Journal of Applied Fluid Mechanics, Vol. 10, No. 5, pp. 1451-1460, 2017.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.73.242.27218

Simulation of Imbibition Phenomena in Fluid Flow through Fractured Heterogeneous Porous Media with Different Porous Materials

H. S. Patel[1†] and R. Meher[2]

Applied Mathematics and Humanities Department, S. V. National Institute of Technology, Surat - 395007, Gujarat, India.

†Corresponding Author Email: hardy.nit@gmail.com

(Received September 19, 2016; accepted April 15, 2017)

ABSTRACT

In this paper, the counter – current imbibition phenomenon in a fractured heterogeneous porous media is studied with the consideration of different types of porous materials like volcanic sand and fine sand and Adomian decomposition method is applied to find the saturation of wetting phase and the recovery rate of the reservoir. A simulation result is developed to study the saturation of wetting phase in volcanic as well as in fine sand with the recovery rate of the oil reservoir with the choices of some interesting parametric value. This problem has a great importance in the oil recovery process.

Keywords: Counter–current imbibition; Fracture porous media; Brooks corey model Adomian decomposition method.

NOMENCLATURE

K	permeability	t	time
ki	relative permeability of each phase i	Vw	seepage velocity of Water
Pc	capillary pressure	Vo	seepage velocity of Oil
Po	pressure of oil		
Pw	pressure of water	ρw	density of water
pd	entry pressure	ρo	density of oil
R∞	ultimate recovery	Φ	porosity, fraction
R	recovery	μw	water viscosity
Se	effective saturation	μo	oil viscosity
Sw	saturation of water, fraction	λ	grain size distribution
So	saturation of oil, fraction		

1. INTRODUCTION

This paper formulates and discusses mathematically the imbibition phenomena in a fractured heterogeneous porous media with the consideration of two different porous materials. Generally Imbibition be initiated due to the viscosity differences between the wetting fluid (water) and non – wetting fluid (oil). Water imbibition is a primary component of fluid transfer from the matrix to the fracture. Many researchers studied this phenomenon with different approaches. (Gunde, Babadagli, Roy and Mitra 2013) discussed the Pore – scale interfacial dynamics and oil - water relative permeabilities of capillary driven counter – current flow in fractured porous media. (Mirzaei-Paiaman 2015) studied the analysis of this phenomenon in presence of resistive gravity forces. (Rezaveisi, Ayatollahi, and Rostami 2012) investigated experimentally the effect of matrix wettability on water imbibition in a fractured artificial porous media. (Patel and Meher 2016a) and (Patel and Meher 2016b) studied the fingering phenomena in fluid flow through fracture porous media with inclination and gravitational effect and counter–current imbibition phenomena in heterogeneous porous media with gravitational and inclination effect and concluded that the initial saturation rate be more for zero inclination and small inclination in homogeneous case as compared to inclined

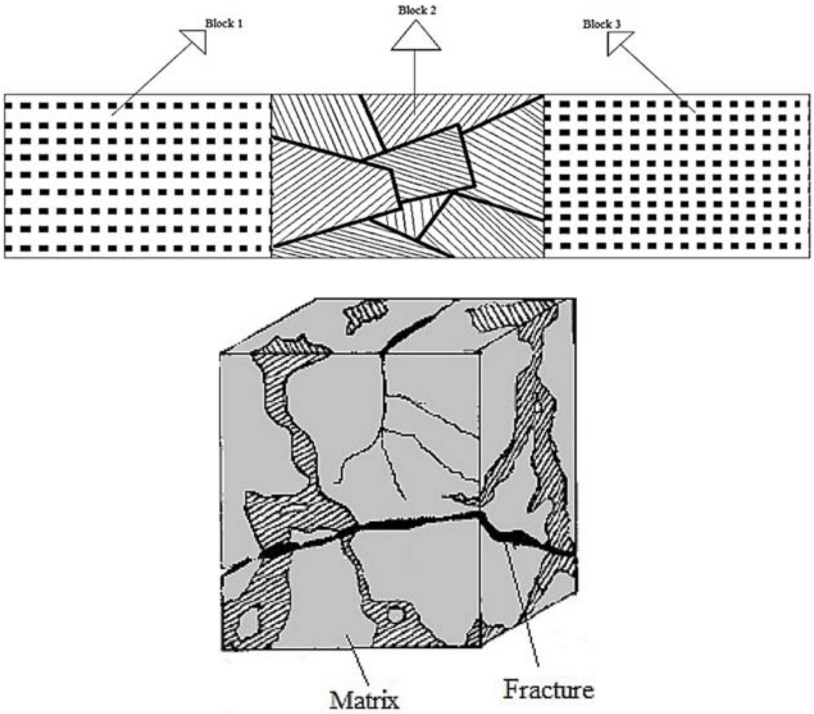

Fig. 1. Block 1 and Block 3 are Matrix blocks and Block 2 is Fracture block.

heterogeneous porous media. (Patel and Meher 2016c) considered corey's model and Scheidegger – Johnson model to simulate the counter-current imbibition phenomenon in heterogeneous porous media and concluded that Corey's model be more appropriate as compared to Scheidegger-Johnson model in order to study the saturation rate. Recently (Patel, Mehta, and Singh 2016) discussed this phenomena in a heterogeneous porous media and concluded that the saturation rate be more in homogeneous as compared to heterogeneous porous matrix. The following empirical function first proposed by (Aronofsky, Masse, Natanson, *et al.* 1958) to study the recovery rate of the reservoir

$$R=R_{\infty}(1-e^{-\gamma T})$$

where $T=\frac{K_C P_d}{\mu_w L^2}t$ be the dimensionless time used to study the recovery rate of the reservoir.

Here we studied the effect of initial water saturation on volcanic sand and fine sand and studied the sensitiveness of imbibition phenomena towards initial water saturation in a heterogeneous fracture porous matrix. Analytical approximate solution for the flow equations is presented here to study the imbibition phenomena in fractures and on heterogeneous porous matrix by using Adomian decomposition method and a simulation result is developed here to study the recovery rate as a function of dimensionless time, T of the reservoir. The effect of permeability, heterogeneity and fractures on saturation rate is simulated by modelled the flow equation in imbibition phenomena.

Here the saturation distribution of the displacing (wetting) fluid in terms of saturations at the interface of the heterogeneous porous matrix as well as in fractures has been obtained. The results obtained here are in perfect agreement with the physical situation. This can be realized by conducting an experiment with the help of fractures and a capillary porous matrix having different porous material filled with oil. When the reservoir oil (non-wetting phase) comes into contact with water (wetting phase) then there is a spontaneous flow of the wetting phase (Water) into the medium and a counter flow of the resident fluid i.e. non wetting phase (oil) from the medium initiated by imbibition. Due to the difference in viscosities of water and oil, the water saturates on the right side of imbibition face and travel only a small distance 'l' due to capillary pressure effect (without external force) initiated by imbibition. The saturation rate for different porous materials and its effect on capillary pressure and relative permeability can be verified from the expression obtained for saturation.it is of great significance in oil recovery, where it can be responsible to increase oil production up to 40% in some cases.

2. MATHEMATICAL MODEL

For the sake of mathematical model: We consider here a piece of porous matrix with fractured block (fig. 1) of an oil formatted region having length 'L' containing viscous oil that is completely surrounded by an impermeable surface except for one end (common interface) which is labelled as the Imbibition face and this end is exposed to an adjacent formation of 'injected' water. Due to the differences

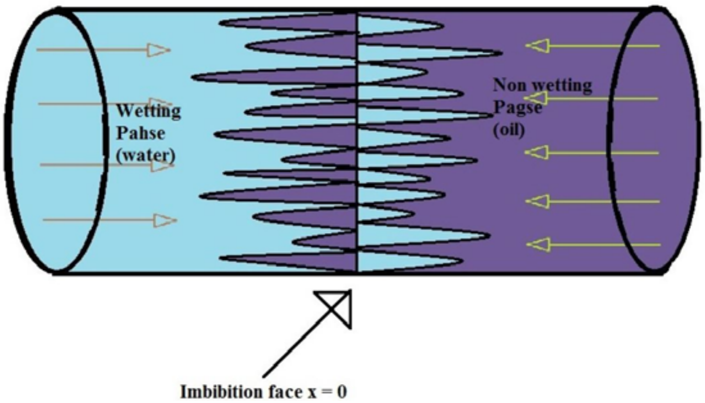

Fig. 2. Schematic diagram of the problem under consideration.

in viscosities of water and oil, the water saturates on the right side of imbibition face and travel only a small distance ‘l’ due to the capillary pressure effect(without external force) as shown in fig. 2.

The conservation equation of mass for two–phase flow can be formulated as

$$\frac{\partial}{\partial t}\left(\phi(x)S_i\rho_i\right)+\nabla.(v_i\rho_i)-\rho_i q_i=0 \tag{1}$$

Where i=o, w, x ∈ ℜ3, t≥0, φ(x) denotes the porosity of the porous medium, Si is the saturation for each phase i, ρi is its specific mass and vi is its volumetric rate which is given by

$$v_i=-K(x)\frac{k_i}{\mu_i}(\nabla\rho_i) \tag{2}$$

Where K(x) denotes the absolute permeability tensor of the porous medium, pi is its pressure, ki is its relative permeability and µi is its viscosity.

If the compressibility of fluid is neglected, then ρis are constant and the conservation equation becomes

$$\frac{\partial}{\partial t}\left(\phi(x)S_i\right)+\nabla.v_i-q_i=0,\quad \text{i=0,w} \tag{3}$$

The imbibition condition for counter – current imbibition can be expressed (Patel, Mehta, and Patel 2013) as

$$v_w=-v_o \tag{4}$$

In porous media, the capillary pressure pc is defined as the pressure difference between the non – wetting phase oil (po) and wetting phase water (po), i.e.

$$p_c=p_o-p_w \tag{5}$$

According to (Oboveanu 1963), the porosity and permeability in heterogeneous porous media can be expressed as

$$(x)=\frac{1}{a(t)-b(t)}$$

$$\text{K(x)}=K_c\phi(x)$$

The most famous pc − Sw relationships which was determined experimentally by (BrooksRH 1964) can be expressed as

$$p_c(S_w)=p_d S_e^{-\frac{1}{\lambda}}=p_d\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{-\frac{1}{\lambda}} \tag{6}$$

The relative permeability, kw of the wetting and non – wetting phases in the domain are governed by the following relation as (BrooksRH 1964)

$$k_w=S_e^{\frac{2+3\lambda}{\lambda}} \tag{7}$$

Combining eq. (2), (4) and (5), we get

$$v_w=k(x)\frac{k_0 k_w}{k_w\mu_0+k_0\mu_w}\left[\frac{\partial p_c}{\partial x}\right] \tag{8}$$

Hence the conservation eq. (3) with eq. (8) can be written as

$$\varphi\frac{\partial S_w}{\partial t}+\frac{\partial}{\partial x}\left[k(x)\frac{k_0 k_w}{k_w\mu_0+k_0\mu_w}\frac{\partial p_c}{\partial S_w}\frac{\partial S_w}{\partial x}\right]=0 \tag{9}$$

Combining eq. (6) and (7) with eq. (9), it yields

$$\frac{\partial S_w}{\partial t}+\frac{k_c p_d}{\mu_w}\frac{\partial}{\partial x}\left[\phi\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{\partial}{\partial S_w}\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial x}\right]-q_w=0 \tag{10}$$

Where $\frac{k_0 k_w}{k_w\mu_0+k_0\mu_w}\approx\frac{k_w}{\mu_w}$ Patel, Mehta, and Patel 2013).

Simplifying eq. (10), it becomes

$$\frac{\partial S_w}{\partial t}+\frac{k_c p_d}{\mu_w}\left[\frac{\partial}{\partial x}\left(\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{\partial}{\partial S_w}\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial x}\right)+\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{\partial}{\partial S_w}\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial x}\frac{1}{\phi}\frac{\partial\phi}{\partial x}\right]-q_w=0 \tag{11}$$

Using the dimensionless variables

$X=\frac{x}{L}$ and $T=\frac{k_c p_d}{\mu_w L^2}$

Simplification of $\frac{1}{\phi}\frac{\partial\phi}{\partial x}$ as

$$\frac{1}{\phi}\frac{\partial\phi}{\partial x}=\frac{\partial}{\partial x}(log\phi)=\frac{\partial}{\partial x}\left[\frac{b}{a}LX-loga\right]$$

(Neglecting higher order term of X) $\frac{b}{a}L$

With the assumption of the source term q_w =

qw(X,T)= $\frac{1}{y+wx}$, v; w _ 1.???

The dimensionless forms of eq. (11) can be written as

$$\frac{\partial S_w}{\partial T}+\frac{\partial}{\partial x}\left[\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{\partial}{\partial S_w}\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial X}\right]+\frac{bL}{a}\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{\partial}{\partial S_w}\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial X}-\frac{D(a-bXL)}{(v+wXL)}=0 \quad (12)$$

Where D=$\frac{\mu_w L^2}{k_c p_d(1-S_{wr})}$

Eq. (12) describes the equation of counter – current imbibition phenomena in a fractured heterogeneous porous media.

3. Analysis of the Method

For the purpose of illustration of the Adomian decomposition method, we consider eq. (12) in an operator form as

$$L_T S_w(X.T)+L_X(NS_w)+\frac{bL}{a}[NS_w]-\frac{D(a-bXL)}{(v+wXL)}=0 \quad (13)$$

Where $NS_w=\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{\partial}{\partial S_w}\left(\frac{S_w-S_{wr}}{1-S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial X}$

and Sw0 can be solved subject to the corresponding initial condition S(X,0) = f (X) = e–X .

Following (Adomian 1994) defined the linear operators $L_T=\frac{\partial}{\partial T}$ and $L_X=\frac{\partial}{\partial X}$ the definite integration inverse operator L_T^{-1} and the nonlinear term as NSw. Operating the inverse operator and following the analysis of Adomian decomposition, we set the recursive relation of eq. (13) as

$$\sum_{n=0}^{\infty}S_{wn}(X,T)=e^{-X}+L_T^{-1}\left[L_X\left(\sum_{n=0}^{\infty}A_n\right)\right]+\frac{bL}{a}L_T^{-1}\left[\sum_{n=0}^{\infty}A_n\right]-\frac{D(a-bXL)}{(v+wXL)} \quad (14)$$

Which gives the recurrence relation as

$$S_{w0}=S_w(X,0)=e^{-X}-\frac{D(a-bXL)}{(v+wXL)}$$

$$S_{w,k+1}=L_T^{-1}[L_X(A_k)]+\frac{bL}{a}(A_k),\ \text{k>0} \quad (15)$$

and the approximate analytical solution of the problem in series form up to three terms can be written as

$$S_w(X,T)=S_{w0}+S_{w1}+S_{w2}+\cdots$$

$$S_w(X,T)=\frac{1}{\lambda^2 a(1-S_{wr})(LwX+v)}\Bigg(\lambda\Big((LwX+v)^3(bL-a)e^{-X}+L^2D(aw+bv)(awLX+av-2aw)\Big)T(wLX+v)\left(\frac{(wXL+v)e^{-X}-D(bXL-a)}{wXL+v}\right)^{\frac{1+2\lambda}{\lambda}}-2\Bigg(\Big(2LD(wXL+v)^2(aw+bv)e^{-X}+(wXL+v)^4e^{-2X}+L^2D^2(aw+bv)^2\Big)T\left(\frac{1}{2}+\lambda\right)\left(\frac{(wXL+v)e^{-X}-D(bXL-a)}{wXL+v}\right)^{\frac{1+\lambda}{\lambda}}-\frac{1}{2}(1-S_{wr})\lambda^2\big((wXL+v)\big)(wXL+v)^3\Bigg)\Bigg) \quad (16)$$

Eq. (16) represents the saturation of wetting phase during counter – current imbibition phenomena in a fractured heterogeneous porous media for two different porous materials.

Table 1 Parametric values of parameters

Property	Volcanic Sand	Fine Sand
λ	2.29	3.70
S_{wr}	0.157	0.167
$p_d(N/m^2)$	16	41
K(μ²)	18	2.5
φ	0.351	0.377

4. Numerical Results and Discussion

4.1 Effect of Fractures and Matrix on Initial Water Saturation

Figure 3 and 4 discusses the variation of initial water saturation in fractures and in heterogeneous porous matrix for volcanic and fine sand. It shows that the initial water saturation rate be more in fractures as well as in volcanic sand compared to porous matrix and fine sand.

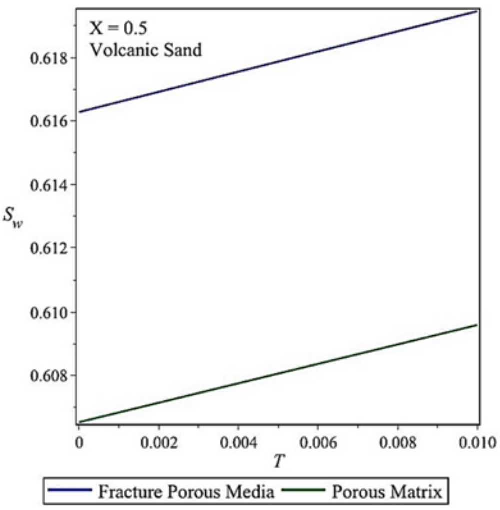

Fig. 3. Comparison for Saturation of water vs. Dimensionless time in Fractured porous media and Porous Matrix for Volcanic Sand.

4.2 Effect of Capillary Pressure on Initial Water Saturation in Volcanic Sand

Figure 5 and 6 discusses the variation of capillary pressure with initial water saturation in fractures and in heterogeneous porous media for volcanic sand. It shows that the capillary pressure be more in matrix porous media as compared to fractures in volcanic sand.

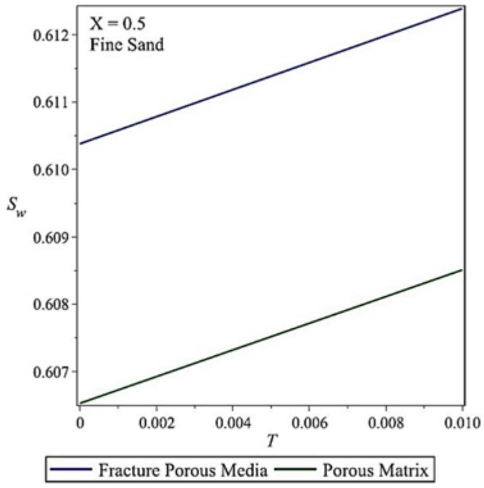

Fig. 4. Comparison for Saturation of water vs. Dimensionless time in Fractured porous media and Porous Matrix for Fine Sand.

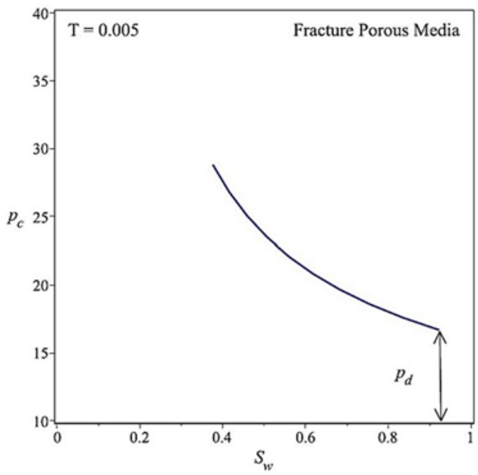

Fig. 5. Capillary pressure vs. Saturation in Fractured Porous Media in Volcanic Sand.

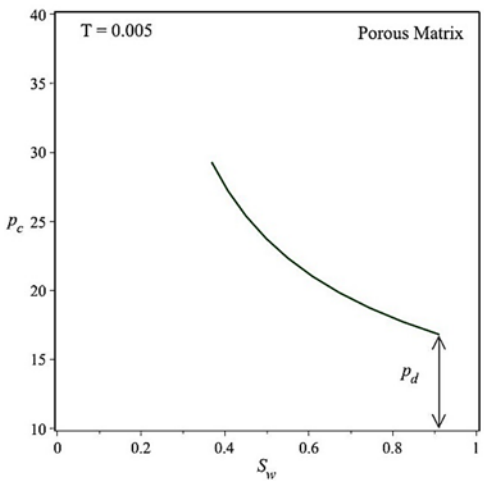

Fig. 6. Capillary pressure vs. Saturation in Porous Matrix in Volcanic Sand.

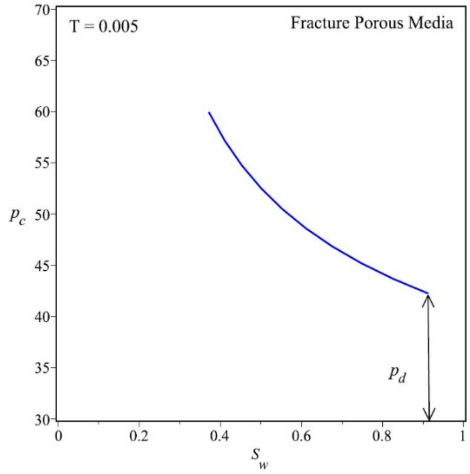

Fig. 7. Capillary pressure vs. Saturation in Fractured Porous Media in Fine Sand.

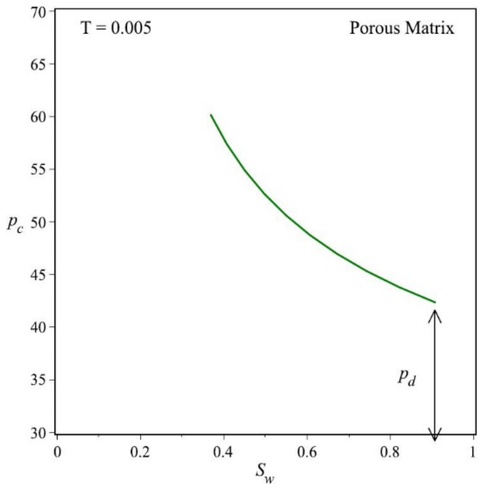

Fig. 8. Capillary pressure vs. Saturation in Porous Matrix in Fine Sand.

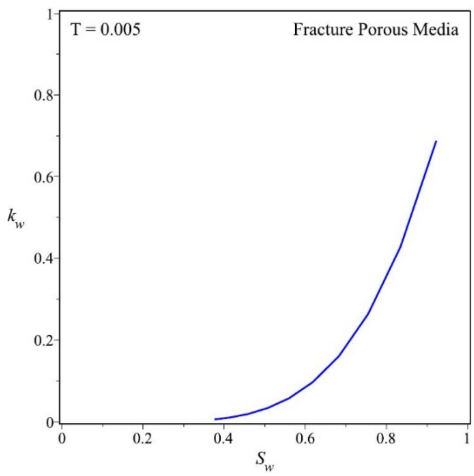

Fig. 9. Relative permeability vs. Saturation in Fractured Porous Media in Volcanic Sand.

4.3 Effect of Capillary Pressure on Initial water Saturation in Fine Sand

Figure 7 and 8 discusses the variation of capillary pressure with initial water saturation in fractures and in heterogeneous porous media for fine sand. It shows that the capillary pressure be more in matrix porous media as compared to fractures in fine sand.

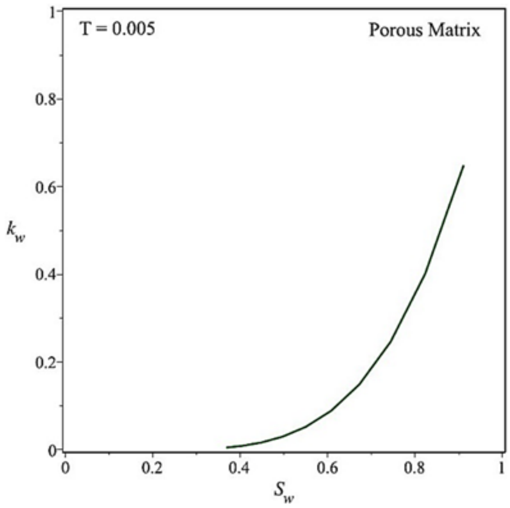

Fig. 10. Relative permeability vs. Saturation in Porous Matrix in Volcanic Sand.

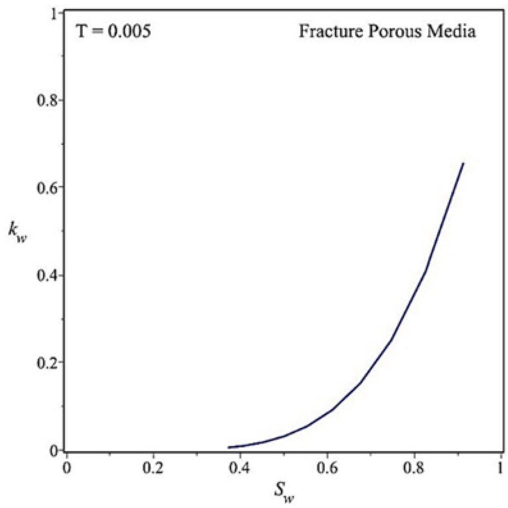

Fig. 11. Relative permeability vs. Saturation in Fractured Porous Media in Fine Sand.

4.4 Effect of Relative Permeability on Initial Water Saturation in Volcanic Sand

Figure 9 and 10 discusses the variation of Relative permeability with initial water saturation in fractures and in heterogeneous porous media for volcanic sand. It shows that the value of Relative permeability be more in fractures as compared to matrix porous media in volcanic sand.

4.5 Effect of Relative Permeability on Initial Water Saturation in Fine Sand

Figure 11 and 12 discusses the variation of Relative permeability with initial water saturation in fractures and in heterogeneous porous media for fine sand. It shows that the value of Relative permeability be more in fractures as compared to matrix porous media in fine sand.

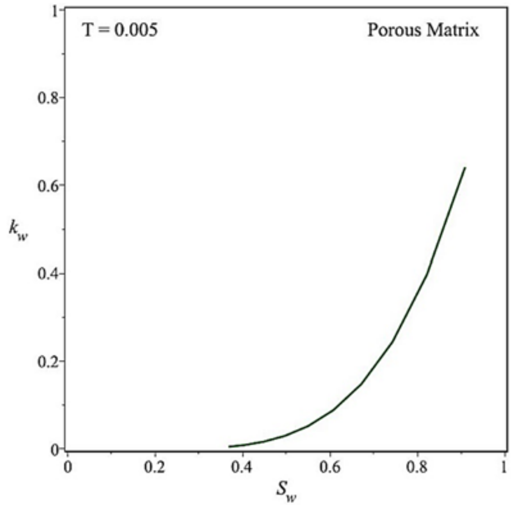

Fig. 12. Relative permeability vs. Saturation in Porous Matrix in Fine Sand.

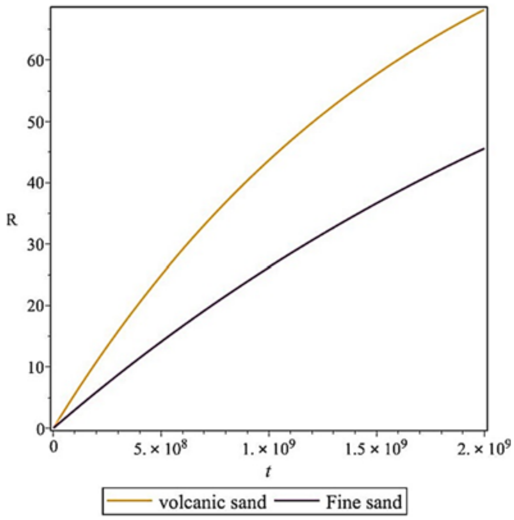

Fig. 13. Comparisons of Recovery rate vs. Dimensionless Time for Volcanic Sand and Fine sand.

5. RECOVERY RATE

It is found here that the dependence of different porous materials on saturation rate rendered the problem highly nonlinear. The significant part of this study is to study the advantage of the proposed mathematical expression in the determination of saturation of wetting phase and the recovery rate of this phenomenon with the inclusion of fractured and porous matrix and with different porous materials with the choices of suitable parametric values. It is found that there is an impact of fractures and types of porous materials on saturation of wetting phase in counter – current imbibition phenomena and it shows that the saturation rate be more in presence of fractures as well as in volcanic sand as compared to fine sand and increases with time provided the

Table 2 Saturation of water in Fractured Porous Media in Volcanic Sand

Volcanic Sand										
X/T	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
0.1	0.917547	0.918795	0.920042	0.921290	0.922537	0.923785	0.925033	0.926280	0.927528	0.928776
0.2	0.830617	0.831503	0.832389	0.833275	0.834161	0.835048	0.835934	0.836820	0.837706	0.838593
0.3	0.752009	0.752638	0.753268	0.753898	0.754527	0.755157	0.755787	0.756416	0.757046	0.757675
0.4	0.680912	0.681359	0.681807	0.682254	0.682701	0.683149	0.683596	0.684043	0.684491	0.684938
0.5	0.616597	0.616915	0.617233	0.617551	0.617869	0.618187	0.618505	0.618822	0.619140	0.619458
0.6	0.558409	0.558635	0.558861	0.559087	0.559313	0.559539	0.559765	0.559991	0.560217	0.560443
0.7	0.505758	0.505919	0.506080	0.506240	0.506401	0.506561	0.506722	0.506883	0.507043	0.507204
0.8	0.458113	0.458227	0.458341	0.458455	0.458569	0.458684	0.458798	0.458912	0.459026	0.459140
0.9	0.414993	0.415074	0.415155	0.415236	0.415318	0.415399	0.415480	0.415561	0.415642	0.415724
1.0	0.375966	0.376024	0.376081	0.376139	0.376197	0.376255	0.376312	0.376370	0.376428	0.376486

Table 3 Saturation of water in Porous Matrix in Volcanic Sand

Volcanic Sand										
X/T	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
0.1	0.906048	0.907259	0.908470	0.909681	0.910892	0.912103	0.913314	0.914525	0.915736	0.916947
0.2	0.819589	0.820448	0.821307	0.822166	0.823024	0.823883	0.824742	0.825601	0.826460	0.827319
0.3	0.741427	0.742036	0.742645	0.743254	0.743863	0.744472	0.745081	0.745690	0.746299	0.746908
0.4	0.670751	0.671183	0.671615	0.672047	0.672479	0.672911	0.673343	0.673775	0.674207	0.674639
0.5	0.606836	0.607143	0.607449	0.607755	0.608062	0.608368	0.608674	0.608981	0.609287	0.609593
0.6	0.549028	0.549246	0.549463	0.549680	0.549897	0.550114	0.550332	0.550549	0.550766	0.550983
0.7	0.496739	0.496893	0.497047	0.497201	0.497355	0.497509	0.497663	0.497817	0.497971	0.498125
0.8	0.449438	0.449547	0.449656	0.449765	0.449875	0.449984	0.450093	0.450202	0.450312	0.450421
0.9	0.406647	0.406724	0.406802	0.406879	0.406957	0.407034	0.407111	0.407189	0.407266	0.407344
1.0	0.367934	0.367989	0.368044	0.368099	0.368154	0.368209	0.368264	0.368318	0.368373	0.368428

Table 4 Saturation of water in Fractured Porous Media in Fine Sand

Volcanic Sand										
X/T	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
0.1	0.910104	0.910845	0.911585	0.912325	0.913066	0.913806	0.914547	0.915287	0.916028	0.916768
0.2	0.823609	0.824143	0.824677	0.825211	0.825746	0.826280	0.826814	0.827348	0.827882	0.828417
0.3	0.745374	0.745760	0.746145	0.746531	0.746916	0.747302	0.747687	0.748072	0.748458	0.748843
0.4	0.674604	0.674882	0.675161	0.675439	0.675717	0.675995	0.676273	0.676551	0.676829	0.677108
0.5	0.610581	0.610782	0.610983	0.611183	0.611384	0.611585	0.611785	0.611986	0.612187	0.612387
0.6	0.552657	0.552802	0.552947	0.553092	0.553237	0.553382	0.553526	0.553671	0.553816	0.553961
0.7	0.500249	0.500353	0.500458	0.500562	0.500667	0.500771	0.500876	0.500980	0.501085	0.501190
0.8	0.452828	0.452903	0.452979	0.453054	0.453130	0.453205	0.453281	0.453356	0.453431	0.453507
0.9	0.409918	0.409973	0.410027	0.410082	0.410136	0.410191	0.410245	0.410299	0.410354	0.410408
1.0	0.371089	0.371129	0.371168	0.371207	0.371246	0.371286	0.371325	0.371364	0.371404	0.371443

Table 5 Saturation of water in Porous Matrix in Fine Sand

Volcanic Sand										
X/T	0.001	0,002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
0.1	0.905569	0.906302	0.907034	0.907766	0.908499	0.909231	0.909963	0.910696	0.911428	0.912160
0.2	0.819258	0.819786	0.820314	0.820842	0.821371	0.821899	0.822427	0.822955	0.823483	0.824011
0.3	0.741198	0.741579	0.741960	0.742341	0.742722	0.743102	0.743483	0.743864	0.744245	0.744625
0.4	0.670594	0.670869	0.671143	0.671418	0.671692	0.671967	0.672241	0.672516	0.672791	0.673065
0.5	0.606728	0.606926	0.607124	0.607322	0.607520	0.607718	0.607916	0.608114	0.608312	0.608510
0.6	0.548954	0.549097	0.549239	0.549382	0.549525	0.549668	0.549810	0.549953	0.550096	0.550239
0.7	0.496688	0.496791	0.496894	0.496997	0.497099	0.497202	0.497305	0.497408	0.497511	0.497614
0.8	0.449403	0.449477	0.449551	0.449625	0.449700	0.449774	0.449848	0.449922	0.449996	0.450071
0.9	0.406623	0.406676	0.406730	0.406783	0.406837	0.406890	0.406944	0.406997	0.407051	0.407104
1.0	0.367918	0.367956	0.367995	0.368033	0.368072	0.368110	0.368149	0.368188	0.368226	0.368265

Table 6 Comparison between capillary pressure vs. Saturation in Fractured and Porous Matrix in Volcanic and Fine Sand

T = 0:005							
Volcanic Sand				Fine Sand			
Fracture Porous Matrix		Porous Matrix		Fracture Porous Matrix		Porous Matrix	
(S_w)	(p_c)	(S_w)	(p_c)	(S_w)	(p_c)	(S_w)	(p_c)
0.922537921	16.68782883	0.910892649	16.79990829	0.913066351	42.23972012	0.908499086	42.30988038
0.834161970	17.60611998	0.823024906	17.73408046	0.825746067	43.68493412	0.821371051	43.76368019
0.754527803	18.59475616	0.743863480	18.74156208	0.746916568	45.21596723	0.742722045	45.30476634
0.682701928	19.66428062	0.672479635	19.83362536	0.675717333	46.84542957	0.671692826	46.94609757
0.617869318	20.82761477	0.608062163	21.02416467	0.611384424	48.58887864	0.607520522	48.70369668
0.559313532	22.10087526	0.549897723	22.33062074	0.553237195	50.46585266	0.549525390	50.59773572
0.506401225	23.50454920	0.497355518	23.77532357	0.500667411	52.50137408	0.497099966	52.65412208
0.458569907	25.06523386	0.449875172	25.38750314	0.453130229	54.72820749	0.449700069	54.90689098
0.415318124	26.81829746	0.406957011	27.20640659	0.410136626	57.19037467	0.406837250	57.40195160
0.376197469	28.81210715	0.368154137	29.28633002	0.371246980	59.94886709	0.368072391	60.20321490

Table 7 Comparison between relative permeability vs. Saturation in Fractured and Porous Matrix in Volcanic and Fine Sand

T = 0:005							
Volcanic Sand				Fine Sand			
Fracture Porous Matrix		Porous Matrix		Fracture Porous Matrix		Porous Matrix	
(S_w)	(k_w)	(S_w)	(k_w)	(S_w)	(k_w)	(S_w)	(k_w)
0.922537921	0.6884266825	0.910892649	0.6487418963	0.913066351	0.6560171672	0.908499086	0.6408001913
0.834161970	0.4280629165	0.823024906	0.4014313664	0.825746067	0.4078215225	0.821371051	0.3975840610
0.754527803	0.2636638688	0.743863480	0.2458990921	0.746916568	0.2508912878	0.742722045	0.2440517581
0.682701928	0.1605535176	0.672479635	0.1487947366	0.675717333	0.1524474629	0.671692826	0.1479169654
0.617869318	0.09642986487	0.608062163	0.08872151277	0.611384424	0.09127955005	0.607520522	0.08830956431
0.559313532	0.05696816319	0.549897723	0.05197496295	0.553237195	0.05370708934	0.549525390	0.05178444178
0.506401225	0.03299336868	0.497355518	0.02980593558	0.500667411	0.03094513617	0.497099966	0.02971934553
0.458569907	0.01865424720	0.449875172	0.01665577842	0.453130229	0.01738432337	0.449700069	0.01661724013
0.415318124	0.01024140339	0.406957011	0.009015936475	0.410136626	0.009468350012	0.406837250	0.008999215947
0.376197469	0.005421429428	0.368154137	0.004690575406	0.371246980	0.004962343821	0.368072391	0.004683545671

Table 8 Comparison of Recovery rate in Fractured Porous Media in Volcanic and Fine Sand

Dimension Time (Second)	Recovery rate (%) for Volcanic Sand	Recovery rate (%) for Fine Sand
1.089×10^8	6.056	3.256
2.179×10^8	11.750	4.612
3.268×10^8	17.094	9.461
4.358×10^8	22.119	12.413
5.447×10^8	26.835	15.266
6.537×10^8	31.270	18.029
7.626×10^8	35.432	20.699
8.716×10^8	39.345	23.285
9.806×10^8	43.021	25.786
1.089×10^8	46.457	28.193

recovery rate be more in volcanic sand as compared to fine sand as shown in fig.13.

5.1 Conclusion

Here we studied the saturation rate as well as the recovery rate in counter – current imbibition phenomenon in a fractured heterogeneous porous media for two types of porous materials like volcanic sand and fine sand. The simulation results for the saturation rate of wetting phase is shown in Table 2, 3, 4 and 5 and with capillary pressure, relative permeability and the recovery rate are shown in Table 6, 7 and 8 with the choices of suitable parametric values which shows that the saturation rate be maximum in fractures as compared to porous matrix implies the recovery rate of oil reservoir be maximum and around 40% in presence of a fractures and in volcanic sand as compared to normal porous matrix and fine sand which is physically consistent with the real world phenomena.

ACKNOWLEDGMENTS

The authors are thankful to S. V. National Institute of Technology, Surat, Gujarat, India for the scholarship and Applied Mathematics and Humanities Department for encouragement and facilities.

REFERENCES

Adomian, G. (1994). Solving frontier problems of physics the decomposition method. Springer.

Aronofsky, J., L. Masse, S. G. Natanson and *et al*. (1958). A model for the mechanism of oil recovery from the porous matrix due to water invasion in fractured reservoirs. trans. *AIME 213*(17), 14.

Brooks R. H, C. (1964). Hydraulicpropertiesof-porousmedia. Colorado State University, *Hydro Paper 3*, 27.

Gunde, A., T. Babadagli, S. S. Roy and S. K. Mitra (2013). Porescale interfacial dynamics and oil–water relative permeabilities of capillary driven counter-current flow in fractured porous media. *Journal of Petroleum Science and Engineering 103*, 106–114.

Mirzaei-Paiaman, A. (2015). Analysis of counter-current spontaneous imbibition in presence of resistive gravity forces: displacement characteristics and scaling. *Journal of Unconventional Oil and Gas Resources 12*, 68–86.

Oboveanu, T. (1963). Scurgerea fluidelor prin medii poroase neomogene. Ed. Academiei, Bucharest 92, 328.

Patel, H. and R. Meher (2016a). Simulation of fingering phenomena in fluid flow through fracture porous media with inclination and gravitational effect. *Journal of Applied Fluid Mechanics 9*(3135–3145).

Patel, H. S. and R. Meher (2016b). Approximate analytical study of counter-current imbibition phenomenon in a heterogeneous porous media. *Applied Mathematical Sciences 10*(14), 673–681.

Patel, H. S. and R. Meher (2016c). A study on recovery rate for counter current imbibition phenomenon with coreys model arising during oil recovery process. Applied Mathematics & *Information Sciences An International Journal 10*(5), 1877–1884.

Patel, K. K., M. Mehta and T. R. Singh (2016). A homotopy series solution to a nonlinear partial differential equation arising from a mathematical model of the countercurrent imbibition phenomenon in a heterogeneous porous medium. *European Journal of Mechanics-B/Fluids 60*, 119–126.

Patel, K. R., M. N. Mehta and T. R. Patel (2013). A mathematical model of imbibition phenomenon in heterogeneous porous media during secondary oil recovery process. *Applied Mathematical Modelling 37*(5), 2933–2942.

Rezaveisi, M., S. Ayatollahi and B. Rostami (2012). Experimental investigation of matrix wettability effects on water imbibition in fractured artificial porous media. *Journal of Petroleum Science and Engineering 86*, 165–171.