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Study of Inertia and Compressibility Effects on the Density Wave Oscillations of Two-Phase Boiling Flows in Parallel Channels

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ABSTRACT

In this research, a theoretical model is presented to investigate the density wave oscillations (DWOs), in two horizontal parallel channels with lumped parameter model based on two phase homogeneous hypothesis. The parallel channel is composed of the entrance section, heating section and outlet section and the model consists of the boiling channel model, pressure drop model, parallel channel model, constructive model and inertia and compressibility effects, while subcooled boiling effect is neglected and the governing equations are solved by Gear method. The model is validated with experimental data of a single channel flow instability experiment. Then the flow instability in twin channel system is studied under different conditions. This model can analyze the effects of external parameters, such as fluid inertia and compressible gases on the stability margins of density wave oscillations. The results show that, the fluid inertia and compressible gases can significantly change the stability margins of two parallel channels; in fact, the stability behavior of two parallel channel system improves with increasing the inlet inertia and outlet compressibility but, increasing the outlet inertia and inlet compressibility have negative effects the system stability.

Keywords: Density wave oscillations; Parallel channels; Fluid inertia effects; Compressibility effects.

NOMENCLATURE

A	coefficients of equations
B	coefficients of equations
C	coefficients of equations
De	hydraulic diameter
G	mass flux
H	enthalpy
h_{fg}	latent heat of evaporation
K	local throttling coefficient
L	length of channel
f	darcy–weisbach friction factor
P	pressure
Q	power
t	time
U	velocity along the channel
V	specific volume
W	mass flow rate
X	quality
x,y,z	cartesian coordinateoutletoutlet section
γ	the ratio of vg to vf
ΔP	pressure drop
Π	coefficients of equations
ρ	density

Superscripts

+	dimensionless
-	average

Subscripts

0	reference
1φ	single phase
2φ	two phase
Acc	acceleration
E	entrance
Ex	exit
f	saturated
f	frictional
f_{fg}	the difference between saturated vapor and liquid properties
g	saturated
g	gravitational
H	heating section
in	inlet parameter
j	the jth channel
outlet	outlet section

Dimensionless groups

$N_{pch}=Q.v_{fg}/(W.h_{fg}.v_f)$	phase change number
$N_{sub}=(h_f - h_{in}).\ v_{fg}/(h_{fg}\ .\ v_f)$	sub cooling number

1. INTRODUCTION

Two phase flow instabilities are undesirable phenomena that can be found in many industrial systems and equipments, such as steam generators, boiling water reactors, reboilers, nuclear power plants and refrigeration plants. There are many studies dealing with these instabilities (Zahn, 1964; Wedekind *et al*. 1986; Mithraratne *et al*., 2001; Beck *et al*., 1986; Wedekind *et al*. 1974; Ibrahim, 2001).

These instabilities can result in mechanical vibration, system control problems, inducing undesirable mechanical stress, changing local heat transfer characteristics, restrict operating parameters and reducing system safety.(Zahn, 1964; Wedekind *et al*., 1986; Mithraratne *et al*., 2001). The appropriate characterization of these instabilities and the condition for their occurrence can determine optimal and safe operation of the involved systems.

An important case of two-phase flow instability is density wave oscillation (DWO). The density wave makes a delay in the local pressure drop that is caused by a change in inlet flow. Because of this delay, the sum of all local pressure drops may result in a total drop that is out-of-phase with the inlet flow (Kakac *et al*., 2008). The basic mechanism causing flow instabilities in BWRs is the density wave. The characteristic periods of these oscillations are associated with the time required for a fluid particle to travel through the entire loop.

Several experimental, theoretical and numerical studies have been performed in the area of two-phase flow instabilities in parallel channels (Libo *et al*., 2014; Aritomi *et al*., 1979; Fukuda *et al*., 1979; Clausse *et al*., 1989; Podowski *et al*., 1990; Guido *et al*., 1991; Xiao *et al*., 1993; Lee *et al*., 1999; Hirayama *et al*., 2006; Yun *et al*., 2008; Zhang *et al*., 2009; Guo *et al*., 2010, Lee *et al*.,2014, zhou *et al*.,2013, Chiapero *et al*.,2013, Paul *et al*.,2014, Hua *et al*.,2015).

Aritomi *et al*. (1977), investigated density wave oscillations in parallel channels by using a nonlinear model. The results were in accordance with experimental data. According to their model the parallel channels with 2, 3 and 4 channels show similar characteristics.

Podowski *et al*. (1997), analyzed density wave oscillations in a single heated channel which was in parallel with a single-phase adiabatic channel. They examined the effect of different models and computational problems on the stability behavior.

Guido *et al*. (1991), analytically studied DWOs in two parallel channels based on the homogenous equilibrium model. Their results showed that there were two primary modes for two parallel channels: in-phase and out-of-phase oscillation.

Lee *et al*. (1999) used Galerkin nodal approximation method based on Clausse and Lahey's theory (1990) to study flow instability in parallel channels. Their results showed that increasing channel numbers makes the system more unstable.

Muñoz-Cobo *et al*. (2002) proposed a model integrated with a high order modal kinetics to investigate the DWO in a BWR. According to their results the period of out-of-phase oscillations was half of pressure drop between plenums.

Guo *et al*. (2008) investigated the DWOs in parallel channels analytically. That model was based on the model of Clausse and Lahey (1990) and Lee and Pan (1999). The effect of system pressure, inlet throttling coefficients, asymmetric heating as well as the inlet and riser sections on the stability of parallel-channel systems were studied. A commercial code like RELAP5, can analyze the flow instabilities in parallel-channel systems. The literature describing the codes such as RELAP5, can be found in the references (Colombo *et al*., 2012; Xia *et al*., 2010, 2012).

Libo Qian *et al*. (2014) investigated DWOs in parallel channels with a lumped mathematical model based on homogenous hypothesis. They studied the effects of pressure P, inlet throttling coefficient, asymmetric heating and throttling, kin and exit throttling coefficient, kout on the stability margins. The results were presented, based on the sub cooling number (Nsub) and phase change number (Npch).

Ruspini(2012) investigated numerically, the effect of external parameters such as the fluid inertia and the presence of compressible gases, on DWOs in a single channel. The physical model used in this study is based on homogenous hypothesis. He showed that fluid inertia and compressible gases in the loop can change stability limits.

According to Ruspini (2012), the effects fluid inertia and compressible gases in the loop will be investigated in the two parallel channels. In most studies so far, the external effects are not investigated in the parallel channels and focus of studies are on the internal parameters, and the external effects are ignored, so in this study, a system which is composed of two parallel channels, two surge tanks, two reservoirs with the piping between them is considered to investigate the role of external parameters on the stability limits.

2. THEORETICAL MODEL AND NUMERICAL SIMULATION

The system used in this study is illustrated schematically in Fig. 1.

The system is composed of two parallel channels, two constant pressure tanks, two different surge tanks, four pipe lines and four localized pressure drops. It is considered that, the surge tanks have constant temperature and the gas inside of them acts as an ideal gas. The parallel-channel system is illustrated schematically in Fig. 2.

In Fig. 2, Each channel is divided into three sections as, entrance section, heating section and outlet section, and the channels are connected between inlet and outlet plenum, which could confirm that the pressure drop between channels is identical all the time. The length of the entrance section, heating section and outlet section are $L_{E,j}$, $L_{H,j}$, L_{outlet}, respectively. In the heating section the boiling

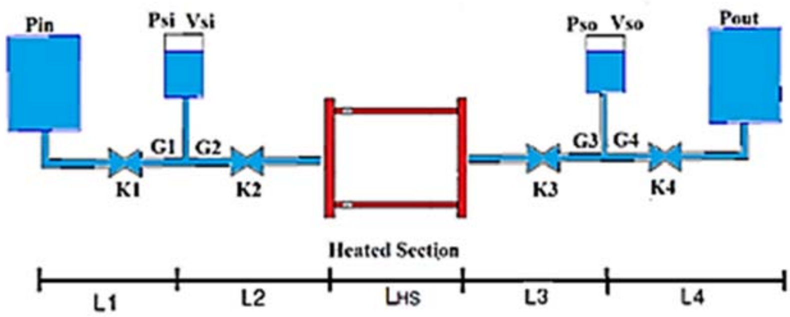

Fig. 1. Schematic of the model.

happens and two phase region appears. The boundary where the boiling starts is presented by λ which shows the onset of boiling.

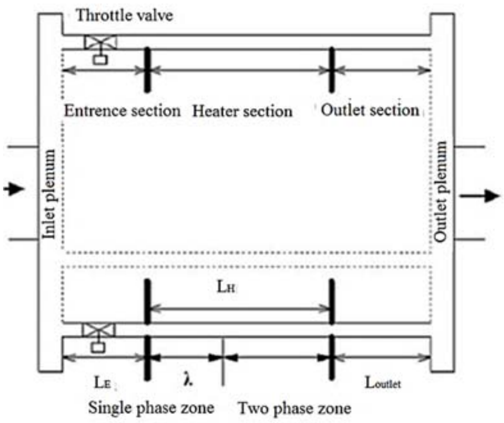

Fig. 2. Schematic of the parallel-channel system.

2.1 Governing Equations

According to Fig. 1, the governing equations can be presented as boiling channel and external system equations. The details are expressed as follow:

2.1.1 Boiling Channel Governing Equations

Boiling channel governing equations are based on the application of conservation equations of mass, momentum and energy in the heated channel with following assumptions:

1. One-dimensional flow.
2. Homogeneous-equilibrium two-phase flow model. In the homogenous-equilibrium model (HEM) the two phases everywhere are assumed to be well mixed, have the same velocity and are in thermal equilibrium.
3. Uniform heating along the heated section.
4. Constant working pressure in the heated channel
5. Constant fluid properties at given system inlet pressure.
6. Sub cooled boiling is neglected

These equations are expressed as follow:

$$\frac{\partial\rho}{\partial t}+\frac{\partial G}{\partial z}=0 \tag{1}$$

$$\frac{\partial G}{\partial t}+\frac{\partial}{\partial z}\left(\frac{G^2}{\rho}\right)+\frac{\partial P}{\partial z}+f\frac{G}{D_H\rho}=0 \tag{2}$$

$$\frac{\partial\rho h}{\partial t}+\frac{\partial Gh}{\partial z}=Q\frac{P_H}{A} \tag{3}$$

2.1.2 External System Governing Equations

In this section the mathematical description of external system is presented.

In order to find the governing equations of the surge tanks, only the continuity equation should be applied to the surge tanks presented in Fig. 1. The results are illustrated as follow:

$$\frac{d}{dt}P_{si}=\frac{P_{si}^2}{P_{soO}V_{soO}}\frac{A}{\rho_l}(G_1-G_2) \tag{4}$$

$$\frac{d}{dt}P_{so}=\frac{P_{so}^2}{P_{soO}V_{soO}}\frac{A}{\rho_l}(G_3-G_4) \tag{5}$$

Finally, application of the momentum equation to the fluid in the pipe results in governing equations. These equations for different pipe parts are presented as follow:

$$\dot{G_1}=\left[P_{in}-P_{si}-K_1\frac{G_1}{2\rho_l}|G_1|\right]\frac{1}{L_1} \tag{6}$$

$$\dot{G_2}=\left[P_{si}-P_2-(K_2+1)\frac{G_2}{2\rho_l}|G_2|\right]\frac{1}{L_2} \tag{7}$$

$$\dot{G_3}=\left[P_3-P_{so}-(K_3-1)\frac{G_3}{2\rho_{out}}|G_3|\right]\frac{1}{L_3} \tag{8}$$

$$\dot{G_4}=\left[P_4-P_{out}-(K_4-1)\frac{G_4}{2\rho_{out}}|G_4|\right]\frac{1}{L_4} \tag{9}$$

2.1.3 Non Dimensional Form of Governing Equations

Using the quantities as given in Table 1, the non-dimensional form of the governing are given below

Table 1 Non-dimensional parameters

Non-dimensional parameters	Definition
Non-dimensional boiling boundary	$\lambda_j^+=\lambda_j/L_H$
Non-dimensional location	$z^+=\frac{z}{L_H}$
Non-dimensional volume	$V^+=\frac{V}{A_j L_H}$
Non-dimensional mass flux	$W^+=W/(G_0.S)$
Non-dimensional mass flux	$G^+=G/G_0$
Non-dimensional enthalpy	$h^+=(h-h_{in})/(h_f-h_{in})$
Non-dimensional differential pressure	$\Delta P^+=\Delta P/(G_0^2/\rho_f)$
Non-dimensional time	$t^+=t/\frac{L_{H,j}\rho_f}{G_0}$
Non-dimensional density	$\rho^+=\rho/\rho_f$
Non-dimensional specific volume	$\upsilon^+=\upsilon/\upsilon_f$
Non-dimensional length	$L^+=L/L_H$

$$\frac{\partial\rho^+}{\partial t^+}+\frac{\partial}{\partial z^+}(G^+)=0 \tag{10}$$

$$\frac{\partial h^+\rho^+}{\partial t^+}+\frac{\partial}{\partial z^+}(G^+h^+)=\frac{N_{pch,j}}{N_{sub,j}} \tag{11}$$

$$\frac{\partial\rho^+}{\partial t^+}+\frac{\partial}{\partial z^+}\left(\frac{{G^+}^2}{\rho^+}\right)=-f\frac{L^+}{2D_e^+}\frac{{G^+}^2}{\rho^+}-\frac{\partial P^+}{\partial Z^+} \tag{12}$$

$$\frac{d}{dt^+}P_{si}^+=\frac{{P^+}_{si}^2}{{P^+}_{si0}{V^+}_{si0}}(G_2^+-G_1^+) \tag{13}$$

$$\frac{d}{dt^+}P_{s0}^+=\frac{{P^+}_{s0}^2}{{P^+}_{s00}{V^+}_{s00}}(G_4^+-G_3^+) \tag{14}$$

$$\dot{G^+}_1=\left[P_{in}^+(G_1,t)-P_{si}^+-K_1\frac{G_1^+}{2}\left|G_1^+\right|\right]\frac{1}{L_1^+} \tag{15}$$

$$\dot{G^+}_2=\left[P_{si}^+-P_2^+-(K_2+1)\frac{G_2^+}{2}\left|G_2^+\right|\right]\frac{1}{L_2^+} \tag{16}$$

$$\dot{G^+}_3=\left[P_3^+-P_{so}^+-(K_3-1)\frac{G_3^+}{2\rho_{out}^+}\left|G_3^+\right|\right]\frac{1}{L_3^+} \tag{17}$$

$$\dot{G^+}_4=\left[P_4^+-P_{out}^+-(K_4-1)\frac{G_4^+}{2\rho_{out}^+}\left|G_4^+\right|\right]\frac{1}{L_4^+} \tag{18}$$

The velocity gradient in the two phase region can be obtained by combining Eqs. (10-11) as follow:

$$\frac{\partial}{\partial z^+}(u^+)=N_{pch,j} \tag{19}$$

2.2 Analytical Lumped Parameter Model

The boiling channel equations (Eqs. 10-12) are set partial differential equations. They depend on the time (t) and axial position (z).

Applying integration to these equations with respect to z, can eliminate the z parameter from the governing equations. As result of this mathematical process the governing equations can be presented as zero-dimensional equations. The method described above is called lumped parameter approach in boiling systems.

Integration of the energy equation over single phase region, leads to an ordinary differential equation for the boiling boundary, integration of the mass conservation equation over the two-phase region yields an equation for the exit quality and finally integration of the momentum equation along channel, yields an ordinary differential equation for the inlet mass flux. In the following the calculation details are presented.

2.2.1 Mass-Energy Equation in the Two Phase Region

Integration of Eq. (19), in the two phase region leads to velocity distribution along the channel as follow:

$$u^+=u_{in}^++N_{pch,j}\,(z^+-\lambda^+) \tag{20}$$

The relation between volumetric and total mass flow rate, in the two phase region can be expressed as:

$$u^+=(\frac{x}{\rho_g^+}+1-x)G_{2\varphi}^+(z^+,t^+). \tag{21}$$

Combining the Eq. (20) and (21), from boiling boundary to any arbitrary point in two phase region yields:

$$G_{2\varphi}^+(z^+,t^+)=\frac{N_{pch,j}.[z^+-\lambda_j^+(t^+)]+G_{in,1}^+(t^+)}{(1+x_j(z^+,t^+)\upsilon_{fg}^+)} \tag{22}$$

According to Eq. (39), the total mass flux at channel exit is:

$$G_{ex,j}^+(t^+)=\frac{N_{pch,j}.[1-\lambda_j^+(t^+)]+G_{in,1}^+(t^+)}{(1+x_{ex}(t^+)\upsilon_{fg}^+)} \tag{23}$$

2.2.3 Boiling Boundary Equation

Integration of the energy equation-Eq. (11)-in the single-phase region leads to differential equation describing boiling boundary dynamics as Eq. (24). In this derivation, it is assumed that enthalpy changes linearly.

$$\frac{d\lambda_j^+(t^+)}{dt^+}=2\left[G_{in,j}^+(t^+)-\frac{N_{pch,j}}{N_{sub,j}}.\lambda_j^+(t^+)\right] \quad (24)$$

2.2.3 Exit Quality Equation

Integration of the mass conservation equation- Eq. (10)- along the channel, leads to exit quality dynamics as follow:

$$\int_0^1 \frac{\partial\rho^+}{\partial t^+}dz^+ + \int_0^1 \frac{\partial G^+}{\partial z^+}dz^+=0$$

$$\Rightarrow \frac{d}{dt^+}\int_0^1 \rho^+\, dz^+ = G^+_{in}-G^+_{ex} \quad (25)$$

In Eq. (25), ρ can be expressed as follows:

- In the single phase region:

$\rho=\rho_f$

- In the two phase region:

$\rho=\rho_f(1-\alpha)+\rho_g\alpha$

Where α is void fraction and has the following relation with quality.

$$\alpha=\frac{\gamma x}{1+(\gamma-1)x} \quad ,\gamma=\frac{\upsilon_g}{\upsilon_f} \quad (26)$$

Combining Eq. (25) and Eq. (26), one has:

$$\frac{dx_{ex}}{dt^+}=b_9{}^+ \quad (27)$$

Where:

$$b_9{}^+ = b_6{}^+ + b_7{}^+ b_8{}^+ \quad (28)$$

$$b_{5,j}^+=\frac{\gamma}{(\gamma-1)^2 x_{ex(t^+)}}[-\frac{(\gamma-1)}{1+(\gamma-1)x_{ex(t^+)}} + \frac{\ln(1+(\gamma-1)x_{ex(t^+)})}{x_{ex}(t^+)}] \quad (29)$$

$$b_{6,j}^+ = \frac{G_{in,j}^+(t^+)\upsilon_{fg}^+ x_{ex,j}(t^+) - N_{pch,j}(1-\lambda_j(t)^+)}{(1-\lambda_j(t)^+)(\rho_g^+-1)(1+\upsilon_{fg}^+.x_{ex}(t^+))b_{5,j}^+} \quad (30)$$

$$b_{7,j}^+ = \frac{\gamma}{(\gamma-1)(1-\lambda_j(t)^+)b_{5,j}^+} .[1-\frac{\ln(1+(\gamma-1)x_{ex},j(t^+))}{(\gamma-1)x_{ex,j}(t^+)}] \quad (31)$$

$$b_{8,j}^+ = 2\left[G_{in}^+, j(t^+) - \frac{N_{pch,j}}{N_{sub,j}}.\lambda_j(t)^+\right] \quad (32)$$

2.2.4 Integral Form of Momentum Equation

Integrating of the momentum equation (Eq. (12)), along the jth channel, yields:

$$\int_0^{L_{E,J}^+ +1+L_{R,J}^+} \frac{\partial G_j^+(t^+)}{\partial t^+}dz^+ = \Delta P^+(t^+)-\Delta P_{acc,j}^+(t^+)-\Delta P_{f,j}^+(t^+) \quad (33)$$

The right hand terms of Eq. (33), are expressed as:

- $\Delta P^+(t^+)$, is the pressure drop between lower and upper plenum, and identical to all channels.
- $\Delta P_{acc,j}^+(t^+)$, is the acceleration pressure drop, and can be presented as:

$$\Delta P_{acc,j}^+(t^+)=\frac{G_{ex,j}^+(t^+)^2}{\rho_{ex,j}^+(t^+)} - G_{in,j}^+(t^+)^2 \quad (34)$$

And the frictional pressure drop $\Delta P_{f,j}^+(t^+)$, is presented as:

$$\Delta P_{f,j}^+(t^+)=f_{1j}\frac{\lambda_j^+(t^+)}{2D_e^+}.G_{in,j}^+(t^+)^2 + f_{1j}\frac{[1-\lambda_j^+(t^+)]}{2D_e^+}\langle\phi^2\rangle\langle G_{2\varphi}^+{}^2\rangle \quad (35)$$

Substituting Eqs. (22) into Eq. (33) and rearranging the results, yields:

$$\Pi_{1,j}^+.\frac{dG_{in,j}^+(t^+)}{dt^+}+\Pi_{2,j}^+.\frac{d\lambda_j^+(t^+)}{dt^+}+\Pi_{3,j}^+.\frac{dx_{ex,j}^+(t^+)}{dt^+} =\Delta P^+(t^+)-\Delta P_{acc,j}^+(t^+)-\Delta P_{g,j}^+(t^+) -\Delta P_{f,j}^+(t^+)-\Delta P_{E,j}^+(t^+)-\Delta P_{R,j}^+(t^+) \quad (36)$$

Where

$$\Pi_{1,j}^+=[\left(\frac{(L_{R,j}^+ + L_{E,j}^+)+x_{ex,j}(t^+).\upsilon_{fg}^+.L_{E,j}^+}{(1+x_{ex,j}(t^+).\upsilon_{fg}^+)}\right) +(\lambda_j^+(t^+)+b_{3,j}^+)] \quad (37)$$

$$\Pi_{2,j}^+=\left[\frac{-N_{pch,j}.L_{R,j}^+}{(1+x_{ex,j}(t^+).\upsilon_{fg}^+)}+(b_{1,j}^+ + G_{in,j}^+(t^+))\right] \quad (38)$$

$$\Pi_{3,j}^+=\left[b_{2,j}^+ - \frac{\upsilon_{fg}^+ L_{R,j}^+.(N_{pch,j}.[1-\lambda_j^+(t^+)]+G_{in,j}^+(t^+))}{(1+x_{ex,j}(t^+).\upsilon_{fg}^+)^2}\right] \quad (39)$$

$$b_{1,j}^+=-\frac{2N_{pch,j}.[1-\lambda_j^+(t^+)]}{(x_{ex,j}(t^+).\upsilon_{fg}^+)}+\frac{\ln(1+x_{ex,j}(t^+).\upsilon_{fg}^+)}{(x_{ex,j}(t^+).\upsilon_{fg}^+)} .\left(\frac{2N_{pch,j}.[1-\lambda_j^+(t^+)]}{(x_{ex,j}(t^+).\upsilon_{fg}^+)}-G_{in,j}^+(t^+)\right) \quad (40)$$

$$b_{2,j}^+=-\frac{N_{pch,j}.[1-\lambda_j^+(t^+)]^2}{(x_{ex,j}(t^+)^2.\upsilon_{fg}^+)}+\frac{[1-\lambda_j^+(t^+)]}{(x_{ex,j}(t^+)^2.\upsilon_{fg}^+)} .\left(\frac{2N_{pch,j}.[1-\lambda_j^+(t^+)]}{(x_{ex,j}(t^+).\upsilon_{fg}^+)}-G_{in,j}^+(t^+)\right).\ln((1+x_{ex,j}(t^+).\upsilon_{fg}^+)) \quad (41)$$

$$+\frac{[1\text{-}\lambda_j^+(t^+)]}{\left(x_{ex,j}(t^+).\left(1+x_{ex,j}(t^+).\upsilon_{fg}^+\right)\right)}(G_{in,j}^+(t^+)\text{-}\frac{N_{pch,j}.[1\text{-}\lambda_j^+(t^+)]}{\left(x_{ex,j}(t^+).\upsilon_{fg}^+\right)})$$

$$b_{3,j}^+=\frac{[1\text{-}\lambda_j^+(t^+)]}{\left(x_{ex,j}(t^+).\upsilon_{fg}^+\right)}.\ln\left(1+x_{ex,j}(t^+).\upsilon_{fg}^+\right) \tag{42}$$

2.3 Parallel Channel Section

In Fig. 2, all the channels have the same inlet and outlet plenum, so they have the same pressure drop, thus:

$$\Delta P_1^+=\Delta P_2^+=P_2^+\text{-}\ P_3^+ \tag{43}$$

Combining Eq. (16) and Eq.(17), for (P_2^+- P_3^+), yields:

$$P_2^+-P_3^+ = -L_2^+\dot{G}^+{}_2-L_3^+\dot{G}^+{}_3+\left(P_{si}^+-\dot{P}^+{}_{so}\right) \text{-}(K_2\text{+}1)\frac{G_2^+}{2}|G_2^+|-(K_3\text{-}1)\frac{G_3^+}{2\rho_{out}^+}|G_3^+| \tag{44}$$

Substituting Eq. (44), Eq. (24) and Eq. (27), into Eq. (36), giving:

$$P_2^+-P_3^+=-L_2^+\left(\dot{G}^+{}_{in,1}+\dot{G}^+{}_{in,2}\right) -L_3^+\left(A_1+A_2+B_1\frac{dG_{in,1}^+(t^+)}{dt^+}+B_2\frac{dG_{in,2}^+(t^+)}{dt^+}\right) +\left(P_{si}^+-P_{so}^+\right)-(K_2\text{+}1)\frac{G_2^+}{2}|G_2^+| -(K_3\text{-}1)\frac{G_3^+}{2\rho_{out}^+}|G_3^+| \tag{45}$$

After some algebra, one has:

$$\frac{dG_{in,1}^+(t^+)}{dt^+}=[(\Pi_{1,2}^++L_2^++L_3^+).(-L_3^+(A_1+A_2)+C_1)-(-L_3^+(A_1+A_2)+C_2)(L_2^++L_3^+B_2)]/[(\Pi_{1,2}^++L_2^++L_3^+)(\Pi_{1,1}^++L_2^++L_3^+B_2)-(L_2^++L_3^+B_2)(L_2^++L_3^+B_1)] \tag{46}$$

$$\frac{dG_{in,2}^+(t^+)}{dt^+}=[(\Pi_{1,1}^++L_2^++L_3^+B_1).(-L_3^+(A_1+A_2)+C_2)-(-L_3^+(A_1+A_2)+C_1)(L_2^++L_3^+B_1)]/[(\Pi_{1,2}^++L_2^++L_3^+)(\Pi_{1,1}^++L_2^++L_3^+B_2)-(L_2^++L_3^+B_2)(L_2^++L_3^+B_1)] \tag{47}$$

At this stage all the required ordinary differential equations are derived. These are Eq. (15), Eq. (18), Eq. (24), Eq. (27), Eq. (46) and Eq. (47). Solving the governing equations simultaneously can find eight unknowns of the parallel channel system.

2.4 Friction Correlations

The correlations of friction coefficient of single- and two-phase flow are shown in Table 2.

Table 2 Friction coefficients

Regions	Correlations
Single phase	
Laminar Re<1000	f=64/Re
1000 <Re< 2300	Linear interpolation between laminar and Blasius equation
Turbulence 2300<Re<10000	equation($f=0.3164/Re^{0.25}$)
10000<Re<100000	Linear interpolation between Blassius and Colebrook equation
Re>100000	Colebrook equation $f=\left(1.8.\log(Re/6.9)\right)^{-2}$
Two phase	
Two phase friction multiplier	$\phi_{l0}^2=\left[1+x(\frac{\upsilon_{fg}}{\upsilon_f})\right]\left[1+x(\frac{\mu_f}{\mu_g}\text{-}1)\right]^{-0.25}$

2.5. Numerical Simulation

In this section the numerical method used to solve the governing equations are expressed.

The governing equations of the system presented in Fig. 1, can be presented as below in general:

$$\frac{d\vec{y}}{dt^+}=\vec{f}(t^+,\vec{y},\vec{y}',x) \qquad ,\vec{y}(t_0^+)=\vec{y}_0$$

Where $\vec{y}$ is the state vector $[\lambda_j^+(t^+, x_{ex}, G_{in,1}^+(t^+), G_{in,2}^+(t^+), P_{si}^+, P_{sO}^+, G_1^+, G_2^+]$ and subscript '0' stands for steady state and x is the input vector. The input vector can be heat power, system pressure, and inlet and exit throttling coefficient, inlet sub cooling, total mass rate, etc.

These equations belong to a specific class of equations called stiff. The Gear multi-value method (1974) is the special algorithm which was designed to solve stiff problems, in this method discretization process of ordinary differential equations has the following form:

$$y_{n+k}\text{-}h\beta_k f(x_{n+k},y_{n+k})=g_{n+k} \qquad ,g_{n+k}=\sum_{j=0}^{j=k-1}\alpha_j y_{n+j}$$

In the Gear method the value of y_{n+k} is related to values of $(y_{n+k-1},y_{n+k-2,}y_n)$ and $f(x_{n+k},y_{n+k})$. The values of α_j,β_k , when the discretization order is taken 3, are as the following Table3:

Table 3 Coefficient of Gear method (Order 4)

β_4	α_0	α_1	α_2	α_3
12/25	-12/100	48/75	-36/25	48/25

The Gear method is in implicit form, so in order to find the value of y_{n+k} , an iteration method, like newton's method should be used, with a suitable initial guess:

$$y^{s+1}_{n+k}=y^{s}_{n+k}-\left[I-h\beta_k\frac{\partial f}{\partial y}(x_{n+k},y^{s}_{n+k})\right]^{-1}\left[y^{s}_{n+k}-h\beta_k f(x_{n+k},y^{s}_{n+k})-g_{n+k}\right]$$

Applying Gear method to the governing equations and considering the initial conditions, the stability test of an interest point can be done, according to Fig. 3. In this algorithm, a small perturbation of the heating power should be considered. If the solution dampens out following the perturbation, the system is stable; otherwise if the solution grows the system is unstable. Between these two stable and unstable conditions, the solution is periodic and the corresponding points are located on the stability margin chart. The frequency of these oscillations is regarded as the frequency of the system. The relative error is considered to 0.00001 which provides the best trade-off between the calculation accuracy and the computation time.

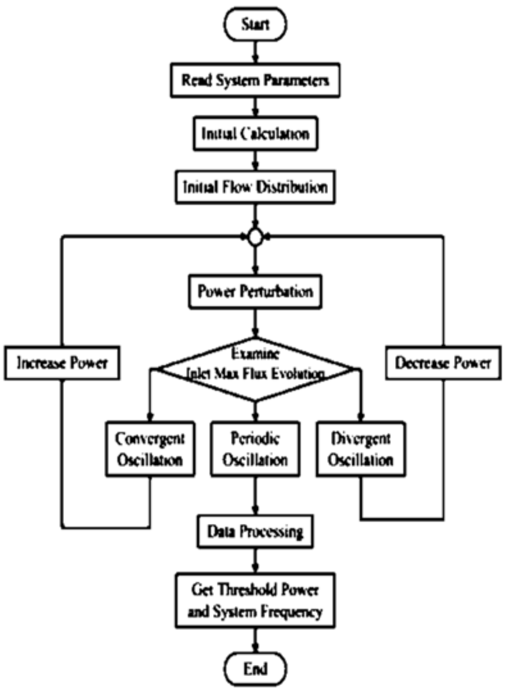

Fig. 3. Calculation algorithm.

3. Results and Discussion

3.1. Validation

The theorical model used to investigate the parallel channels flow field in this study, is validated by Libo *et al*., (2014). In Fig. 4, the result of theoretical model and available experimental data, are presented. According to Fig. 4, It is found that present theoretical model provides a good agreement with the experimental data especially in low sub cooling region.

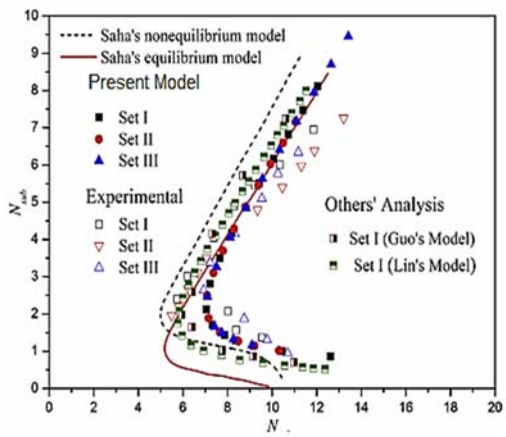

Fig. 4. Validation of theorical model (Libo *et al*.,2014).

3.2 Results for the Parallel Channels

Based on the non-dimensional parameters, the results of the stability analysis can be presented as function of non-dimensional parameters. These parameters are N_{pch}, N_{sub}, P^{+}_{inlet}, L_1^{+}, L^{+}_2, L_3^{+}, L_4^{+}, $V^{+}_{si,0}$, $V^{+}_{so,0}$. The first three parameters control the operational effects like, heat flux, sub cooling and fluid properties. The last six parameters reflect the influence of external parameters on the stability margins. In fact $L_1^{+}, L_2^{+}, L_3^{+}, L_4^{+}$ are representive for external inertia effects while, V^{+}_{si0}, V^{+}_{sO0} simulate the external compressibility effects. So according to these non-dimensional parameters, in the following, the effects of fluid inertia of external pipes and compressible volumes on the stability limits of parallel channels have been investigated. Water is selected as the working fluid. In the whole simulations the pipe diameter and heated section length are 0.0128 and 3.16 meters, respectively.

In whole simulations, only in phase oscillations were found and it is found that inertia and compressibility effects cannot produce out of phase oscillations.

3.2.1 Inertia Analysis

In this section the effect of increasing the inlet and outlet inertia on the stability margin of a reference case is studied. The reference case has the following characteristics:

$L_1= L_2= L_3= L_4= 1$m

$K_1=5$, $K_2=0$, $K_3=0$, $K_1=2$

$Vsi= Vso=0$

This analysis is done at three pressure levels and the pressure ranges from 3 to 7MPa.

The Fig. (5-10) show the stability margins when the inlet and outlet inertia of reference case is increased. The results show that in all of simulations, increasing the inlet inertia makes the system more stable while increasing the outlet inertia destabilize the system.

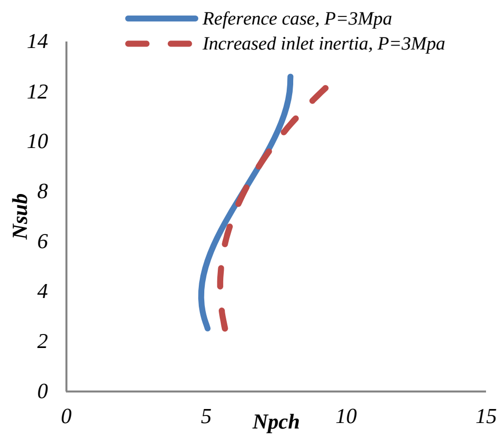

Fig. 5. The effect of increasing inlet inertia on the stability margins at P=3Mpa.

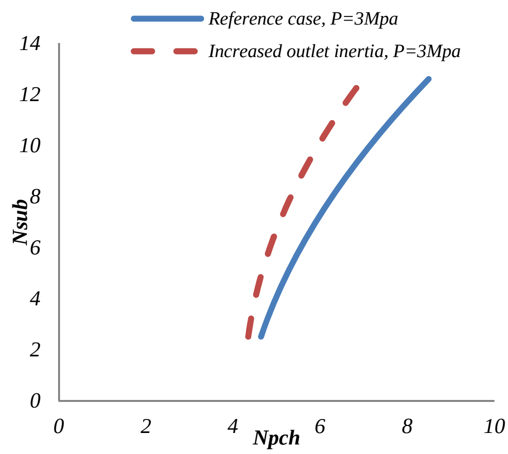

Fig. 6. The effect of increasing outlet inertia on the stability margins at P=3Mpa.

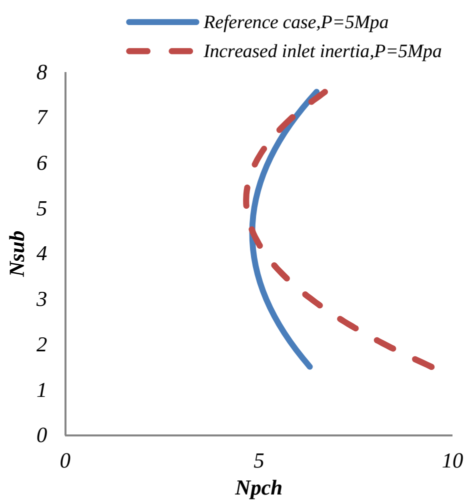

Fig. 7. The effect of increasing inlet inertia on the stability margins at P=5Mpa.

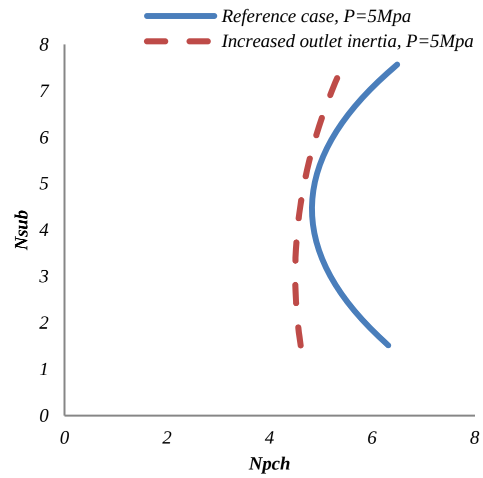

Fig. 8. The effect of increasing outlet inertia on the stability margins at P=5Mpa.

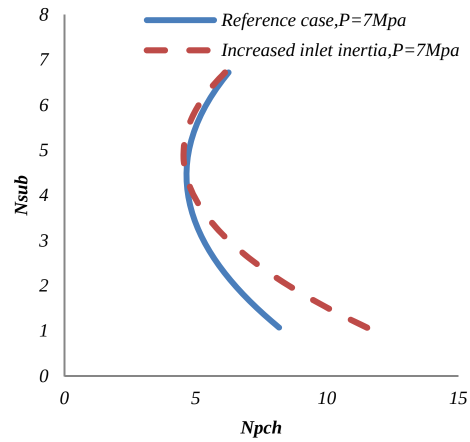

Fig. 9. The effect of increasing inlet inertia on the stability margins at P=7Mpa.

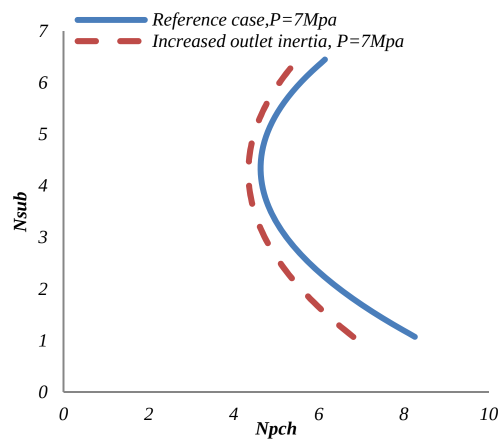

Fig. 10. The effect of increasing outlet inertia on the stability margins at P=7Mpa.

These results can be explained according to the idea of Guo *et al.* (2008).That idea has two aspects. First part of that idea says, when a phenomenon can increase the single phase pressure drop, it can increase damping effect and stabilizes the system. For example, increasing the inlet throttling can increase single phase pressure drop, so it stabilizes the system. The second part says when, a phenomenon can increase the two phase pressure drop, it can decrease damping effect and destabilizes the system more, so , increasing the exit throttling , can increase the two phase pressure drop which leads to decrease system stability.

It should be noted that external inertia can be considered as a dynamic resistance. In fact, inertia acts like a valve and imposes a resistance on the heated section, so according to the idea of Guo *et al*. (2008), the effect of inlet and outlet inertia on the heated section is increasing inlet and outlet pressure drops, respectively. So increasing inlet inertia can increase system stability while increasing outlet inertia can decrease system stability.

3.2.2 Compressibility Analysis

Inlet and outlet compressibility can change the stability margin, so in this section the effect of increasing the inlet and outlet compressibility of parallel channel is investigated. This analysis is performed at three pressure levels and in each case; the amount of compressible volume is increased from 10 liters to 20 liters. The results, for each pressure level, are illustrated in Fig. (11-16). The results of simulations show that increasing inlet compressibility will decrease the system stability while increasing outlet compressibility can increase system stability.

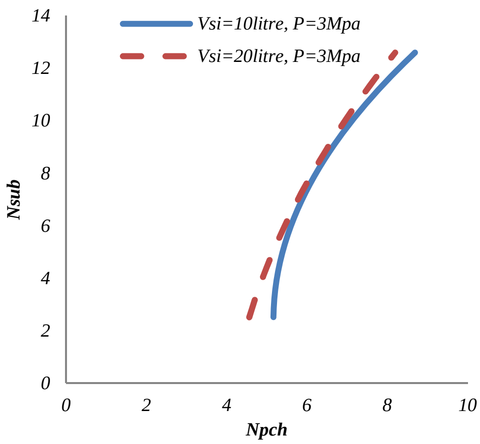

Fig. 11. The effect of increasing inlet compressibility on the stability margins at P=3Mpa.

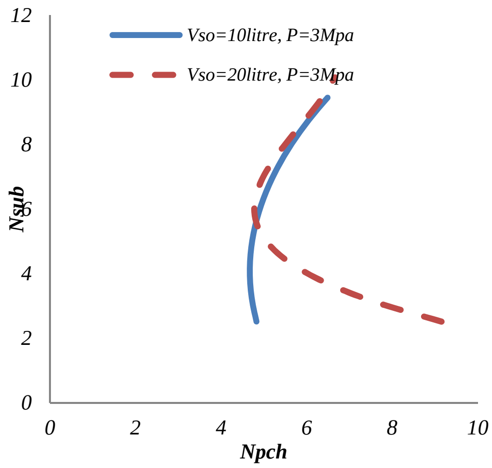

Fig. 12. The effect of increasing outlet compressibility on the stability margins at P=3Mpa.

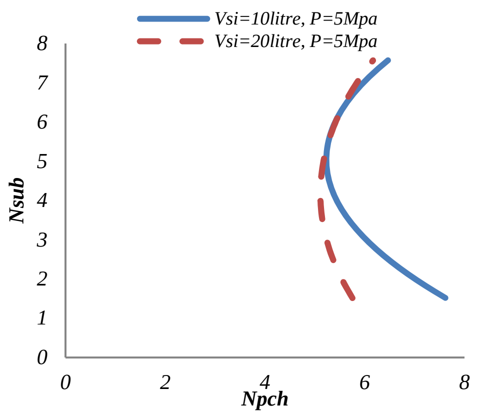

Fig. 13. The effect of increasing inlet compressibility on the stability margins at P=5Mpa.

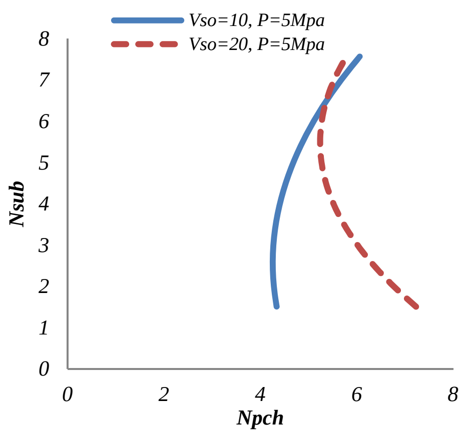

Fig. 14. The effect of increasing outlet compressibility on the stability margins at P=5Mpa.

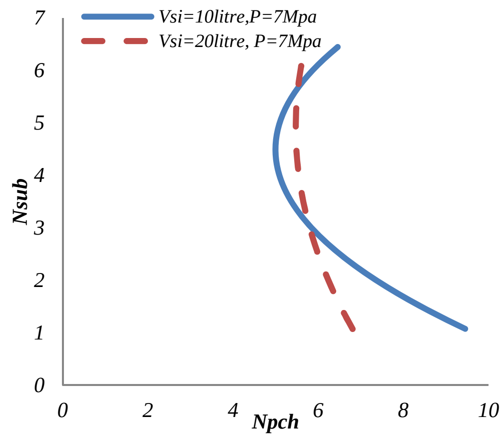

Fig. 15. The effect of increasing inlet compressibility on the stability margins at P=7Mpa.

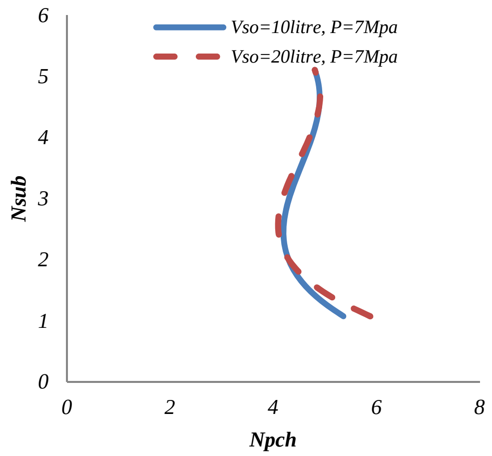

Fig. 16 . The effect of increasing outlet compressibility on the stability margins at P=7Mpa

In order to explain the effect of compressible volumes, it should be noticed that, compressible volumes can decrease the imposed resistance on the heated section, so according to the idea of Guo *et al.* (2008), increasing the inlet compressibility, makes the system unstable while increasing the outlet compressibility increase stability margin.

4. CONCLUSION

A mathematical model has been developed to study density wave oscillations (DWOs) of flow instability in parallel channels. The model is adopted to include the effects of external fluid inertia and compressible volumes on the stability margins of two parallel channels. According to the results the following conclusions are obtained:

- In whole simulations, only in phase oscillations were found and it is found that inertia and compressibility effects cannot produce out of phase oscillations.
- Increasing the inlet inertia makes the system more stable while increasing the outlet inertia destabilizes the system.
- Increasing the inlet compressible volume will decrease system stability while increasing outlet compressible volume will increase the system stability.

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REFERENCES

Aritomi, M., S. Aoki and A. Inoue (1977). Instabilities in parallel channel of forced convection boiling up flow system, (I) mathematical model. *Journal of Nuclear Science and Technology*. 14, 22–30.

Aritomi, M., S. Aoki and A. Inoue (1979). Instabilities in parallel channel of forced convection boiling up flow system, (iii) – system with different flow conditions between two channels. *Journal of Nuclear Science and Technology*. 16, 343–355.

Beck, B. T. and G. L. Wedekind (1986). On the mean period of dry out point fluctuations. *Trans ASME, Journal of Heat Transfer*. 108,988–90.

Chiapero, E. M., M. Fernandino and C. A. Dorao (2013). Numerical analysis of pressure drop oscillations in parallel channels. *International Journal of Multiphase Flow*. 56, 15–24.

Clausse, A. and R. T Lahey (1990). An investigation of periodic and strange attractors in boiling flows using chaos theory. *In: Proceedings of the 9th International Heat Transfer Conference*. Jerusalem, Israel. 3–8.

Clausse, A., R. T. Lahey and M. Podowski (1989). An analysis of stability and oscillation modes in boiling multichannel loops using parameter perturbation methods. *International Journal of Heat and Mass Transfer*. 32, 2055–2064.

Colombo, M., A. Cammi, D. Papini and M. E Ricotti (2012). RELAP5/MOD3.3 study on density wave instabilities in single channel and two parallel channels. *Progress in Nuclear Energy*. 56, 15–23.

Fukuda, K. and T. Kobori (1979). Classification of two-phase flow instability by density wave oscillation model. *Journal of Nuclear Science and Technology* 16, 95–108.

Gear, A. C. (1974). *Ordinary Differential Equation System Solver.* Lawrence Liivarmore Laboratory.

Guido, G., J. Converti and A. Clausse (1991). Density-wave oscillations in parallel channels – an analytical approach. *Nuclear Engineering and Designs*. 125, 121–136.

Guo, Y., J. Huang, G. Xia and H. Zeng (2010). Experiment investigation on two-phase flow instability in a parallel twin-channel system. *Annals of Nuclear Energy* 37, 1281–1289.

Guo, Y., S. Z Qiu, G. H Su and D. N Jia (2008). Theoretical investigations on two-phase flow instability in parallel multichannel system. *Annals of Nuclear Energy* 35, 665–676.

Hirayama, M., H. Umekawa and M. Ozawa (2006). Parallel-channel instability in natural circulation system. *Multiphase Science Technology* 18, 305–337.

Hua, L., D., Y. Chena, D. Huangb, Y. Yuanb, S. Wangb and L. Pana (2015). Numerical investigation of the mechanism of two-phase flow instability in parallel narrow . *Nuclear Engineering and Design* 287 ,78–89.

Ibrahim, G. A. (2001). Effect of sudden changes in evaporator external parameters on a refrigeration system with an evaporator controlled by a thermostatic expansion valve. *International Journal of Refrigeration* 24,566–76.

Kakac, S. and B. Bon (2008). Review of two-phase flow dynamic instabilities in tube boiling systems. *International Journal of Heat and Mass Transfer* 51, 399–433.

Lee, J. D. and P. Pan (2014). The complex nonlinear dynamics in the multiple boiling channels coupling with multi-point reactors with constant total flow rate. *Annals of Nuclear Energy*. 71 , 174–189.

Lee, J. D. and C. Pan (1999). Dynamics of multiple parallel boiling channel systems with forced flows. *Nuclear Engineering and Design*. 192, 31–44.

Libo, Q., S. Ding and S. Qiu (2014). Research on two-phase flow instability in parallel rectangular channels. *Annals of Nuclear Energy*.65,47–59.

Mithraratne P. and N. E Wijeysundera (2001). An experimental and numerical study of the dynamic behavior of counter-flow evaporators *International Journal of Refrigeration* 24,554–65.

Muõz-Cobo, J. L., M. Z Podowski and S. Chiva (2002). Parallel channel instabilities in boiling water reactor systems: boundary conditions for out of phase oscillations. *Annals of Nuclear Energy* 29, 1891–1917.

Paul, S. and S. Singh (2014). A density variant drift flux model for density wave oscillations, *International Journal of Heat and Mass Transfer* 69 ,151–163.

Podowski, M. Z. and M. P Rosa (1997). Modeling and numerical simulation of oscillatory two-phase flows, with application to boiling water nuclear reactors. *Nuclear Engineering and Design*. 177.

Podowski, M. Z., J. R. Lahey, R. T. Clausse and A. Desanctis (1990). Modeling and analysis of channel-to-channel instabilities in boiling systems. *Chemical Engineering. Community* 93, 75–92.

Ruspini, L. C. (2012). Inertia and compressibility effects on density waves and Ledinegg phenomena in two-phase flow systems. *Nuclear Engineering and Design* 250, 60– 67.

Wedekind, G. L. and B. T Beck (1974). Theoretical model of the mixture–vapor transition point oscillation associated with two-phase evaporating flow instabilities. *Trans ASME, Journal of Heat Transfer* 96,138–44.

Wedekind, G. L. and W. F Stoecher (1986). Theoretical model for predicting the transient response of the mixture–vapor transition point in horizontal evaporating flow. *Trans ASME, Journal of Heat Transfer* 90,165–74.

Xia, G., M. Peng and Y. Guo (2012). Research of two-phase flow instability in parallel narrow multichannel system. *Annual of Nuclear Energy* 48, 1–16.

Xia, G. L., M. J. Peng and Y. Guo (2010). Analysis of Instability in Narrow Annular Multichannel System Based on RELAP5 Code Zero-Carbon. *Energy Kyoto 2009.* Springer, Japan292–299.

Xiao, M., X. Chen, M. Zhang, T. Vezirogˆlu and S. Kakaç (1993). A multivariable linear investigation of two- phase flow instabilities in parallel boiling channels under high pressure. *International Journal of Multiphase Flow*. 19, 65–77.

Yun, G., S. Qiu, G. Su and D. Jia (2008). Theoretical investigations on two-phase flow instability in parallel multichannel system. *Annals of Nuclear Energy* 35, 665–676.

Zahn, W. R. (1964). A visual study of two-phase flow while evaporating in horizontal tubes. *Trans ASME, Journal of Heat Transfer* 86, 417–29.

Zhang, Y., G. Su, X. Yang and S. Qiu (2009). Theoretical research on two-phase flow instability in parallel channels. *Nuclear Engineering and Design* 239, 1294–130.

Zhang, Y., H. Li, L. Li and X. Lei (2014). Wang, Study on two-phase flow instabilities in internally-ribbed tubes by using frequency domain method, *Applied Thermal Engineering*. 65 ,1-13.

Zhou, Y., Z. Zhang , M. Lin , D. Hou and X. Yan (2013). Capability of RELAP5 MOD3.3 code to simulate density wave instability in parallel narrow rectangular channels. *Annals of Nuclear Energy* 60 , 256–266.