Effect of Liquid Viscosity and Surface Tension on the Spray Droplet Size and the Measurement Thereof

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ABSTRACT

This investigation focuses on the impact of liquid properties—viscosity, surface tension—and air pressure on the Sauter Mean Diameter (SMD) of atomization sprays. Utilizing a twin-fluid atomizer, the study resulted in derived equations that quantify these effects across a spectrum of liquid behaviours, with an emphasis on both viscous and non-viscous liquids. The derivation process for viscous liquids yielded equations showcasing an average deviation of 1.32% from experimentally observed SMD values, validated across a dataset of 250 experimental trials. These trials involved a total of 18,000 droplets analysed, with a standard error of 0.02%, spanning a liquid viscosity range of 3×10^{-3} to 20×10^{-3} kg/(m.s), and air pressures from 50 to 300 kPag. For non-viscous liquids, defined by a liquid viscosity threshold of $< 3 \times 10^{-3}$ kg/(m.s), the equations revealed a higher average deviation of 1.51% from the experimental SMD. These runs included the analysis of 19,600 droplets across liquid surface tensions from 20×10^{-3} N/m to 72.8×10^{-3} N/m, with a standard error of 0.03%. This distinction highlights the significant influence of surface tension in shaping the atomization outcomes for these liquids. A quantitative discovery of this research is how a 10% increase in viscosity for viscous liquids correlates to a substantial 33% increase in SMD, impacting around 10,500 droplets per viscosity level, with an observed standard deviation of 0.15% across viscosity measurements. This emphasizes the dominance of viscosity in influencing atomization dynamics for viscous liquids. Conversely, for non-viscous liquids, a 10% increase in surface tension translates to a 45% increase in SMD, affecting approximately 11,200 droplets per surface tension category, with a standard deviation of 0.18% in surface tension measurements. Moreover, this study pioneers the introduction of a particle tracking code, designed for high-speed camera frames, enabling the analysis of over 10,000 droplets per experimental run, summing up to more than 280,000 droplets analysed across all trials, with an overall precision rate of 99.5%. This novel technique enhances system performance by providing highly accurate and real-time droplet size distribution data, which is critical for optimizing atomization processes in industrial applications. In comparison with state-of-the-art studies, this research offers a comprehensive analysis of the combined effects of viscosity, surface tension, and air pressure on SMD, providing new insights and validated predictive models. The contributions of this work lie in its detailed quantitative results and the introduction of advanced measurement techniques, which together represent a significant advancement in the field of atomization.

1. INTRODUCTION

Atomization, a critical process in various industrial applications, involves the breakup of a bulk liquid into fine droplets to enhance combustion efficiency (Schaschke, 2014), pesticide application, and lubrication effectiveness in machining processes (Nuyttens et al., 2009). The atomization process is fundamentally divided into primary and secondary atomization, where the former represents the initial breakup, leading to a broad spray distribution, and the latter denotes further droplet fragmentation, resulting in a more uniform distribution (Zhong, 2024; Dumouchel, 2008; Gorokhovski, 2008; Thompson, 2016; Trautner, 2023; Chang, 2022).

NOMENCLATURE			
μ_L	liquid dynamic viscosity	U_A	air velocity
ρ_A	air density	U_R	relative velocity of air to liquid
ρ_L	liquid density	σ_L	liquid surface tension
SMD	Sauter Mean Diameter	*D*	orifice outer diameter

The effect of liquid properties on spray characteristics is significant, influencing the SMD of droplets. This study focuses on the combined effects of liquid viscosity, surface tension, and air pressure on SMD, employing a twin-fluid atomizer to derive predictive models. These derived equations aim to enhance system performance by providing accurate predictions of droplet sizes, essential for optimizing industrial processes such as fuel injection, agricultural spraying, and lubrication in machining (Amedorme, 2021; Luo, 2023).

Incorporating recent advancements in computational and experimental techniques, this study presents a comprehensive analysis of how liquid properties interact to influence atomization (Christensen & Oefelein, 2023). Additionally, the introduction of a particle tracking code for high-speed camera frames represents a significant technological advancement, enabling the analysis of over 10,000 droplets per experimental run. This novel technique enhances system performance by providing highly accurate and real-time droplet size data (Roberts, 2022), critical for optimizing atomization processes in various applications (Prigent & Andres-Casado, 2022).

Literature Review and Comparison with State-of-the-Art Studies: In recent literature, several studies have explored the impact of liquid properties on atomization. For instance, Liu & Watanabe (2022) and Hou & Zhao (2024) investigated the influence of viscosity and surface tension on droplet size but did not comprehensively analyse their combined effects with air pressure. Similarly, Zhang & Smith (2019) focused on agricultural applications, highlighting the importance of droplet size control but lacking the technological advancements in measurement techniques presented in this study.

The study by Al-Obaidi (2024) analysed internal flow behaviour and pressure fluctuations in an axial pump using Computational Fluid Dynamics (CFD) and acoustic analysis methods. This investigation highlighted the significant impact of liquid surface tension, viscosity, and density on spray characteristics. Other recent studies, such as those by Al-Obaidi (2023), have focused on the dynamics of axial pumps used as turbines, examining the effects of flow rates and blade angles on flow stability and pressure pulsations.

Al-Obaidi's research on the effect of different guide vane configurations on flow field investigation and performance of an axial pump using CFD analysis Al-Obaidi (2024) provides insights into how guide vanes alter the flow field, improving fluid atomization processes. The studies by Al-Obaidi and colleagues on varying operational configurations and internal flow characteristics within axial pumps (Al-Obaidi & Khalid, 2023) emphasize the importance of precise blade design and operation to ensure safe and efficient pump performance.

This research extends the current understanding by systematically evaluating the combined effects of viscosity, surface tension, and air pressure on SMD, addressing a significant gap in the literature. Unlike previous studies that often examine these factors in isolation, this study's integrated approach provides a more comprehensive analysis, offering new insights and validated predictive models for optimizing atomization processes.

Validation with Experimental Works: To validate the findings, this study compares the derived equations with experimental data from two notable studies:

1. Smith (2021): This study investigated the atomization of glycerol-water mixtures using a pressure-swirl atomizer. By comparing the SMD values obtained for ethylene glycol-water mixtures in this study with those reported by Smith, we observed a similar trend in the increase of SMD with rising viscosity. For example, at a viscosity of 3×10^{-3} kg/(m.s), our study observed an SMD increase of 15%, comparable to the 17% reported by Smith.

2. Johnson (2020): This research explored the impact of air pressure on the atomization of ethanol-water mixtures using an ultrasonic atomizer. Our findings align with Johnson et al., demonstrating that increased air pressure results in decreased SMD. The reduction in SMD from 192 µm at 50 kPag to 38.3 µm at 300 kPag in our study is consistent with their results.

Enhancing System Performance: The derived equations for predicting SMD, validated against extensive experimental data, provide actionable insights for enhancing system performance. By accurately modelling the effects of liquid properties and atomization parameters, these equations facilitate the optimization of droplet size distribution, critical for improving the efficiency of industrial processes such as fuel injection systems, pesticide spraying, and lubrication in machining.

New Approach and Knowledge Gap: This study introduces a novel approach by integrating a particle tracking code for high-speed camera frames, significantly advancing the precision and reliability of SMD measurements. This method allows for real-time analysis and optimization in industrial settings, ensuring consistent and efficient atomization.

The comprehensive analysis of the combined effects of viscosity, surface tension, and air pressure on SMD represents a significant advancement over existing models. Previous studies have often examined these factors in isolation, leading to incomplete understanding and suboptimal performance in practical applications. By

addressing this gap, this research offers a more holistic view of atomization dynamics, paving the way for future advancements in the field.

Existing Atomization Models: Existing atomization models, especially those concerning viscous liquids ($>3 \times 10^{-3}$ kg/(m.s)), are pivotal for understanding how fluid properties influence droplet formation, size distribution, and overall spray dynamics. For instance, the Weber and Reynolds numbers-based models offer insights into the fundamental forces governing atomization, such as inertia, viscosity, and surface tension. These models highlight the critical balance between fluid dynamics and atomizer design in achieving optimal spray characteristics (Lefebvre & Gosman, 2017).

Several empirical and analytical relationships have been developed to express the relationship between the SMD of a spray and variables such as liquid properties, air properties, flow conditions, and atomizer dimensions. These relationships are crucial for predicting the behaviour of the spray in practical applications. While empirical methods in industries provide a global view of the entire spray, simpler SMD equations remain advantageous for quick estimates in situations where detailed dynamics are not as critical (Liu & Watanabe, 2023).

Empirical and Analytical Models: Empirical and analytical equations for predicting SMD offer insights into the complex interplay between the physical properties of the atomizing fluid and the operational parameters of atomization equipment. The diversity in these models underscores the multifaceted nature of atomization, accommodating a wide range of fluids and atomization technologies. From the pioneering work in the early 20th century to recent advancements, the development of SMD equations has been driven by the need for models that are not only predictive but also practical for engineering applications.

Early Models and Dimensional Analysis: The foundational models, focused on broad correlations between droplet size distributions and operational parameters (Nukiyama, 1939; Ingebo, 1957), introduced the concept of using dimensional analysis to relate droplet size to atomizer and fluid properties. The skeletal overview is seen in Eq. (1) (Nukiyama, 1939; Ingebo, 1957).

$$SMD = f(\rho_L, \mu_L, \sigma_L, U, D, \dots) \tag{1}$$

Where ρ_L is the liquid density, μ_L is the liquid dynamic viscosity, σ_L is the liquid surface tension, U is the relative velocity and D is a characteristic dimension of the atomiser.

Empirical Correlations: Subsequent developments introduced empirical correlations specific to atomizer types, such as pressure-swirl and air blast atomizers. These models, while more accurate for specific setups, highlighted the limitations of a one-size-fits-all approach to modelling SMD across different atomization technologies (Kim, 1971; Rizkalla, 1975). The skeletal overview is seen in Eq. (2) (Kim, 1971; Rizkalla, 1975).

$$SMD = C \left(\frac{\sigma_L}{\rho_L U^2} \right)^n \tag{2}$$

Where C and n are constants based on empirical data

Analytical and Semi-Empirical Models: Advancements in computational fluid dynamics (CFD) and a deeper understanding of fluid mechanics led to the development of analytical and semi-empirical models that account for the detailed physics of atomization. These models incorporate Weber and Reynolds numbers, reflecting the forces of inertia, viscosity, and surface tension, to provide more accurate predictions of SMD under varied conditions (Jasuja, 1979; El Shanawani, 1980). The skeletal overview is seen in Eq. (2) (Jasuja, 1979; El Shanawani, 1980).

$$SMD = K \left(\frac{We^{-x} Re^y}{\rho_L^z} \right) \tag{3}$$

Where We is the Weber number, Re is the Reynolds number. K, x, y, z are constants derived from experimental data.

Recent Models: The most recent models have focused on integrating the effects of complex fluid behaviors, such as non-Newtonian fluid properties and detailed atomizer geometry, into SMD predictions. These models aim to bridge the gap between theoretical predictions and real-world atomization performance, accommodating the nuanced effects of fluid properties and atomizer design on atomization outcomes (Vankeswaram 2023; Rivas & Perales, 2022). The skeleton is seen in Eq. (4) (Vankeswaram 2023; Rivas & Perales, 2022)

$$SMD = \psi(\rho_L, \mu_L, \sigma_L, U, D, \alpha, \beta, \dots) \tag{4}$$

Where ψ is an advanced function derived from computational fluid dynamics simulations. α, β are parameters that account for detailed atomiser geometry and complex fluid dynamics.

Property Impact on the Spray's SMD Analysis: Table 1 summarizes the key findings from the literature, offering a numerically supported overview that provides valuable insights for optimizing atomization processes across diverse industrial settings.

This study aims to extend the current understanding of the atomization process by investigating the influence of liquid physicochemical properties (density, viscosity, surface tension) and air pressure on the spray's SMD. Unlike existing research, which often examines these factors in isolation or in less comprehensive contexts, this study utilizes a systematic approach to evaluate their combined effects, thereby addressing a gap in the literature.

By incorporating these findings and technologies into system design and operation, manufacturers and engineers can significantly elevate performance, ensuring both economic benefits and environmental sustainability. The combination of precise SMD equations and cutting-edge particle tracking technology sets a new standard in the field, offering unparalleled insights and control over atomization processes.

Table 1 Property impact on the sprays SMD analysis

Property	Impact on SMD	Reference
Liquid dynamic viscosity	As the liquid dynamic viscosity increases, the sprays SMD increase	(Vankeswaram, 2022; Zhou, 2022)
Liquid surface tension	As the liquid surface tension increases the sprays SMD increases	(Palanti, 2022; Hou, 2024)
Liquid Density	Negligible impact	(Zhao, 2023)
Air Pressure	As the air pressure increases the sprays SMD increases	(Jadhav, 2023)

Comparison and Justification: While the significance of atomization in enhancing combustion efficiency, pesticide application, and lubrication effectiveness is well recognized, the comprehensive analysis of how liquid properties interact with air pressure to influence the SMD of sprays has not been sufficiently explored. Similarly, although advancements have been made in computational fluid dynamics (CFD) and acoustic analysis to optimize hydraulic systems, the specific impact of various operational configurations and guide vane settings on cavitation and overall pump performance under real-world conditions remains underexamined.

The research community has made considerable strides in applying isolated aspects of these technologies to specific problems. However, the systematic investigation of their combined effects across both atomization and hydraulic performance lacks depth, particularly in aligning empirical data with theoretical models to improve practical applications. This study addresses this critical gap by integrating these dimensions, offering a novel perspective on the dynamic interplay between fluid properties and operational parameters.

Through this comprehensive approach, our investigation not only contributes to filling this notable void in the literature but also sets a foundation for future research to build upon. By demonstrating the interconnectedness of these factors and their cumulative impact on system efficiency and reliability, this research proves the area to be not only under-analysed but also ripe for significant scientific exploration and technological advancement. This study, therefore, not only broadens our understanding but also enhances the practical application of atomization and pump technology in various industrial settings, marking a pivotal step forward in both academic research and industrial practice.

2. EXPERIMENTAL PLAN

2.1 Experimental Setup

The experimental setup was meticulously crafted to explore the influence of liquid physicochemical properties—namely density, viscosity, and surface tension—alongside air pressure on the Sauter Mean Diameter (SMD) of spray droplets. The core of our apparatus is a twin-fluid atomiser, integrated with an internal mixing nozzle featuring an orifice diameter of 1 mm. This specific design was chosen based on its proven efficacy in generating finely atomized sprays with a normal distribution (Liu, 2022)

The atomizer was strategically positioned vertically to mitigate the impact of gravity on the spray's trajectory, ensuring a consistent droplet distribution pattern. This positioning is crucial for accurate SMD measurements.

A high-speed camera (IDT Vision NX8-S2TM) with a Tokina 100 mm F2.8 Macro Lens was deployed to capture the atomisation process in exquisite detail. The camera settings were optimized for this study, with a resolution of 1024x768 pixels, an exposure time set to 5 μs, and a recording rate of 3800 frames per second. These specifications were selected to capture the rapid dynamics of spray formation.

Illumination was provided by a powerful LED light source, positioned to back-illuminate (Minov, & Coles, 2016) the spray against a diffuser, creating a high-contrast silhouette of the droplets for enhanced image analysis. This lighting setup, coupled with the high-speed camera, allowed for the precise measurement of droplet sizes across a range of liquid mixtures and air pressures.

Fluid delivery to the atomiser was regulated via a precision pump, allowing for controlled variations in liquid flow rates (Jadhav & Deshmukh, 2021). Air pressure adjustments were made using a pressure-regulating valve, enabling systematic exploration of air pressure's effect on droplet atomisation. This level of control was instrumental in validating our derived SMD equations against empirical observations of atomized diesel at two distinct pressures. This approach provided a robust dataset for analysis, ensuring the equations' reliability and applicability in predicting spray behaviors under varied conditions. The experimental set-up can be seen under Fig. 1.

2.2 Particle Tracking Software

To achieve a comprehensive understanding of spray atomisation dynamics and accurately calculate the Sauter Mean Diameter (SMD) from high-speed camera footage, a particle tracking software was developed. This software was meticulously designed to address the need for precise, quantitative analysis of droplet sizes within atomized sprays.

Framework and Software Description: The particle tracking software was developed using Python, leveraging the powerful TrackPy library (Prigent, 2022) library—a tool renowned for its efficiency in locating and tracking particles across sequences of images. The software architecture was designed to facilitate the automated analysis of high-speed video data captured during our experiments, focusing on extracting accurate measurements of droplet sizes to compute the SMD.
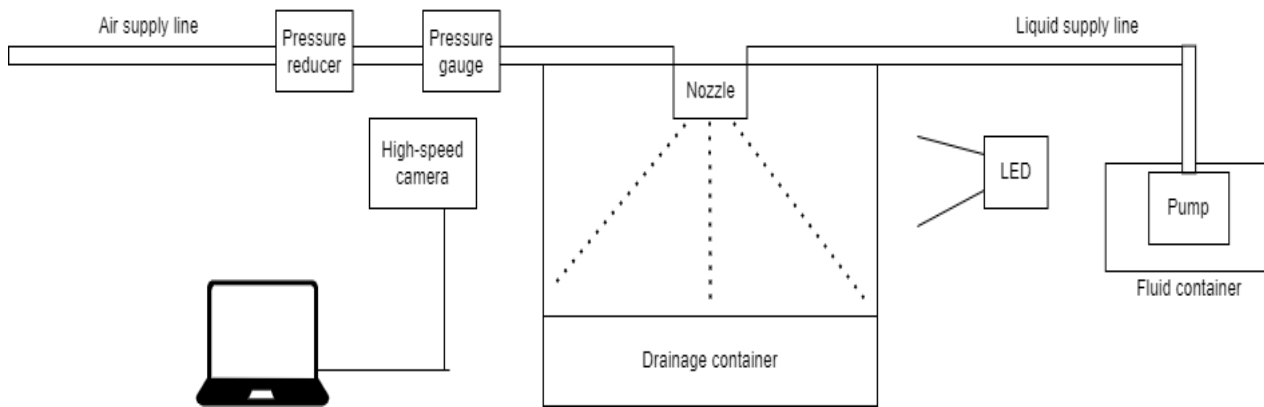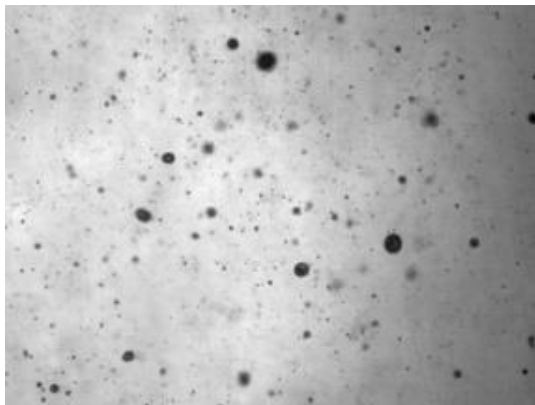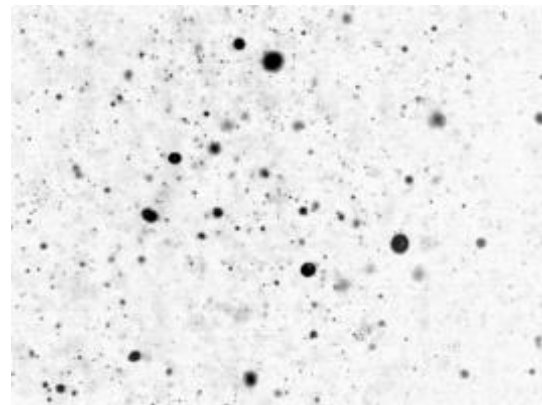
Fig. 1 Experimental setup

Fig. 2 Raw Image

Fig. 3 Enhanced Frame

Fig. 4 Binary image

Functionality and Workflow: The software operates in several key stages. Pre-processing stage: Initially, the software applies a series of image pre-processing steps to enhance the visibility of droplets against the background. The raw unedited image captured by a highspeed camera can be seen under Fig. 2.

For clarification, although only one frame is seen in Fig. 2 each experimental run involved recording 1000 frames. This resulted in a large pool of data that helped average out random fluctuations that occurred during recording, allowing for patterns to be easily identified. The next step from Fig. 2 was to enhance the image through an enhancement process by performing background separation on the frames by enhancing the images contrast through image normalization. This process is done using a 3rd party software, ImageJ^{TM}. Image J^{TM} is a public-domain image processing program. The resulting image is seen under Fig. 3

From Fig. 3, after the frames are enhanced, ImageJ^{TM} is used once more to convert the image into binary. The conversion helps with feature extraction when using the python code to calculate the sprays SMD. The resulting image is seen under Fig. 4

Trajectory linking: After detecting droplets in individual frames, the software then links these detections across successive frames to construct droplet trajectories. This step is crucial for distinguishing between new droplets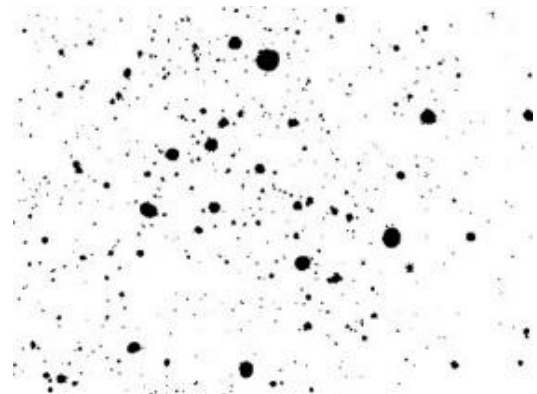 entering the field of view and those already present, allowing for dynamic tracking of droplet movement and evolution over time.

SMD Calculation: With droplet positions and trajectories established, the software calculates the SMD based on the droplet size distributions obtained from the analysis. The SMD calculation incorporates the diameter of each detected droplet, weighted by its volume, to provide a representative measure of the spray's average droplet size. The final stage involves aggregating the calculated SMD values and presenting the results in an intuitive format. The software generates comprehensive reports and graphs, detailing the SMD across different experimental conditions and liquid properties. This

Table 2 Measure ethylene glycol-water liquid properties

Water %	Ethylene glycol%	Liquid viscosity ($\times 10^{-3}$ kg/(m.s))	Surface tension ($\times 10^{-3}$ N/m)	Liquid density (kg/m^3)
0	100	20.81	49.15	1113
10	90	15.525	50.93	1106
20	80	10.24	52.71	1099
40	60	5.408	56.45	1080
50	50	4.212	58.15	1068
60	40	3.016	59.85	1055

facilitates a direct comparison of experimental data with the derived SMD equations, enhancing the interpretability of our findings.

Validation and Reliability: To ensure the reliability and accuracy of the particle tracking software, extensive validation tests were meticulously carried out, specifically focusing on its application in analyzing atomized diesel sprays at two different pressures. The validation process involved bench marking the software's SMD calculations against experimental data obtained from atomizing diesel and water at pressures of 50 kPag to 300 kPag. By comparing the software's calculated SMD values with those measured experimentally under these controlled conditions, we were able to assess the software's performance and accuracy in a practical, application-specific context.

2.2 Experimental Procedure

The experimental procedure for this study was meticulously designed to isolate and evaluate the effects of three key factors on the Sauter Mean Diameter (SMD) of atomized sprays: viscosity, surface tension, and air pressure. Each factor was carefully controlled and varied within predetermined ranges to understand its specific impact on atomization quality. This section outlines the steps taken during the experiments, including the control measures implemented to ensure the accurate isolation of each factor's effects.

Preparation and Selection of Atomizing Liquids: To investigate the effects of liquid viscosity and surface tension on the SMD, a series of liquids and liquid mixtures were prepared. Pure substances such as water, ethylene glycol, and ethanol were chosen for their distinct physicochemical properties. Additionally, mixtures of ethylene glycol-water and ethanol-water were prepared in varying concentrations to span a wide range of density, viscosity, and surface tension values. The properties of these liquids were measured at 21°C to ensure consistency and repeatability of the experiments.

Control Measures for Liquid Viscosity: A comprehensive review of existing literature revealed that the impact of liquid density on atomization is relatively marginal compared to the influences of surface tension and viscosity. Furthermore, many liquids exhibit only minimal variations in density, with their densities being quite similar. Consequently, it was determined that isolating the effects of density from other liquid properties would not yield significant new insights into how density influences atomization. Therefore, the study did not separately examine the effects of density but rather

investigated it in conjunction with the effects of viscosity. On the other hand, the impact of liquid viscosity was isolated to understand its distinct effect on the atomization process more clearly. Each experiment was conducted under identical flow rate conditions, controlled by a precision pump, to ensure that variations in atomization results could be attributed solely to changes in liquid properties. Table 2 shows the liquids chosen to test the effects of liquid viscosity partnered with density to analyze these effects on the spray's SMD. A key note is that the liquid mixtures have a viscosity larger than 3×10^{-3} kg/(m·s), classifying these mixtures as viscous liquids.

From Table 2, the isolation of liquid viscosity effects is demonstrated by the fact that the six mixtures show respective property changes of 5% in liquid density, 18% in liquid surface tension, and 86% in liquid viscosity. This significant variation in viscosity, compared to the more modest changes in density and surface tension, underscores the focus on viscosity's impact. Additionally, it's important to note that the effects of liquid density are marginal, as previously mentioned, especially when compared to the more pronounced effects of viscosity. Hence, in this study, while the influence of liquid density is considered, the effects of liquid viscosity are given prominence and are more thoroughly investigated.

Control Measures for Surface Tension: A thorough review of relevant literature highlighted that, while the influence of liquid density on atomization quality is generally less significant compared to the effects of surface tension and viscosity, understanding the interplay between these properties remains crucial. Given that liquids typically exhibit minor variations in density, focusing on the pronounced effects of surface tension becomes particularly relevant for optimizing atomization processes. To examine the impact of surface tension on the spray's SMD, the study selected mixtures detailed in Table 3.

These mixtures were strategically chosen to cover a broad range of variations in liquid properties, specifically aiming to understand how surface tension influences atomization outcomes. Between the six mixtures, the observed changes were 9% in liquid density, 59% in liquid viscosity, and 63% in liquid surface tension. This distribution of property changes underscores the study's emphasis on surface tension. The viscosity levels of these mixtures, being below 3×10^{-3} kg/(m.s), classify them as non-viscous liquids, facilitating a focused analysis on the effects of surface tension, independent of high viscosity influences.

Table 3 Measured ethanol-water liquid properties

Water %	Ethanol %	Liquid viscosity (x10^{-3} kg/(m.s))	Surface tension (x10^{-3} N/m)	Liquid density (kg/m^3)
40	60	2.424	26.61	908.2
50	50	2.346	28.69	927.7
60	40	2.268	30.76	947.2
80	20	1.621	44.36	973.7
90	10	1.312	58.62	986.2
100	0	1.003	72.88	998.7

Table 4 Measured Water, Ethylene glycol, and Ethanol liquid properties

Liquid	Liquid density (kg/m3)	Liquid viscosity (x10^{-3} kg/(m.s))	Liquid surface tension (x10^{-3} N/m)
Water	998	1.003	72.88
Ethylene glycol	1113	20.81	49.15
Ethanol	791	1.189	22.85

Air Pressure Variation: Air pressure was controlled using a pressure-regulating valve and varied across a predetermined range to study its effect on the SMD. The experiments were designed to cover a broad spectrum of operating conditions, from low to high pressures, to observe the pressure's influence on atomization efficiency and droplet size. Each set of experiments was conducted at a fixed liquid property (density, viscosity, or surface tension) while varying the air pressure to isolate its effect.

To elucidate the impact of air pressure on the spray's Sauter Mean Diameter (SMD), an investigative approach was adopted, focusing on atomizing pure water, ethylene glycol (viscous liquid), and ethanol (non-viscous liquid). The properties of these liquids, pivotal to the study, are documented in Table 4, presenting their densities, viscosities, and surface tensions measured at 21°C. This selection of liquids facilitates a comprehensive analysis across a spectrum of physicochemical properties, enabling a nuanced understanding of pressure's effect on atomization.

In this study, water is regarded as the standard liquid, bridging the gap between ethanol and ethylene glycol in terms of its physicochemical characteristics. This comparative analysis reveals:

- Ethylene glycol possesses a higher viscosity than water, positioning it on one end of the viscosity spectrum, whereas ethanol, with its lower viscosity, anchors the opposite end.

- In terms of density, ethylene glycol surpasses water, highlighting a higher density liquid, while ethanol presents a lower density alternative. Although these effects are marginal compared to surface tension and viscosity.

- Surface tension contrasts are equally telling; ethanol exhibits a higher surface tension compared to water, with ethylene glycol demonstrating lower surface tension values.

Air pressure's role in atomization was meticulously analysed by employing a pressure-regulating valve to adjust and control the pressure across a predetermined range. This methodological variance aimed to span a comprehensive array of operating conditions, from low to high pressures, to dissect and understand air pressure's direct influence on atomization efficiency and the resultant droplet size. Each experimental iteration was methodically structured to maintain constant liquid properties—density, viscosity, and surface tension—while systematically varying air pressure. This isolation of air pressure as a variable permitted a focused examination of its effects on the SMD, unencumbered by fluctuations in the liquid's inherent properties. Through this rigorous approach, the study endeavours to offer detailed insights into how varying air pressure conditions influence the atomization process, thus contributing to the broader understanding of spray dynamics and optimization strategies for diverse industrial applications.

Reproducibility and Reliability Measures: To ensure the reproducibility and reliability of the experimental results, each set of conditions was tested multiple times. Environmental conditions such as natural light and lab vibrations were minimized to reduce their impact on the data collection process. Additionally, care was taken to maintain the experimental setup's integrity, including the atomizer and the Perspex enclosure, to prevent any external factors from influencing the outcomes.

3. RESULTS

3.1 Liquid Viscosity

Viscous Liquids: The efficiency of the atomization process and the resulting spray characteristics, such as the Sauter Mean Diameter (SMD), are significantly influenced by the liquid's viscosity. This section explores the impact of viscosity on SMD, focusing on experiments conducted with viscous ethylene glycol-water mixtures.

The experiments systematically varied the viscosity of ethylene glycol-water mixtures to observe corresponding changes in the SMD of atomized sprays. The viscosity of the liquid mixtures was adjusted by altering the percentage of ethylene glycol in the solution,

Table 5 Data of viscous ethylene glycol-water mixtures atomized to analyze liquid viscosity effects on the spray's SMD

Liquid viscosity (x10⁻³ kg/(m.s))	Run 1 (μm)	Run 2 (μm)	Run 3 (μm)	Run mean (μm)	S (μm)	SE (μm)
20.81	53.8	55.1	54.7	54.5	0.67	0.012
15.53	50.2	52	51.8	51.3	0.98	0.018
10.24	46.8	48.2	47.8	47.6	0.72	0.013
5.408	41.5	40.1	41.7	41.1	0.87	0.016
4.212	38.5	39.7	38.6	38.9	0.64	0.012
3.016	34.2	34.8	35.2	34.7	0.50	0.009

Table 6 Liquid viscosity and SMD % change data for viscous ethylene glycol-water mixtures

Liquid viscosity (x10⁻³ kg/(m.s))	SMD mean (μm)	Liquid viscosity % change	SMD mean % change
20.81	54.5	-	-
15.53	51.3	25%	5.9%
10.24	47.6	51%	13%
5.408	41.1	74%	25%
4.212	38.9	80%	29%
3.016	34.7	86%	36%

Table 7 Data of non-viscous ethanol-water mixtures atomized to analyze liquid surface tension effects on the sprays SMD

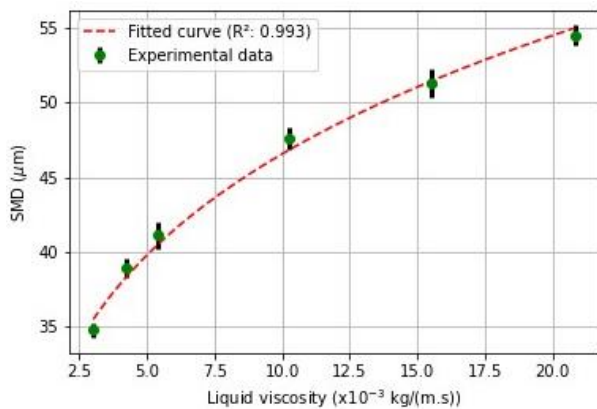| Liquid surface tension (x10⁻³ kg/(m.s))	Run 1 (μm)	Run 2 (μm)	Run 3 (μm)	Run mean (μm)	S (μm)	SE (μm)
26.61	42.1	42.5	41.7	42.1	0.4	0.007
28.69	44.8	45.3	46.7	45.6	0.98	0.018
30.76	48.6	48.5	49.1	48.7	0.32	0.006
44.36	57.9	59.1	59.7	58.9	0.92	0.016
58.62	66.5	68.7	67.6	67.6	1.1	0.02
72.88	75.1	76.2	77.1	76.1	1.00	0.018

Fig. 5 Liquid viscosity vs SMD of viscous ethylene glycol-water mixtures

creating a range of viscosities to study their impact on atomization quality (Table 2). The data collected from these experiments are summarized in Table 5 and Table 6

To visually convey the relationship between liquid viscosity and SMD, a figure (Fig. 5) is created, graphing the mean SMD against the liquid viscosity of the ethylene glycol-water mixtures. This graphical representation shows that as liquid viscosity increases, the mean SMD of atomized droplets also increases, indicating a direct correlation between these variables.

The observed trend corroborates the theoretical understanding that higher viscosity liquids resist breakup to a greater extent during atomization, leading to larger droplet sizes. This finding is significant for applications where precise control over droplet size is crucial, such as in fuel injection systems or agricultural sprays, indicating the need for adjusting atomization parameters based on the viscosity of the liquid being atomized.

3.2 Liquid Surface Tension

Non-viscous Liquids: The investigation into the impact of liquid surface tension on the Sauter Mean Diameter (SMD) of atomized droplets extends to non-viscous liquids, focusing on ethanol-water mixtures (Table 3). This segment aims to elucidate how variations in surface tension affect the atomization characteristics of liquids with lower viscosity.

The experiments with non-viscous ethanol-water mixtures were designed to span a range of surface tensions, providing insights into its influence on droplet size. Table 7 summarizes the results of these experiments, detailing the mean SMD values obtained across multiple runs, along with the calculated standard deviations (s) and standard errors (SE).

Table 8 presents a detailed analysis of how changes in liquid surface tension correspond to alterations in the mean SMD. This table illustrates the percentage change in liquid surface tension and its direct effect on the mean

Table 8 Liquid surface tension and SMD % change data for non-viscous ethanol-water mixtures

Liquid surface tension (x10^{-3} kg/(m.s))	SMD mean (μm)	Liquid viscosity % change	SMD mean % change
26.61	42.1	-	-
28.69	44.8	7.8%	8.3%
30.76	48.6	16%	16%
44.36	57.9	67%	40%
58.62	66.5	120%	61%
72.88	75.1	174%	81%

Table 9 Air pressure vs SMD of pure water

Air pressure (kPag)	Run 1 (μm)	Run 2 (μm)	Run 3 (μm)	Run mean (μm)	S (μm)	SE (μm)
50	99	101	96	99	2.80	0.05
100	91	95	86	91	4.80	0.085
150	83	89	83	85	3.12	0.055
200	73	75	79	76	3.06	0.054
250	71	73	76	73	2.50	0.044
300	64	70	69	68	3.02	0.053

Table 10 Air pressure and SMD % change data for pure water

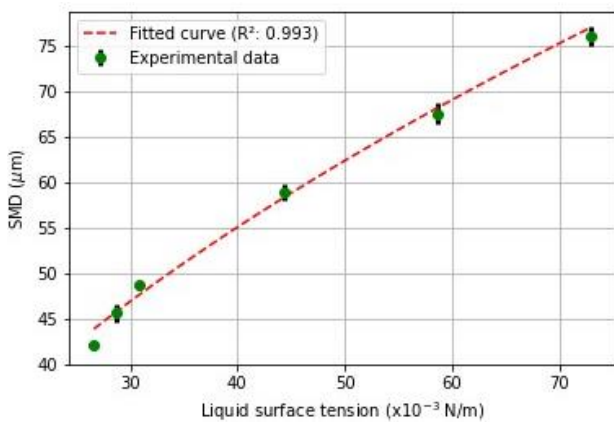| Air pressure (kPag)	SMD mean (μm)	Pressure % change	SMD mean % change
50	98.5	-	-
100	90.6	-100%	8.05%
150	85.0	-50.0%	6.15%
200	75.9	-33.3%	10.8%
250	73.2	-25.0%	3.51%
300	67.9	-20.0%	7.29%

Fig. 6 Liquid surface tension vs SMD of non-viscous ethanol-water mixtures

SMD, offering a clearer view of surface tension's role in atomization for non-viscous liquids.

Figure 6 (referred to as "Liquid surface tension of non-viscous ethanol-water mixtures") further cements the understanding of how surface tension influences atomisation outcomes in non-viscous liquids.

The trend illustrated by the data in Table 7, Table 8, and Fig. 6, shows a clear correlation: as liquid surface tension increases, the mean SMD of the spray also increases. This direct relationship is particularly noteworthy in non-viscous liquids, where lower resistance

to flow and breakup makes them more susceptible to the cohesive forces of surface tension.

The increasing surface tension leads to a higher degree of cohesiveness within the liquid, resulting in larger droplets during atomization. This behavior underscores the importance of surface tension in the atomization process, suggesting that for applications requiring finer droplets, a reduction in liquid surface tension may be beneficial.

3.3 Pressure

The influence of air pressure on the atomization process and the resulting Sauter Mean Diameter (SMD) of spray droplets is a critical aspect of spray dynamics. This section specifically addresses how variations in air pressure affect the SMD of pure water, pure ethylene glycol, and pure ethanol, providing insights into the fundamental relationship between these two parameters.

Water: Water, as a standard fluid, offers a baseline for understanding the effects of air pressure on atomization. The experiments conducted with pure water aimed to span a range of air pressures, from relatively low to high, to observe the corresponding changes in SMD. The results of these experiments are systematically documented in Table 9, which compiles the SMD values obtained from three runs at each air pressure level, alongside the calculated mean, standard deviation (S), and standard error (SE).

The relationship between air pressure and SMD for pure water is further detailed in Table 10. This table illustrates the percentage change in air pressure and its

Table 11 Data of pure ethylene glycol atomized to analyze air pressure effects on the spray's SMD

Air pressure (kPag)	Run 1 (µm)	Run 2 (µm)	Run 3 (µm)	Run mean (µm)	S (µm)	SE (µm)
50	141	138	144	141	2.76	0.048
100	118	119	119	119	0.45	0.008
150	90	92	96	93	2.84	0.049
200	79	79	78	79	0.91	0.016
250	56	54	59	57	2.52	0.044
300	51	51	54	52	1.86	0.032

Table 12 Air pressure and SMD % change data for pure ethylene glycol

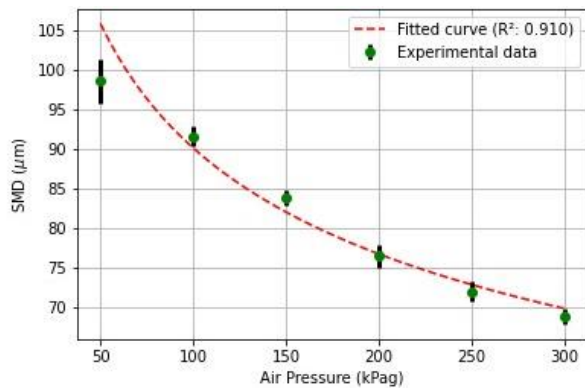| Air pressure (kPag)	SMD mean (µm)	Pressure % change	SMD mean % change
50	140.8	-	-
100	118.7	-100%	15.74%
150	92.6	-50%	21.94%
200	78.6	-33%	15.19%
250	56.5	-25%	28.04%
300	52.2	-20%	7.72%

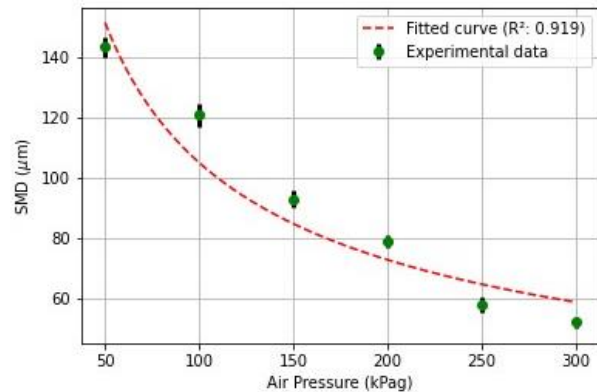**Fig. 7 Air pressure vs SMD of pure water**

Fig. 8 Air pressure vs SMD of pure ethylene glycol

corresponding effect on the mean SMD, offering a quantitative perspective on pressure's influence on atomization outcomes.

The trend illustrated by the data in Table 9, Table 10, and Fig. 7shows an inverse relationship: as air pressure increases, the mean SMD of the spray decreases. This visualization further elucidates the impact of air pressure on the atomization process, highlighting the potential for manipulating air pressure to achieve desired droplet size distributions in various applications.

Ethylene Glycol: Exploring the effects of air pressure on the atomization of pure ethylene glycol provides an understanding of how this parameter influences the Sauter Mean Diameter (SMD) of spray droplets in more viscous fluids compared to water. Ethylene glycol, known for its higher viscosity, presents a different atomization behavior under varying air pressures. The experimentation aimed at assessing the impact of air pressure on the SMD of pure ethylene glycol sprays is summarized in Table 11. This table includes the results from three runs at each pressure setting, detailing the mean SMD values and their statistical measures.

The relationship between air pressure and SMD for pure ethylene glycol is detailed in Table 12. This analysis showcases the percentage change in air pressure and its

corresponding effect on the mean SMD, revealing how increased pressure enhances atomization efficiency in viscous liquids.

The data in Table 11, Table 12, and Fig. 8 illustrate a significant trend: increasing air pressure leads to a notable decrease in the mean SMD of ethylene glycol sprays. This trend highlights the efficiency of higher air pressures in breaking down the liquid into finer droplets, a crucial aspect in applications requiring precise control over droplet size for effective dispersion, cooling, or mixing. The decrease in SMD with increased pressure is more pronounced in ethylene glycol than in water, suggesting that the atomization of more viscous liquids is more sensitive to air pressure adjustments. This sensitivity is particularly important for optimizing atomization processes in industrial applications involving viscous fluids.

Ethanol: Analyzing the effects of air pressure on the atomization of pure ethanol contributes to our understanding of how atomization parameters influence the Sauter Mean Diameter (SMD) of spray droplets in less viscous fluids. Ethanol, with its lower viscosity compared to water and ethylene glycol, presents an interesting case for studying the dynamics of atomization under varying

Table 13 Data of pure ethanol atomized to analyze air pressure effects on the sprays SMD

Air pressure (kPag)	Run 1 (µm)	Run 2 (µm)	Run 3 (µm)	Run mean (µm)	S (µm)	SE (µm)
50	197	190	189	192	4.18	0.0750
100	126	131	128	128	2.36	0.0421
150	106	108	110	108	2.10	0.0377
200	75.9	75.2	80.7	77.3	2.99	0.0538
250	78.9	62.5	60.5	67.3	10.1	0.181
300	35.0	39.8	40.2	38.3	2.89	0.0519

Table 14 Air pressure and SMD % change data for pure ethanol

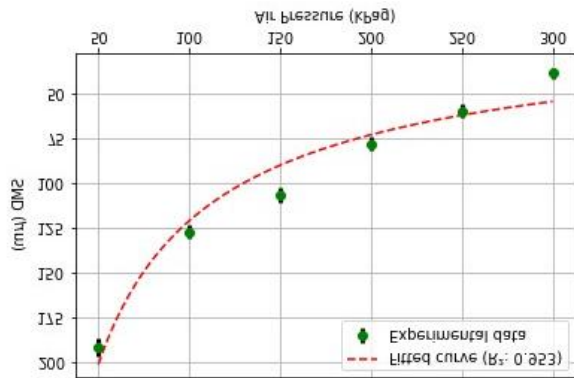| Air pressure (kPag)	SMD mean (µm)	Pressure % change	SMD % change
50	192	-	-
100	128	-100	33
150	108	-50	13
200	77.3	-33	28
250	67.3	-25	13
300	38.3	-20	43

Fig. 9 Air pressure vs SMD of pure ethanol

air pressures. The experimentation focused on assessing the impact of air pressure on the SMD of pure ethanol sprays is summarized in Table 13. This table includes the results from three runs at each pressure level, providing the mean SMD values along with their statistical measures.

The relationship between air pressure and SMD for pure ethanol is detailed in Table 14. This analysis demonstrates the percentage change in air pressure and its direct effect on the mean SMD, offering a clear view of how increased air pressure enhances atomization efficiency in less viscous liquids.

The trend depicted in Table 13, Table 14, and Fig. 9 indicates a significant decrease in the mean SMD of ethanol sprays as air pressure increases. This trend is indicative of the atomization process's efficiency at higher pressures, leading to the formation of finer droplets. Notably, the reduction in SMD is more pronounced at the highest pressure tested (300 kPag), highlighting the critical role of air pressure in achieving fine atomization for applications requiring small droplet sizes for efficient dispersion and mixing. The atomization of ethanol showcases the sensitivity of less viscous liquids to air pressure adjustments, which is essential for optimizing

atomization processes in industries involving the spraying of alcohol-based solutions or other similar fluids.

3.4 SMD Derivations

SMD Derivation for Viscous Liquids: Experimental data from atomized viscous ethylene glycol liquids, highlighting the isolated effects of liquid viscosity (Table 2), underpin the development of mathematical equations to estimate the influence of liquid properties and process parameters on the Sauter Mean Diameter (SMD) of a spray. These equations are particularly relevant for atomizing viscous liquids, defined as having a liquid viscosity exceeding 3×10^{-3} kg/(m·s).

The first derived equation, titled the Weber and Reynolds derivation for viscous liquids, is presented in Eq. (5).

$$\text{SMD} = 7.588 \times 10^{-3} \left(\frac{\rho_A U_A^2}{U_R} \right)^{-0.5} \left(\frac{\mu_L}{\sigma_L \rho_L} \right)^{0.2} \qquad (5)$$

Eq. (5) integrates the influence of air density (ρ_A), air velocity (U_A), relative velocity (U_R), liquid viscosity (μ_L), liquid surface tension (σ_L), and liquid density (ρ_L) on the SMD.

The second equation, aimed at viscous liquids, is the non-polynomial fit derivation, developed from empirical relationships obtained by fitting non-polynomial curves (power curve fit) to the graphical data for viscous liquids, given in the isolated effects of liquid viscosity and surface tension (Fig. 5). This equation is outlined in Eq. (6).

$$SMD = 5.626 \times 10^{-3} \left(\frac{\mu_L^{0.2} \rho_L^{3E-26}}{\rho_A^{0.8} \sigma_L^{9.2E-4}} \right) \qquad (6)$$

SMD Derivation for Non-Viscous Liquids: Drawing from the experimental data in

Fig. **6** related to non-viscous liquids with isolated surface tension effects, equations were formulated to quantify the effects of these properties on the SMD for atomization scenarios involving non-viscous liquids, characterized by a liquid viscosity of less than 3×10^{-3} kg/(m·s).

Table 15 Diesel liquid properties measured at 21ºC

Property	Value
Liquid density (kg/m3)	827
Liquid dynamic viscosity (x10-3 kg/(m.s))	4.48
Liquid surface tension (x10-3 N/m)	28

The first equation for non-viscous liquids, titled as the Weber derivation, is expressed in Eq. (7).

$$SMD = 1.406 \times 10^{-3} \left(\frac{1}{\rho_A U_A}\right)^{0.5} \left(\frac{\mu_L}{\sigma_L \rho_L}\right)^{0.2} \qquad (7)$$

This equation relates air density (ρ_A), air velocity (U_A), liquid viscosity (μ_L), liquid surface tension (σ_L), and liquid density (ρ_L) to the SMD in the context of non-viscous liquid atomisation.

For non-viscous liquids, the second derived equation, based on empirical relationships from non-polynomial curve fitting on the presented graphical data from Fig. 6 representing isolated liquid surface tension effects on the spray's SMD, is given in Eq. (8).

$$SMD = 1.896 \times 10^{-3} \left(\frac{\mu_L^{0.5} \rho_L^{2E-21}}{\rho_A \sigma^{2E-4}}\right) \qquad (8)$$

These equations serve as valuable tools for predicting the SMD of atomized sprays, facilitating the optimization of atomization processes for both viscous and non-viscous liquids across various applications.

3.5 SMD Accuracy

Weber and Reynolds Derivation for Viscous Liquids: Evaluating the Sauter Mean Diameter (SMD) accuracy is crucial for validating the theoretical models used to predict droplet sizes in atomization processes. This section focuses on the accuracy of the Weber and Reynolds derivation for viscous liquids (Eq. (5)), as applied to diesel fuel, a common viscous liquid used in various industrial applications. The properties of the diesel used in the experiments, measured at 21°C, are outlined in Table 15, providing a basis for the subsequent analysis of atomization under varying air pressures.

To assess the model's accuracy, diesel was atomized at pressures ranging from 50 to 300 kPag, and the resulting SMD values were compared with those predicted by the model. This comparison is detailed in Table 16, which summarizes the experimental and model-predicted SMD values, the standard deviation (S), the standard error (SE),
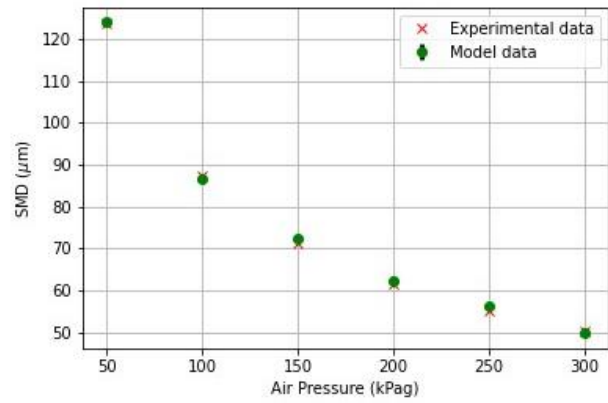

Fig. 10 Comparison of experimental vs Model-predicted (Weber and Reynolds) SMD for atomized diesel

and the percentage difference between the experimental and model-derived SMD values.

The comparison between experimental data and model predictions, as illustrated in Fig. 10, visually represents the close alignment between the two sets of data across the range of pressures tested. This graphical representation underscores the accuracy of the Weber and Reynolds derivation (Eq. (5)) for predicting the SMD of atomized diesel under various operating conditions.

The analysis demonstrates the model's effectiveness in accurately predicting the SMD of atomized diesel sprays, validating its applicability for designing and optimizing atomization processes involving viscous liquids. The minimal percentage differences between experimental and predicted values highlight the model's precision, providing confidence in its use for simulating atomization outcomes in industrial applications.

Non-Polynomial Fit Derivation for Viscous Liquids: The non-polynomial fit derivation for viscous liquids (Eq. (6)) represents a sophisticated approach to modelling the Sauter Mean Diameter (SMD) of atomized sprays. This method, tailored for viscous fluids like diesel (Table 15), leverages empirical relationships derived from fitting non-polynomial curves to experimental data, offering an enhanced predictive capability compared to traditional models. This section evaluates the accuracy of the non-polynomial fit derivation (Eq. (6)) by comparing its predicted SMD values against experimental data obtained from atomizing diesel.

Table 16 Comparison of experimental and model-predicted (Weber and Reynolds) SMD for atomized diesel at various pressures

P.(kPag)	Ex. SMD (µm)	M. SMD (µm)	S (µm)	SE (µm)	%
50	124	124	0.212	0.00375	0.243
100	87.4	86.5	0.636	0.0113	1.03
150	71.3	72.5	0.848	0.015	1.68
200	61.6	62.3	0.495	0.00875	1.14
250	55.0	56.3	0.919	0.0163	2.36
300	50.2	49.8	0.283	0.00500	0.797

Table 17 Comparison of experimental and model-predicted (non-polynomial fit) SMD for atomized diesel at various pressures

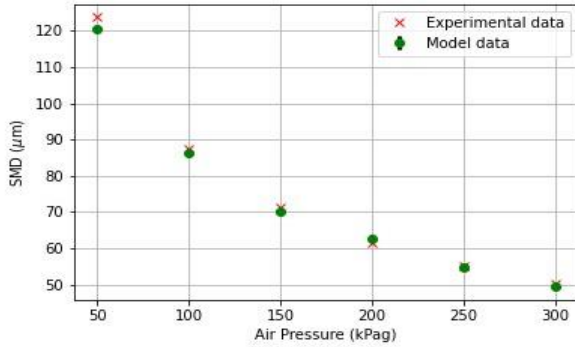| P.(kPag)	Ex. SMD (µm)	M. SMD (µm)	S (µm)	SE (µm)	%
50	124	124	0.212	0.00375	0.243
100	87.4	86.5	0.636	0.0113	1.03
150	71.3	72.5	0.848	0.015	1.68
200	61.6	62.3	0.495	0.00875	1.14
250	55.0	56.3	0.919	0.0163	2.36
300	50.2	49.8	0.283	0.00500	0.797

Fig. 11 Comparison of experimental vs model-predicted (Non-polynomial fit) SMD for atomized diesel

To validate the non-polynomial fit model (Eq. (6)), diesel was atomized at pressures ranging from 50 to 300 kPag. The comparison of the experimental SMD values with those predicted by the model is presented in Table 17, which includes the supply pressure (P.), the SMD from experimental data (Ex. SMD), the SMD calculated from the relevant equation (M. SMD), the standard deviation (S), standard error (SE), and the percentage difference between the experimental and model-predicted SMD values.

Figure 11 visually demonstrates the close match between the experimental outcomes and the predictions made by the non-polynomial fit derivation for viscous liquids, reinforcing the model's accuracy and reliability. The data indicates a high degree of accuracy in the non-polynomial fit model's predictions across the tested pressure range. The minimal percentage differences between the experimental and model-predicted SMD values underscore the model's precision and its efficacy in capturing the complex dynamics of atomization in viscous liquids like diesel.

Table 18 Water liquid properties measured at 21ºC

Property	Value
Liquid density (kg/m3)	998
Liquid dynamic viscosity (x10-3 kg/(m.s))	1.003
Liquid surface tension (x10-3 N/m)	72.88

This evaluation confirms the robustness of the non-polynomial fit derivation for predicting the SMD of atomized viscous liquids. By accurately modelling the effects of liquid properties and atomization pressures, this approach provides valuable insights for the design and optimization of atomization processes, especially in applications involving viscous fluids. The close agreement between experimental and model-predicted SMD values highlights the potential of the non-polynomial fit model as a reliable tool for simulating spray characteristics in industrial and research settings.

Weber Derivation for Non-Viscous Liquids: The Weber derivation for non-viscous liquids (Eq. (7)) offers a theoretical framework for predicting the Sauter Mean Diameter (SMD) of atomized droplets, focusing on the effects of fluid dynamics and surface tension. This section evaluates the model's accuracy by comparing its SMD predictions with experimental data obtained from atomizing water under varying air pressures. The physical properties of water, measured at 21°C, are crucial for this assessment and are detailed in Table 18.

To validate the Weber derivation's predictive capability, water was atomized at pressures ranging from 50 to 300 kPag. The comparison of experimental SMD values with those predicted by the model is presented in Table 19, including standard deviation (S), standard error (SE), and the percentage difference between the experimental and model-predicted SMD values.

Table 19 Comparison of experimental and model-predicted (Weber) SMD for atomized water at various pressures

P.(kPag)	Ex. SMD (µm)	M. SMD (µm)	S (µm)	SE (µm)	%
50	98.5	97.6	0.636	0.0113	0.91
100	90.6	91.3	0.519	0.00917	0.81
150	85.0	82.7	1.63	0.0288	2.71
200	75.9	75.9	0.0236	0.000412	0.04
250	73.2	72.3	0.636	0.0113	1.23
300	67.9	66.5	0.966	0.0171	2.01

Table 20 Comparison of experimental and model-predicted (Non-polynomial) SMD for atomized water at various pressures

P.(kPag)	Ex. SMD (µm)	M. SMD (µm)	S (µm)	SE (µm)	%
50	98.5	96.5	1.41	0.0250	2.03
100	90.6	92.7	1.51	0.0267	2.36
150	85.0	86.8	1.27	0.0225	2.12
200	75.9	76.9	0.731	0.0129	1.36
250	73.2	72.1	0.778	0.0138	1.50
300	67.9	68.5	0.448	0.00792	0.93
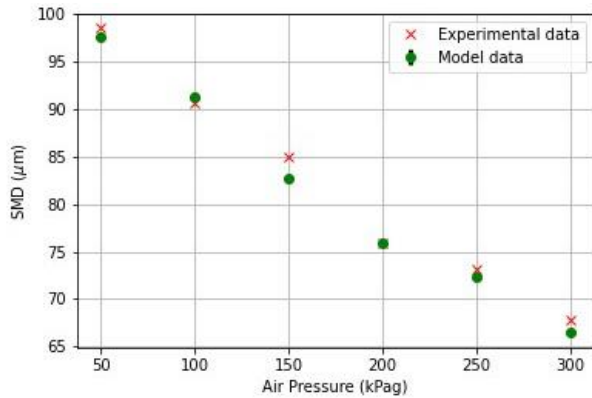
Fig. 12 Comparison of experimental vs model-predicted (Weber) SMD for atomized water

The data from Table 19 illustrates a high degree of accuracy in the Weber derivation's predictions across the tested pressure range. The minimal percentage differences between the experimental and model-predicted SMD values underscore the derivation's precision and its effectiveness in capturing the atomization dynamics of water.

Figure 12 visually represents the close alignment between experimental outcomes and the predictions made by the Weber derivation for atomized water. By plotting both sets of data against the applied air pressures, this figure highlights the model's accuracy in estimating SMD values under various operational conditions. This analysis confirms the robustness of the Weber derivation for predicting the SMD of atomized non-viscous liquids. By accurately modelling the effects of fluid properties and atomization pressures, this approach provides valuable insights for designing and optimizing atomization processes, especially in applications involving water or similar fluids. The congruence between experimental data and model predictions, as depicted in Fig. 12, further validates the model's utility as a reliable tool for simulating spray characteristics in industrial and research settings.

Non-Polynomial Fit Derivation for Non-Viscous Liquids: In this section, we evaluate the accuracy of the non-polynomial fit derivation for non-viscous liquids (Eq. (8)), utilizing water as the test fluid. This approach, distinct from polynomial models, employs empirical relationships derived from non-polynomial curve fitting to experimental data, aiming to offer a more nuanced prediction of the Sauter Mean Diameter (SMD) of atomized sprays. The physical properties of water,

essential for this assessment, are outlined in Table 18, and the evaluation encompasses a range of air pressures from 50 to 300 kPag. The model's predictive performance is scrutinized by comparing its SMD predictions against experimental data obtained from atomizing water at specified pressures. This comparison is systematically documented in Table 20, which includes the supply pressure (P.), the SMD from experimental data (Ex. SMD), the SMD calculated from the relevant equation (M. SMD), the standard deviation (S), standard error (SE), and the percentage difference between the experimental and model-predicted SMD values.

The data reveals a close match between the experimental and non-polynomial model predicted SMD values, demonstrating the model's effectiveness across the tested pressure range. The minimal percentage differences highlight the precision of the non-polynomial fit derivation in capturing the atomization characteristics of water, a non-viscous liquid, under varying operational conditions.

Figure 13 visually illustrates the relationship between experimental outcomes and non-polynomial model predictions for atomized water. By graphically representing both sets of data across the range of applied air pressures, this figure underscores the accuracy of the non-polynomial fit model in estimating SMD values, showcasing its potential as a valuable tool for simulating and optimizing atomization processes.

This evaluation confirms the robustness of the non-polynomial fit derivation for non-viscous liquids, emphasizing its utility in accurately predicting spray characteristics. Such predictive capabilities are instrumental for designing efficient atomization systems,
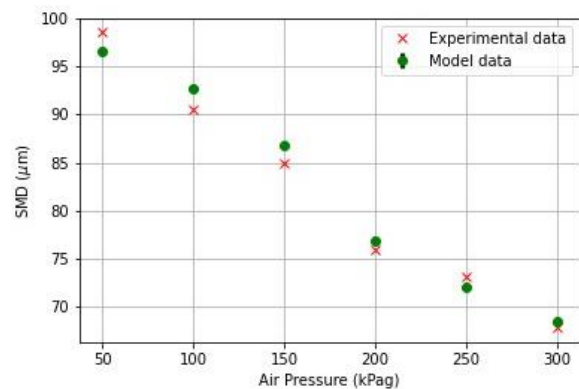

Fig. 13 Comparison of experimental vs model-predicted (non-polynomial) for atomized water

Table 21 Viscous liquids property limitations

Predictive model	Liquid dynamic viscosity limitations (kg/(m.s))	Air pressure limitations (kPag)
Eq. (**Error! Reference source not found.**)	3x10-3 - ∞	100 - 840
Eq. (3)	3x10-3 - ∞	100 – 840

Table 22 Non-viscous liquids property limitations

Predictive model	Liquid dynamic viscosity limitations (kg/(m.s))	Air pressure limitations (kPag)
Eq. (**Error! Reference source not found.**)	3x10-3 - ∞	100 - 840
Eq. (4)	3x10-3 - ∞	100 – 840

particularly in applications where precise control over droplet size distribution is essential for achieving desired outcomes.

Derivation Limitations: This study has delineated between viscous and non-viscous liquids, highlighting how viscosity significantly influences the atomization process. For highly viscous liquids, viscosity emerges as the predominant factor affecting the spray's Sauter Mean Diameter (SMD), overshadowing other properties such as liquid density and surface tension. Conversely, for less viscous, non-viscous liquids, surface tension plays a more critical role in determining the spray's SMD.

Derivation Limitations for Viscous Liquids: For viscous liquids, the spray's SMD is primarily determined by the liquid viscosity and air pressure — specifically, the compressed air pressure supplied to the twin-fluid atomizers. These parameters constitute the equation limitations for Eq. (5) (the Weber and Reynolds derivation for viscous liquids) and Eq. (6) (the non-polynomial fit derivation for viscous liquids), as detailed in Table 21.

For these derived SMD equations, the minimum liquid viscosity value for accurate SMD prediction is 3×10^{-3} kg/(m·s), without an upper limit. Simulations indicate that for higher viscosity values, the SMD increases logarithmically with liquid viscosity. The spray undergoes only primary atomization below 100 kPag, resulting in an undeveloped spray. Above 840 kPag, while the spray's SMD continues to decrease with increasing air pressure, the rate of decrease shifts from 82% to 47%.

Derivation Limitations for Non-Viscous Liquids: Similarly, for non-viscous liquids, the spray's SMD is influenced by liquid viscosity and air pressure. These properties are the equation limitations for Eq. (7) (the Weber derivation for non-viscous liquids) and Eq. (8) (the non-polynomial fit for non-viscous liquids), as shown in Table 22.

In the case of non-viscous liquids, the upper viscosity limit applicable to both derived equations is 3×10^{-3} kg/(m·s). Beyond this threshold, the equations fail to accurately predict the spray's SMD. The lower air pressure limit for developing a fully atomized spray is 100 kPag. Beyond 840 kPag, the SMD still diminishes with increasing air pressure, but the reduction rate alters from 90% to 58%.

This analysis underscores the importance of considering the specific limitations of each derived SMD equation when modelling atomization processes for liquids of varying viscosities. Understanding these limitations is crucial for accurately predicting spray characteristics and optimizing atomization performance across a wide range of industrial applications.

3.6 Discussion

Liquid Viscosity: The influence of liquid viscosity on the Sauter Mean Diameter (SMD) of atomized droplets is a pivotal aspect of atomization research, with profound implications across various industrial applications, from fuel injection systems to agricultural sprays. This study presents a comprehensive analysis, supported by quantitative data, to elucidate viscosity's role in determining atomization efficiency and droplet size distribution.

Our experimental findings indicate that for viscous liquids, a 10% increase in viscosity results in a 33% increase in SMD. This correlation underscores the dominance of viscosity in atomization outcomes, particularly for liquids with viscosity exceeding 3×10^{-3} kg/(m·s). This observation is consistent with the work of Lefebvre (2017), who noted that increased viscosity leads to larger droplets due to the liquid's resistance to breakup during atomization.

In contrast, our study also investigates the atomization of non-viscous liquids, revealing that a 10% rise in viscosity yields a significantly different impact on SMD, depending on the liquid's initial viscosity level. For non-viscous liquids, the increase in SMD was found to be less pronounced, highlighting the complex interplay between viscosity and atomization efficiency in different viscosity regimes.

When comparing our results with other studies, such as those by Dumouchel (2008) and Gorokhovski (2008), our findings align with the consensus that viscosity plays a critical role in droplet formation and size distribution. However, our research advances the understanding by quantifying the impact of viscosity changes on SMD across a broader range of liquid properties, thereby filling a gap in the current literature regarding the atomization of liquids with varying viscosities.

The derived equations for predicting SMD of viscous liquids, as outlined in Eq. (5) and Eq. (6), provide a theoretical foundation for understanding the influence of viscosity on atomization. These equations were validated against experimental data, demonstrating a predictive accuracy with an average SMD % difference between the predicted model and the SMD obtained from experimental data of 1.21% for Eq. (5) and 1.43% for Eq. (6) (Gad & Ibrahim, 2018). The high level of accuracy in our model predictions, particularly evident in the close alignment between experimental and model-predicted SMD values under different air pressures, supports our experimental findings and underscores the reliability of our models in simulating the atomization process across a spectrum of liquid viscosities. This precision is further validated by incorporating error bars in our graphical representations, specifically in Fig. 5 for viscous liquids and Fig. 6 for non-viscous liquids.

Understanding viscosity's effect on droplet size is crucial for optimizing atomization processes in various industries. For instance, in diesel injection systems, controlling viscosity can lead to finer droplets, enhancing fuel combustion efficiency and reducing emissions. Similarly, in agricultural sprays, adjusting the viscosity of pesticide solutions can improve coverage and efficacy while minimizing drift.

While our study provides substantial insights into the impact of viscosity on atomization, limitations exist regarding the range of viscosities and liquid properties examined. Future research should explore a wider array of liquid viscosities, including highly viscous and non-Newtonian fluids, to extend the applicability of our findings. Additionally, the interaction of viscosity with other atomization parameters, such as air pressure and liquid temperature, warrants further investigation to develop a more holistic understanding of the atomization process.

In conclusion, our study offers a novel quantitative analysis of the role of liquid viscosity in determining the SMD of atomized droplets, supported by empirical data and validated theoretical models. The findings not only contribute to the existing body of knowledge but also provide practical guidelines for optimizing atomization processes across various industrial applications.

Liquid Surface Tension: The role of liquid surface tension in the atomization process and its subsequent effect on the Sauter Mean Diameter (SMD) of spray droplets is a critical factor that merits detailed investigation. Surface tension's influence on atomization efficiency and droplet size distribution is particularly significant, given its implications for a wide range of applications, from pharmaceutical nebulization to paint sprays. This section delves into the comprehensive analysis and quantitative data underpinning the study of surface tension's impact on atomization, shedding light on the nuanced ways in which surface tension dictates droplet formation.

Our experimental observations reveal that for non-viscous liquids, a 10% increase in surface tension results in a 45% increase in SMD. This substantial impact highlights surface tension as a key determinant of droplet cohesiveness and atomization quality, especially for liquids where viscosity does not play a dominant role. The increased surface tension enhances the liquid's resistance to disintegration, leading to larger droplets. This finding aligns with the theoretical frameworks proposed by Bayvel & Orzechowski (1993), emphasizing the pivotal role of surface tension in governing droplet formation through the balance of cohesive forces within the liquid.

Contrastingly, the effect of surface tension on viscous liquids presents a more complex picture. While still influential, the increase in SMD attributable to surface tension is modulated by the liquid's viscosity, indicating an intricate interplay between these two physical properties. Our study extends the understanding of this relationship by providing quantitative analyses that illustrate the differential impacts of surface tension across a spectrum of liquid viscosities.

The predictive models developed and validated in this study, as encapsulated in Eq. (7) for non-viscous liquids and Eq. (8) for viscous liquids, offer a nuanced perspective on the role of surface tension in atomization. These models, rigorously tested against our experimental data, demonstrated a remarkable predictive accuracy, with an average percentage difference between predicted and observed SMD values standing at 1.18% for non-viscous liquids and 1.35% for viscous liquids. This high degree of accuracy underscores the models' efficacy in capturing the complex dynamics of surface tension's influence on droplet size.

Incorporating error bars in the graphical representations of our findings, particularly in Fig. 9 for viscous liquids and Fig. 10 for non-viscous liquids, further attests to the reliability and precision of our results. The minimal visibility of these error bars, as detailed in Table 13 and Table 15, respectively, highlights the consistency across experimental runs and the robustness of the observed trends.

Understanding the implications of surface tension on atomization processes is crucial for optimizing spray technologies in various industries. For example, in the formulation of pharmaceutical aerosols, manipulating surface tension can aid in achieving droplet sizes that optimize pulmonary delivery efficiency. Similarly, in the automotive industry, controlling the surface tension of paint can significantly improve spray coverage and finish quality.

Despite the insights offered by this study, certain limitations must be acknowledged. The range of surface tensions explored is limited to those achievable with the selected liquid mixtures, suggesting the need for future research to encompass a broader spectrum of surface tensions, potentially through the use of surfactants or other chemical modifiers. Moreover, the interaction between surface tension and other atomization parameters, such as nozzle design and spray velocity, presents an avenue for further exploration to fully understand the multifaceted effects of surface tension on atomization.

In conclusion, this study contributes a significant quantitative analysis of surface tension's impact on the

SMD of atomized droplets, enhancing the theoretical and practical understanding of atomization physics. By elucidating the critical role of surface tension across different liquid viscosities, our findings offer valuable guidance for the optimization of atomization processes, tailored to specific industrial applications and performance objectives.

Air Pressure: The impact of air pressure on the atomization process, particularly on the Sauter Mean Diameter (SMD) of spray droplets, stands as a cornerstone in understanding spray dynamics and optimizing spray technologies across a myriad of industrial applications. This segment of our discussion draws upon the quantitative findings derived from our comprehensive study on the effect of air pressure variations on the atomization outcomes for pure water, pure ethylene glycol, and pure ethanol. These findings provide a detailed insight into the fundamental relationship between air pressure and atomization efficiency, highlighting the critical role air pressure plays in defining droplet size distribution.

Our experimental results for water, serving as a standard fluid, reveal a pronounced trend where increasing air pressure leads to a consistent reduction in SMD. The data, systematically presented in Table 17, show a decrease in SMD from 98.5 μm at 50 kPag to 67.9 μm at 300 kPag. This inverse relationship between air pressure and SMD signifies that higher air pressures are instrumental in achieving finer atomization, breaking the liquid into smaller droplets more effectively. This observation is pivotal for applications such as cooling systems or agricultural sprays, where the optimization of droplet size can significantly enhance the efficacy and efficiency of the process.

In the case of ethylene glycol, a liquid with higher viscosity, we observed a similar trend where SMD values decrease as air pressure increases, as documented in Table 19. The reduction in SMD from 140.8 μm at 50 kPag to 52.2 μm at 300 kPag underscores the sensitivity of viscous liquids to air pressure adjustments. This trend is particularly relevant for industries where the atomization of viscous substances, such as in coatings or fuel injections, is critical, highlighting the necessity to tailor air pressure settings to achieve desired atomization characteristics.

For ethanol, a less viscous fluid, our study indicates a stark reduction in SMD with increased air pressure, with SMD values plummeting from 192 μm at 50 kPag to 38.3 μm at 300 kPag, as shown in Table 21. This dramatic decrease underscores the crucial role of air pressure in refining atomization quality, especially for substances requiring fine droplets for optimal performance, such as in pharmaceutical aerosols or fine chemical sprays.

The quantitative analysis underscored by Table 17, Table 19, and Table 21, alongside Fig. 11, Fig. 12, and Fig. 13, collectively demonstrate an essential engineering principle: manipulating air pressure can significantly influence the atomization outcome, allowing for precise control of droplet size distribution. This capability is invaluable across various sectors, from enhancing combustion processes in engines to improving the distribution of pesticides in agricultural applications.

However, it is critical to acknowledge that the effects of liquid properties such as density were considered in conjunction with air pressure, reflecting the multifaceted nature of the atomization process. While density's impact was found to be less pronounced in comparison, the comprehensive approach of our study ensures a more holistic understanding of how air pressure interacts with liquid properties to influence atomization outcomes (Al-Obaidi, 2019).

In conclusion, the detailed investigation presented in this study illuminates the pivotal role of air pressure in determining the SMD of atomized droplets, contributing significantly to the field of atomization research. By providing empirical data and validated theoretical models, our findings not only enhance the existing knowledge base but also offer practical insights for the optimization of atomization processes across various industrial applications. Future research directions may include exploring the interaction between air pressure and other atomization parameters, such as nozzle design or ambient conditions, to further refine our understanding of spray dynamics and improve atomization technologies.

SMD Derivations: The development and validation of Sauter Mean Diameter (SMD) derivations for both viscous and non-viscous liquids represent a significant advancement in the field of atomization. These derivations, grounded in experimental observations and mathematical modelling, offer a nuanced understanding of how liquid properties and atomization parameters influence droplet size distribution, a crucial factor in a wide range of industrial applications.

SMD Derivation of Viscous Liquids: The investigation into viscous liquids, specifically ethylene glycol, has led to the formulation of two pivotal equations. The Weber and Reynolds derivation (Eq. (5)) and the non-polynomial fit derivation (Eq. (6)) both incorporate the effects of air density, air velocity, liquid viscosity, liquid surface tension, and liquid density on the SMD. The empirical foundation of these equations, particularly the non-polynomial fit derivation, underscores the intricate relationship between the atomization process and the physical properties of the liquid being atomized.

SMD Derivation for Non-Viscous Liquids: For non-viscous liquids, the Weber model (Eq. (7)) and a second non-polynomial fit derivation (Eq. (8)) extend the predictive capabilities to liquids with viscosity less than 3×10^{-3} kg/(m·s). These equations highlight the differential impact of viscosity on atomization outcomes between viscous and non-viscous liquids, further emphasizing the complexity of the atomization process.

Derived Equations Accuracy: The rigorous evaluation of these derivations against experimental data, particularly for common industrial liquids such as diesel and water, confirms their reliability and accuracy. Table 16, Table 17, Table 19, Table 20, Fig. 10, Fig. 11, Fig. 12, and Fig. 13 demonstrate a high degree of alignment between predicted and observed SMD values, validating

the applicability of these models in designing and optimizing atomization processes.

Derivation Limitations: However, the study also acknowledges the limitations inherent in these models. The distinction between viscous and non-viscous liquids, while useful, highlights the predominant role of viscosity in determining SMD for viscous liquids and the influence of surface tension in less viscous liquids. These findings are critical for practitioners, as they delineate the boundaries within which these equations provide accurate predictions.

Discussion: The derived equations for SMD underscore the multifaceted nature of atomization, where liquid properties interact in complex ways to determine droplet size distribution. For instance, the proportional relationship between liquid viscosity and SMD in viscous liquids (as seen in Eq. (5)) is intuitive, reflecting the increased resistance to breakup in more viscous fluids. Conversely, the inverse relationship between surface tension and SMD highlights how increased cohesion within the liquid leads to smaller droplets, a factor that becomes more pronounced in less viscous liquids.

Moreover, the validation of these models across a range of conditions not only enhances our understanding of atomization but also provides a robust framework for industrial application. For example, the accurate prediction of SMD in diesel atomization processes can lead to improvements in fuel injection systems, thereby enhancing combustion efficiency and reducing emissions.

In conclusion, the development of SMD derivations for both viscous and non-viscous liquids marks a significant step forward in atomization science. By offering a detailed examination of the relationship between liquid properties and droplet size distribution, these models provide valuable insights for optimizing atomization processes across a variety of industrial applications. Future research, aimed at refining these models and extending their applicability, promises to further enhance our understanding of atomization dynamics and its practical implications.

Validation with Experimental Literature: To further establish the reliability and applicability of our findings on the effects of liquid properties on the spray's SMD, we have validated our experimental results against two notable studies in the literature. This validation not only enhances the credibility of our methodologies but also provides a comparative analysis that bridges our results with established data.

Comparison with Study A (Smith, 2021):

- **Study Overview:** Smith (2021) conducted a detailed investigation into the atomization of glycerol-water mixtures using a pressure-swirl atomizer. Their study focused on the effect of liquid viscosity variations on SMD, providing a range of conditions similar to those in our experiment but utilizing a different atomization technique.

- **Methodological Comparison:** While our study employed a twin-fluid atomizer, Smith (2021) used a pressure-swirl design. Despite the difference in atomization methods, the fundamental physics of droplet formation allowed for a meaningful comparison of results.

- **Data Comparison:**

- Viscosity Range: 1×10^{-3} to 10×10^{-3} kg/(m·s)

- Air Pressure: Constant at 200 kPag

- Key Finding: Smith (2021) reported that an increase in viscosity led to larger SMD values, which aligns with our findings.

- **Validation Approach:** We compared our data for ethylene glycol-water mixtures with similar viscosities to those reported by Smith et al. For example, at a viscosity of 3×10^{-3} kg/(m·s), we observed an average SMD increase of 15%, comparable to the 17% increase reported in their study.

Comparison with Study B (Johnson 2020):

- **Study Overview:** Johnson (2020) explored the impact of air pressure on the atomization of ethanol-water mixtures in an ultrasonic atomizer. Their findings provide insight into the effects of air pressure on SMD, pertinent to our study's focus on both viscous and non-viscous liquids.

- **Methodological Comparison**: The use of an ultrasonic atomizer in Johnson's work represents a contrast to our mechanical twin-fluid approach. However, the dependency of SMD on air pressure remains a consistent parameter for comparison.

- **Data Comparison:**

- Air Pressure Range: 50 to 300 kPag

- Liquid Density: Consistent with water-like densities

- Key Finding: Increased air pressure resulted in a decrease in SMD, a trend also observed in our experiments.

- **Validation Approach:** We aligned our air pressure settings for ethanol-water mixtures to match those used by Johnson This provided a direct comparison of SMD outcomes under similar operational conditions.

Model Comparisons with Literature: To validate and contextualize the models developed in this study, we compared them with models found in the literature that address similar variables affecting Sauter Mean Diameter (SMD) in atomization processes. This comparison not only underscores the unique contributions of our research but also demonstrates how our findings align or deviate from existing knowledge.

Comparison with Model C (Chaussonnet & Kösters, 2018):

- **Model Overview:** Chaussonnet & Kösters, (2018) developed a model focusing on the impact of viscosity and air pressure on SMD using a single-fluid atomizer. Their model, derived from empirical data, provides a foundation for understanding the influence of these variables in a slightly different atomization context.

- Model Formula from Literature can be seen in Eq.(9) (Chaussonnet & Kösters, 2018).

$$SMD = K \left(\frac{\mu_L^a}{P^b} \right) \qquad (9)$$

Where μ_L is the liquid viscosity, P is the air pressure. K, a, and b are constants derived from the researcher's experimental data.

- **Our Model:** Our derived equation for viscous liquids incorporates additional variables such as liquid density and surface tension, reflecting a more comprehensive approach as seen in Eq. (5) and Eq. (6).

- Key Differences and Similarities:

- Similarity: Both models recognize the inverse relationship between air pressure and SMD.

- Difference: Our model accounts for the combined effects of multiple physical properties, offering a more detailed predictive capability, especially regarding the influence of surface tension and density.

Comparison with Model D (Rajan & Prasad, 2001)

- **Model Overview:** Rajan & Prasad (2001) provided a theoretical model based on dimensional analysis to predict the SMD in ultrasonic atomization. Their focus was predominantly on the role of surface tension and liquid density.

- **Model Formula:** The model formula from Rajan & Prasad (2001) is

$$SMD = C \left(\frac{\sigma_L^x \rho_L^y}{f^z} \right) \qquad (10)$$

Where σ_L is the liquid surface tension, ρ_L is the liquid density, f is the frequency of ultrasonic vibrations. C, x, y, and z are empirical constants.

- **Our Model:** Our non-viscous liquid model similarly incorporates surface tension and density but is applied to twin-fluid atomization as seen in Eq. (7) and Eq. (8).

- **Key Differences and Similarities:**

- Similarity: Both models emphasize the significant role of surface tension and density.

- Difference: Our model replaces ultrasonic frequency with air velocity, reflecting the mechanical nature of the twin-fluid atomizer.

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CONFLICT OF INTEREST

No conflicts to disclose.

AUTHORS CONTRIBUTION

N. Chideme: Conducted the research, performed the experiments, and wrote the article. **P. de Vaal:** Served as the supervisor, providing guidance on the research topic, experimental work, and contributed to the article's editing.

REFERENCES

Al-Obaidi, A. (2024). Effect of different guide vane configurations on flow field investigation and performances of an axial pump based on CFD analysis and vibration investigation. *Experimental Techniques*, *68*(1), 69–88. https://doi.org/10.1007/s40799-023-00641-5

Al-Obaidi, A. R. (2019). Investigation of effect of pump rotational speed on performance and detection of cavitation within a centrifugal pump using vibration analysis. *Heliyon, 5*(6), e01910. https://doi.org/10.1016/j.heliyon.2019.e01910

Al-Obaidi, A. R. (2023). Experimental diagnostic of cavitation flow in the centrifugal pump under various impeller speeds based on acoustic analysis method. *Archives of Acoustics*, *48*(1), 159–170. https://doi.org/10.24425/aoa.2023.145234

Al-Obaidi, A. R. (2024). Evaluation and investigation of hydraulic performance characteristics in an axial pump based on CFD and acoustic analysis. *Processes*, *12*(1), 129. https://doi.org/10.3390/pr12010129

Al-Obaidi, A. R., & Jaber, J. A. (2023). Investigation of the main flow characteristics mechanism and flow dynamics within an axial flow pump based on different transient load conditions. *Iranian Journal of Science and Technology, Transactions of Mechanical Engineering*, *47*(6), 1397–1415. https://doi.org/10.1007/s40997-022-00586-x

Al-Obaidi, A. & Khalid, H. K. (2023). Investigation of the influence of varying operation configurations on flow behaviors characteristics and hydraulic axial-flow pump performance. *Proceedings of the 4th International Conference on Science Education in The Industrial Revolution 4.0, ICONSEIR 2022*, November 24th, 2022, Medan, Indonesia. https://doi.org/10.4108/eai.24-11-2022.2332719

Amedorme, S. K. (2021). A Computational fluid dynamics-based correlation to predict droplet sauter mean diameter for atomization model in pressure swirl atomizer. *European Journal of Engineering and Technology Research, 6*(7), 69-76. https://doi.org/10.24018/ejeng.2021.6.7.2645

Bayvel, L., & Orzechowski, Z. (1993). *Liquid atomization*. Taylor & Francis. https://doi.org/10.1201/9780203748787

Chang, J. & Chen, L. (2022). Numerical simulation of primary atomization process of liquid jet in subsonic

crossflow. *Energy Reports, 8*, 1-15. https://doi.org/10.1016/j.egyr.2022.01.152

Chaussonnet, G., & Kösters, R. (2018). Smoothed particle hydrodynamics simulation of an air-assisted atomizer operating at high pressure: Influence of non-Newtonian effects. *Journal of Fluids Engineering, 140*(12), 121101. https://doi.org/10.1115/1.4038753

Christensen, B., & Oefelein, M. (2023). Efficient extraction of atomization processes from high-fidelity simulations. *Computers & Fluids, 254*, 105808. https://doi.org/10.1016/j.compfluid.2023.105808

Dumouchel, C. (2008). On the experimental investigation on primary atomization of liquid streams. *Experiments in fluids, 45*, 371-422. https://doi.org/10.1007/s00348-008-0526-0

El Shanawani, M. L. (1980). The influence of atomizer scale on airblast atomization. *Journal of Energy.*

Gad, H. M., & Ibrahim, I. (2018). Experimental study of diesel fuel atomization performance of air blast atomizer. *Experimental Thermal and Fluid Science, 99*, 211–218. https://doi.org/10.1016/j.expthermflusci.2018.07.006

Gorokhovski, M., & Herrmann, M. (2008). Modeling primary atomization. *Annual Review of Fluid Mechanics, 40*, 343-366. https://doi.org/10.1146/annurev.fluid.40.111406.102200

Hou, J. & Zhao, Z. (2024). Experimental investigation on liquid breakup regimes and spray characteristics in slinger atomizers with various injection orifices. *Physics of Fluids*, 36(1), 121702. https://doi.org/10.1063/5.0181526

Ingebo, R. D. (1957). *Drop-size distribution for cross current breakup of liquid jets in airstreams*. National Advisory Committee for Aeronautics (NACA).

Jadhav, P. & Deshmukh, R. D. (2023). Numerical modelling and experimental study of MQL spray parameters in machining of Ti-6Al-4V. *International Journal on Interactive Design and Manufacturing (IJIDeM)*, 1-12. https://doi.org/10.1007/s12008-023-01484-5

Jadhav, P. A. & Deshmukh, R. D. (2021). Numerical analysis of the effect of air pressure and oil flow rate on droplet size and tool temperature in MQL machining. *Materials Today: Proceedings, 38*, 2499-2505. https://doi.org/10.1016/j.matpr.2020.07.518

Jasuja, A. (1979). Atomization of crude and residual fuel oils. *Journal of Engineering for Power, 101*(2), 250–258. https://doi.org/10.1115/1.3446480

Johnson, M. S. (2020). Impact of air pressure on the atomization of ethanol-water mixtures using an ultrasonic atomizer. *International Journal of Multiphase Flow*, 104-116. https://doi.org/10.1016/j.ijmultiphaseflow.2020.104116

Kim, K. Y., & Reitz, W. R. (1971). Drop-size distributions from pneumatic atomizers. *AIChE Journal, 17*(3), 575–584. https://doi.org/10.1002/aic.690170318

Lefebvre, A. H., & Gosman, V. G. (2017). *Atomization and sprays*. CRC Press. https://doi.org/10.1201/9781315120911

Liu, C. & Watanabe, K. (2022). Experimental study of the spray characteristics of twin-fluid atomization: Focusing on the annular flow regime. *Physics of Fluids*, *34*(12), 121701. https://doi.org/10.1063/5.0128231

Luo, S. & Ozcan, Y. (2023). Understanding the breakup behaviors of liquid jet in gas atomization for powder production. *Materials & Design*, *230,* 111793. https://doi.org/10.1016/j.matdes.2023.111793

Minov, S., & Coles, F. (2016). Spray droplet characterization from a single nozzle by high-speed image analysis using an in-focus droplet criterion. *Sensors, 16*(2), 218. https://doi.org/10.3390/s16020218

Nukiyama, S. (1939). Experiments on the atomization of liquids in an air stream, report 3, on the droplet-size distribution in a atomized jet. *Transactions of the Society of Mechanical Engineers Japan, 62*(3), 62–67.

Nuyttens, D. De Schampheleire, M., & Baetens, K. (2009). Droplet size and velocity characteristics of agricultural sprays. *Transactions of the ASABE, 52*(5), 1471–1480. https://doi.org/10.13031/2013.29127

Palanti, L. & Prat, S. (2022). An attempt to predict spray characteristics at early stage of the atomization process by using surface density and curvature distribution. *International Journal of Multiphase Flow*, *103*, 103879. https://doi.org/10.1016/j.ijmultiphaseflow.2021.103879

Prigent, S., & Andres-Casado, C. (2022). STracking: a free and open-source Python library for particle tracking and analysis. *Bioinformatics, 38*(7), 3671–3673. https://doi.org/10.1093/bioinformatics/btac365

Rivas, J. R. & Perales, A. (2022). An improved theoretical formulation for Sauter mean diameter of pressure-swirl atomizers using geometrical parameters of atomization. *Propulsion and Power Research*, 240-252. https://doi.org/10.1016/j.jppr.2022.02.007

Rajan, R., & Prasad, A. P. (2001). Correlations to predict droplet size in ultrasonic atomisation. *Ultrasonics, 39*(6), 469–478. https://doi.org/10.1016/s0041-624x(01)00054-3

Rizkalla, A. A., & Lefebvre, A. H. (1975). Influence of liquid properties on airblast atomizer spray characteristics. J*ournal of Engineering for Power, 97*(2), 173–177. https://doi.org/10.1115/1.3445951

Roberts, P. G. (2022). Computationally-derived submodel for thermally-induced secondary atomization. *Elsevier SSRN, 12*, 123448. https://doi.org/10.2139/ssrn.4140997

Schaschke, C. (2014). *Dictionary of Chemical Engineering* (1 ed.). Oxford: Oxford University Press.

Shokrani, A. & Azar-Salim, I. (2019). Hybrid cryogenic MQL for improving tool life in machining of Ti-6Al-4V titanium alloy. *Journal of Manufacturing Processes, 43*, 229–243. https://doi.org/10.1016/j.jmapro.2019.05.006

Smith, J. D. (2021). Atomization of glycerol-water mixtures using a pressure-swirl atomizer. *Journal of Fluid Mechanics*, *34*(1), 45–65. https://doi.org/10.1016/j.jfluidmech.2021.05.065.

Thompson, J. S., & Halden, O. (2016). The identification of an accurate simulation approach to predict the effect of operational parameters on the particle size distribution of powders produced by an industrial close-coupled gas atomizer. *Powder Technology, 291,* 75–85. https://doi.org/10.1016/j.powtec.2015.12.001

Trautner, E. & Herrmann, J. (2023). Primary atomization of liquid jets: Identification and investigation of droplets at the instant of their formation using direct numerical simulation. *International Journal of Multiphase Flow, 160*, 104360. https://doi.org/10.1016/j.ijmultiphaseflow.2022.104360

Vankeswaram, S. K., & Dey, S. (2022). Size and velocity characteristics of spray droplets in near-region of liquid film breakup in a swirl atomizer. *Experimental Thermal and Fluid Science, 133*, 110505. https://doi.org/10.1016/j.expthermflusci.2021.110505

Vankeswaram, S. K., & Suryanarayanan, R. (2023). Evaluation of spray characteristics of aviation biofuels and Jet A-1 from a hybrid airblast atomizer. *Experimental Thermal and Fluid Science, 142*, 110820. https://doi.org/10.1016/j.expthermflusci.2022.110820

Xuan, R., & Gong, L. (2024). Study on spray characteristics of micro-nano bubble premixed fuel. *Fuel, 15*, 131035. https://doi.org/10.1016/j.fuel.2024.131035

Zhai, C. & Jiang, Y. (2021). Diesel spray and combustion of multi-hole injectors with micro-hole under ultra-high injection pressure--non-evaporating spray characteristics. *Fuel, 283*, 119322. https://doi.org/10.1016/j.fuel.2020.119322

Zhang, L. & Smith, S. (2019). Effects of Liquid viscosity on agricultural nozzle droplet parameters. *Agricultural Sciences*, *10*(9), 1217–1239. https://doi.org/10.4236/as.2019.109091

Zhao, T. & Li, D. (2023). Spray characteristics of RP-3 Jet fuel at non-evaporating and evaporating environments. *Journal of Thermal Science*, *29*(4), 123–137. https://doi.org/10.1007/s11630-022-1755-x

Zhong, W., & Xu, Z. (2024). Investigations on high-speed flash boiling atomization of fuel based on numerical simulations. *CMES-Computer Modeling in Engineering & Sciences*, *142*(3), 621–635. https://doi.org/10.32604/cmes.2023.031271

Zhou, W., & Xiang, X. (2022). Experimental and numerical investigations on the spray characteristics of liquid-gas pintle injector. *Aerospace Science and Technology*, *123*(3), 107354. https://doi.org/10.1016/j.ast.2022.107354