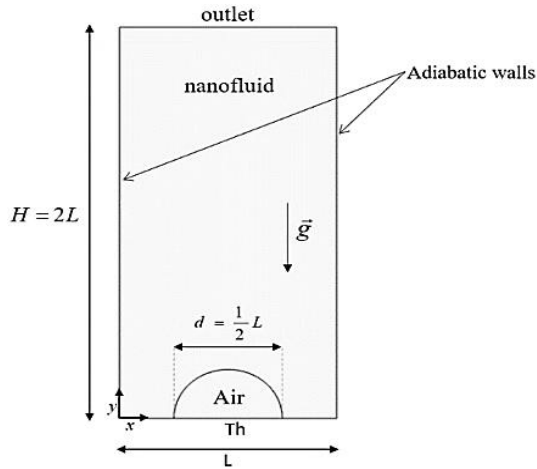| Proprieties	Al₂O₃ (nanoparticles)	Cu (nanoparticles)	H₂O (base fluid)
k (w/m.K)	37.17	401	0.593
ρ (kg/m³)	3960.14	8954	998.2
Cp (J/Kg.K)	761.6	385	4181.8

Fig. 1 Illustration of the physical model's domain

potential technical applications. Table 1 presents a detailed overview of the thermophysical attributes of key constituents, such as water, copper, and aluminum trioxide.

The geometric configuration under consideration assumes a channel of length 'L = 1 m', spanning half the height 'H', with an initial bubble located at the lower wall, characterized by a diameter denoted as 'd'. The lower wall is maintained at an elevated temperature, indicated as T_h, while the upper wall serves as the outlet and is kept at a lower temperature, represented as T_c. Additionally, the adiabatic condition applies to both sides (right and left) of the channel.

3. MODELING AND SIMULATION

Employing fluid dynamics principles, the approach incorporates laminar and incompressible flow assumptions to streamline the complexity of mathematical models and equations. By making these assumptions, the intricate influences of turbulence and density fluctuations that could otherwise introduce significant complications into the analysis and calculations are effectively mitigated.

With the simplifications outlined above, the continuity equation, recognized as the principle of mass conservation, can be mathematically expressed as follows:

$$\nabla . (\rho \vec{u}) = 0 \tag{1}$$

The principle governing the conservation of momentum can be articulated in the following manner:

$$\frac{\partial}{\partial t}(\rho . \vec{u}) + \nabla . (\rho . \vec{u} . \vec{u}) \tag{2}$$
$$= -\nabla p + \nabla(\mu \nabla \vec{u}) + \rho \vec{g} + \vec{F}$$

Where, the vector $\vec{u}$ denotes the velocity of motion for the two fluids. The variable t represents time, and p means static pressure. The surface tension force is defined as $\vec{F}$.

The equation governing the conservation of energy is as follows:

$$\frac{\partial}{\partial t}(\rho . Cp . T) + \nabla . (\rho . u . Cp . T) = -\nabla . (k \nabla T) + q \tag{3}$$

In this context:

ρ represents the combined density of the liquid-vapor mixture.

μ indicates the viscosity of the liquid-vapor mixture.

T stands for temperature.

Cp denotes the specific heat of the mixture.

k signifies the thermal conductivity of the mixture.

q represents the heat source in the energy conservation equation.

This study aims to develop familiarity with the Phase Field Method, which simulates intricate microstructural evolution and phase transitions in multiphase systems, facilitating the investigation of diverse phases within a mixture. Fundamental properties such as ρ, μ, Cp, and k govern the behavior of the mixture under varying conditions, thereby influencing interactions and interfaces.

$$\rho = \rho_l . \alpha_l + \rho_v . \alpha_v \tag{4}$$

$$\mu = \mu_l . \alpha_l + \mu_v . \alpha_v \tag{5}$$

$$Cp = Cp_l . \alpha_l + Cp_v . \alpha_v \tag{6}$$

$$k = k_l . \alpha_l + k_v . \alpha_v \tag{7}$$

$$\alpha_l + \alpha_v = 1 \tag{8}$$

The equations below determine fluid characteristics when nanoparticles are added, incorporating factors such as size, concentration, and interaction with molecules (Pak & Cho, 1998; Hemmati-Sarapardeh et al., 2018; Rudyak & Minakov, 2018). Analyzing these equations helps understand how nanoparticles affect fluid properties, including viscosity, thermal conductivity, and flow behavior.

$$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_{np} \tag{9}$$

$$Cp_{nf} = (1 - \varphi)Cp_f + \varphi Cp_{np} \tag{10}$$

$$\mu_{nf} = \mu_f(1 + 2.5\varphi) \tag{11}$$

$$\frac{k_{nf}}{k_f} = \frac{k_{np} + 2k_f + 2(k_{np} - k_f)\varphi}{k_{np} + 2k_f - (k_{np} - k_f)\varphi} \tag{12}$$
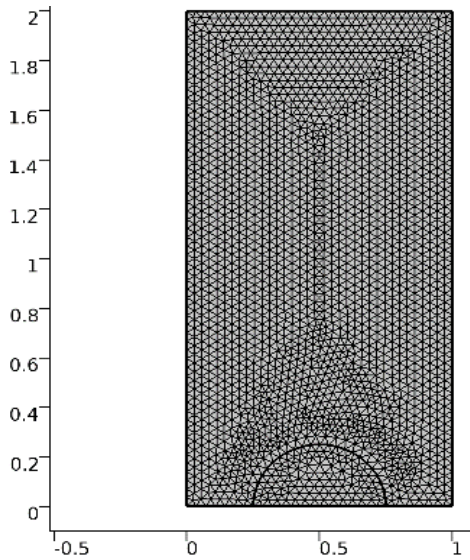
Fig. 2 Mesh domain depiction of the physical model

$$\varphi = \frac{V_s}{V_s + V_f} \tag{13}$$

In this context, the governing equations are formulated as follows:

$$\nabla.\left(\rho_{nf}\vec{u}\right) = 0 \tag{14}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\rho_{nf}.\vec{u}\right) + \nabla.\left(\rho_{nf}.\vec{u}.\vec{u}\right) \\ = -\nabla p + \nabla\left(\mu_{nf}\nabla\vec{u}\right) + \rho_{nf}\vec{g} \\ + \vec{F}\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\rho_{nf}.Cp_{nf}.T\right) + \nabla.\left(\rho_{nf}.u.Cp_{nf}.T\right) \\ = -\nabla.\left(k_{nf}\nabla T\right) + q\end{aligned} \tag{16}$$

This strategic simplification enables a concentration on the foundational principles that govern fluid behavior, thereby enhancing the precision of predictions across a diverse array of applications. To facilitate this process, the selection of the PRISDO solver was informed by its comprehensive and user-friendly features, as well as its well-documented success in analogous industries. In Fig. 2, the utilization of a triangular mesh with an element size of 0.0013 is evident.

4. FINDINGS AND DISCUSSIONS

To ensure the accuracy of the simulation results, a thorough validation process was conducted, involving a meticulous examination of relevant numerical studies available in the literature. This validation effort encompassed multiple comparisons with the findings presented by Hysing et al. (2009) and Verma et al. (2017), utilizing consistent parameters identified within these studies.

Notably, the bubble velocity profiles derived from the current prediction were extensively compared with those observed by Hysing et al. (2009), as visually depicted in Fig. 3 and succinctly outlined in Table 2.
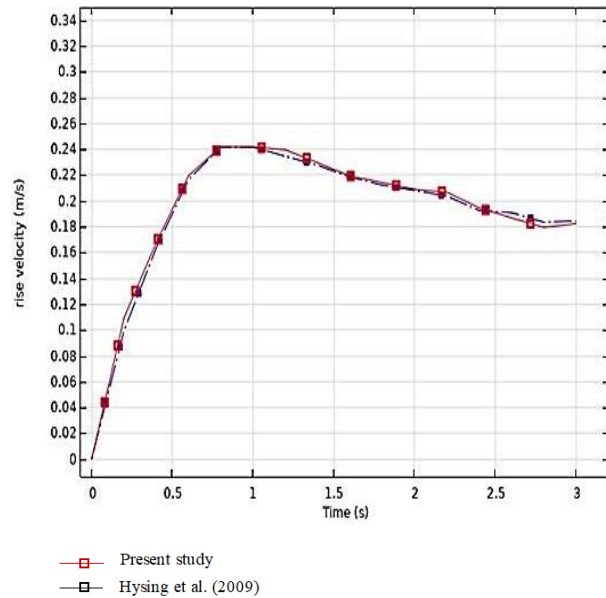

Fig. 3 Assessing bubble rise velocity in relation to the study of Hysing et al. (2009)

Table 2 Validation of the model by comparing maximum rise velocity with literature data

	Present study	Data from Hysing et al. (2009)
Maximum rise velocity (m/s)	0.2416	0.2418
t **(s)**	0.9053	0.9141

Impressively, the numerical simulation yielded bubble velocity results closely aligned with those documented by Hysing et al. (2009), demonstrating a remarkable error margin of less than 5%.

Furthermore, the analysis extended to examining the volume of fluid representation, as illustrated through contours, aligning with the investigation led by Verma et al. (2017), as shown in Fig. 4.

Remarkably, a substantial agreement was observed between the outcomes of both studies. These comprehensive comparisons involved a meticulous scrutiny of the fundamental parameters and variables employed in the respective simulations.

Through meticulous examination of both the parallels and disparities between our findings and those of the referenced authors (Hysing et al., 2009; Verma et al., 2017), a confident affirmation can be made regarding the accuracy and fidelity of the simulation outcomes. This emphasizes the notion that the numerical simulation adeptly captures the dynamics of bubble velocity, substantiating its reliability and robustness.

Figure 5 provides a comprehensive portrayal of the intricate effects stemming from nanoparticle infusion into the liquid medium on heat transfer dynamics. Figure 5(a) meticulously compares the heat flux profiles between air-nanofluid (Cu) and pure air-fluid at φ of 6%. Meanwhile,

Present study

(a) $t = 0\ s$ (b) $t = 1.5\ s$

(c) $t = 3\ s$

Data from Verma et al. (2017)

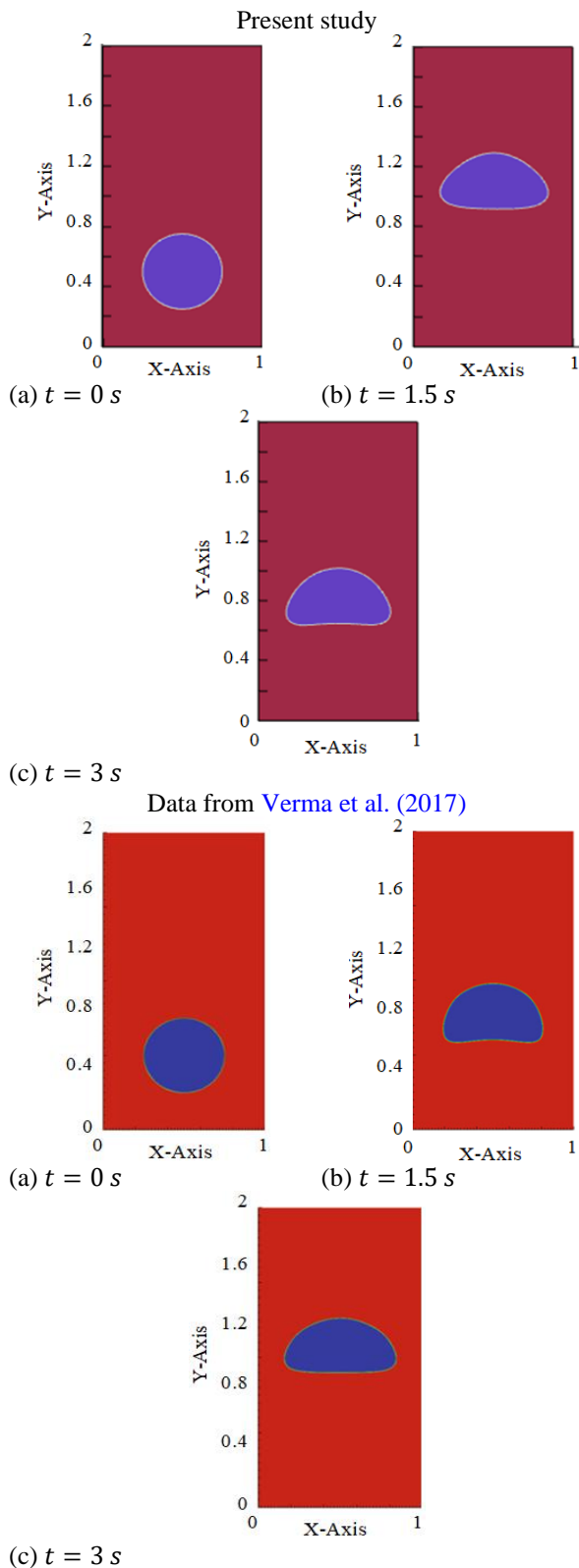(a) $t = 0\ s$ (b) $t = 1.5\ s$

(c) $t = 3\ s$

Fig. 4 Verification of volume of fluid contour accuracy via a cross-reference with the research of Verma et al. (2017)

Figure 5(b) delves deeper into this comparison, focusing on the maximum heat flux attainable under similar conditions.

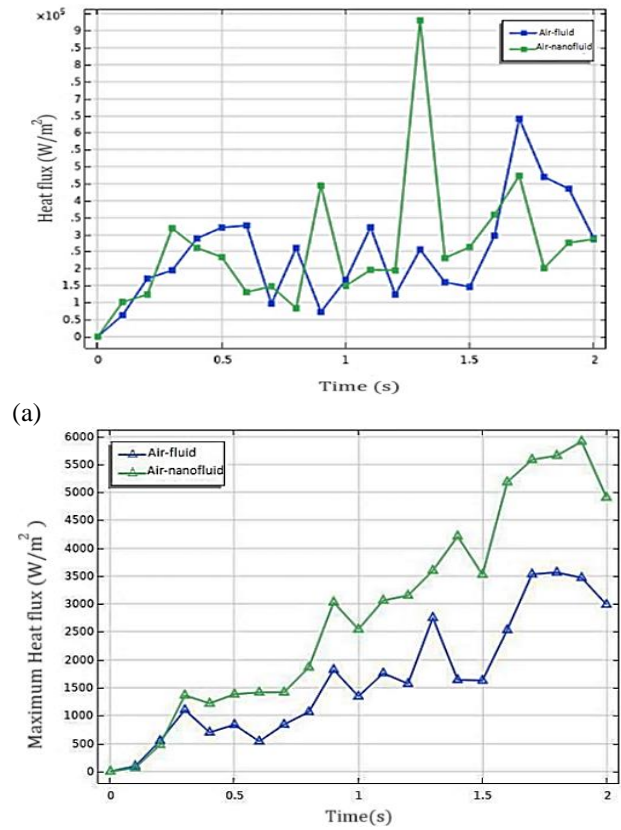The findings from Fig. 5 underscore a significant enhancement in heat transfer coefficients within the

(a)

(b)

Fig. 5 Comparing (a) heat flux and (b) maximum heat flux between air–nanofluid (Cu) and air-fluid at $\varphi = 6\%$

nanofluid system compared to its pure fluid counterpart. This enhancement suggests that the presence of nanoparticles alters the thermal conductivity and convective heat transfer properties of the fluid, leading to improved heat transfer performance. Additionally, the conspicuous increase in the rate of bubble formation within the nanofluid signifies a complex interplay between nanoparticle dispersion and the boiling behavior of the liquid.

In Fig. 6(a), the contours of the volume of fluid unveil the intricate dynamics of the air-fluid system, while Fig. 6(b) extends this analysis to the air-nanofluid (Cu) configuration at $\varphi = 8\%$. The visual representations not only delineate the spatial distribution of the two-phase flow but also illuminate significant alterations in the number, morphology, and relative sizes of bubbles within the channel. These observations resonate with the findings presented in Fig. 5, emphasizing the profound impact of nanoparticle infusion on bubble dynamics and phase transition phenomena. The introduction of nanoparticles initiates a compelling transformation of the liquid phase towards a gaseous state, as evidenced by the evolving bubble structures and distribution patterns observed in the visualizations. This phenomenon underscores the intricate interplay between nanomaterial properties and heat transfer phenomena, highlighting the potential for enhancing heat transfer efficiency through nanoparticle manipulation. Furthermore, forthcoming simulations will conduct a comprehensive evaluation of
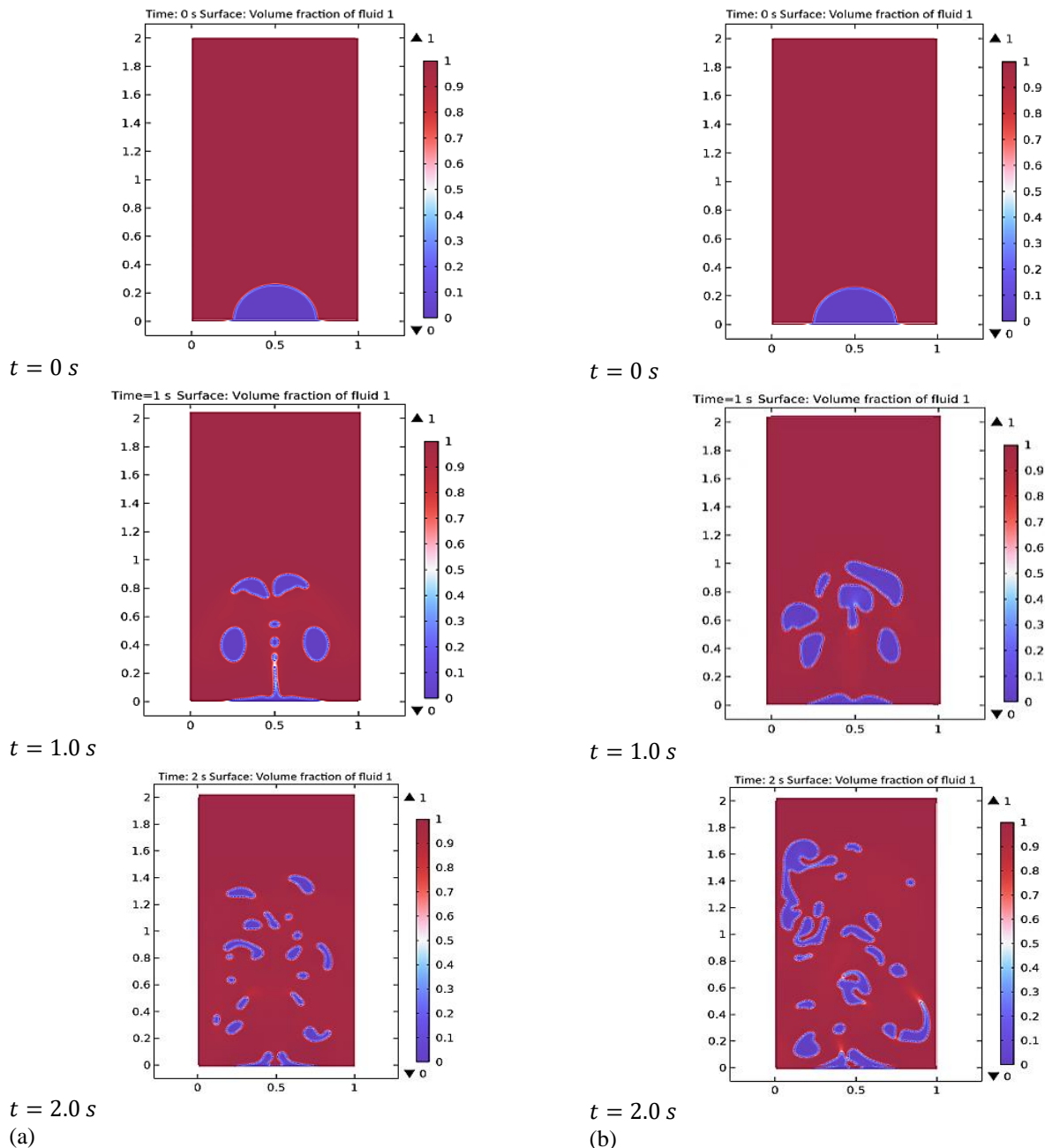
$t = 0\ s$

$t = 1.0\ s$

$t = 2.0\ s$

(a)

$t = 0\ s$

$t = 1.0\ s$

$t = 2.0\ s$

(b)

Fig. 6 Contours of volume of fluid for (a) air-fluid and (b) air-nanofluid (Cu) at $\varphi = 8\%$

the diverse effects of nanoparticles on the overall efficiency and effectiveness of the heat transfer process.

This endeavor aims to unravel the complex interplay between nanomaterials and heat transfer phenomena, offering valuable insights to enhance heat transfer processes across diverse engineering applications.

Simulations were conducted to compare the thermal performance of two types of nanoparticles, namely Cu and Al_2O_3, using identical concentration ratios and maintaining consistent channel geometry, initial conditions, and boundary conditions. Figure 7(a) provides a comparison of heat flux profiles between air-nanofluid (Cu) and air-nanofluid (Al_2O_3) at $\varphi = 6\%$, while Fig. 7(b) compares maximum heat flux profiles under the same conditions. Analysis of the plots in Fig. 7 reveals that the choice of nanoparticles significantly influences thermal transfer efficiency and maximum

values.

Cu nanoparticles, characterized by their elevated ρ and exceptional k, substantially enhance heat transfer efficiency by accelerating the dissipation of heat. Additionally, their smaller size contributes to enhanced heat transfer capabilities due to an increased surface area-to-volume ratio, facilitating improved dispersion within the material and ensuring uniform thermal dissipation throughout the system.

In contrast, while Al_2O_3 nanoparticles also exhibit enhanced dispersion and uniform thermal dissipation, their thermal conductivity is generally lower than that of Cu nanoparticles, resulting in comparatively lower heat transfer efficiency.

Overall, Cu nanoparticles emerge as the preferred choice for applications requiring superior thermal transfer and maximum values due to their combined
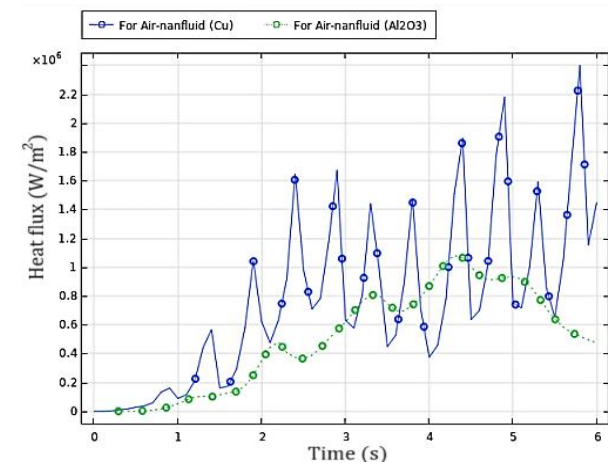
(a)
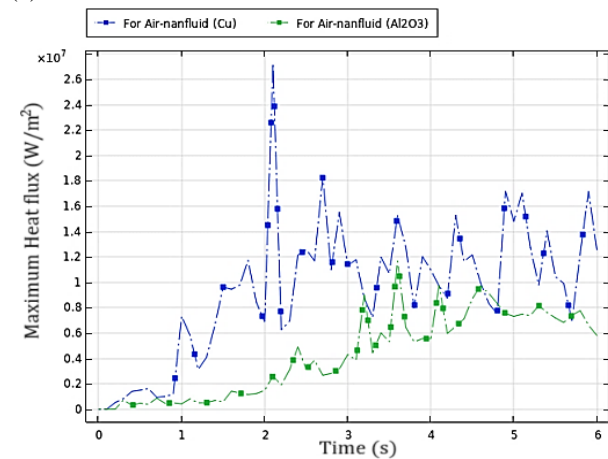
(b)

Fig. 7 Comparing (a) heat flux and (b) maximum heat flux between air–nanofluid (Cu) and air-nanofluid (Al₂O₃) at φ = 6%

Fig. 8 Velocity magnitude comparison between air-nanofluid (Cu) and air-nanofluid (Al₂O₃) at φ = 6%

attributes of high ρ, exceptional k, and enhanced dispersion characteristics.

Figure 8 presents a comparison of velocity magnitudes between air-nanofluid (Cu) and air-nanofluid (Al₂O₃) at φ of 6%. The figure reveals a noticeable discrepancy in bubble velocities between the Cu and Al₂O₃ nanofluids. Specifically, Cu nanofluid exhibits significantly higher bubble velocities compared to its Al₂O₃ counterpart, indicating the superior ability of Cu nanoparticles to enhance bubble motion within the fluid.

This variation underscores the substantial impact of nanofluid composition on fluid dynamics and heat transfer performance. By promoting more significant bubble motion, Cu nanoparticles contribute to enhanced mixing and convective heat transfer within the fluid, highlighting the importance of careful nanoparticle material selection for optimizing heat transfer processes in various engineering applications.

The influence of nanoparticle types goes beyond simple heat transport, profoundly affecting global mass transfer processes. This influence is illustrated by transformations in the physical state of the liquid and the morphology and dynamics of bubbles within the channel.

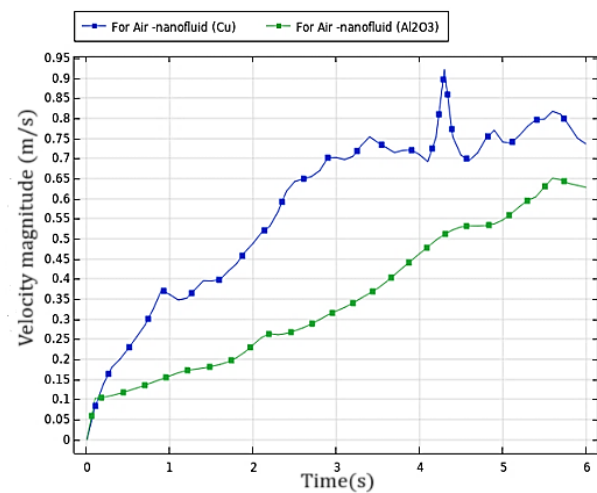These transformative effects are clearly depicted in

the comprehensive visual representations of Fig. 9. Here, field variables such as volume of fluid contours (Fig. 9a), pressure distributions (Fig. 9b), streamlines (Fig. 9c), and isotherms (Fig. 9d) collectively provide invaluable insights into the intricate fluid distribution within the vertical channel. These visualizations not only corroborate earlier conclusions but also offer deeper understanding into the complex interplay between nanoparticle types and fluid dynamics.

Furthermore, the observed modifications in the liquid state and bubble behavior provide significant insights into the overall influence of various types of nanoparticle types on fluid dynamics at large. This multi-faceted exploration enables us to better understand the impact of nanomaterials on fluid behavior and emphasizes the interconnected nature of heat and mass transfer phenomena within the system.

This information is essential for advancing the design and optimization of nanofluid-based systems in various engineering disciplines.

5. CONCLUSION

Rigorous validation processes were conducted by comparing simulation results with data from relevant prior studies, mainly focusing on bubble velocity. The comparison yielded highly encouraging results, with a consistency within an error margin of less than 5%, improving the accuracy and reliability of our simulation results.

Introducing nanoparticles into nanofluids resulted in significant improvements in heat transfer coefficients and an increase in bubble formation rates. These alterations in fluid properties were visually represented, emphasizing the transformative potential of nanomaterials in enhancing heat transfer efficiency.

The choice of nanoparticle type was critical in determining thermal transfer and maximum values, with copper nanoparticles emerging as the preferred option for superior thermal transfer and maximum performance.
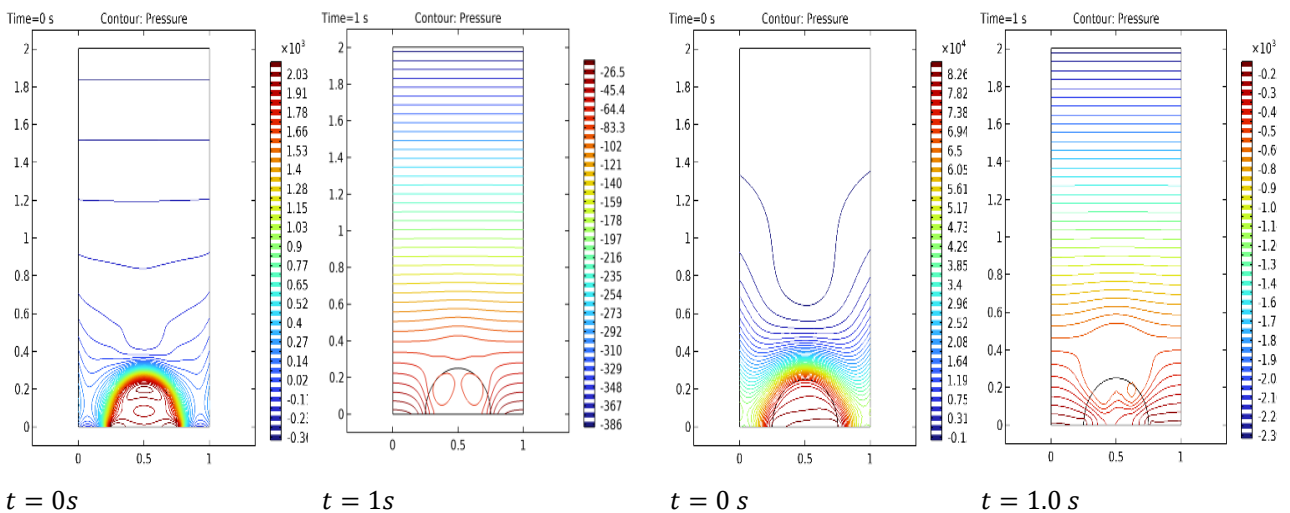
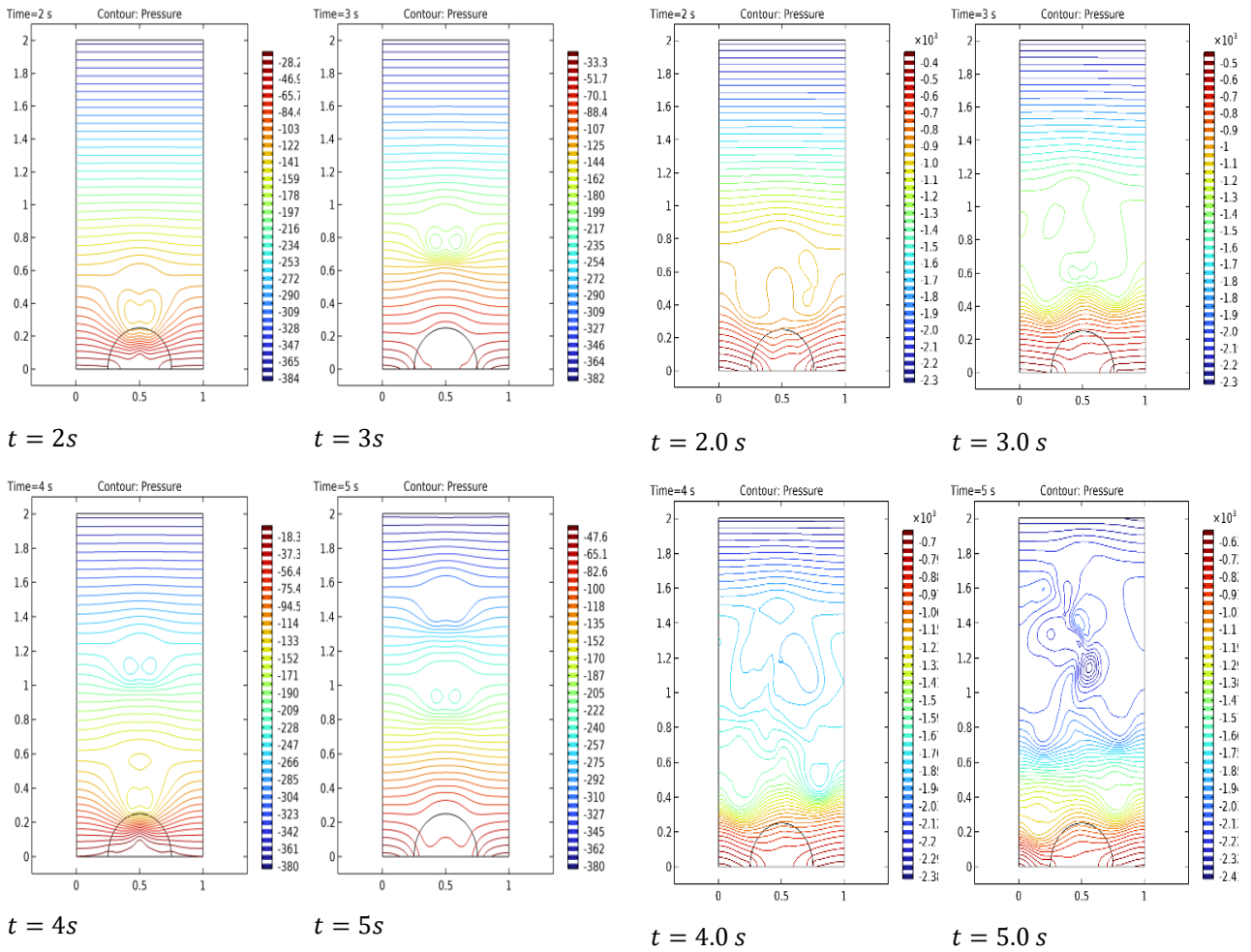**Air-nanofluid (Al₂O₃)**   **Air-nanofluid (Cu)**
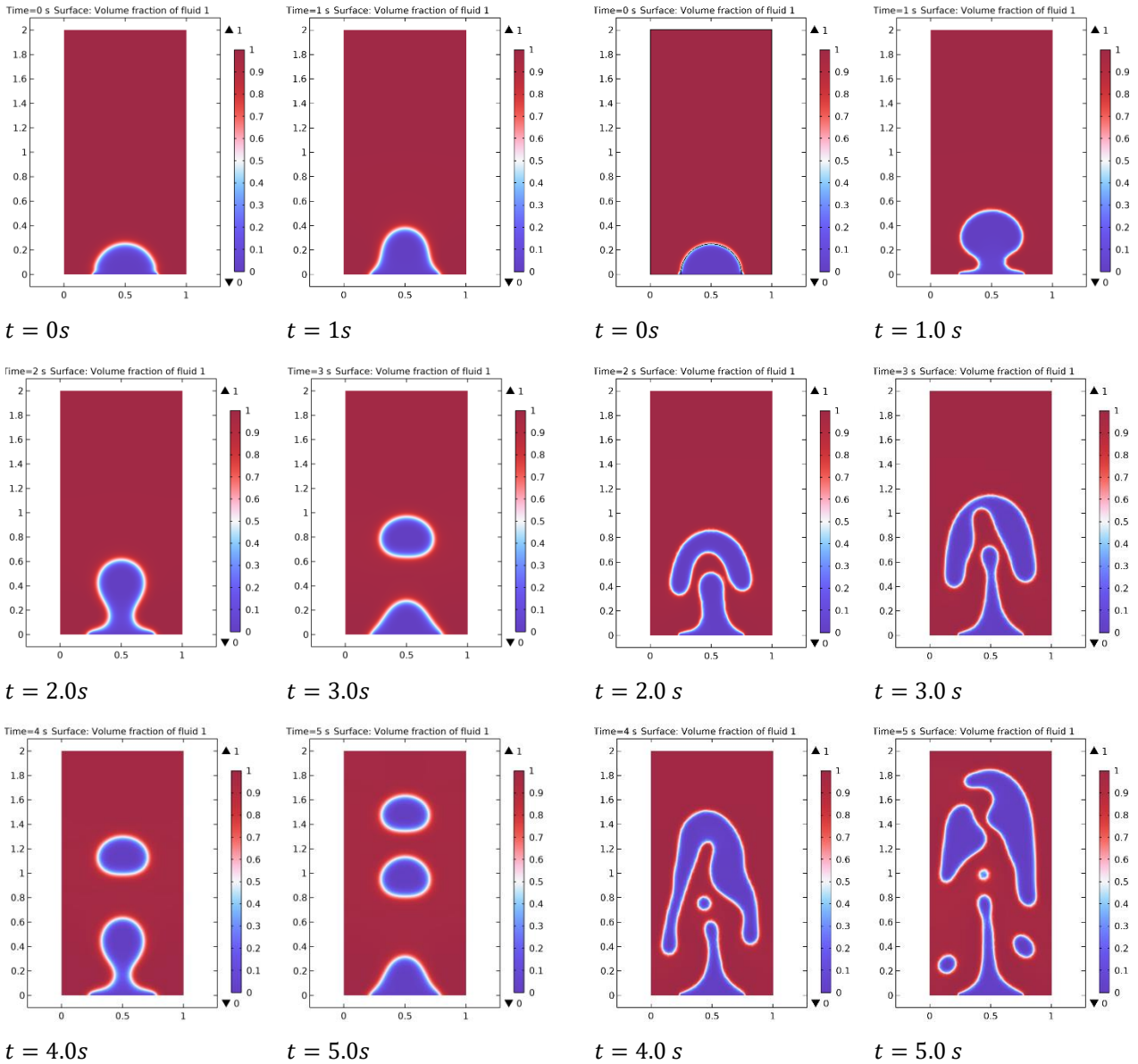
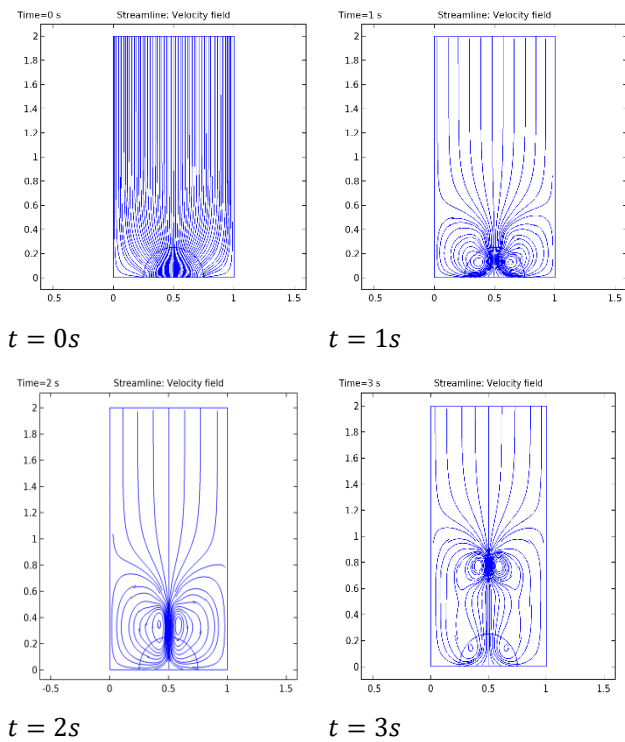(a) Volume of fluid contours

Air-nanofluid (Al₂O₃)   **Air-nanofluid (Cu)**
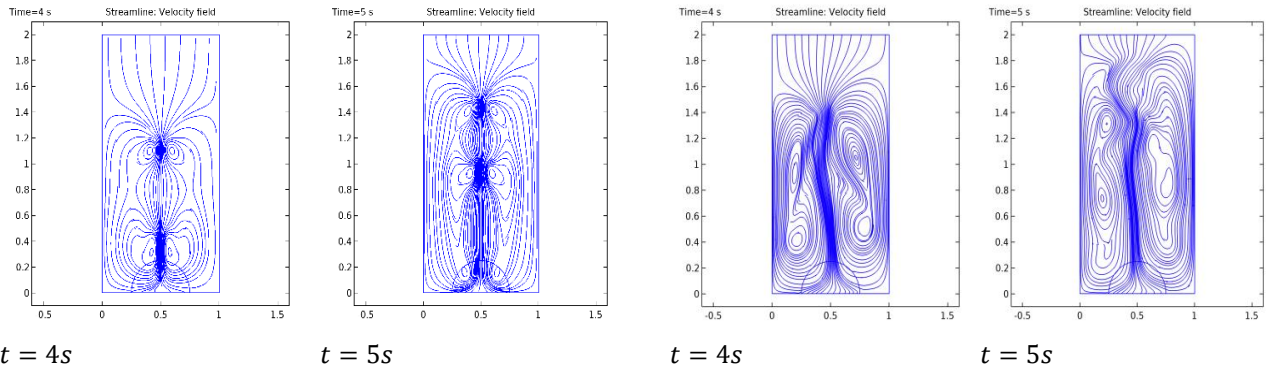
(b) Pressure contours

Air-nanofluid (Al₂O₃)   **Air-nanofluid (Cu)**

$t = 4s$ $t = 5s$ $t = 4s$ $t = 5s$
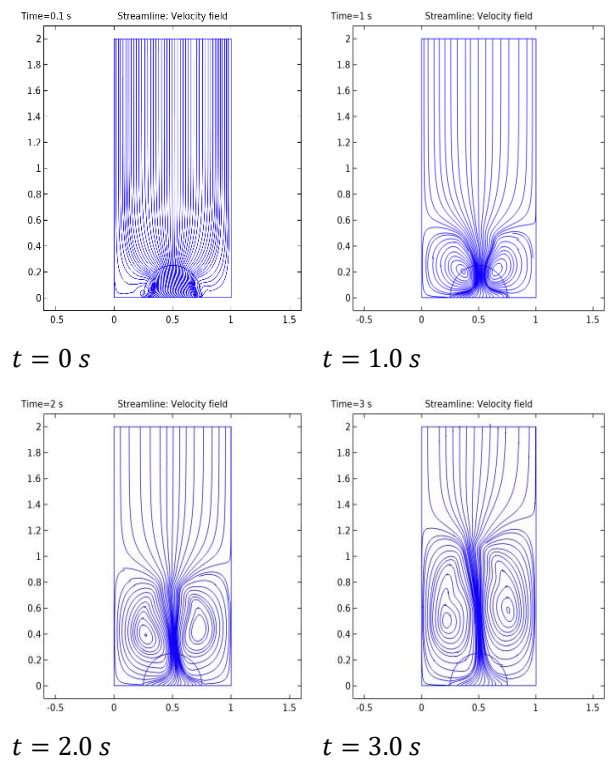
(c) Streamlines

Air-nanofluid (Al₂O₃)   **Air-nanofluid (Cu)**

$t = 0\ s$ $t = 1.0\ s$ $t = 0\ s$ $t = 1.0\ s$

$t = 2.0\ s$ $t = 3.0\ s$ $t = 2.0\ s$ $t = 3.0\ s$

$t = 4s$ $t = 5.0\ s$ $t = 4s$ $t = 5s$
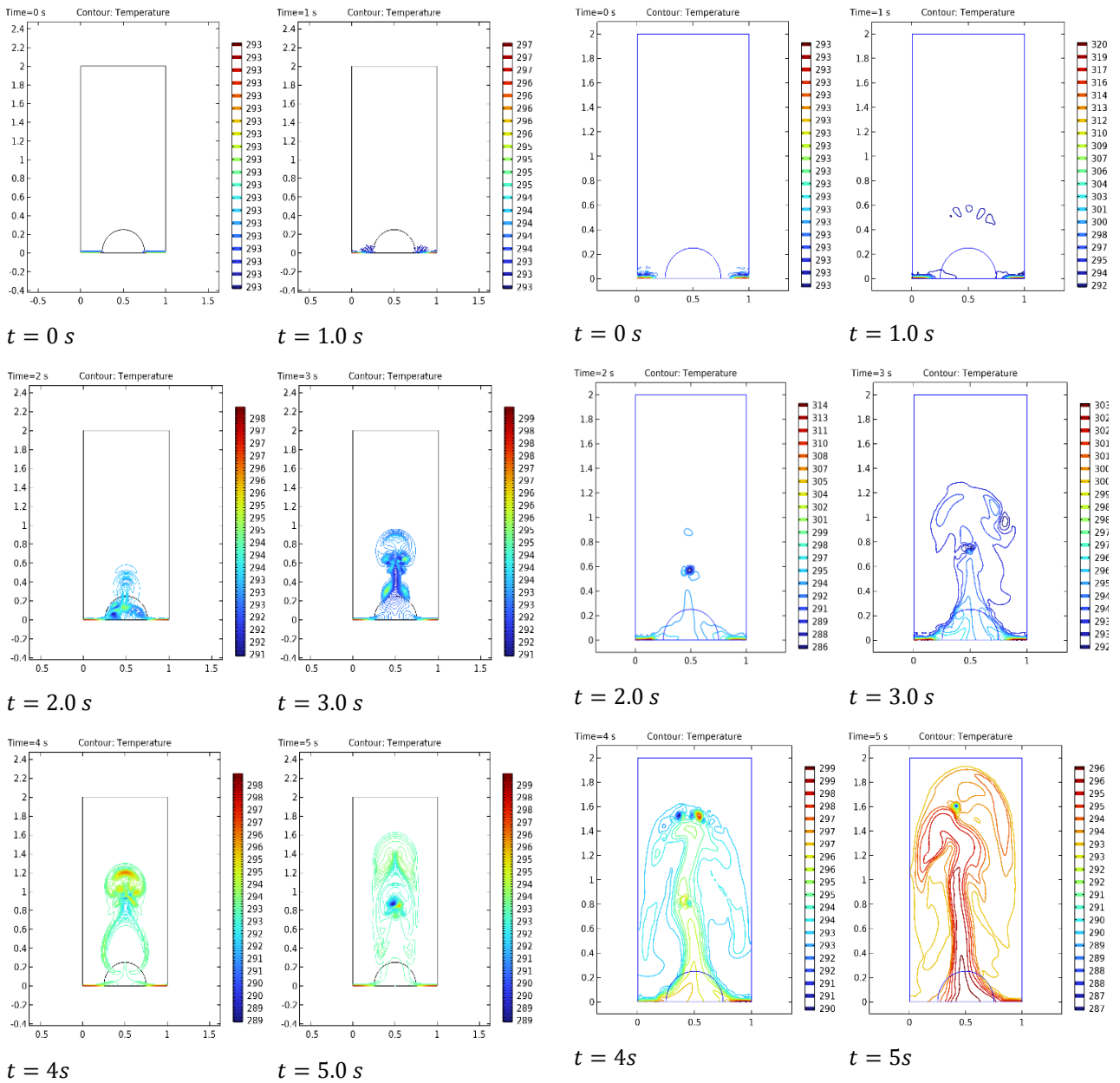
(d) Isotherms

Fig. 9 Visualization of volume of fluid contours, pressure contours, streamlines and isotherms for both cases studied at $\varphi = 0.06$

Nanoparticle types were found to intricately impact mass transfer within the system, leading to changes in the physical state of the liquid, shifts in bubble morphology and behavior, and variations in pressure distribution within the channel. These findings underscored the interconnected relationship between heat and mass transfer phenomena influenced by distinct nanomaterials.

This research provides valuable information on the interaction between nanoparticle-infused nanofluids and heat transfer, bubble dynamics and mass transfer. It emphasizes the potential of nanoparticles to enhance these phenomena and stresses the importance of carefully selecting nanoparticles for specific applications. These findings lay the foundation for future research and innovative strategies for strengthening thermal and mass transfer processes in various engineering applications.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

AUTHORS CONTRIBUTION

I. Aliouane provided conceptualization, framing the overarching ideas and objectives, and conducted simulation of the problem. **E. Benachour** meticulously designed the methodology, providing the structured framework for data collection and analysis. **M. Hasnat** led the numerical investigation, delving into the intricacies of the mathematical aspects of the study. **M.A. Almajed** undertook the responsibility of numerical validation, ensuring the accuracy and reliability of the computational results. **M. S. Alhassan**, with a keen eye for detail, conducted a thorough review and contributed to the editing process, enhancing the overall quality of the manuscript. Lastly, **Y. Menni** took charge of the preparation and supervision, guiding the team through the research process.

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