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Scattering of Fexural Gravity Waves by a Two-Dimensional Thin Plate

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ABSTRACT

An approximate analysis based on standard perturbation technique together with an application of Green's integral theorem is used in this paper to study the problem of scattering of water waves by a two dimensional thin plate submerged in deep ocean with ice cover. The reflection and transmission coefficients upto first order are obtained in terms of the shape function describing the plate and are studied graphically for different shapes of the plate.

Keywords: Water wave scattering; Two dimensional thin plate; Nearly vertical plate; Reflection coefficient; Transmission coefficient.

1. INTRODUCTION

The study of ocean wave interactions with a thin, floating elastic plate has gained immense importance since last decade as it can be used to model a wide range of physical systems. One of its important applications consists in modelling a very large floating structure (VLFS), that is used in ocean space utilization for the construction of megafloats such as floating airports, offsh ore runways, floating restaurant etc. It is a technology that allows these megafloats, which are considered to be artificial lands to float on rising sea level and has a minimal effect on marine habitat, natural and tidal current flow (cf Wang *et al*. 2010; Wadhams 1978). Owing to the large surface area and relatively small depth, VLFS behaves elastically under wave action (cf Wang *et al*. 2010). In the polar region, surface gravity waves propagate from the open ocean into ice-covered seas. Understanding the modus operandi of formation of sea ice and its distribution is imperative to explain the geophysical phenomena occurring in the polar regions and in the marginal ice zone. A precinct between ocean and atmosphere, the sea ice arrests the escape of heat from the ocean to the air above. Consequently it plays a crucial role in conservation of marine life. An uninterrupted expanse of unbroken ice over a vast stretch in the polar region often encounters waves propagating at free surface. It is well known that waves may weaken and rupture the continuous sea ice causing fissures which may lead to melting of sea ice. This phenomena is an indicator of global climatic change. The amplitude of the waves travelling beneath the ice needs to be studied as it causes the ice-cover to bend. The bending of ice-cover is attributed to its elastic property. A mathematical model for treating the ice sheet as floating thin elastic plate is well known and a significant research has been carried out using this model to study the problems related to ocean wave interaction with sea ice (cf. Fox and Squire 1994; Squire 2007; Chung and Fox 2002; Linton and Chung 2003; Chakrabarti 2000; Gayen *et al*. 2005). In order to minimize the impact of wave action on a VLFS or ice sheet, various anti motion structures and devices such as break-waters, submerged plates, oscillating water column breakwater, air cushion, curtain pile breakwater are designed (cf Wang 2010; Tari and Ohkubo 2000). Also, a number of experiments measuring wave propagation through marginal ice zone have been reported of which first measurement was carried out by a ship borne wave recorder (cf Kohout and Meylan 2008). Later, measurements were carried out by a echo sounder from a submerged hovering submarine, acoustic Doppler Current Profiler mounted on an autonomous under water vehicle (cf Kohout and Meylan 2008, Wadhams 1978). Thus the study of the waves in presence of thin plate under ice cover or VLFS is important. Mathematically, the boundary value problem (BVP) related to study of water waves in ocean with ice-cover, involves fifth order derivative of the potential function in the boundary condition on ice cover whereas the governing partial differential equation is of second order. The literature concerning the study of ocean wave interaction in ocean with ice-cover in the presence of a body submerged beneath the ice-cover floating in a deep water is rather limited, although the study of ocean wave interaction with structures present in the ocean with free surface under linearised theory has been a subject of interest since early twentieth century. A number of researchers contributed significantly to this topic, although the closed form solution to these problems are available only when the structure is in form of a thin rigid vertical plate and that too for the two dimensional motion in water. Diffraction problems involving nearly vertical barriers are more general than vertical barrier. One such problem of water waves scattering by a nearly vertical plate partially immersed in deep water was considered by Shaw (1985). He used a perturbation analysis that involved solution of singular integral equation. Later Mandal and Chakrabarti (1989) and Mandal and Kundu (1990) considered the problems of water waves scattering by a nearly vertical barrier and utilized a perturbation analysis different from Shaw (1985) to handle the problems. The problem of water wave diffraction by a symmetric two dimensional thin slender was plate mentioned briefly by Shaw (1985) although the first order correction to reflection and transmission coefficients are not given there explicitly. Later Kundu (1997), Kundu and Saha (1998) considered the problem of water wave scattering by a thin two dimensional slender body either partially immersed or completely submerged or submerged in deep water. They used the perturbation technique described in Mandal and Chakrabarti (1989) to obtain first order correction to reflection and transmission coefficients in terms of the shape functions of two sides of the slender barrier. All the above mentioned wave structure interaction problems were considered when the water region is covered by a free surface. In recent past Das and Mandal (2007) investigated the problem of ocean water and sea ice interaction in presence of a long horizontal cylinder. Maiti and Mandal (2010), Maiti *et al*. (2011) studied the ocean wave interaction with a thin vertical barrier present in ocean with ice cover. They used Green's integral theorem to reduce the corresponding boundary value problem to a hyper-singuler integral equation which was then solved by collocation method. In the present paper we have studied the problem of scattering of ocean waves by two dimensional thin plate submerged in ocean with ice cover. Using the perturbation analysis as given by Mandal and Chakrabarti (1989), together with application of Green's integral theorem, the first order correction to the reflection and transmission coefficient are obtained in terms of the shape function describing the shape of two sides of the plate, which are then studied graphically for various values of wave number and different values of ice cover parameter. The reflection and transmission coefficients up to first order due to the two dimensional thin barrier are compared with those due to one dimensional nearly vertical barrier submerged in ocean with ice-cover. It was also observed that when the value of ice-cover parameters are small, the reflection and transmission coefficients matches with the results in 1998 when the ocean is covered by a free surface. It is observed that unlike nearly vertical plate, the thickness of the symmetric two dimensional plate has some effect on the reflection and transmission coefficient. From the graph it is noted that in presence of a symmetric two dimensional plate, the long waves do not feel the presence of ice-cover as they are confined towards the bottom of the ocean. However, the short waves which are confined near the ice-cover surface are affected by the presence of ice-cover. It is observed that long plate induces more reflection of wave energy and less transmission. Also for a particular length of the plate, the increase in ice cover parameter induces more transmission of wave energy.

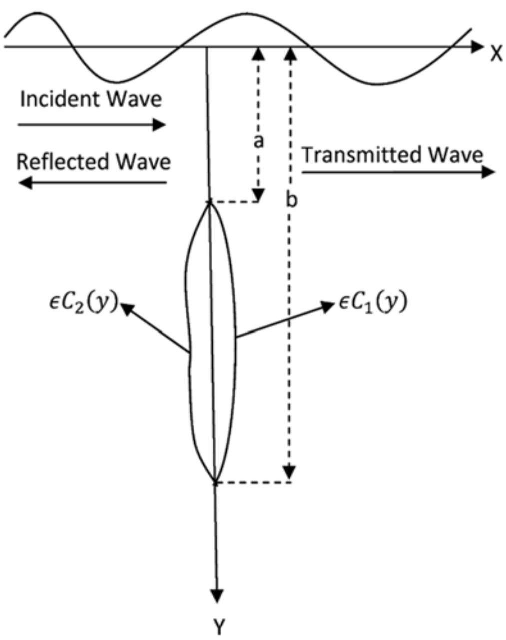

Fig. 1. Schematic sketch of two dimensional thin plate.

2. FORMULATION OF THE PROBLEM

We consider two dimensional irrotational motion in ocean with ice cover due to scattering of time harmonic incident wave by a two dimensional thin barrier submerged in infinitely deep ocean. We choose a rectangular cartesian coordinate system in which y axis is vertically downwards into fluid region y ≥ 0 and x axis is along rest position of lower part of ice-cover. Ice-cover is modelled as a thin elastic plate of thickness h_1 and density ρ_1 with flexural rigidity $L = \dfrac{Eh_1^3}{12(1-\nu^2)}$ where E is the

Young's modulus, ν is the Poisson ratio of the elastic material of the ice-cover. A thin two dimensional rigid plate described by $S = S_1 \cup S_2$ where S_1 and S_2 denote two sides of two dimensional thin plate given respectively by $x = \varepsilon C_1(y)$ and $x = \varepsilon C_2(y), a \le y \le b$ (cf figure 1).

Here ε is a non dimensional small parameter which can be regarded as a measure of thinness of the plate and $C_i(y)$, $i = 1,2$ are bounded continuous function of y with $C_i(a) = C_i(b) = 0, i = 1,2.$. A train of time harmonic waves represented by velocity potential $\mathrm{Re}\left\{\phi^{inc}(x,y)e^{-i\sigma t}\right\}$ where σ is the circular frequency, is incident upon the barrier from negative infinity and is partially reflected by the barrier and partially transmitted over and below the barrier.

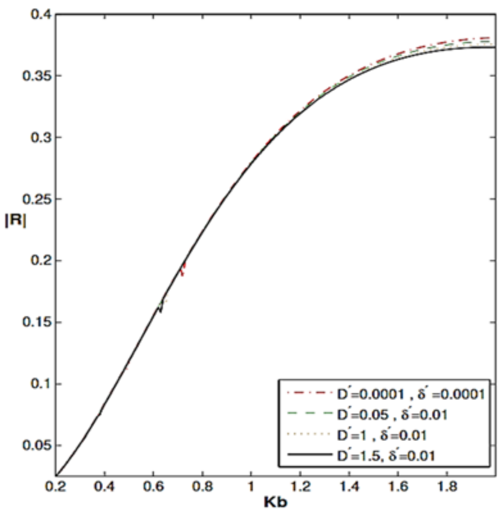

Fig. 2. |*R*| versus *Kb* for different values of *D'* (*u* = 0.01 and ε = 0.005).

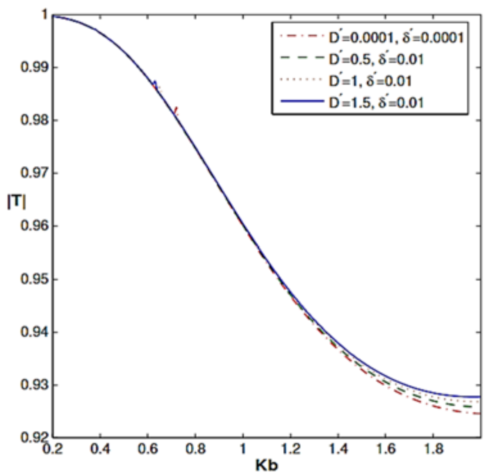

Fig. 3. |*T*| versus *Kb* for different values of *D'* (*u* = 0.01 and ε = 0.005).

Assuming linearised theory, the motion is described by the velocity potential $\mathrm{Re}\left\{\phi(x,y)e^{-i\sigma t}\right\}$ where φ satisfies,

$$\nabla^2\phi = 0,\ y \ge 0, \tag{1}$$

the linearised ice cover condition (cf Landau and Lipschitz (1970) pp 44),

$$(D\frac{\partial^4}{\partial x^4} - \delta K + 1)\phi_y + K\phi = 0 \text{ on y=0,} \tag{2}$$

the condition on the barrier,

$$\begin{aligned} &\frac{\partial\phi}{\partial n}(x,y) = 0 \text{ on } x = \varepsilon C_1(y), \quad a < y < b \\ &\frac{\partial\phi}{\partial n}(x,y) = 0 \text{ on } x = \varepsilon C_2(y) \quad a < y < b, \end{aligned} \tag{3}$$

where $\frac{\partial}{\partial n}$ denotes the normal derivative to the barrier,

the edge condition,

$$r^{\frac{1}{2}}\nabla\phi \text{ is bounded as } r \to 0, \tag{4}$$

the bottom condition

$$\nabla\phi \to 0 \quad \text{as y} \to \infty \tag{5}$$

Also φ(x,y) satisfies the far field conditions given by

$$\phi(x,y) \sim \begin{cases} \phi^{inc}(x,y) + R\phi^{inc}(-x,y) & \text{as } x \to -\infty, \\ T\phi^{inc}(x,y) & \text{as } x \to -\infty. \end{cases} \tag{6}$$

Here *R* and *T* are the reflection and transmission coefficients. Also,

$$\phi^{inc}(x,y) = e^{-\lambda Ky + i\lambda Kx}, \tag{7}$$

where $k = \lambda K$ is the unique positive root of the dispersion relation

$$\left(Dk^4 + 1 - \delta K\right)k - K = 0, \tag{8}$$

Here $K = \frac{\sigma^2}{g}$, $g =$ acceleration due to gravity, $D = \frac{L}{\rho g}$, L is the flexural rigidity of ice-cover as mentioned before, ρ is the density of fluid and $\delta = \frac{\rho_1}{\rho}h_1$. .

The other roots of (2) are $\lambda_1 K$, $\overline{\lambda_1}K$, $\lambda_2 K$ and $\overline{\lambda_2}K$ with $\mathrm{Re}(\lambda_1) > 0$ and $\mathrm{Re}(\lambda_2) < 0$. A detailed discussion on the roots of dispersion relation (2)is given in Chakrabarti *et al*. (2003).

Assuming that the parameter ε > 0 is very small and neglecting o (ε^2) terms, the boundary condition (3) can be approximately written as

$$\phi(x,y)\sim\begin{cases}\left.\dfrac{\partial\phi}{\partial x}\right|_{(+0,y)}=\varepsilon\dfrac{d}{dy}\{C_1(y)\phi_y(+0,y)\}\\ \qquad a<y<b,\\ \left.\dfrac{\partial\phi}{\partial x}\right|_{(-0,y)}=\varepsilon\dfrac{d}{dy}\{C_2(y)\phi_y(-0,y)\}\\ \qquad a<y<b.\end{cases}\tag{9}$$

3. METHOD OF SOLUTION

The form of the approximate boundary condition given by (9) suggest that we may expand the function $\phi(x,y)$ and the two unknown physical constant R and T in terms of the small parameter ε as

$$\phi(x,y)=\phi_0(x,y)+\varepsilon\phi_1(x,y)+o(\varepsilon\hat{}2),\tag{10}$$

$$R=R_0+\varepsilon R_1+o(\varepsilon^2),\tag{11}$$

$$T=T_0+\varepsilon T_1+o(\varepsilon^2).\tag{12}$$

Substituting ϕ , *R, T* from (10), (11), (12) into (1),(2), (5) to (7) and equating the coefficients of ε^0 and ε^1 from both sides of the equations we find that ϕ_0 and ϕ_1 satisfies the following two boundary value problems(*BVPs*).

BVP − 0: The function ϕ_0 satisfies

$$\nabla^2\phi_0=0,\ \ y\ge 0,\tag{13}$$

$$(D\frac{\partial^4}{\partial x^4}-\delta K+1)\phi_{0y}+K\phi_0=0\quad\text{on}\quad y=0,\tag{14}$$

$$\phi_{0x}=0,\quad\text{on}\ \ x=0,\quad a<y<b,\tag{15}$$

$$r^{\frac{1}{2}}\nabla\phi_0\ \text{is bounded as}\ r\to 0,\tag{16}$$

$$\nabla\phi_0\to 0\quad\text{as}\quad y\to\infty,\tag{17}$$

$$\phi_0(x,y)\sim\begin{cases}\phi^{inc}(x,y)+R_0\phi^{inc}(-x,y) & \text{as}\ x\to-\infty\\ T_0\phi^{inc}(x,y) & \text{as}\ x\to\infty.\end{cases}\tag{18}$$

BVP − 1: The function ϕ_1 satisfies

$$\nabla^2\phi_1=0,\ \ y\ge 0,\tag{19}$$

$$(D\frac{\partial^4}{\partial x^4}-\delta K+1)\phi_{1y}+K\phi_1=0\quad\text{on}\quad y=0,\tag{20}$$

$$\left.\frac{\partial\phi_1}{\partial x}\right|_{(+0,y)}=\frac{d}{dy}\{C_1(y)\phi_{0y}(+0,y)\}\ a<y<b,$$
$$\left.\frac{\partial\phi_1}{\partial x}\right|_{(-0,y)}=\frac{d}{dy}\{C_2(y)\phi_{0y}(-0,y)\}\ a<y<b\tag{21}$$

$$r^{\frac{1}{2}}\nabla\phi_1\ \text{is bounded as}\ r\to 0,\tag{22}$$

$$\nabla\phi_1\to 0\quad\text{as}\quad y\to\infty,\tag{23}$$

$$\phi_1(x,y)\sim\begin{cases}R_1\phi^{inc}(-x,y) & \text{as}\ x\to-\infty,\\ T_1\phi^{inc}(x,y) & \text{as}\ x\to\infty.\end{cases}\tag{24}$$

The function $\phi_0(x,y)$ satisfying BVP-0 which describes the problem of scattering of time harmonic wave by a thin rigid vertical barrier submerged in ocean with ice-cover. The explicit solution of BVP-0 is known from Maiti *et al*. (2011) and is given by

$$\phi_0(\xi,\eta)=\phi^{inc}(\xi,\eta)-\frac{1}{2\pi}\int_a^b\Psi(y)\frac{\partial G}{\partial x}(0,y;\xi,\eta)dy,\tag{25}$$

$$\Psi(y)=\phi_0(+0,y)-\phi_0(-0,y),\tag{26}$$

$$\text{with}\ \ \Psi(a)=\Psi(b)=0\tag{27}$$

and $G(x,y;\xi,\eta)$ is the source potential due to presence of a line source at point (ξ,η), where *G* satisfies the following BVP:

$$\nabla^2G=0\ \text{except at}\ (\xi,\eta),\tag{28}$$

$$G\sim\ln\rho\ \text{as}\ \rho=(\xi,\eta)\to 0,\tag{29}$$

$$\{D\frac{\partial^4}{\partial x^4}-(1-\varepsilon K)\}G_y+KG=0\quad\text{on}\ y=0,\tag{30}$$

$$\nabla G\to 0\ \text{as}\ y\to\infty,\tag{31}$$

G behaves as outgoing waves as $|x-\xi|\to\infty$. The expression for G(x,y;ξ,η) is given by (cf. Maiti *et al*. (2011))

$$\begin{aligned}G(x,y;\xi,\eta)=\\ &-2\int_0^\infty\frac{L(k,y)L(k,\eta)}{k\{k^2(1-\delta K+DK\hat{}4)+K^2\}}e^{-k|x-\xi|}dk\\ &-2\pi i\frac{1}{\lambda(5DK^4\lambda^4-\delta K+1)}e^{-K\lambda(y+\eta)+i\lambda k|x-\xi|}\\ &-2\pi i\frac{1}{\lambda_1(5DK^4\lambda_1^4-\delta K+1)}e^{-K\lambda_1(y+\eta)+i\lambda_1 k|x-\xi|}\\ &+2\pi i\frac{1}{\bar{\lambda}_1(5DK^4\bar{\lambda}_1^4-\delta K+1)}e^{-K\bar{\lambda}_1(y+\eta)-i\bar{\lambda}_1 k|x-\xi|}.\end{aligned}\tag{32}$$

Here $\Psi(y)$ is unknown function which satisfies a hypersingular integral equation. The detailed derivation of the hypersingular integral equation in $\Psi(y)$ and its solution is given in the Appendix A.

The physical quantities R_1 and T_1 can be obtained from BVP-1 by a judicious application of Green's integral theorem described below.

Determination of R_1 : To find R_1 applying Green's

Table 1 $|R_F|$ and $|R|$ vs *u* for for Kb=1.4, Ɛ=0.001

u	D′ = 10−4, δ′ = 10−4		D′ = 0, δ′ = 0	
	|R| (NVP)	|R| (S2DTP)	|RF | (NVP)	|RF| (S2DTP)
0.01	0.655315	0.654387	0.66893	0.655811
0.02	0.586879	0.586307	0.593985	0.586969
0.05	0.466643	0.466492	0.46955	0.466607
0.1	0.349997	0.350059	0.351313	0.349903
0.25	0.170384	0.17045	0.170689	0.170291
0.5	0.0489999	0.0490209	0.0490455	0.0489674
0.75	0.00837073	0.0083743	0.00837353	0.00836262

Table 2 $|R|$ vs *u* for *Kb* = 1.4, *D′* = 1, δ′ = 0.01

u	ε = 0.001, ε = 0.005		|*R*| (NVP), |*R*| (S2DTP)	
	|R| (NVP)	|R| (S2DTP)	|RF | (NVP)	|RF| (S2DTP)
0.01	0.655262	0.651633	0.655263	0.637301
0.02	0.586716	0.584379	0.586717	0.575092
0.05	0.46599	0.465225	0.465991	0.462174
0.1	0.347933	0.347909	0.347938	0.347793
0.25	0.158326	0.158385	0.158326	0.153954
0.5	0.0320719	0.0320925	0.0320719	0.0294683
0.75	0.00416634	0.00407001	0.00416634	0.00384524

Table 3 $|R|$ vs *u* for *Kb* = 1, *D′* = 1.5, δ′ = 0.01

u	ε = 0.001, ε = 0.005		|*R*| (NVP), |*R*| (S2DTP)	
	|R| (NVP)	|R| (S2DTP)	|RF | (NVP)	|RF| (S2DTP)
0.01	0.655231	0.649617	0.655232	0.627587
0.02	0.586643	0.58301	0.586644	0.56862
0.05	0.465706	0.464499	0.465707	0.459684
0.1	0.347071	0.346976	0.347071	0.347793
0.25	0.153695	0.153747	0.153695	0.153954
0.5	0.029366	0.0293865	0.029366	0.0294683
0.75	0.00382713	0.00383075	0.00382713	0.00384524

integral theorem to the function $\phi_0(x,y)$ and $\phi_1(x,y)$ in the region Ω bounded by the lines

$y = 0, -X \le x \le X;\ x = -X,\ 0 \le y \le Y;\ y = Y,\ -X \le x \le X;$

$x = X,\ 0 \le y \le Y;\ x = 0+,\ a < y < b;\ x = 0-,\ a < y < b;$

and circles c_1, c_2 of small radius δ_0 with center at (0,*a*) and (0,*b*) where $X,Y > 0$. Making $X,Y \to \infty$, $\delta 0 \to 0$ and noting that $C_j(a) = C_j(b) = 0$, j = 1,2 we have

$$iR_1 = \int_a^b [\phi_{0y}^2(0^+,y)C_1(y) - \phi_{0y}^2(0^-,y)C_2(y)]dy. \quad (33)$$

For nearly vertical plate $C_1(y) = C_2(y) = C(y)$

$$R_1 = -i\int_a^b C(y)(\phi_{0y}^2(0^+,y)C_1(y) - \phi_{0y}^2(0^-,y))dy. \quad (34)$$

For symmetric two dimensional thin plate $C_1(y) = -C_2(y) = C(y)$, so that

$$R_1 = -i\int_a^b [\phi_{0y}^2(0^+,y)(y) + \phi_{0y}^2(0^-,y)(y)]C(y)dy. \quad (35)$$

Determination of T1: Applying Green's theorem to the functions $\phi_0(-x,y)$ and $\phi_1(x,y)$ in the region Ω, after simplifying we have

$$iT_1 = \int_a^b \phi_{0y}(0^-,y)\phi_{0y}(0^+,y)[C_1(y) - C_2(y)]dy. \quad (36)$$

For nearly vertical plate $C_1(y) = C_2(y) = C(y)$,

$$T_1 = 0. \quad (37)$$

For symmetric two dimensional thin plate $C_1(y) = -C_2(y) = C(y)$

$$T_1 = -2i\int_a^b \phi_{0y}(0^-,y)\phi_{0y}(0^+,y)C(y)dy. \quad (38)$$

4. NUMERICAL RESULTS

The interaction of surface waves with a two dimensional thin plate submerged in deep ocean with ice-cover is characterised by reflection and transmission coefficients. The reflection coefficient $|R| = |R_0 + \varepsilon R_1|$ and transmission coefficient $|T| = |T_0 + \varepsilon T_1|$ are computed up to first order of ε for different values of non dimensional ice cover parameter $D' = \frac{D}{a}$, $\delta' = \frac{\delta}{a}$ and wave number Kb and the ratio $u = \frac{a}{b}$. The different integrals in the numerical computation are evaluated by using Mathematica.

In Tables 1-3, $|R|$ is presented against u for a nearly vertical plate (NVP) described by $C_1(y) = C_2(y) = C(y)$ and for a symmetric two dimensional thin plate (S2DP) described by $C_1(y) = -C_2(y) = C(y)$ where $C(y) = \frac{(y-a)(b-y)}{(b-a)}, a < y < b.$

In figures 2-11, $|R|$ and $|T|$ up to first order of ε are presented graphically for a symmetric two dimensional plate described by the shape function $C_1(y) = -C_2(y) = C(y) = a\sin\frac{\pi(y-a)}{(b-a)},\ a < y < b.$

In figures 2 and 3 $|R|$ and $|T|$ are presented respectively against wave number Kb for $u = 0.1$, $\varepsilon = 0.005$ and for various values of ice-cover parameter D′. It is observed that for $u = 0.1$ $|R|$ almost coincide for different values of D' for wave number $Kb < 1.2$. However for $Kb > 1.2$, for any fixed value of Kb, $|R|$ diminishes as D' increases although this change in $|R|$ for different values of D' is not very significant. From figure 3, it is observed that increase of ice-cover parameter increases $|T|$ for a fixed length of plate although the change in $|T|$ is not much significant.

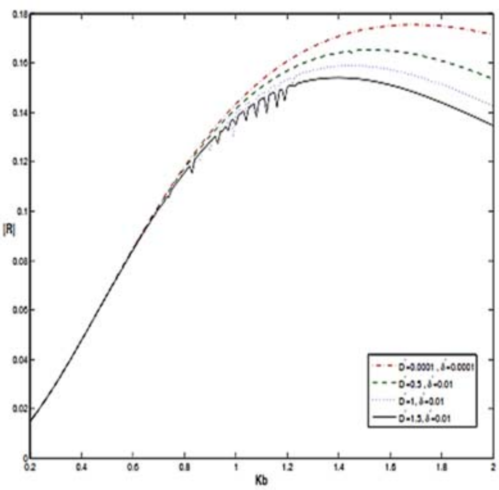

Fig. 4. |R| versus Kb for different values of *D′* (*u* = 0.25 and ε = 0.005).

Figures 4 and 5 shows the behavior of $|R|$ and $|T|$against Kb for $u = 0.25$, $\varepsilon = 0.005$ and the different values of D'. It is observed from figure 4, that $|R|$ coincides for different values of ice-cover parameter D' when $Kb < 0.7$. However, for $Kb > 0.7$, for any fixed Kb, $|R|$ decreases as D' increases.

From figure 5 it is found that $|T|$ increases as D′increases. The change in $|R|$ and $|T|$ for $Kb > 0.7$ is significant. Also, it is observed from figures 4 and 5 that $|R|$ and $|T|$ shows oscillatory behaviour for $0.8 < Kb < 1.2$, for $D' = 1,1.5$. However, for smaller values of D' this oscillation in $|R|$ and $|T|$ is not significant.

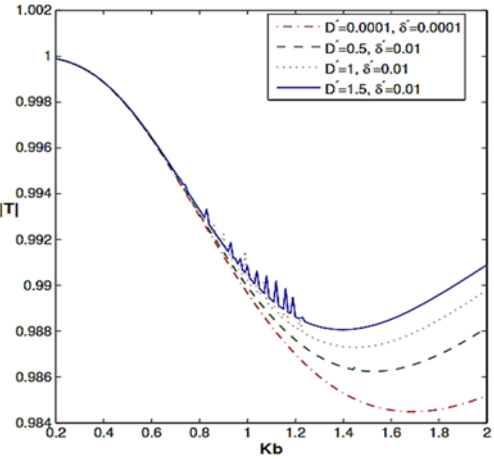

Fig. 5. |*T*| versus *Kb* for different values of *D′* (u = 0.25 and ε = 0.005).

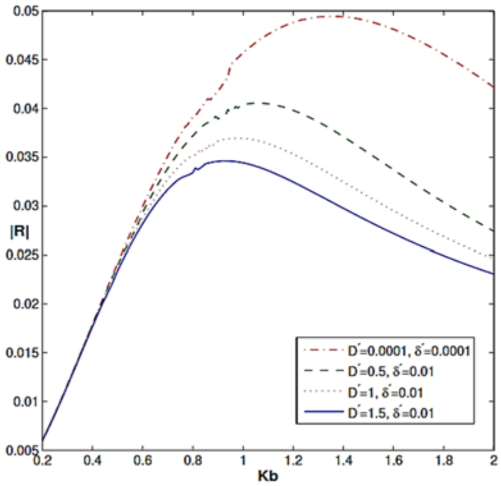

Fig. 6. |*R*| versus *Kb* for different values of *D′* (*u* = 0.5 and ε = 0.005).

Again from figures 6 and 7, $|R|$ and $|T|$ are plotted against Kb for $u = 0.5$, $\varepsilon = 0.005$ for different values D'. It is seen that for $Kb > 0.5$, $|R|$ decreases and $|T|$ increases for a particular Kb as D' increases.

Thus it is observed from figures 2-7, that for a particular length of plates, the long waves, which corresponds to small wave number, do not feel the presence of ice-cover as they are confined towards the bottom of the ocean. However, the short waves which are confined near the ice-cover surface are affected by the presence of ice-cover. It is observed that as the ice-cover parameter D' increases, $|R|$ diminishes and $|T|$ increases for a particular length of the plate. This may be attributed to the elastic property of the ice cover. Also, this change in $|R|$ and $|T|$ is significant as the length of the plate diminishes.

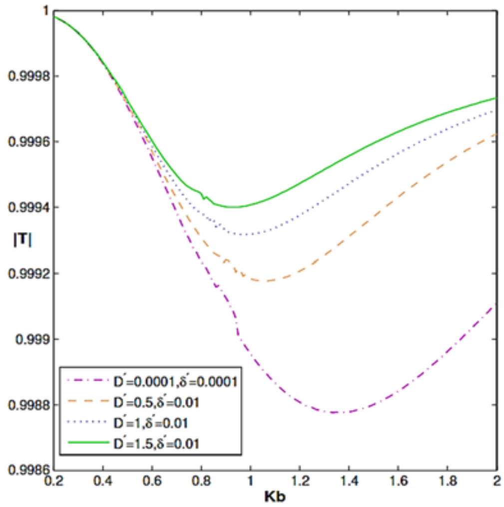

Fig. 7. $|T|$ **versus** Kb **for different values of** D' **(**u **= 0.5 and** ε **= 0.005).**

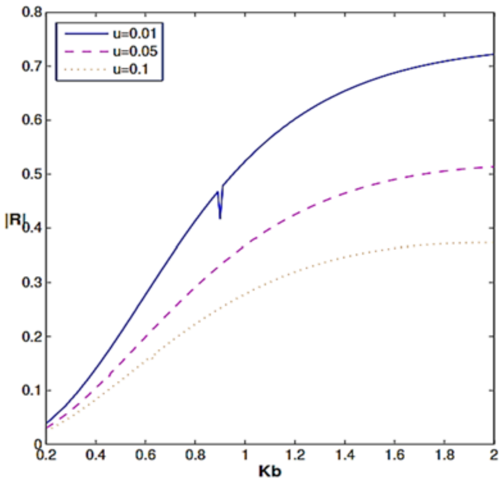

Fig. 8. $|R|$ **versus** Kb **for small values of** u **(**D' **= 1.5,** δ' **= 0.01).**

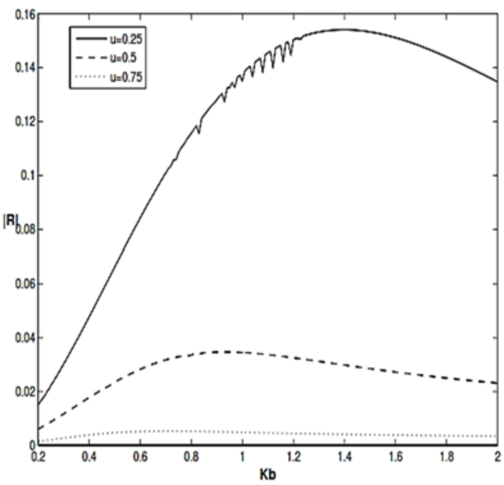

Fig. 9. $|R|$ **versus** Kb **for large values of of** u **(**D' **= 1.5,** δ' **= 0.01).**

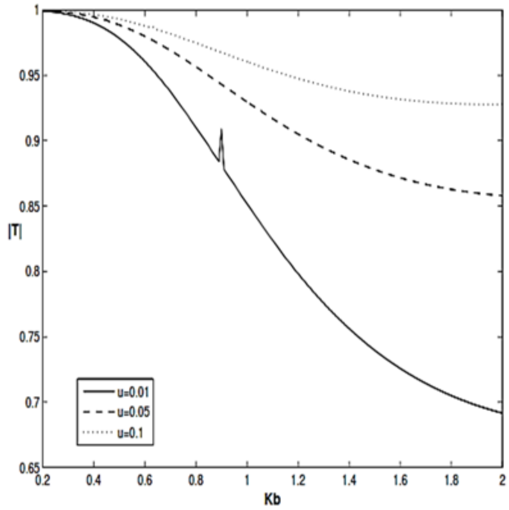

Fig. 10. $|R|$ **versus Kb for small values of** u **(**D' **= 1.5,** δ' **= 0.01).**

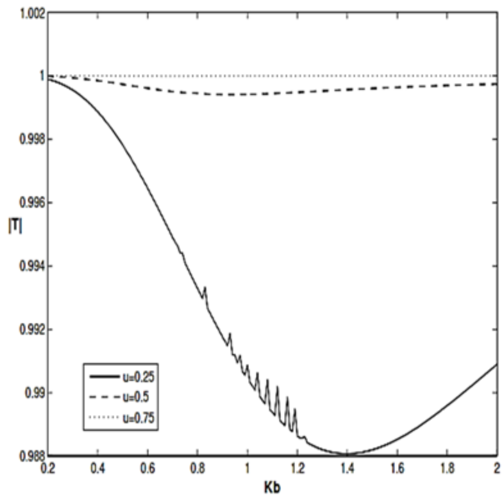

Fig. 11. $|T|$ **versus** Kb **for large values of** u **(**D' **= 1.5,** δ' **= 0.01.).**

In figures 8-11, $|R|$ and $|T|$ are plotted against Kb for $D' = 1.5$, $\varepsilon = 0.005$ for various lengths of the plate. It is observed from figures 8 and 9 that $|R|$ decreases as the length of the plate decreases while figures 10 and 11 show that $|T|$ increases with the decrease in the length of the plate for a fixed value of D' and ε. Thus long plate induces more reflection of wave energy. Also for a particular length of the plate, the increase in ice cover parameter induces more transmission of wave energy.

5. CONCLUSION

The problem of scattering of water waves by a two dimensional thin plate submerged in deep ocean with ice cover is observed. The reflection and transmission coefficients up to first order are compared with those due to one dimensional nearly vertical barrier submerged in ocean with ice-cover. From numerical results it is noted that in presence of a symmetric two dimensional plate, the long waves do not feel the presence of ice-cover as they are confined towards the bottom of the ocean. However, the short waves which are confined near the ice-cover surface are affected by the presence of ice-cover. It is observed that long plate induces more reflection of wave energy and less transmission. Also for a particular length of the plate, the increase in ice cover parameter induces more transmission of wave energy.

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APPENDIX A

Equation (27) involves the unknown function $\Psi(y) = \phi_0(+0, y) - \phi_0(-0, y)$, where $\phi_0(x, y)$ is the velocity potential describing the wave motion due to scattering of an incident wave by a thin vertical plate submerged in deep ocean with ice cover. This problem was studied by Maiti *et al*. (2011). Here we present in brief the methodology used in Maiti *et al* . (2011) to find $\phi_0(x, y)$.

Let us consider the function $\Psi(x, y)$ as

$$\Psi(x, y) = \phi_0(x, y) - \phi^{inc}(x, y) \qquad \text{A1}$$

where $\phi_0(x, y)$ satisfies BVP0 and $\phi^{inc}(x, y)$ is given by (7).

Then $\Psi(x, y)$ satisfies

$$\nabla^2\Psi = 0 \text{ on } y \geq 0 \quad \text{A2}$$

$$\{D\frac{\partial^4}{\partial x^4} + (1-\delta K)\}\Psi_y + K\Psi = 0 \text{ on } y=0 \quad \text{A3}$$

$$\Psi_x = -\frac{\partial}{\partial x}\phi^{inc}(0,y), \qquad a < y < b \quad \text{A4}$$

$$r^{\frac{1}{2}}\nabla\phi \text{ is bounded as } r \to 0 \quad \text{A5}$$

Also

$$\Psi(x,y) \to \begin{cases} R\phi^{inc}(-x,y) & \text{as } x \to -\infty, \\ (T-1)\phi^{inc}(x,y) & \text{as } x \to -\infty. \end{cases} \quad \text{A6}$$

Let G(x,y;ξ,η) be the source potential which describes the motion in water covered with ice due to presence of a line source at (ξ,η). The expression for G(x,y;ξ,η) is given by (32).

We now use Green's integral theorem to the harmonic function G(x,y;ξ,η) and Ψ(x,y) in the region bounded by the lines y = 0, −X ≤ x ≤ X; *x* =±*X, 0 ≤ y ≤ Y; y = Y, −X ≤ x ≤ X; x = 0*±*, a ≤ y ≤ b* and a circle of radius ε_0 with centre at (ξ,η) and ultimately we make *X, Y* → ∞ and $\varepsilon_0 \to 0$ to get

$$2\pi\Psi(\xi,\eta) = -\int_a^b \Psi(y)\frac{\partial G}{\partial x}(0,y;\xi,\eta)dy. \quad \text{A7}$$

Noting (A1), Eq. (A7) transforms to

$$\phi_0(\xi,\eta) = \phi^{inc}(\xi,\eta) - \frac{1}{2\pi}\int_a^b \Psi(y)\frac{\partial G}{\partial x}(0,y;\xi,\eta)dy. \quad \text{A8}$$

Now using the third condition in BVP-0 in (A8) and using the relation (7), we obtain the following hypersingular integral equation after some simplifications

$$X\int_a^b [\frac{1}{(y-\eta)^2} + \frac{1}{(y+\eta)^2} + \frac{2K^2\pi i\lambda e^{-\lambda K(y+\eta)}}{5D(\lambda K)^4 + 1 - \delta K}$$
$$-2K\int_0^{\lambda K}\frac{ke^{-\lambda k(y+\eta)}}{k(Dk^4+1-\delta K)-K}dk$$
$$+2K(\lambda K)^2 \quad \text{A9}$$
$$\int_0^1 \frac{e^{-\frac{\lambda k}{x}(y+\eta)x^2}}{\lambda K(D\lambda^4k^4 + x^4 - \delta Kx^4) - Kx^5}dx]\Psi(y)dy$$
$$= -2\pi i\lambda K e^{-\lambda K\eta} \qquad a < \eta < b.$$

To solve the above hypersingular integral equation we put

$$\left.\begin{aligned} y &= \frac{b+a}{2} + \frac{b-a}{2}t \\ \eta &= \frac{b+a}{2} + \frac{b-a}{2}u. \end{aligned}\right\} \quad \text{A10}$$

into (A9), to obtain

$$X\int_{-1}^1\left[\frac{1}{(t-u)^2} + L(u,t)\right]F(t)dt = h(u), \qquad -1 < u < 1 \quad \text{A11}$$

Where

$$L(u,t) = \frac{(b-a)^2}{4(b+a)^2 + 4(b^2-a^2)(t+u) + (b-a)^2(t+u)^2}$$
$$+\frac{(b-a)^2}{2}\frac{K^2\pi i\lambda e^{-\lambda K(y+\mu)}}{5D(\lambda K)^4 + 1 - \delta K} \quad \text{A12}$$
$$-\frac{(b-a)^2}{4}\int_0^{\lambda K}\frac{2Kk^{-k\mu}}{k(Dk^4+1-\delta K)-K}$$
$$-(\lambda K)^2\frac{(b-a)^2}{2}K\int_0^1\frac{e^{-\mu\frac{\lambda K}{x}x^2}}{\lambda K(D\lambda^4k^4 + x^4 - Kx^5)}$$

$$\mu = b + a + \frac{b-a}{2}(t+u) \quad \text{A13}$$

$$F(t) = \Psi\left(\frac{b+a}{2} + \frac{b-a}{2}t\right), \qquad -1 < t < 1, \quad \text{A14}$$

$$F(\pm 1) = 0, \quad \text{A15}$$

and

$$h(u) = -2\pi i\lambda K(\frac{b-a}{2})e^{-K\lambda(\frac{b+a}{2}+\frac{b-a}{2}u)} \qquad -1 < u < 1. \quad \text{A16}$$

Following the methodology used by Parson's and Martin (1992) we assume

$$F(t) = (1-t^2)H(t) \quad \text{A17}$$

so that $F(\pm 1) = 0$.

We now approximate *H*(*t*) as

$$H(t) \approx \sum_{n=0}^{N} a_n U_n(t), \qquad -1 < u < 1, \quad \text{A18}$$

where Un(t) is a Chebyshev polynomial of the second kind and $a = a_n (n = 0,1,2,...N)$ are unknown constants. Using the expansion (A18) in (A17) and substituting in (A11) we obtain

$$\sum_{n=0}^{N} a_n A_n(u) = h(u), \qquad -1 < u < 1 \quad \text{A19}$$

Where

$$A_n(u) = -\pi(n+1)U_n(u) + \int_{-1}^1 (1-t^2)^{\frac{1}{2}} U_n(t)L(u,t)dt. \quad \text{A 20}$$

To find the unknown constants $a = a_n (n = 0,1,2,...N)$, we put $u = u_j (j = 0,1,...,N)$, − 1 < uj < 1 in the relation (A19) to obtain the linear system

$$\sum_{n=0}^{N} a_n A_{nj} = h_j,\ j = 0,1,2,...,N \quad \text{A21}$$

Where

$$A_{nj} = A_n(u_j), \quad h_j = h(u_j) \quad \text{A22}$$

The collocation points u_j can be chosen suitably. Here we have chosen

$$u_j = \cos\frac{(j+1)}{N+2}, \quad j = 0,1,2,...,N. \quad \text{A23}$$

The linear system (A22) is solved by any standard method to obtain the constants $a = a_n (n = 0,1,2,...N)$. Knowing a_n's, $H(t)$ can be obtained from (A18) and hence $F(t)$ from (A17). Knowing $F(t)$, $\Psi(t)$ can be obtained from (A14). Using $\Psi(t)$ from (A14) to (A8) we find the general expression for $\phi_0(\xi,\eta)$.