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Prediction of Velocity-Dip-Position at the Central Section of Open Channels using Entropy Theory

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ABSTRACT

An analytical model to predict the velocity-dip-position at the central section of open channels is presented in this study. Unlike the previous studies where empirical or semi-empirical models were suggested, in this study the model is derived by using entropy theory. Using the principle of maximum entropy, the model for dip-position is derived by maximizing the Shannon entropy function after assuming dimensionless dip-position at the central section as a random variable. No estimation of empirical parameter is required for calculating dip-position from the proposed model. The model is able to predict the location of maximum velocity at the central section of an open channel with any aspect ratio. The developed model of velocity-dip-position is tested with experimental data from twenty-two researchers reported in literature for a wide range of aspect ratio. The model is also compared with other existing empirical models. The present model shows good agreement with the observed data and provides least prediction error compared to other models.

Keywords: Velocity-dip-phenomenon; Shannon entropy; Maximum entropy; Lagranges multiplier; Open channel turbulent flow.

NOMENCLATURE

a_0, a_1, b_0	parameters
Ar	aspect ratio
Ar_c	critical aspect ratio
b	width of the open channel constraints
D_*	maximum value of ξ_d
f	probability density function(PDF)
F	cumulative distribution function (CDF)
G	a function
h	flow height
H	entropy function
L_0	Lagrange's function
L	model index parameter
m	a parameter
M	entropy parameter
N	total number of data points
r	average percentage relative error
s_1	sum of square relative error
s_2	sum of logarithmic deviation error
$\bar{u}$	streamwise depth mean velocity
u_{max}	streamwise maximum velocity
y	vertical co-ordinate
y_d	location of u_{max} from bed
z	lateral co-ordinate
λ_0, λ_1	Lagrange multipliers
ξ_d	dimensionless dip-position
ξ_*	minimum value of ξ_d
$\xi_{d,c}$	calculated value of ξ_d
$\xi_{d,0}$	observed value of ξ_d
φ	a function

1. INTRODUCTION

The location of the maximum velocity from channel bed, has special interest among civil engineers. It has great application to find the stream wise velocity distribution. For more than a century ago, scientists Francis (1878), Stearns (1883), Murphy (1904), Gibson (1909) and Vanoni (1946) have found the position of the maximum mean velocity below the water surface. This phenomenon is known as velocity-dip-phenomenon and the location of maximum velocity from channel bottom is known as velocity-dip-position. The maximum velocity in an open-channel at any cross section may occur up to 55% of the flow height from channel bed. Even in the large river like the Mississippi River, the maximum velocity appears at two-third of the water depth from the channel bottom (Gordon 1992). From many experimental results and analysis it is widely accepted that at the central section of any open channel the velocity-dip-phenomenon occurs if the aspect ratio of the channel *Ar* (defined as the ratio of channel width b to flow depth h), is less than a certain value, called the critical aspect ratio Ar_c (Nezu and Rodi 1985; Hu and Hui 1995; Guo and Julien 2008; Guo 2013; Kundu and Ghoshal 2013). From their experimental observations Nezu and Rodi (1985) proposed that $Ar_c \approx 5$. Whereas though at the central section of a wide open channel (where $Ar>Ar_c$) maximum velocity appears at the free surface, but near to the sidewall region the dip phenomenon appears (Vanoni 1941). Recent experiments of Wang and Cheng (2005) shows that for wide open channels, dip-phenomenon may appears at the central section due to the variation of bed roughness or bed elevation along lateral *z* direction. It is important to mention here that though dip-position may occur near to side wall region, this study focuses only on the prediction of dip-position at the central section (Fig. 1).

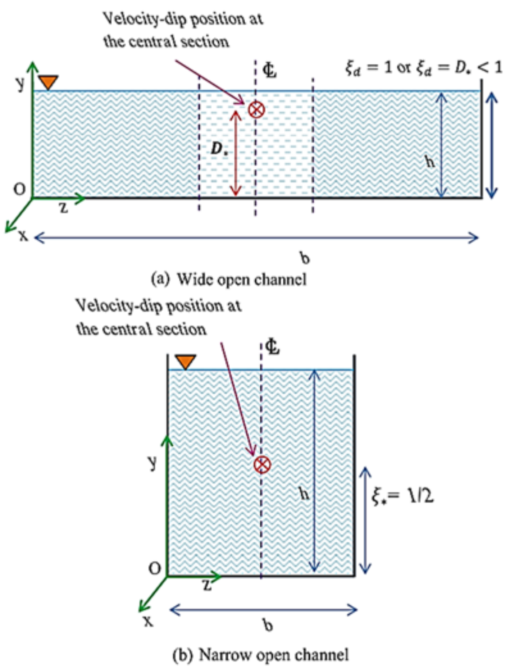

Fig. 1. Variation of velocity-dip-position for (a) wide and (b) narrow open channels at the central section.

It is a challenging task to scientists and engineers to predict the velocity-dip-position at the central section open channel flows. According to the best of the knowledge of the author, till now no theoretical model has been developed for predicting dip-position. Different analytical (empirical or semi-empirical) models are reported by several investigators. First Wang *et al. (*2001) proposed a relation for yd (the location of velocity-dip-position from channel bottom or bed) as a function of aspect ratio at the central section by observing the pat-tern of measured data obtained by Nezu and Rodi (1986) and other eight researchers. Their empirical model is applicable only to narrow open channels. Yang *et al.* (2004) analyzed data from five researchers and proposed another empirical model for dip position at the central section. This modelis valid both in narrow and wide open channels.

They also found that velocity dip may occur very close to sidewall region even when channel aspect ratio is large. Absi (2011) modified this model to predict the velocity-dip-position for validating his proposed velocity model full dip-modified-logwakelaw (fDMLW-law). Bonakdari *et al.* (2008) critically analyzed both the models of Wang *et al.* (2001) and Yang *et al.* (2004) for small channel aspect ratio. They found that both of these models overestimate the experimental results when Aris small. On the basis of experimental observation from five researchers, Bonakdari *et al.* (2008) proposed a sigmoid model for smooth open channel flows which is a ratio of two different functions of the aspect ratio *Ar*. The model is applicable to both wide and narrow open channel flows with smooth boundaries. It can be observed from the model (Table 2) that for $Ar \rightarrow 0$, ξ_d $(= y_d /h) \rightarrow 0.448$, which indicates that it does not satisfy the asymptotic boundary conditions: $Ar \rightarrow 0$, $\xi_d \rightarrow 0.5$ and for $Ar \rightarrow \infty$, $\xi_d \rightarrow 1$ (Hu and Hui 1995). Guo (2013) studied velocity profile for smooth rectangular open channel flows. He found that the velocity-dipposition shifts exponentially from the water surface to half flow depth as the channel aspect ratio decreases from infinity to zero. He proposed an empirical model for dip position at the channel central section. Besides this, Guo (2013) analyzed the models of Wang *et al.* (2001), Yang *et al.* (2004)and Bonakdari *et al.* (2008) and found that none of these models satisfy the asymptotic condition $0.5 \leq \xi_d \leq 1$ consistent with Hu and Hui (1995)observations. In this same year, Pu (2013) proposed an empirical model for velocity-dip-position at the central section of open channels. This model satisfies both the asymptotic boundary conditions. Pu (2013) validated his model for both wide and narrow open channel flows with rough and smooth beds. As a consequence this model is universal to use for any kind of open channel flows with rough and smooth boundaries.

Apart from these empirical and analytical models, many researchers proposed numerical models and methods to determine the velocity-dip-position. Wang and Cheng (2005) proposed a model for turbulent shear stress over the whole cross section of any open channel. From this model, velocity dip-position can be computed numerically by applying the zero shear stress condition at the location of maximum velocity. Sarma *et al.* (2000) proposed a binary law which combines log law in inner region and a parabolic law in the outer region. They found that the junction point of log law and parabolic law is 0.5 when there is no dipphenomenonand the value decreases as $\xi_d/2$ when the dip-phenomenon occurs. This relation can beused to develop a method for finding velocity-dip-position. The method will fail if no satisfactory tangential parabola be obtained. Later, Guo and Julien (2008) describes a method to determine the dip position by fitting a parabola to the velocity data near the free surface region.

Since the development of the entropy theory by Shannon (1948) and of the principle of maximum entropy (POME) by Jaynes (1957a) there has been a number of applications of entropy theory in hydrological and environmental sciences. Chiu and his associates (Chiu 1987; Chiu 1989; Chiu and Tung 2002) have derived the probability density function for velocity and using the POME, they derived the models for mean velocity distribution, turbulent shear stress distribution and particle suspension concentration distribution. Later on, Singh (1997), Singh (1998), Singh (2010), Singh (2011), Luo and Singh (2011) and Singh and Luo (2011) performed a lot of studies on velocity of an open channel flow on the basis of Shannon entropy and Tsallis entropy theory developed by Tsallis (1988) and Gell-Mann and Tsallis (2004). Recently Kumbhakar and Ghoshal (2016a) and Kumbhakar and Ghoshal (2016b) studied one and two dimensional velocity distributions in open channels using Renyi entropy theory respectively. Till now no studies on dip-position using entropy theory has been reported in literature. However Chiu and Tung (2002) studied the maximum velocity and regularities in open channel flows. They derived an empirical model using regression technique for dip-position which is expressed as

$$\xi_d = 1 + 0.2\ln\frac{G(M)}{58.3}, G(M) = \frac{(e^M - 1)^2}{(M-1)e^M + 1} \quad (1)$$

where M is the entropy parameter (Chiu 1987). The computation of velocity-dip-position from this model requires the knowledge of M. Chiu and Tung (2002) computed the value of M from the average value $\bar{u} - u_{max}$ relation. Therefore it gives an average relation between ξ_d and M which may indicate that exact value may fluctuate or deviate above or below that proposed relation. Though an explicit model for velocity-dip-position has been developed using the entropy concept, but the computation of dip-position requires the knowledge of velocity distribution. Therefore a more rigorous and simple general model is still lacking in the literature.

Our motivation for the present study stems from the fact that an analytical expression of velocity-dip-position at the central section of open channels based on a theoretical approach is not proposed yet. Though empirical analytical and numerical models are found in literature, but most of these models are not satisfying the asymptotic boundary condition and applicable to limited data sets. Therefore, in this study we make an attempt to propose a simple entropy based model for velocity-dip-position for the central section of open channel flow.

2. THEORETICAL FORMULATION USINGENTROPY THEORY

It is observed by several investigators, that at the central section of a wide open channel velocity-dip-position yd , appears at free surface and it gradually decreases with decrease of aspect ratio (Hu and Hui 1995; Yan *et al.* 2011; Guo 2013). In their experiment, Hu and Hui (1995) found that for open channel flows, dip-position may appears up to 50% height of flow from bed. These facts suggest that velocity-dip-position at central section of open channels changes from lowest value *0.5h* to highest value *h* with aspect ratio. Therefore the dip-position y_d can be considered as random variable having some probability distribution. For the application of the entropy theory, dimensionless dip-position ξ_d (= y_d /h) at the central section is assumed to be a random variable.

2.1 Shannon Entropy for Dip-Position

To find the probability density function $f(\xi_d)$ of ξ_d , in this study the Shannon entropy for dimension-less random variable ξ_d is considered as (Shannon 1948; Shannon and Weaver 1949).

$$H(\xi_d) = -\int_{\xi_*}^{D_*} f(\xi_d)\ln[f(\xi_d)]d\xi_d \quad (2)$$

where ξ_* and D_*(= D/h) are the lower and upper bounds of ξ_d respectively. Equation (2) defines a measure of uncertainty of the function $f(\xi_d)$. To find $f(\xi_d)$ the principle of maximum entropy (POME) developed by Jaynes (1957a), Jaynes (1957b), Jaynes (1982) is applied, which includes the specification of certain information, called constraints, on velocity-dip-position. According to the POME, to get the least biased probability of the random variable, we maximize the entropy function H subject to some specific constraints.

2.2 Specification of Constraints

If the dip-position data are available, one way to express information is in forms of the constraints. To define the constraints, the total probability law must be satisfied for the probability density function $f(\xi_d)$. Therefore the first constraint is given as

$$C_1 = \int_{\xi_*}^{D_*} f(\xi_d)d\xi_d = 1 \quad (3)$$

The other constraint is taken as the mean of ξ_d

$$C_2 = \int_{\xi_*}^{D_*} \xi_d f(\xi_d)d\xi_d = \overline{\xi_d} \quad (4)$$

Equation (4) gives the mean of the values of ξ_d. In this study the mean value ξ_d is calculated from the available data found in literature.

2.3 Maximization of Entropy Function $H(\xi_d)$

In order to get the least biased probability density function $f(\xi_d)$, the Shannon entropy function given by Eq. (2) is maximized by POME subject to the constraints given by Eqs. (3) and (4). The method of Lagrange multiplier is employed here to maximize the function $H(\xi_d)$. The Lagrange maximization function is defined after neglecting the integration signs as

$$L_0 = -f(\xi_d)\ln[f(\xi_d)] + \lambda_0 f(\xi_d) + \lambda_1\xi_d f(\xi_d) \quad (5)$$

in which λ_0 and λ_1 are Lagrange multipliers. In order to obtain $f(\xi_d)$ which maximizes L_0, the Euler-Lagrange equation of the calculus of variation is applied to Eq. (5) which gives

$$\frac{\partial L_0}{\partial f} = 0 = -\ln[f(\xi_d)] - 1 + \lambda_0 + \lambda_1\xi_d \quad (6)$$

Rearranging Eq. (6), the probability density function $f(\xi_d)$ for velocity-dip-position containing the Lagrange multipliers is expressed as

$$f(\xi_d) = \exp(\lambda_0 - 1)\exp(\lambda_1 \xi_d) \quad (7)$$

Therefore the cumulative distribution function $F(\xi_d)$ of ξ_d is obtained by using Eq. 7 as

$$F(\xi_d) = \int_{\xi_*}^{\xi_d} f(\xi_d) d\xi_d = \frac{\exp(\lambda_0 - 1)}{\lambda_1}\left[\exp(\lambda_1 \xi_d) - \exp(\lambda_1 \xi_*)\right] \quad (8)$$

Similarly the Shannon entropy function is obtained by inserting Eq. (7) into Eq. (2) as

$$H(\xi_d) = \frac{\exp(\lambda_0 - 1)}{\lambda_1}\left[e^{\lambda_1 D_*}\varphi(D_*) - e^{\lambda_1 \xi_*}\varphi(\xi_*)\right] \quad (9)$$

where $\varphi(D*)$ and $\varphi(\xi*)$ are given as

$$\varphi(D_*) = 2 - \lambda_0 - \lambda_1 D_* \quad (10)$$

$$\varphi(\xi_*) = 2 - \lambda_0 - \lambda_1 \xi_* \quad (11)$$

One can observe from Eqs. (7)-(9) that the probability density function, cumulative distribution function and the Shannon entropy function depend on the value of Lagrange multipliers λ_0 and λ_1. Therefore to get the complete understanding of these functions, determination of these parameters are required.

2.4 Determination of Lagrange Multipliers

Two unknown Lagrange multipliers λ_0 and λ_1 are determined in the following way. Substitution of Eq. (7) into Eq. (3) gives

$$\frac{\exp(\lambda_0 - 1)}{\lambda_1}\left[\exp(\lambda_1 D_*) - \exp(\lambda_1 \xi_*)\right] = 1 \quad (12)$$

Similarly, inserting Eq. (7) into Eq. (4) gives the other equation as

$$\frac{\exp(\lambda_0 - 1)}{\lambda_1^2}\left[\exp(\lambda_1 D_*)(\lambda_1 D_* - 1) - \exp(\lambda_1 \xi_*)(1 - \lambda_1 \xi_*)\right] = \overline{\xi_d} \quad (13)$$

Equations (12) and (13) constitute a system of two non-linear equations of the unknown Lagrange multipliers λ_0 and λ_1. These equations can be solved numerically to get the values of the multipliers. It can be observed from these equations that to compute the unknown multipliers, the value of $\xi*$ and $D*$ are required. There are no formulae available in literature to determine those values. Following the analysis of Hu and Hui (1995) and Guo (2013) the value of $\xi*$ and $D*$ are taken as 0.5 and 1 respectively in this study. This system is solved in MATLAB by using non-linear equation solver and the values are obtained as $\lambda_0 = 0.5849$ and $\lambda_1 = 1.4487$.

2.5 Cumulative Distribution Function (CDF)

To establish the relation between the probability domain and the flow domain, a hypothesis on the cumulative distribution function $F(\xi_d)$ in the flow do-main needs to be formulated. In Fig. 2 the computed values of the CDF are plotted from Eq. (8). From the figure it can be observed that dip-position exponentially decreases with the decrease of aspect ratio. Similar conclusion also obtained by Guo (2013). Therefore the CDF model in flow domain is proposed as

$$F(\xi_d) = 1 - \exp\left[-b_0 (Ar)^m\right] \quad (14)$$

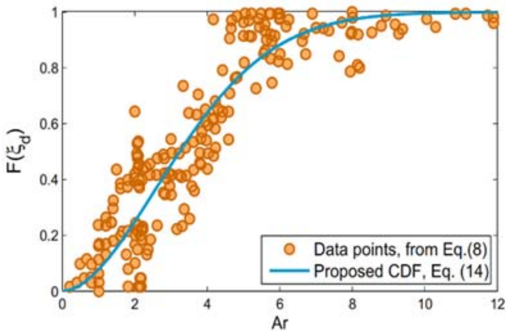

Fig. 2. Validation of the proposed CDF model (Eq. (14)).

Table 1 Details of selected data set

Data	Aspect ratio range
Hu(1985)	1.6642 ~11.1319
Sarma et al.(1983)	2.0240~7.9835
Coleman(1986)	2.0578~2.1317
Knight and Macdonald (1979)	1.0120~1.012
Rajaratnam and Muralidhar (1969)	3.2834~10.8396
Yan et al. (2011)	2.3058~6.9988
Cardoso et al. (1989)	4.6729~7.3529
Vanoni (1946)	4.9973~11.8951
Wang and Qian (1989)	3~3.75
Murphy (1904)	0.1552~1.3460
Nezu and Rodi (1986)	0.1888~5.42
Gibson (1909)	0.256~2
Wang and Fu (1991)	4.1664~4.2913
Zippe and Graf (1983)	6.1728~7.6336
Kironoto and Graf (1994)	2.069~6.8966
Wang and An (1994)	3.7975~4.1667
Guy et al. (1966)	7.94~8.54
Larrarte (2006)	1.7562~2.9248
Nezu and Rodi (1985)	1.0086~6.0011
Tominaga et al. (1989)	2.0054~7.9946
Song and Graf (1994)	3.0039~4.5878
Montes and Ippen (1973)	5.5056~7.6563

Where Ar denotes the aspect ratio of channel, b_0 is a parameter and the exponent m is a fitting parameter which describes the rate of declination of the CFD curve with decrease of aspect ratio. It is to be mentioned here that the proposed model for $F(\xi_d)$ lies in the range (0,1). When $Ar \rightarrow 0$, $exp[-b_0(Ar)m] \rightarrow 1$ and consequently $F(\xi_d) \rightarrow 0$. Also when $Ar \rightarrow \infty$, $exp[-b_0(Ar)m] \rightarrow 0$ and consequently $F(\xi_d) \rightarrow 1$. The validation of the proposed CDF model i.e. Eq. (14) is presented in Fig. 2 by comparing with the experimental data of 22 different researchers given in Table 1. The values of b_0 and the exponent m are obtained as 0.0698 and 1.8767 respectively after fitting Eq. (14) with the experimental data. From the Fig. 2 one can observe that the proposed cumulative distribution function agrees well with the experimental data for large range of aspect ratio.

3. PROPOSED MODEL OF VELOCITY-DIP-POSITION FOR CENTRAL SECTION

Substituting Eq. (12) into Eq. (8), the CDF of dip-position is further expressed as

$$F(\xi_d) = \frac{\exp(\lambda_1 \xi_d) - \exp(\lambda_1 \xi_*)}{\exp(\lambda_1 D_*) - \exp(\lambda_1 \xi_*)} \quad (15)$$

Equating Eq. (14) and Eq. (15) the dimensionless velocity-dip-position model is obtained as

$$\xi_d = \frac{1}{\lambda_1} \ln\left[a_0 + a_1\left\{1 - \exp\left(-0.07 Ar^{1.88}\right)\right\}\right] \quad (16)$$

where

$a_0 = exp(\lambda_1 \xi_*)$ and $a_1 = exp(\lambda_1 D_*) - a_0$ (17)

The velocity-dip-position can be calculated from Eq. (16) at the central section of open channels with any given aspect ratio.

3.1 Re-Parametrization of the Model

To represent the proposed model in a more appropriate form, a dimensionless parameter L is introduced herein as

$$L = \lambda_1 (D_* - \xi_*) \quad (18)$$

Therefore using Eq. (18) the model for dip-position can be expressed in terms of the parameter L as

$$\xi_d = \xi_* + \frac{1}{2L} \ln\left[1 + \left(e^L - 1\right)\left\{1 - e^{-0.07 Ar^{1.88}}\right\}\right] \quad (19)$$

From Eq. (19) it can be observed that dip-position changes only with the parameter L and therefore it can be used as an index for determining the distribution of dip position. For given values of the parameters $\xi_* = 0.5$, the variation of the dip-position with parameter L is presented in the Fig. 3 for six values of $L = -1$, −0.9, −0.8, 0.8, 0.9 and 1. From the figure it is observed that dip-position decreases with increase of L from −1 to −0.8 without changing the pattern and dip-position increases with in-crease of L from 0.8 to 1. It is also observed that the variation becomes significant when Ar lies in the range 0 to 8. It is important to mention here that the model can also be applied to those flows through wide open channels where dip-phenomenon occurs at the central section. In this case, the value of L could be modified from Eq. (18).

It is observed from Eq. (19) that the proposed model satisfies the asymptotic boundary conditions: $\xi_d \rightarrow 0.5$ when $Ar \rightarrow 0$ and $\xi_d \rightarrow 1$ when $Ar \rightarrow \infty$ as mentioned by Hu and Hui (1995) and Guo (2013). To discuss the asymptotic behavior, first we consider that $Ar \rightarrow 0$. Therefore one can observe that

$$\exp\left[-0.07 Ar^{1.88}\right] \rightarrow 1 \quad (20)$$

Therefore from Eq. (19) one gets

$$\xi_d = \xi_* = 0.5 \quad (21)$$

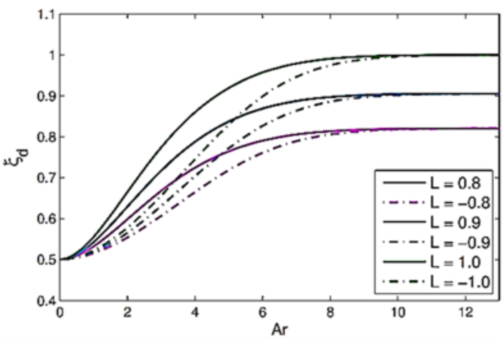

Fig. 3. Variation of dip-position (Eq. (19)) with the parameter L.

Similarly, as a second case we consider $Ar \rightarrow \infty$.

Then

$$\exp\left[-0.07 Ar^{1.88}\right] \rightarrow 0 \quad (22)$$

and consequently

$$\xi_d \rightarrow \xi_* + \frac{1}{2} = 1 \quad (23)$$

This suggests that the proposed model satisfies both the asymptotic boundary conditions.

4. COMPARISON WITH EXPERIMENTAL DATA AND OTHER EXISTING MODELS

To test the validity of the entropy based proposed model for a wide range of experimental data, existing data reported in literature from 22 researchers are considered. All the details of the data sets are given in Table 1. Including all the data sets, the aspect ratio has a wide range from 0.1552 to 12.

The validity of the proposed model (Eq. (16) or (19)) with the experimental data is plotted in Fig. 4. The value of the model parameter L is computed from Eq. (18) after computing the Lagrange multiplier λ_1 solving Eqs. (12) and (13) using

Matlab nonlinear solver. From the figure it is observed that the proposed model based on the entropy theory agrees well with the experimental data over a large range of channel aspect ratio. The R_2 value for this modelis obtained as 0.84. Due to the scatteredness of the experimental data points when $Ar< 5$, the low R^2value is obtained. Also to get a quantitative idea about the overall goodness of fitting of all the models, five error terms (Mean absolute standard error, Average percentage relative error, Sum of squared relative error, Sum of logarithmic deviation error and Root mean square error) are calculated which are discussed in details in the next section.

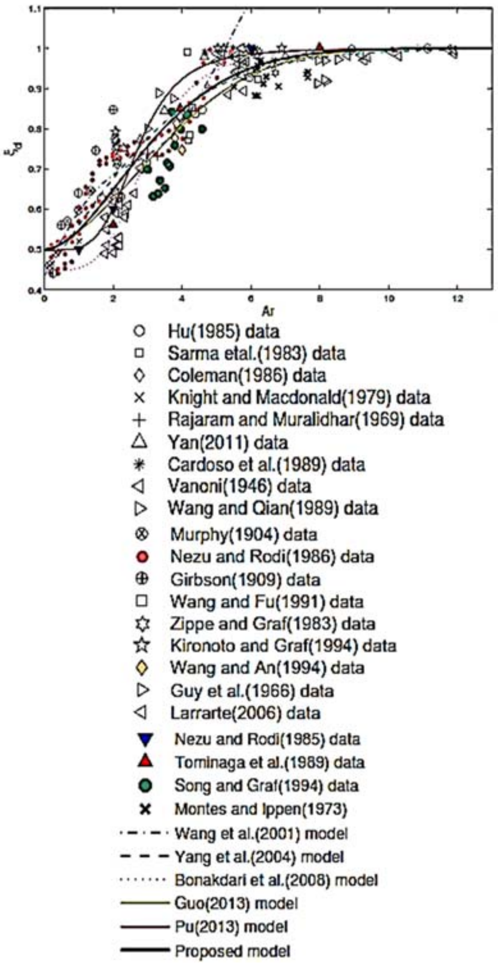

Fig. 4. Validation of the proposed entropy model with experimental data.

Proposed model (Eq. (19)) is also compared with other existing models from Wang *et al.* (2001), Yang *et al.* (2004), Bonakdari *et al.* (2008), Guo (2013) and Pu (2013) at the central section for different open channels. The explicit form of the models for all aforementioned researchers are shown in Table 2. All these selected models are also plotted in Fig. 4. From the figure it is clear that computed values of velocity-dip-position by all these models are comparable to each other except for the model of Wang *et al.* (2001) for entire range of aspect ratio. It is found from the figure that the model of Wang *et al.* (2001) is applicable to narrow open channels only. To measure the accuracy of these models five different aforementioned error terms are also computed and shown in Table 2. From the table it is seen that prediction accuracy of the pro-posed model is good than all other existing models. This results suggest that dip-position is best described by an exponential decay type model. It is important to mention here that though Yang *et al.* (2004), Guo (2013) proposed similar type models but no theoretical basis was mentioned. Therefore this study not only provides good model for dip-position but also gives a theoretical base which was lacking in the literature till now.

5. ERROR ANALYSIS

To compare the proposed model with other existing models in literature, detail error analysis has been carried out. Instead of calculating a single error term and determining the result, in this study we consider five different statistical parameters in or-der to get a detail idea about the comparison of all the models. These five different statistical quantities are: (i) Mean absolute standard error (MASE),(ii) Average percentage relative error *(r%),* (iii) Sum of squared relative error (s_1), (*iv*) Sum of logarithmic deviation error (s_2) and (*v*) Root mean square error (RMSE). The detail definition of these errors are as follows:

Mean absolute standard error (MASE) is de- fined as

$$MASE = \frac{1}{N}\sum_{i=1}^{N} M_i \tag{24}$$

where Mi is given as

$$M_i = \begin{cases} \dfrac{\xi_{d,c}}{\xi_{d,0}} \, if \xi_{d,c} \rangle \xi_{d,0} \\ \dfrac{\xi_{d,0}}{\xi_{d,c}} \, if \xi_{d,c} \langle \xi_{d,0} \end{cases}$$

(i) Average percentage relative error is denoted by r and is defined as

$$r = \frac{1}{N}\sum_{i=1}^{N} \frac{\left|\xi_{d,c} - \xi_{d,0}\right|}{\xi_{d,0}} \times 100(\%) \tag{25}$$

(ii) Sum of squared relative error is denoted by s_1 and is defined as

$$s_1 = \sum_{i=1}^{N} \frac{\left(\xi_{d,c} - \xi_{d,0}\right)^2}{\xi_{d,c}^{\;2}} \tag{26}$$

(iii) Sum of logarithmic deviation error defined using the logarithmic value of dip position is de-noted by s_2 and is expressed as

$$s_2 = \sum_{i=1}^{N} \left(\log\left|\xi_{d,c}\right| - \log\left|\xi_{d,0}\right|\right)^2 \tag{27}$$

And

(iv) The root mean square error (RMSE) is defined as

Table 2 Previous and present model on velocity-dip-position applicable at the central section of any open channel (∗ corresponds to minimum error)

NO.	Investigators	Proposed formula	Prediction error				
			MASE	r(%)	RMSE	s1	s2
1	Wang *et al.* (2001)	$\xi_d = 0.44 + 0.212\left(\frac{Ar}{2}\right) + 0.05Sin\left(\frac{2\pi}{2.6}\frac{Ar}{2}\right)$	1.1422	14.0451	9.1920	6.6540	0.1773
2	Yang *et al.* (2004)	$\xi_d = \left[1 + 1.3\exp\left(-\frac{Ar}{2}\right)\right]^{-1}$	1.0775	7.4840	2.5493	2.2220	0.0672
3	Bonakdari *et al.* (2008)	$\xi_d = \frac{42.4 + Ar^{4.2}}{94.7 + Ar^{4.2}}$	1.1187	9.8232	4.2367	5.3431	0.0957
4	Absi (2011)	$\xi_d = \left[1 + 1.3\exp\left(-\frac{Ar}{2}\right)\right]^{-1}$	1.0775	7.4840	2.5493	2.2220	0.0672
5	Guo (2013)	$\xi_d = \left[1 + \exp\left\{-\left(\frac{Ar}{\pi}\right)^{1.5}\right\}\right]^{-1}$	1.0793	7.3184	2.2700	2.2357	0.0683
6	Pu (2013)	$\xi_d = \frac{40.1 + Ar^{4.4}}{80.5 + Ar^{4.4}}$	1.1006	9.2778	3.4655	3.4906	0.0861
7	Proposed model	$\xi_d = \xi_* + \frac{1}{2L}\ln\left[1 + \left(e^L - 1\right)\left\{1 - e^{-0.07Ar^{1.88}}\right\}\right]$	1.0770*	7.2980*	2.1564*	2.0858*	0.0655*

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\xi_{d,c} - \xi_{d,0}\right)^2} \quad (28)$$

where in all these error terms N denotes the total number of data points, $\xi_{d,c}$ and $\xi_{d,o}$ denote the computed and observed or experimental values of velocity-dip-position.

The computed values of all these errors for the models (detail formulas are given in Table 1) are presented in the Table 1 for all the selected experimental data. In the table, star (∗) denotes the least error among all these models. From the error results in this table one can observe that the proposed model has the least average percentage relative error of 7.2980% which gives the best representation of the experimental measurements. From the table it can also be observed that in most of the cases the least value of all these error corresponds to the proposed model. This comparison and error results show the good prediction accuracy of the proposed entropy based model.

6. CONCLUSIONS

Applying the entropy theory based on probability distribution of random variable and using the principle of maximum entropy, an analytical model to predict velocity-dip-position at the central section of open channels is derived in this study. The model has been developed on theoretical basis. The following conclusions are obtained from this study.

1. The model is able to predict the velocity-dip-position at the central section of any open channel with given aspect ratio.
2. No estimation of parameter is required to compute the dip-position from the proposed model for any given aspect ratio.
3. For general applicability of the model, the model is expressed using one parameter L introduced in this study. The variation of dip-position with aspect ratio for different L is also shown in this study. Results show that the model can also be applied in those wide open channel flows where dip-position occurs at the central section by suitable choice of L.
4. Finally the model is tested and validated over a large number of twenty-four different experimental data sets published in literature and is also compared with all other possible existing models found in literature.
5. To get an idea about the accuracy of these models, five different errors are calculated for all these selected data sets. The obtained results of error analysis show that proposed model gives least error compared to other models.
6. Furthermore this study can be extended to find a more general model for velocity-dip position which will be applicable over the entire cross section.

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