

On Dispersion in Oscillatory Annular Flow Driven Jointly by Pressure Pulsation and Wall Oscillation

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ABSTRACT

Longitudinal dispersion of solute released in an unsteady flow between two coaxial cylinders is re-examined in the presence of first order chemical kinetics in the bulk flow. The flow unsteadiness is caused by the oscillation of the outer tube around its axis as well as by a periodic pressure gradient. Unlike some previous works, the gap width of the annular tube is used as the typical length scale which is physically meaningful to a greater extent. In order to employ the method of moment, a finite difference implicit scheme has been adopted to solve the Aris integral moment equations arising from the unsteady convective diffusion equation for all time periods. The individual and combined effects of different velocity components resulting from steady and time-dependent parts of the driving forces are examined and they are identified based on their functionality. In any flow situation, wall factor is found to have a larger contribution in velocity as well as in dispersion compared to the pressure factor. The behaviour of dispersion coefficient with the variation of radius ratio, bulk flow reaction parameter, and frequency parameters have been examined. Dispersion coefficient is found to diminish with the increase of the reaction-rate in the bulk flow, whereas the effect of the radius ratio on the dispersion coefficient is fixed by the form of the velocity distribution. The axial distributions of mean concentration are approximated using Hermite polynomial representation from the first four central moments for a range of different reaction-rate parameters. It has been found that, irrespective of the flow situation, the peak of the concentration distribution decreases with the increase in reaction rate parameter.

Keywords: Dispersion coefficient; Axial Reynolds number; Concentration distribution; Radius ratio; Poiseuille number; Bulk-flow reaction.

NOMENCLATURE

C	concentration	u	axial velocity
D_α	dispersion coefficient	α_p	frequency of pressure pulsation
i	time index during navigation	α_w	frequency of Wall oscillation
j	space index	ϵ_p	amplitude of pressure pulsation
P	Poiseuille number	ϵ_w	amplitude of wall oscillation
r	radial coordinate	η	aspect ratio
Re	axial Reynolds number	ν	kinetic viscosity
r_i	internal radius	ρ	density
r_o	external radius	δ	direct delta function
Sc	Schmidth number	κ	bulk flow reaction constant
t	time		

1. INTRODUCTION

Dispersion is the mechanism that controls the rate of spreading of a cloud contaminant in a flowing stream. Due to its numerous application in chemical, environmental and bio-medical processes, considerable attention has been given on the axial dispersion of tracer. When Taylor (1953) initiated the study of dispersion, it was his

anticipation that in shear flow, additional longitudinal diffusion of matters could result from the combined action of lateral diffusion and velocity shear. Aris (1956) generalized Taylors conceptual model by removing restrictions imposed by Taylor to include longitudinal diffusion and developed an approach viz. method of moments whose main scope is to analyze the asymptotic behaviour of the second moment of the

distribution of solute about the mean. Certain technical difficulties in Aris (1956) method was resolved by Barton (1983) and obtained the solutions of second and third-order moment equations of the distribution of solute which are valid for all time. Flow unsteadiness is one of the key factors having a heavier impact on the dispersion phenomena. For laminar flow, the case of an oscillatory axial flow in a uniform tube was first studied by Aris (1960) by the moment method. He studied the effect of flow pulsation with the consideration that the velocity at a point as a periodic function of time. An exact solution of the diffusion equation was acquired by Chatwin (1970) to study the dispersion by considering solute concentration as a harmonic function of time. The idea proposed by Chatwin was utilized by Watson (1983) to dissect the mass transfer of a diffusing substance through a pipe in oscillatory flow, which was extended to an annular flow by Tsangaris and Athanassiadis (1985).

The effect of wall absorption on dispersion in an oscillatory flow through a pipe was explained by Mazumder and Das (1992). Mondal and Mazumder (2005) studied the tracer dispersion in an annular pipe with reactive boundary. Solute transport in oscillatory flow through an annulus was investigated by Sarkar and Jayaraman (2004) and Mazumder and Mondal (2005) and also explained the application of their study to a catheterized artery. Ng (2006) and Mazumder and Paul (2012) examined dispersion process in presence of reversible and irreversible reactions in the boundary. In spite of the fact that there exists a number of attempts where boundary reaction is considered in the study of dispersion process to analyze the impact of the former on the later, but very scant attention has been given to study the behaviour of dispersion in the presence of homogeneous reaction in the bulk flow though it has abundant applications in chemical and biomedical engineering, e.g. hydrolysis of ester, gas absorption in an agitated tank with chemical reaction and so on. Cleland and Wilhelm (1956), Gupta and Gupta (1972), Kumar, Umavathi, and Basavaraj (2012) are a few who threw some light on the effect of first order reaction in the bulk flow.

For flow through a channel, quite a number of efforts have been launched to investigate dispersion phenomena under the pulsation of walls. Few studies in this field may include Secomb (1978), Hydon and Pedley (1993), Waters (2001) etc. Dispersion process through a channel forced by unsteady pressure gradient has been studied by Paul and Mazumder (2009), Mazumder and Mondal (2005). Literature suggests that there exist a number of studies on dispersion under the sole influence of either pressure pulsation or boundary oscillation, but very few studies considered both pressure pulsation or boundary oscillation.

In two successive attempts, Paul and Mazumder (2008) and Paul (2009) has given weightage to both types of driving forces in order to study their combined effect on the dispersion process. While the first attempt was focused on channel-flow, it

was extended to flow through an annular tube in the subsequent attempt where the radius of the outer cylinder was considered as the typical length scale which is based on weak physical background. The same physically insignificant non-dimensionalisation procedure was adopted in the work of Mazumder and Mondal (2005) also. The radial diffusion processes like viscous and concentration diffusion, are expected to be over the gap width of the annular tube which was violated in those studies. Time was also scaled on the same faulty platform. The typical diffusion time should never be over a length scale with outer radius. These types of errors lead the authors to have some erroneous results. The unexpected presence of Schmidt number in the velocity profile, for example, is the outcome of the slips done in those works. As the feedback of concentration gradients on the flow was explicitly excluded, therefore the flow is not affected by the concentration field, so the ratio of viscosity and concentration diffusion are not relevant to the flow indicating the Schmidt number independence of the velocity distribution.

In the present work, attempts are made to re-investigate the dispersion process through an annular tube in the presence of two types of driving forces, of course, by avoiding the slips already pointed out. Like the work of Paul (2011), the gap width of the annular tube is considered as the typical length scale. It is assumed that the solute chemically reacts with the liquid in which it is dispersed, the rate of reaction being first order. The main objective is to explain the behaviour of the dispersion coefficient due to the intricate distribution of velocity resulting from the interactions of the two driving forces and to solve the present problem we consider Aris-Barton method, which allows to simplify the convection-diffusion equation into moment equation and solve numerically, as it is pretty complicated when $p > 1$. There are some analytical and semi-analytical methods, viz., Perturbation method (Purtell 1981), Multiple scale analysis method (Paul and Mazumder 2009) and Homotopy analysis method (He 2000; Tufail, Butt, and Ali 2016), etc., by which one can solve the advection-diffusion paradigm directly and eventually able to calculate the apparent diffusion coefficient. But these methods have certain limitation such as perturbation method is physically sound for weak physical parameter again multiple scale analysis could not produce time dependent behaviour as the method based on averaging the time, The Homotopy analysis method overcome these difficulty but very difficult to implement. There are some non-perturb analytic method such as Weighted linearization method (Agrwal and Denman 1985), Adomian decomposition (Adomian 1988; El-Danaf, Ramadan, and Alaal 2005) method, Laplace Transform method (Alia, Sheikha; Saqiba, and Khanb 2017) vibrational iteration method (He, Wan, and Guo 2004), δ -expansion method (Awrejcewicz, Andrianov, and Manevitch 2012) however all these customary strategies can't guarantee about the convergence of solution series and also those methods cannot be

are easily implemented. The present work may excel for the following aspects:

First - To the best of our knowledge it is one of the very few works which takes into account both of the driving forces, specially when the flow is through an annulus.

Second - It removes the technical difficulties of some previous works in this field that occurred due to the consideration of the outer radius as the typical length scale.

Third - It is probably a fresh attempt where two driving forces and bulk flow reactions receiving parallel attention in order to find out the complex interactions of the two forces with the reaction parameter.

For simplicity, only first order chemical kinetics have been considered here. The outer wall of the annulus is considered to execute oscillatory motion in its own plane keeping the inner one stationary. To find out the aggregate effects due to the presence of both the driving forces as compared with the isolated effects due to either of the oscillations alone, velocity is dissected into multiple parts using an additive model from where the participation of the velocity terms in the dispersion process can be made out. Results are obtained to find the effects of the radius ratio of the annular tube, the frequencies of oscillations of the forces and bulk flow reaction parameter on the spreading of tracers. The movement of the center of mass and the patterns of the mean concentration distributions in presence of both driving forces are also discussed. In a wide variety of problems of chemical engineering, diffusion of a solute takes place in oscillatory flows with simultaneous chemical reactions. Many industrial processes, such as the transport of oil, gas, water and foodstuffs through pipes, are directly or indirectly related with the dispersion subject to this type of physical situation. The study may have a significant contribution to the understanding of pulsatile flow through catheterized artery where bulk flow reaction may be considered as mandatory due to injection as a part of medication.

2. MATHEMATICAL MODEL

An unsteady fully developed, axi-symmetric laminar flow of a homogeneous, incompressible viscous fluid is considered through the annular gap of two coaxial infinitely long cylinders having a and b their external and internal radii respectively, $d = a - b$ being the annular gap between the cylinders. The geometry of the annulus is maintained by the radius ratio (ratio of the inner radius to the outer radius) $\eta =$ of the annulus. Due to the infinite axial extend of the system, the aspect ratio, ratio between the axial length L and the gap width d , is infinite in this study. In the cylindrical coordinate system used, the radial and axial co-ordinates are \hat{r} and \hat{x} respectively, where hat represents dimensional quantities. The flow through the above geometry is strictly one-dimensional and the Navier-Stokes equation

becomes

$$\frac{\partial \hat{u}}{\partial \hat{t}} = -\frac{1}{\rho} \frac{\partial \hat{p}}{\partial \hat{x}} + \nu \frac{1}{\hat{r}} \frac{\partial}{\partial \hat{r}} \left(\hat{r} \frac{\partial \hat{u}}{\partial \hat{r}} \right) \quad (1)$$

where ρ, ν and \hat{p} are the density, the kinematic viscosity and the pressure of the fluid respectively.

The flow is driven by the combined action of periodic pressure gradient and pulsation of the outer tube around its axis, both with non-zero means. While the axial periodic pressure gradient is given by

$$-\frac{1}{\rho} \frac{\partial \hat{p}}{\partial \hat{x}} = P_x [1 + e_p \text{Re}(e^{i\omega_p \hat{t}})] \quad (2)$$

the velocity of the outer wall of the annular tube is prescribed by,

$$\hat{u}(a, \hat{t}) = U [1 + e_w \text{Re}(e^{i\omega_w \hat{t}})], \quad (3)$$

where \hat{t} is the time, U is the steady component of velocity of the outer cylinder, e_p and e_w are factors representing respectively the amplitude of the pressure pulsation and that of the wall oscillation, ω_p is the frequency of the pressure pulsation and ω_w of the wall oscillation. $\text{Re}(\cdot)$ represents the real part of the complex number.

As the inner wall of the annulus is stationary, no-slip condition holds for the inner wall, i.e.,

$$\hat{u}(b, \hat{t}) = 0 \quad (4)$$

3. CONVECTION-DIFFUSION EQUATION

If a solute is injected in the above discussed flow situation through the annular gap of the cylinders, the concentration $C(t, r, x)$ of the solute as a function of axial distance x , radial distance r and time t , satisfies the non-dimensional convective-diffusion equation,

$$\frac{\partial C}{\partial t} + u(r, t) \frac{\partial C}{\partial x} = \frac{1}{Sc} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) + \frac{\partial^2 C}{\partial x^2} \right] - \kappa C \quad (5)$$

with dimensionless quantities

$$C = \frac{\hat{C}}{C_0}, \quad r = \frac{\hat{r}}{a}, \quad r = \frac{\hat{x}}{d}, \quad t = \frac{\nu \hat{t}}{d^2},$$

$$Sc = \frac{\nu}{D}, \quad u = \frac{\hat{u}d}{\nu}, \quad \kappa = \frac{\hat{\kappa}d^2}{\nu}.$$

Here the velocity $u(r, t)$ is composed of steady components $u_{sp}(r)$ & $u_{sw}(r)$ and the unsteady components $u_{op}(r, t)$ & $u_{ow}(r, t)$ respectively due to the periodic pressure gradient and oscillatory motion of the outer cylinder, both with non zero means. C_0 is the reference concentration, D is the molecular diffusion coefficient (assumed constant) and κ is the first-order reaction rate constant so that the last term κC represents the volume rate of disappearance of the solute due to chemical reaction. Sc is the Schmidt number (the ratio of viscous diffusion to molecular diffusion). The discharged material is assumed to be so dilute in concentration that its presence does not

materially effect the flow of the carrying fluid.

The initial and boundary conditions for the concentration distributions are

$$\left. \begin{aligned} C(0, r, x) &= B(r)\delta(x), \quad (r_i \leq r \leq r_o) \\ \frac{\partial C}{\partial r} &= 0 \quad \text{at } r = r_i \\ \frac{\partial C}{\partial r} &= 0 \quad \text{at } r = r_o \\ C &\text{ is finite at all point,} \\ \frac{\int_{r_i}^{r_o} \int_0^{2\pi} \int_{-\infty}^{\infty} rC(0, r, x) dr d\theta dx}{\pi(r_o^2 - r_i^2)} &= 1 \end{aligned} \right\} \quad (6)$$

where B(r) is a function of δ is the Dirac delta function, $r_i (= \frac{\eta}{1-\eta})$ and $r_o (= \frac{\eta}{1-\eta})$ are the dimensionless inner and outer radius of the annulus respectively. The first condition depicts the initial condition at $t=0$. Conditions second and third describe the no flux boundary condition at the inner and outer wall of the annular tube respectively whereas the last condition of equation (6) tells that the total amount of material inside the annulus is unity at $t = 0$. This amount will be depleted over time because of the reaction in the bulk flow.

4. VELOCITY DISTRIBUTION

Using the following non-dimensional quantities

$$P = \frac{P_x d^3}{\nu^2}, R_e = \frac{Ud}{\nu}, \alpha_p = d\sqrt{\frac{\omega_p}{\nu}}, \alpha_w = d\sqrt{\frac{\omega_w}{\nu}}$$

the Navier-Stokes equation along with boundary

conditions i.e., the system of equations (1) – (4) reduce to the following form:

$$\frac{\partial u}{\partial t} = P + \varepsilon_p Re(e^{i\alpha_p^2 t}) + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \quad (7)$$

with

$$u = \begin{cases} 0 & \text{at } r = r_i \\ R_e + \varepsilon_w Re[i\alpha_w^2 t] & \text{at } r = r_o \end{cases} \quad (8)$$

where P is the Poiseuille number, $\varepsilon_p (= \varepsilon_p P)$ is the non-dimensional amplitude of the pressure pulsation and $\varepsilon_w (= \varepsilon_w R_e)$ that of the outer-cylinders axial oscillation which is determined by the axial Reynolds number R_e .

The velocity profile in (5) can be readily found from the solution of equation (7) with boundary conditions (8) and is given in dimensionless form as

$$u(r, t) = u_{sp}(r) + u_{sw}(r) + u_{op}(r, t) + u_{ow}(r, t) \quad (9)$$

where

$$u_{sp}(r) = \frac{P}{4} \left[\frac{1}{1-\eta^2} - r^2 - \frac{1+\eta}{1-\eta} \frac{\log[r(1-\eta)]}{\log \eta} \right] \quad (10)$$

$$u_{sw}(r) = \frac{R_e}{\log \eta} \log \left[\frac{\eta}{r(1-\eta)} \right] \quad (11)$$

$$u_{op}(r, t) = \frac{i\varepsilon_p e^{i\alpha_p^2 t}}{\alpha_p^2} \left[C_1 J_0(i\sqrt{i}\alpha_p r) + C_2 Y_0(i\sqrt{i}\alpha_p r) - 1 \right] \quad (12)$$

$$u_{ow}(r, t) = \varepsilon_w e^{i\alpha_w^2 t} \left[C_3 J_0(i\sqrt{i}\alpha_w r) - C_4 Y_0(i\sqrt{i}\alpha_w r) \right], \quad 0 \leq \eta \leq 1 \quad (13)$$

The constants C_i 's are given by where

$$C_1 = \frac{Y_0(i\sqrt{i}\alpha_p r_i) - Y_0(i\sqrt{i}\alpha_p r_o)}{Y_0(i\sqrt{i}\alpha_p r_i)J_0(i\sqrt{i}\alpha_p r_o) - Y_0(i\sqrt{i}\alpha_p r_o)J_0(i\sqrt{i}\alpha_p r_i)}$$

$$C_2 = \frac{J_0(i\sqrt{i}\alpha_p r_i) - Y_0(i\sqrt{i}\alpha_p r_i)}{Y_0(i\sqrt{i}\alpha_p r_i)J_0(i\sqrt{i}\alpha_p r_i) - Y_0(i\sqrt{i}\alpha_p r_i)J_0(i\sqrt{i}\alpha_p r_i)}$$

$$C_3 = \frac{Y_0(i\sqrt{i}\alpha_w r_i)}{Y_0(i\sqrt{i}\alpha_w r_i)J_0(i\sqrt{i}\alpha_w r_o) - Y_0(i\sqrt{i}\alpha_w r_o)J_0(i\sqrt{i}\alpha_w r_i)}$$

$$C_4 = \frac{J_0(i\sqrt{i}\alpha_w r_i)}{Y_0(i\sqrt{i}\alpha_w r_i)J_0(i\sqrt{i}\alpha_w r_o) - Y_0(i\sqrt{i}\alpha_w r_o)J_0(i\sqrt{i}\alpha_w r_i)}$$

Here J_0, Y_0 are the Bessel functions of first and second kind respectively. The dimensionless frequency parameters α_p, α_w used here are the measures of ratio of the time $\left(\frac{d^2}{\nu}\right)$ required for viscosity to smooth out the transverse variation in vorticity to the periods of oscillations $\left(\frac{1}{\omega_p}\right)$ and $\left(\frac{1}{\omega_w}\right)$ respectively. The first term in the expression for $u(r, t)$ (Eq.9) results from the steady part of the pressure force and the second term from the steady part of the wall force. Last two terms originate from the periodic time dependent parts of the two driving forces respectively.

5. MOMENT EQUATIONS

Following the method of integral moment as proposed by Aris (1956), the p th moment of the distribution of the solute in the filament through r at time t is defined as,

$$C_p(t, r) = \int_{-\infty}^{+\infty} x^p(t, r, x) dx \quad (14)$$

and the concentration distribution of the solute over the crosssection of the annulus is given by,

$$\bar{C}_p(t) = \frac{2}{r_o^2 - r_i^2} \int_{r_i}^{r_o} r C^p(t, r) dr \quad (15)$$

where over-bar denotes the cross-sectional average.

Using equation (14), the diffusion equation (5) subject to the initial and boundary conditions can be written as

$$\frac{\partial C_p}{\partial t} - \frac{1}{Sc} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C_p}{\partial r} \right) = p u(r, t) C_{p-1} + \frac{1}{Sc} p(p-1) C_{p-2} - k C_p \quad (16)$$

with

$$C_p(0, r) = \begin{cases} B(r) & \text{for } p = 0 \\ 0 & \text{for } p > 0 \end{cases} \quad (17)$$

$$\left. \begin{aligned} \frac{\partial C_p}{\partial r} &= 0 \quad \text{at } r = r_i \\ \frac{\partial C_p}{\partial r} &= 0 \quad \text{at } r = r_o, \end{aligned} \right\}$$

where $B(r)=1$ for $r_i \leq r \leq r_o$.

Averaging over the annular cross-section, Eq. (16) subject to the conditions (17) reduces to,

$$\frac{d\bar{C}_p}{dt} = p \overline{u(r, t)C_{p-1}} + \frac{1}{Sc} p(p-1)\bar{C}_{p-2} - k\bar{C}_p \quad (18)$$

and

$$\begin{aligned} \bar{C}_p(0) &= 1 \quad \text{for } p = 0 \\ &= 0 \quad \text{for } p > 0 \end{aligned} \quad (19)$$

The p th order central moment of the concentration distribution about the mean can be defined as

$$\mu_p(t) = \frac{\int_{r_i}^{r_o} \int_0^{2\pi} \int_{-\infty}^{+\infty} r(x-x_g)^p C \, dr \, d\theta \, dx}{\int_{r_i}^{r_o} \int_0^{2\pi} \int_{-\infty}^{+\infty} r C \, dr \, d\theta \, dx} \quad (20)$$

where

$$x_g = \frac{\iiint x C \, dV}{\iiint C \, dV} = \frac{\bar{C}_1}{\bar{C}_0} \quad (21)$$

is the centroid or first moment of the solute and \bar{C}_0 represents the total mass of the solute in the whole volume of the annulus. which decays gradually with time due to the reaction in the bulk flow. The expressions for the central moments can then be obtained from (20) as

$$\left. \begin{aligned} \mu_0(t) &= 1 \\ \mu_1(t) &= 0 \\ \mu_2(t) &= \frac{\bar{C}_2}{\bar{C}_0} - x_g^2, \\ \mu_3(t) &= \frac{\bar{C}_3}{\bar{C}_0} - 3x_g\mu_2 - x_g^3, \\ \mu_4(t) &= \frac{\bar{C}_4}{\bar{C}_0} - 4x_g\mu_3 - 6x_g^2\mu_2 - x_g^4, \end{aligned} \right\} \quad (22)$$

Each of the integral moments of concentration defined in (22) has an important contribution for predicting dispersion phenomenon. The integral moments serve as simple and physically meaningful descriptors of the overall behaviour of the slug -(i) The zeroth moment gives the total area under the distribution curve, which corresponds to the total mass of the solute. (ii) The first moment x_g measures the location of the center of gravity of the slug movement with the mean velocity of the fluid, initially located at the source. (iii) The second central moment μ_2 represents the variance of the distribution about the mean position whose rate of change gives the dispersion coefficient. [Aris \(1956\)](#) showed that the rate of change of variance is proportional to the sum of molecular diffusion coefficient along the axial direction and apparent dispersion coefficient

(Taylor dispersion coefficient). Since the axial diffusion is negligible compared to the lateral diffusion, the apparent dispersion coefficient Da can be written as

$$Da = \frac{d\mu_2}{dt} \quad (23)$$

The skewness factor $v_1 \left(= \frac{\mu_3}{\mu_2^{3/2}} \right)$, and the flatness factor $v_2 \left(= \frac{\mu_4}{\mu_2^2} - 3 \right)$ are also important factors during the initial stage of matter dispersion.

6. NUMERICAL SOLUTION

In the present paper, a finite difference implicit scheme has been applied due to the complexity of the moment equation when $p > 1$ and for that we have partitioned the entire annular region in $(M-1)$ equal part of width Δr represented by grid point j , where the initial grid $j = 1$ indicates the inner wall and $j = M$, the outer wall. The index i represents the time in which $i = 1$ indicates the initial time $t = 0$. The subsequent time grid is obtained from the relationship $t = \Delta t \times (i-1)$, where Δt is the time increment. The resulting finite difference equation becomes a system of linear algebraic equation with tri-diagonal coefficient matrix,

$$\begin{aligned} P_j C_p(i+1, j+1) + Q_j C_p(i+1, j) \\ + R_j C_p(i+1, j-1) = S_j \end{aligned} \quad (24)$$

where P_j, Q_j, R_j and S_j are the matrix elements and $C_p(i, j)$ are the corresponding value of C_p at the grid point (i, j) . The finite difference form of the initial condition is,

$$\begin{aligned} C_p(1, j) &= 1 \quad \text{for } p = 0 \\ &= 0 \quad \text{for } p > 0 \end{aligned}$$

and that of boundary conditions are

$$C_p(i+1, 0) = C_p(i+1, 2)$$

(at the surface of the inner cylinder)

$$C_p(i+1, M+1) = C_p(i+1, M-1)$$

(at the surface of the outer cylinder)

The tri-diagonal system is solved by Thomas algorithm ([L. Anderson, Tannehill, and Pletcher 1986](#)) with the help of above mentioned initial and boundary conditions, accordingly a matlab code is developed to perform the action. The computational steps are as follows:

(i) Time dependent axial velocity u is computed first from Eq.(7);

(ii) the concentration C_p is then calculated from Eq.(16) as the value of u at each of the grid point (i, j) is already calculated in step (i);

(iii) finally the value of C_p is calculated from Eq.(18) by applying Simpson's one-third rule, with the known values of $u(r, t)$ and C_p .

Although the present scheme is linearly stable for

Table 1 Various combination of velocity components

Velocity component	Explanation
u_{sp}	Flow driven by only the steady part of the pressure gradient i.e., $F_p = P$; $F_w = 0$ where F_p is the driving force exerted by the pressure gradient and F_w that by the boundary movement
u_{sw}	Flow driven by only the steady part of the wall movement i.e., $F_p = 0$, $F_w = R_e$
$u_{sp}+u_{sw}$	Flow driven by the combined action of both steady parts i.e., $F_p = P$, $F_w = R_e$
u_{op}	Flow driven by only the periodic part of the pressure gradient i.e., $F_p = \varepsilon_p R_e \left[e^{i\alpha_p^2 t} \right]$, $F_w = 0$
u_{ow}	Flow driven by only the periodic part of the wall oscillation i.e., $F_p = 0$; $F_w = \varepsilon_w R_e \left[e^{i\alpha_w^2 t} \right]$,
$u_{op}+u_{ow}$	Flow driven by the joint action of both periodic parts i.e., $F_p = \varepsilon_p R_e \left[e^{i\alpha_p^2 t} \right]$, $F_w = \varepsilon_w R_e \left[e^{i\alpha_w^2 t} \right]$,
$u_{sp}+u_{op}$	Flow driven by the steady and periodic part of the pressure gradient i.e., $F_p = \varepsilon_p R_e \left[e^{i\alpha_p^2 t} \right]$, $F_w = 0$
$u_{sp}+u_{sw}+u_{op}$	Flow driven by the steady part of the wall oscillation combined with steady and oscillatory pressure gradient i.e., $F_p = \varepsilon_p R_e \left[e^{i\alpha_p^2 t} \right]$, $F_w = R_e$
$u_{sp}+u_{op}+u_{ow}$	Flow driven by the steady part of the pressure gradient combined with steady and oscillatory boundary movement (case studied by Paul [23]) i.e., $F_p = P + \varepsilon_p R_e \left[e^{i\alpha_p^2 t} \right]$ $F_w = R_e + \varepsilon_w R_e \left[e^{i\alpha_w^2 t} \right]$,
$u_{sw}+u_{op}+u_{ow}$	Flow driven by the oscillatory part of the pressure gradient combined with steady and oscillatory wall oscillation i.e., $F_p = \varepsilon_p R_e \left[e^{i\alpha_p^2 t} \right]$, $F_w = R_e + \varepsilon_w R_e \left[e^{i\alpha_w^2 t} \right]$,
$u_{sw}+u_{op}$	Flow driven by the steady part of the wall oscillation combined with oscillatory pressure gradient i.e., $F_p = \varepsilon_p R_e \left[e^{i\alpha_p^2 t} \right]$, $F_w = R_e$
$u_{sp}+u_{ow}$	Flow driven by the steady part of the pressure gradient combined with oscillatory wall movement i.e., $F_p = P$; $F_w = \varepsilon_w R_e \left[e^{i\alpha_w^2 t} \right]$,
$u_{sp}+u_{sw}+u_{op}+u_{ow}$	Flow is supported by the full participation of both the driving forces i.e., $F_p = P + \varepsilon_p R_e \left[e^{i\alpha_p^2 t} \right]$, $F_w = R_e + \varepsilon_w R_e \left[e^{i\alpha_w^2 t} \right]$,

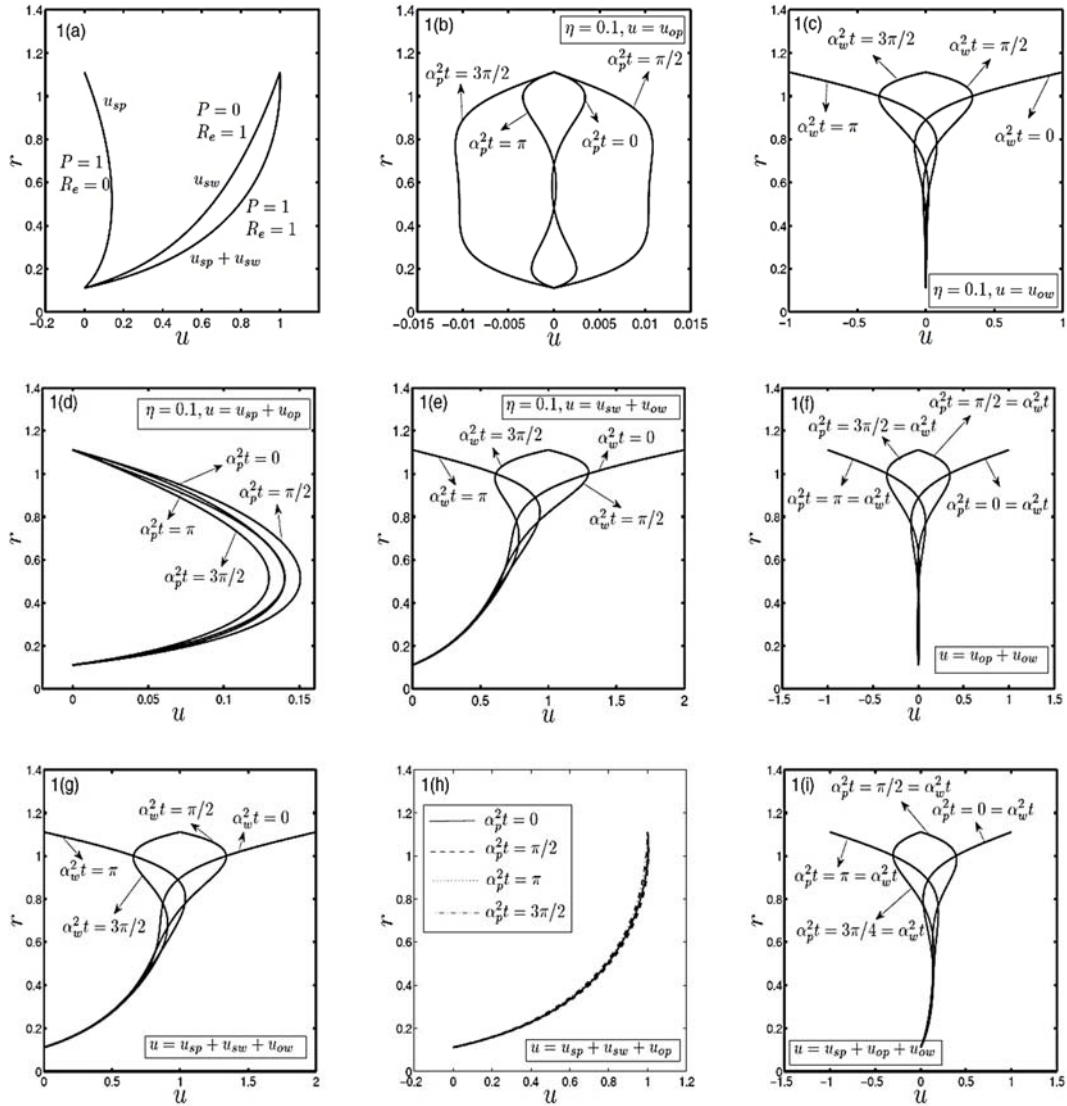


Fig. 1. Various combination of velocity components depending on the physique of the driving forces.

any finite values of $\Delta t/(\Delta r)^2$, sufficiently small mesh size have been taken to obtain the results upto the desired accuracy. In all the cases we have taken $\epsilon_p = \epsilon_w = 1, \text{Re} = P = 1, \text{Sc} = 10^3$.

7. DISTRIBUTIONS OF MEAN CONCENTRATION

Behaviour of the concentration distribution may be obtained from the knowledge of the first four central moments of the distribution. Using these four moments, it is possible to compute the mean axial concentration distribution $C_m(x,t)$ of tracers with the help of Hermite polynomial representation for non-Gaussian curves (Mehta, Merson, and McCoy 1974) and is given by,

$$C_m(z,t) = \bar{C}_0(t) e^{-z^2} \sum_{n=0}^{\infty} a_n(t) H_n(z) \quad (25)$$

where $z = \frac{x-x_g}{(2\mu_2)^{1/2}}, x_g = \frac{\bar{x}_1}{C_0}$ and H_i , the Hermite

polynomials, satisfy the recurrence relation with $H_0(z)=1$ as

$$H_{i+1}(z) = 2zH_i(z) - 2iH_{i-1}(z), \quad i = 0, 1, 2, \dots$$

The coefficients a_i 's are

$$a_0 = \frac{1}{\sqrt{2\pi\mu_2}}, a_1 = a_2 = 0, a_3 = \frac{\sqrt{2}a_0v_2}{24}, a_4 = \frac{a_0v_3}{96}$$

Therefore, given the statistical parameters (21), the concentration distribution can be estimated from (25) at any given location in the axial direction and time.

8. RESULTS AND DISCUSSION

In the absence of the term u_{op} in the velocity, the flow is caused by the constant pressure gradient and periodic movement of the outer cylinder with non-zero mean, a case considered by Paul (2011) for passive solute. Results of the present study are in excellent agreement with Paul under similar background. In the limiting case when $\eta \rightarrow 0$, the

Table 2 Interpretation of results from Fig. 1

Figures	Conclusion
1(a)	Steady part of the wall force dominates over that of the pressure force when the flow is completely time-independent i.e., $u_{sw} > u_{sp}$ when $u = u_{sp} + u_{sw}$.
1(a,b,d)	For flow driven by pressure only, steady part has larger weightage than the periodic part i.e., $u_{sp} > u_{op}$ when $u = u_{sp} + u_{op}$.
1(b,c,f)	Time dependent part of the wall force dominates over that of the pressure force when the flow is completely periodic i.e., $u_{ow} > u_{op}$ when $u = u_{op} + u_{ow}$.
1(a,g,h)	Oscillatory part of the wall force, when combined with the steady flows, receives greater weightage than the pressure force.
1(f,i,j)	Steady part of the wall force again is more influential than that of the pressure force when attached with the periodic flows
1(g,h,k)	For the total flow, unsteady part of the wall force is more effective than that of the pressure force i.e., $u_{ow} > u_{op}$ when $u = u_{sp} + u_{sw} + u_{op} + u_{ow}$.
1(l,j,k)	Steady part of the wall force again is more influential than that of the pressure force when the total flow is considered i.e., $u_{sw} > u_{sp}$ when $u = u_{sp} + u_{sw} + u_{op} + u_{ow}$.

findings of this work can be supported by the existing literature on tube flow.

Let us first analyze the velocity components. The flow is driven by the combined action of the periodic pressure gradient and axial oscillation of outer cylinder in its own plane, both having non-zero means, i.e., both of the driving forces consists of a steady part and a periodic time dependent part. As a result, as we have seen already, the velocity $u(r,t)$ is composed of four components, two being steady and two unsteady. They can be introduced as (i) the steady component $u_{sp}(r)$ arising due to the steady part of the pressure gradient, (ii) the steady component $u_{sw}(r)$ derived from the steady part of the wall motion, (iii) the oscillatory component $u_{op}(r,t)$ due to the time dependent part of the pressure fluctuation and (iv) another oscillatory part $u_{ow}(r,t)$ resulting from the unsteady part of the boundary pulsation. These four parts may be combined with each other in the following way depending upon the build of the driving forces and as velocity distribution is one of the main factor controlling dispersion, the coefficient of dispersion will respond accordingly which can be seen later on To find out the contribution of the four components in the velocity, we have diagrammed some possible combinations of the velocity components in Fig.1. Fig.1(a) shows the variation of the steady components of velocity w.r.t radial coordinate. Velocity components, and are shown in the figure as a function of. In other words, the figure depicts the combined as well as the sole influences of the moving boundary and pressure factor on the steady part of the velocity. In the absence of pressure factor, the velocity is pushed in the positive direction from zero by the outer wall movement and naturally the deviation is growing in the direction of the outer wall. If, on the other hand, boundary wall does not contribute in the flow velocity, the flow profile takes the well known parabolic shape due to constant pressure gradient. The effects of both driving forces can be seen in the velocity when the two parts

combine with each other.

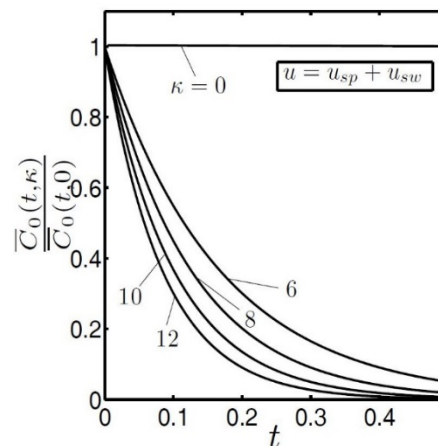


Fig. 2. Solute residual with time due to bulk-flow reaction.

Periodic components of the velocity due to pressure pulsation and wall oscillation are shown in Fig. 1(b) and 1(c) respectively for different phase angles. While the pressure gradient creates disturbances over the whole domain of the flow, the effects of wall oscillation is prominent in the vicinity of the outer wall. Both steady and periodic parts of the pressure gradient and wall force are combined in Fig. 1(d) and 1(e) respectively. Comparison of Fig.1(a) 1(b) and 1(d) reveals that steady part of the pressure gradient has larger contribution than that of the oscillatory part in the velocity profile. If we compare Fig. 1(e) with 1(a) and 1(c), the main function of the steady part u_{sw} will translate the flow near the outer wall in the direction of flow, though the periodic part is found to have larger contribution in the total flow. Both the oscillatory parts of the driving forces ($u = u_{op} + u_{ow}$) combine in Fig. 1(f).

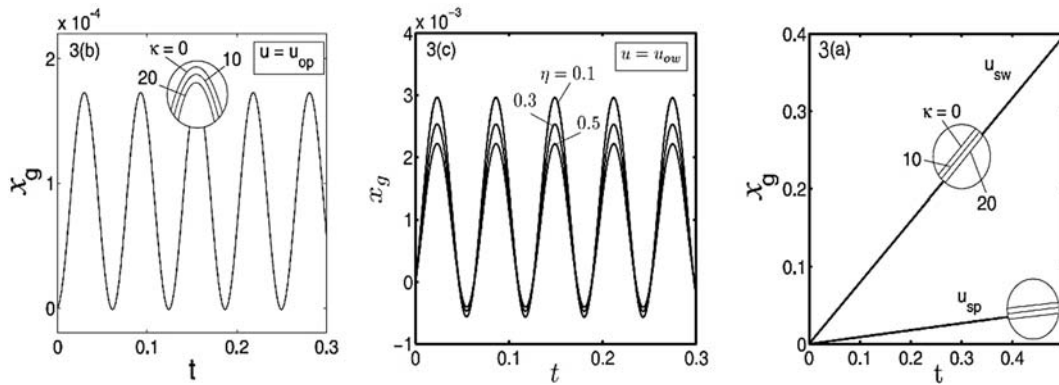


Fig. 3. Temporal variation of the centroid displacement (a) for different values of the reaction parameter when $u = u_{sp}$ & $u = u_{sw}$ and (b) when $u = u_{op}$ (c) for different values of the radius ratio η when $u = u_{ow}$.

Fig. 1(b), 1(c) and 1(f) shows that when both periodic parts joins in the total flow, the part arising from boundary movement has greater contribution, the feedback from the pressure factor seems to be rather small. Fig. 1(g) and 1(h) describes the velocity profile in the absence of periodic pressure gradient ($u = u_{sp} + u_{sw} + u_{ow}$) and boundary movement ($u = u_{sp} + u_{sw} + u_{op}$) respectively. The same is shown in Fig. 1(i) and 1(j) where steady components of the pressure gradient ($u = u_{sw} + u_{op} + u_{ow}$) and boundary movement ($u = u_{sw} + u_{op} + u_{ow}$) are respectively absent. Velocity profile with all components ($u = u_{sp} + u_{sw} + u_{op} + u_{ow}$) can be seen in Fig. 1(k). Comparison of Fig. 1(g) and 1(h) with 1(k) establishes the fact that the absence of periodic wall oscillation is more influential than the periodic pressure gradient. Same conclusions can be drawn about the steady parts also by considering the figures 1(i), 1(j) and 1(k). The greater effect of the periodicity of the boundary movement can also be apprehended by the comparison of the Fig. 1(a), 1(g), 1(h) whereas Fig. 1(f), 1(i), 1(j) brought forward the leading role of the steady part of boundary force over that of the pressure force. As wall factor puts down the pressure factor for both steady and periodic nature of flows (when considered individually), consequently this dominance further enhances in the total flow (Fig. 1(d), 1(e), 1(k)). Steady part of one driving force is combined with the periodic part of the other force in Fig. 4(l) which again proves the superiority of the wall force. Some of the important findings about the dominance of the velocity components over each other are listed in Table 2. To see the depletion of the solute w.r.t time due to the homogeneous reaction with the solvent, Eq. (18) is called upon. When $p = 0$, Eq. (18) becomes

$$\bar{C}_0(t, \kappa) = e^{-\kappa t} \quad (26)$$

Equation (26) represents the total mass of the solute, which is function of κ and t . We observe here how the total amount of tracer material is depleted over time for a given reaction parameter. When $\kappa = 0$, $\bar{C}_0(t, \kappa) = 1$, which represents a constant mass in the whole annular gap w.r.t time. As expected, dimensionless mass $x_g = \frac{\bar{C}_0(t, \kappa)}{\bar{C}_0(t, 0)}$ is a decreasing function of κ and t (Fig. 2).

Centroid displacement of the solute, estimated by the normalized first order moment $x_g = \frac{C_1}{C_0}$ has been studied for steady flows u_{sp} , u_{sw} and periodic flows u_{op} , u_{ow} with different values of the reaction parameter κ and radius ratio η . When the flow is steady, x_g is found to increase linearly with time (Fig. 3(a)) and it advances for a given time t . It is observed that, for periodic flow, the center of gravity of the slug (x_g) proceeds with periodic oscillations over time (Fig. 3(b,c)). In any case, the amplitude of oscillation decreases with the increase in reaction parameter κ , though the effect of radius ratio η is found to be flow dependent. While for pressure driven oscillatory flow, the amplitude increases with increase of η , opposite tendency cropped up for wall driven flow.

Temporal variation of the dispersion coefficient D_a with respect to time is shown in Fig. 4 for various possible combinations of velocity components as described in Table 1. While for most of the figures reaction parameter is considered to vary, the variation of radius ratio and frequency of oscillation are shown in Fig. 4(e) and 4(g) respectively, for the sake of completeness. It can be seen from Fig. 4(a) that for steady flow, dispersion coefficient increases almost linearly with time. D_a is found to raise in the absence of pressure gradient whereas absence of wall movement makes the dispersion coefficient to fall dramatically. For all time, D_a decreases with the increase of κ , the parameter for first order reaction in the bulk flow. This decrease of the dispersion coefficient with the increase in the reaction rate constant is based on sound physical ground. Increase in κ leads to the growth in the number of moles of solute undergoing chemical reaction resulting in a drop in dispersion coefficient (Gupta and Gupta 1972). It can be mentioned that, for absorption in the boundary also similar trend can be seen (Mazumder and Das 1992)

Fig. 4(b) shows the variation of the dispersion coefficient with time when the flow is only due to the periodicity of the pressure gradient. The same is shown in Fig. 4(c) for flow due to unsteady boundary movement only. It is seen that the dispersion coefficient moves with periodic oscillations over time. While for flow due to

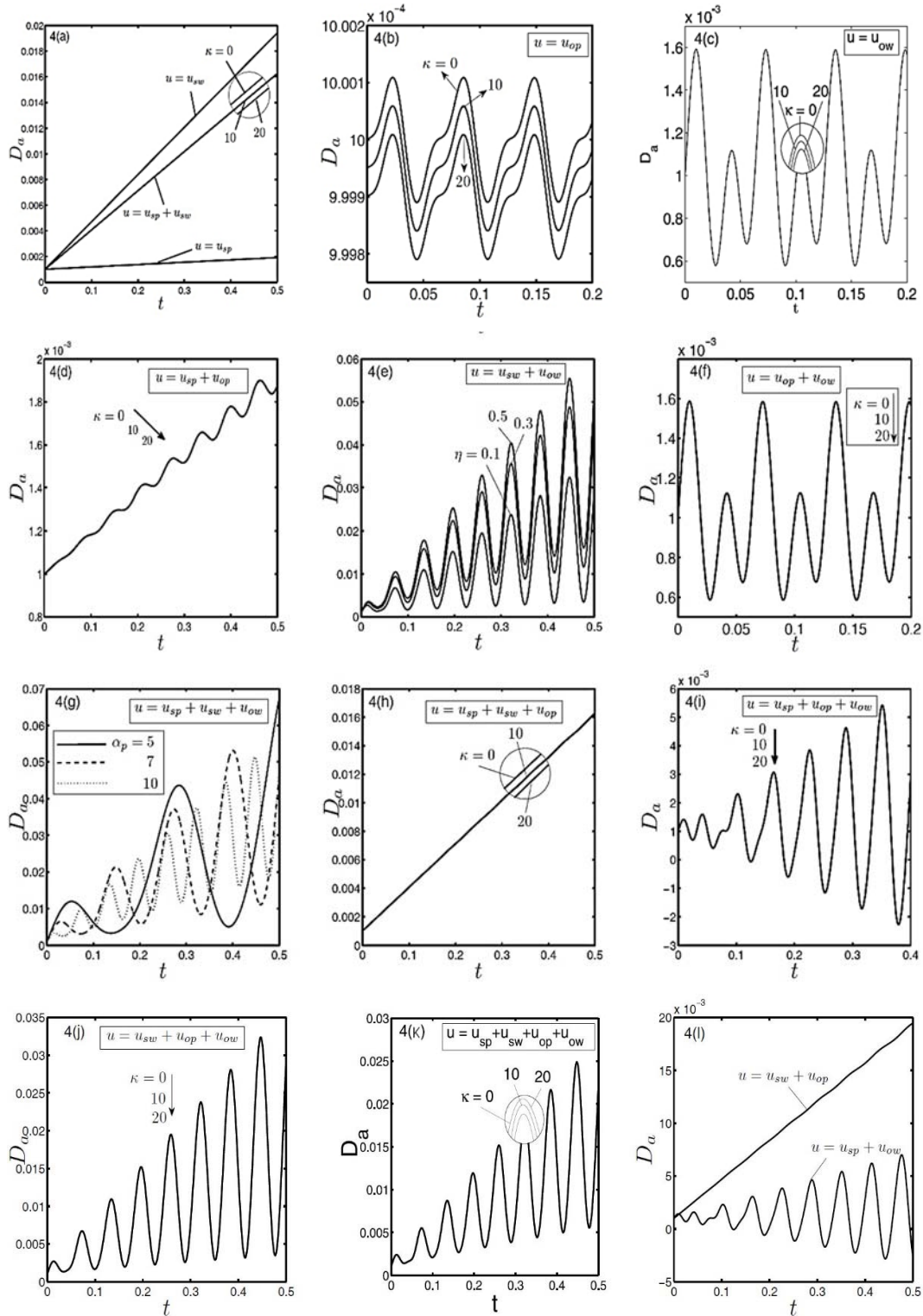


Fig. 4. Dispersion coefficient D_a against time due to various form of the velocity distribution as in Fig 1.

boundary oscillation, the double frequency period in the dispersion coefficient is prominent, it is suppressed in case of pressure fluctuation. Wall oscillation is found to produce more dispersion than the pressure gradient (Paul and Mazumder 2008). Also D_a in the oscillatory current is much lower than D_a in the steady current (Mazumder and Das 1992).

Actually, when the period of oscillatory current is much smaller than the characteristic time of lateral diffusion, the shear effect due to the periodic flow becomes asymptotically smaller than that of a steady current (Okubo 1967). For flow caused by wall factor, amplitudes of the oscillations in dispersion coefficient is many times higher compared to flow

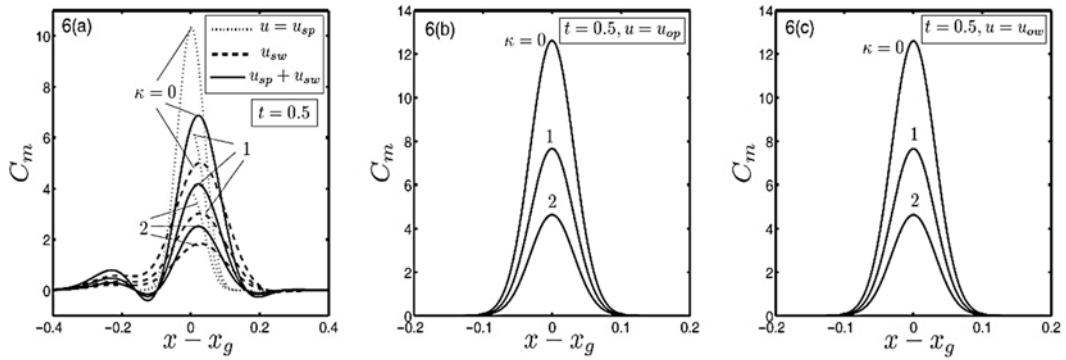


Fig. 5. Distribution of mean concentration against the axial distance for different values of the reaction parameter (a) when $u = u_{sp} + u_{sw}$ (b) when $u = u_{op}$ and (c) when $u = u_{ow}$.

due to pressure factor. When the both oscillatory parts combine together (Fig. 4(f)), dispersion coefficient almost behaves like as if it is due to the boundary oscillation only, the presence of the pressure factor seems to have negligible effect. Therefore, as pointed out by Paul and Mazumder (2008), boundary oscillation has larger contribution in the dispersion coefficient than the pressure pulsation. It is quite expected since the velocity, a major factor controlling dispersion, shows similar biasness towards the wall movement. In all cases, the dispersion coefficient decreases with the increase of the reaction parameter.

Dispersion coefficient due to the steady and oscillatory parts of the pressure gradient and that of the wall movement are shown in Fig. 4(d) and 4(e) respectively. It can be seen that the effect of steady flow is more prominent during initial time. Actually the controlling parameters like reaction parameter κ , radius ratio η , frequencies of oscillations (ω_p and ω_w) etc used to take some time to set themselves in motion. That's why dispersion coefficient usually shows weak dependence on those parameters during initial time. Steady flow is found to have a better control over its unsteady counter part in producing dispersion when the driving force is pressure gradient. For dispersion due to wall vibration, the control is comparatively much weaker. Wall vibration moves many times ahead than the pressure gradient in producing dispersion when steady part is added to its periodic counter part, which is quite natural since for both steady and periodic flow, pressure factor is preceded by wall factor in the run for dispersion.

The effect of radius ratio of the annular tube on the dispersion coefficient is quite interesting. For flow under the sole influence of the periodic pressure gradient, it is found that the dispersion coefficient increases with the increase of radius ratio whereas the trend is reversed in the sole presence of periodic boundary movement. On physical ground, low radius ratio provides greater room for the solute to disperse giving rise to D_a . When the driving force is the periodic pressure gradient, its strength becomes weaker if spread over larger area (i.e., smaller η) resulting a fall in dispersion coefficient. When steady counter part is added to its periodic part, D_a responds

in just a opposite way with the variation of the radius ratio. Due to the dominance of the wall factor, nature of D_a in u_{ow} and $u_{op} + u_{ow}$ and in u_{sw} and $u_{sp} + u_{sw}$ are same. Nature of D_a with respect to the velocity components are shown in the Table 3. For the sake of space figure (Fig. 4(e)) is provided only for a single case.

Table 3 Nature of D_a as a function of η depending on the velocity distribution

Velocity component	Nature of D_a
u_{sp}	Decreasing
u_{sw}	Increasing
u_{op}	Increasing
u_{ow}	Decreasing
$u_{sp} + u_{op}$	Decreasing
$u_{sw} + u_{ow}$	Increasing (Fig. 4(e))
$u_{sp} + u_{sw}$	Increasing
$u_{op} + u_{ow}$	Decreasing

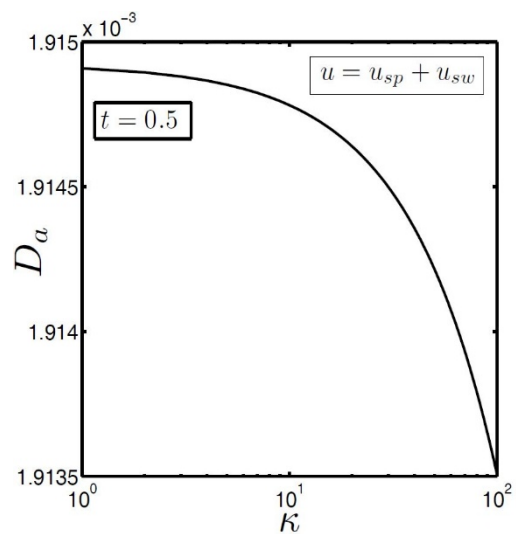


Fig. 6. Variation of the dispersion coefficient with the reaction parameter when the flow is steady ($u = u_{sp} + u_{sw}$).

Dispersion coefficient due to the combined effort of all components of the velocity is shown in Fig. 4(k). It can be concluded that, for steady ($u = u_{sp} + u_{sw}$, Fig. 4(a)) as well as for combined flow ($u = u_{sp} + u_{sw} + u_{op} + u_{ow}$, Fig. 4(k)), the effect of the pressure gradient is to diminish the dispersion coefficient whereas wall movement favors to raise it. The dominance of the wall factor in case of steady flow and periodic flow is already established in Fig. 4(a) and 4(c, f) respectively. Now the same is established for combined flow also in Fig. 4(e,k). Like the case of steady and periodic flow, dispersion coefficient shows biasness towards the wall force compared to the pressure force in case of combined flow. In the case of periodic flow, the supremacy of the wall factor is not only qualitative, but quantitative also (Fig. 4(c, f)).

Comparison of Fig. 4(a,g,h) bring forward the strength of the periodicity of the wall force compared to the pressure force. When the component u_{ow} is introduced, it makes a radical change in the dispersion coefficient whereas the introduction of u_{op} seems to have no noticeable effect. It can also be explained from the Fig. 4(g,h,k). The withdrawal of u_{op} from the total flow seems less effective compared to the withdrawal of u_{ow} which shows a revolutionary effect leaving the velocity in a form as if it is time independent. The effect of frequency of oscillation on D_a is explained in Fig. 4(g). The effect is found to be same irrespective of the combination of the velocity components. In any case, the increase of the frequency parameter leads to a decrease of the dispersion coefficient. The superiority of the steady part of the wall force compared to the pressure force can be inferred from the Fig. 4(i, j,k). While absence of u_{sw} makes a significant change in the profile, the feedback in the absence of u_{sp} is comparatively negligible. Fig. 4(l) shows that the oscillatory part of the pressure force is not stronger enough to produce any significant change in u_{sw} , but substantial effect of u_{ow} on u_{sp} can be seen.

Fig. 6 shows the variation of the dispersion coefficient against the bulk flow reaction parameter κ when the flow is steady ($u = u_{sp} + u_{sw}$). The expected decrement of the dispersion coefficient with the increase of κ can be seen here. It is remarkable that the rate of decrement becomes sharper with the increase of κ .

The variations of mean concentration distribution $C_m(x,t)$ has been presented in Fig. 4(a – d) against the axial distance ($x - x_g$) for different components of velocity. The increase of the reaction parameter κ ensures the depletion of the reactive material, and therefore the peak of the mean concentration distribution gradually decreases in any flow situation. It follows from Fig. 5(a) that steady component of the pressure force acts in favour of rising the peak of the concentration curve, whereas steady wall force attempts to make the concentration distribution curve to spread over larger portion forbidding the growth of the peak of the concentration curve.

9. CONCLUSION

In this work, longitudinal dispersion of reactive solute is examined in presence of homogeneous first order irreversible reaction between the solvent and the solute. The flow is considered to be driven by the joint action of two driving forces consisting of a steady and periodic part resulting in a tetra parted velocity distribution. The role played by each of the individual components of velocity in the dispersion process is sorted out. The following general conclusion can be made from the study:

- (a) The dispersion coefficient decreases with an increase in the reaction rate constant, the effect similar to irreversible reaction in the boundary.
- (b) Flow dependence of the effect of radius ratio is established through this analysis.
- (c) An absolute potency of the wall driven force over the pressure force on the velocity distribution as well as on dispersion coefficient is remarkable.
- (d) Pressure force is found to act in favour of decreasing the dispersion coefficient, whereas wall force against it.
- (e) Because of the depletion of the contaminant, peak of the mean concentration distribution has been seen to decrease with the increase of bulk-flow reaction rate constant.

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