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# Study of Effect of Flow Parameters on Base Pressure in a Suddenly Expanded Duct at Supersonic Mach Number Regimes using CFD and Design of Experiments

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## ABSTRACT

Effectiveness of active control of micro jets has been examined by conducting experiments through an abruptly expanded axi-symmetric duct in a view to control base pressure. For this purpose, 1mm orifice diameter micro jets have been deployed at an interval of $90^0$ along the exit diameter of the nozzle. The experiments have been conducted by considering three flow parameters at three levels. Mach number (M), length to diameter (L/D) ratio and area ratio (AR) are the three parameters used to conduct and analyze the flow experiments. Base pressure is considered to be the response variable. The experimentation has been carried out for two cases, i) without active control; ii) with active control. An $L_9$ orthogonal array has been implemented to plan the experiments. It is observed that the control becomes effective for lower area ratios when compared to the higher ones. In addition to this, at high area ratios suction at the base decreases and hence base pressure continuous to diminish with increasing L/D until it reaches a value of L/D=6. The obtained experimental results are subjected to multiple linear regression analysis and Analysis of variance (ANOVA). The performances of the two linear regression models were tested for their prediction accuracy with the help of 15 random test cases. It is observed that, both linear regression models for base pressure without and with control are statistically adequate and capable of making accurate predictions. Furthermore, this work also concludes that, Mach number is the most significant factor affecting base pressure followed by area ratio and L/D ratio for both cases of experimentation. The obtained experimental results are further validated by CFD analysis and are found to be in good concurrence with each other.

**Keywords:** Base pressure; Mach number; Area ratio; Length to diameter ratio; Analysis of variance.

## NOMENCLATURE

| | |
|---|---|
| A | exit area of the nozzle |
| $A^*$ | throat area of the nozzle |
| AR | Area Ratio |
| $C_p$ | specific heat of air constant pressure |
| F-value | fisher statistic (ratio of variances) |
| MS | Mean of Squares |
| M | Mach number at the nozzle exit |
| NPR | Nozzle Pressure Ratio |
| P-value | probability of the statistical model |
| P | static pressure |
| $P_b$ | base pressure |
| $P_a$ | ambient pressure |
| R | ideal gas constant $8.314\times10^3$ |
| S/N | signal to noise |
| SS | sum of Squares |
| T | temperature of Ideal gas |
| U | local flow velocity with respect to the boundaries |
| X | distance for measurement of wall pressure along duct |
| $\gamma$ | ratio of specific heat at constant pressure to constant volume (1.4 for air) |

## 1. INTRODUCTION

The discipline of “Base Flow Aerodynamics” has been capturing a lot of attention in the recent past. Base flow plays a very important role in deciding the performance relative to the external flow. One such instance is in aerodynamic vehicles such as missiles, rockets, and projectiles which tend to develop a low-pressure recirculation region near to its base. This low pressure at the base is

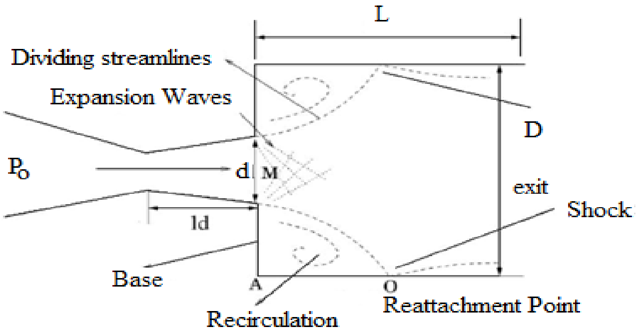


**Fig. 1. Suddenly expanded flow field**.

significantly lesser than the free atmospheric pressure. This pressure difference causes base drag which happens to be one of the most important aspects of base flow aerodynamics. In view of this, the concept of high Mach number largely influences the base pressure and has been greatly emphasized for development of future aerodynamic engines. Thus the performance of such engine tends to be highly reliant on nozzle aerodynamic design with the main parameters of interest being area ratio and length to diameter ratio, therefore is examined in par with the requirements in the present study. Flow separation zone which leads to the development of low pressure circulation zone in the vicinity of the base has to be known in order to examine the base flow in aerodynamic vehicles. In the case of rockets, the efficiency of projectiles is diminished by the rockets' exhaust which interacts with the external flow in the supersonic regime, the reason for triggering it being base flow. Thus base flow is an important study to predict the performance with regard to external flows, for nozzle mechanics endured for future advancement like spike nozzle. However a number of experiments have been carried out in order to facilitate the flow parameters with base flow aerodynamics and choose new applications. For instance, in case of Transonic Mach numbers, a noticeable pressure difference is observed in the case of flow separation which precisely affects the total drag. Similarly for subsonic flows, the pressure difference causing base drag observes to be 10% of the skin friction drag with a wave drag of zero. To further build the base pressure which diminishes the base drag, one can consider distinctive geometric shapes like boat tails, ribs, vented cavities or base bleed applications and combustion at the base. However use of active controls for base drag reduction hasn't been explored much, hence we shall study with the help of internal flows. Plentiful approaches have been dissected to control flow division, either by avoiding or preventing its effects thus reducing base drag. Numerous passive techniques like splitter plate have been employed at the base of the enlarged duct for controlling base pressure, but very few researches have been conducted with regard to active controls (microjets). Thus the current work is conducted to study base pressure variation with the help of microjets for favorable and unfavorable pressure gradient at high supersonic Mach numbers. The ideally expanded case for flow through a nozzle is as shown in Fig. 1.

Taguchi approaches are essentially used for the designing systems of high quality. These designs can optimize system attributes by balancing of the design parameters and decrease the effectiveness of the system execution to the source of deviation (Basavarajappa *et al*. 2009). In a view of trying to achieve effectual designs to curtail drag, control the attachment length, and understanding of complex flow fields, the previous two decades have seen a lot of methods that have been employed to control flow strategies (Gad-el Hak *et al*. 1998). The major parts of the aerodynamic forces are controlled by a partitioned posterior slope, two longitudinal vortices that tend to develop on the sides as well as the base region (Rouméas *et al.* 2008, Boucinha *et al.* 2008). Suddenly expanded air into the ducts leads to the outcome of base pressure and noise. The base pressure should possess a minimum value for the flow to remain attached and this in turn depends on area ratio and nozzle geometry. A combustion burner employed in industries works out as a specific application for such kind of jet flow configurations (Green, 1995). In order to achieve the above said goals numerous passive (Elavarasan *et al.* 2002) and active (Alvi *et al.* 2003) control techniques have been endorsed to alleviate the challenging factors. Thus the present study endeavors to determine the effectiveness of both these methods in controlling the base pressure field thereby proving its versatility and cogency. Issue of base pressure in the transonic and supersonic stream was examined by (Korst, 1956) for cases which developed a sonic and supersonic stream near the base after the wake. Depending on the basis of communication between the dissipative and the neighboring free stream along with conservation of mass in the wake, a physical flow model was developed. Base pressure and noise delivered due to sudden expansion of air into a cylindrical duct was studied by (Anderson *et al.* 1968) where minimum jet pressure relatively equal to what is required to produce minimum base pressure was demonstrated by the overall noise plot. (Wood, 1964) studied the influence of base bleed on a periodic wake. He opined that base bleed decreases the drag of an aerofoil, by aversion of the onset of instability in separated shear layers. The optimum bleed developed was given by a bleed coefficient of 0.125. This decrease in profile drag caused by the base bleed is associated with a decrease in the strength of the vortex street. Theoretical and experimental investigations were conducted on compressible flows through sudden enlargement in a pipe by (Hall *et al.* 1955). They developed a theory that would predict the Mach number in a particular location downstream of the sudden enlargement with known values of Mach number at the exit of the inlet tube. (Khan *et al.* 2002) conducted experiments to investigate the effect of microjets on base pressure in a suddenly expanded duct. The Mach numbers employed in the study were 2.0, 2.5 and 3.0. The experiments were been conducted for a level of overexpansion of ($P_e/P_a$=0.277). It was found that microjets can be used as active operators for handling base pressure. The work further concluded that, for a given M and NPR, the L/D ratio can be identified for describing the maximum increase or decrease of base pressure. (Khan *et al.* 2003) carried out experimental investigations in order to study active control of base pressure with microjets for Mach numbers 1.87, 2.2 and 2.58. The experiments were conducted for nozzle pressure ratios of 3, 5, 7, 9 and 11 respectively. Upto 95 percent escalation with regard to base pressure was observed for a definite set of parameters. Khan *et al.* (2004a) considered M= 1.25, 1.3, 1.48, 1.6, 1.8 and 2.0 for controlling expanded flows through microjets. An under expansion level of $P_{exit}/P_a$=1.5 was maintained during the experimentation. The studies found that micro jets were found to be effective whenever the nozzles were under expanded. Also, the control effectiveness will be at its best whenever there exists a favorable pressure gradient. (Khan *et al.* 2004b) examined the effect of microjets for suddenly expanded flows for nozzles subjected to correct expansion. It was found that the microjets were not effective for Mach numbers 1.25, 1.3, 1.48, 1.6, 1.8 and 2.0. The base pressure values experienced a marginal change. The apprehension was due to a weak wave that was present at a position prior to the nozzle lip. Here, the microjets when activated are not able to bring about changes in the base pressure due to the presence of a weak wave. Also yet another important observation made was that the correctly expanded flows were dominated by waves. Control effectiveness and expansion in suddenly expanded flows for Mach numbers 1.25, 1.3, 1.48, 1.6, 1.8, 2.0, 2.5 and 3.0 was studied by (Khan *et al.* 2006). The experiments were conducted for nozzle pressure ratios of 3, 5, 7, 9 and 11. It was concluded that the values of base pressure increased with increase in area ratio for a given Mach number, L/D ratio and nozzle pressure ratio. This increase in base pressure was observed due to the relief available for the flow due to increase in area ratio. Reduction of axisymmetric base drag at Mach 2.0 by use of passive devices was studied by (Viswanath *et al.* 1990). Base cavities and ventilated cavities were the devices that were being examined. The results showed significant base drag reduction for ventilated cavities. As far as 3 to 5% net reduction in base drag and 50 percent increase in base pressure were observed for Mach numbers in the supersonic regime for a body of revolution. (Rathakrishnan *et al.* 1984) conducted experiments on flows in pipe with sudden expansion. It was concluded that base pressure is a strong function of area ratio, nozzle pressure ratio and L/D ratio. It was observed that an optimal L/D ratio can be identified that can result in maximum total pressure due to enlargement at the nozzle exit on the symmetric axis for a given value of area ratio and nozzle pressure ratio. For an ideal execution of flow through channels with sudden augmentation, it is not adequate if the base pressure minimization alone is considered. The total pressure loss should likewise be considered. (Mathur *et al.* 1996) studied the close wake stream of a cylindrical after-body at Mach 2.5 which was impacted by base bleed. With increase in the base bleed, the base pressure was found to increase initially, attain a peak and then decrease again with further increase in bleed. The condition for optimum base bleed was found to be at a bleed flow rate (I) = 0.0148 which was characterized by weak corner expansion, minimum free shear layer angle, and disappearance of recirculation region along the near wake. Based on the literature cited above, the present research endeavor is one such attempt to i) study the effect of Mach number, area ratio and L/D ratio on the variation of base pressure; ii) Validate the experimental results by predicting them through mathematical models developed by linear regression analysis; iii) determine the concurrence of experimental results with the obtained CFD results. The results obtained from this work can be applied in the right way pertinent to applications such as minimizing base pressure in the combustion chamber for maximizing mixing and maximizing base pressure in cases of rockets and missiles in order to minimize base drag.

## 2. MATHEMATICAL FORMULATION

Nozzles come up in a vast range of applications. Obvious ones are the thrust nozzles of rocket and jet engines. Converging-diverging ducts also come up in aircraft engine inlets, wind tunnels and in all sorts of piping systems designed to control gas flow. The flows associated with volcanic and geyser eruptions are influenced by converging-diverging nozzle geometries that arise naturally in geological formations. From area-averaged equations of motion (Cantwell, 1996) by neglecting the shear stresses and heat fluxes, the governing equations together with the perfect gas law are given by

$$d(\rho U A) = 0 \qquad (1)$$

$$dP + (\rho U dU) = 0 \qquad (2)$$

$$Cp\, dT + (UdU) = 0 \qquad (3)$$

$$P = \rho R T \qquad (4)$$

Now, introducing Mach number

$$U^2 = \gamma R T M^2 \qquad (5)$$

Equations (1),(2),(3) and (4) can be expressed in fractional differential form as

$$\frac{d\rho}{\rho} + \frac{dU^2}{2U^2} + \frac{dA}{A} = 0 \qquad (6)$$

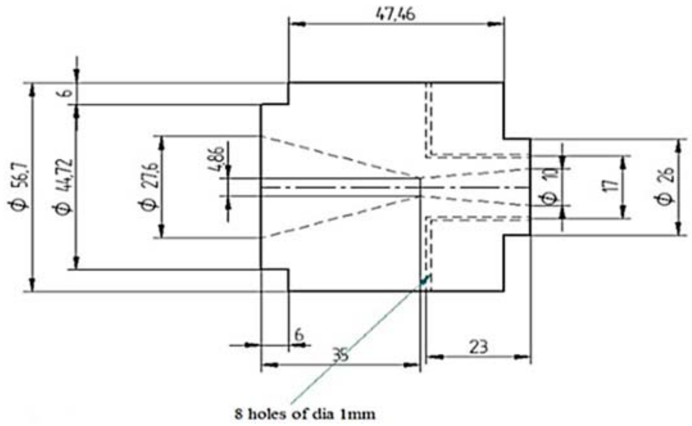


**Fig. 2. Nozzle designed for machining. (Mach 3.0).**

$$\frac{dP}{P}+\frac{\gamma M^2}{2}\frac{dU^2}{U^2}=0^0 \tag{7}$$

$$\frac{dT}{T}+\frac{(\gamma-1)M^2}{2}\frac{dU^2}{U^2}=0 \tag{8}$$

$$\frac{dP}{P}=\frac{d\rho}{\rho}+\frac{dT}{T} \tag{9}$$

Eq. (5) can be expressed in fractional differential form as

$$\frac{dU^2}{U^2}=\frac{dM^2}{M^2}+\frac{dT}{T} \tag{10}$$

By using the equations for mass, momentum and energy to replace the terms in the equation of state, we get

$$-\frac{\gamma M^2}{2}\frac{dU^2}{U^2}=-\frac{dU^2}{2U^2}-\frac{dA}{A}-\frac{(\gamma-1)M^2}{2}\frac{dU^2}{U^2} \tag{11}$$

Solving for $\frac{dU^2}{U^2}$, we get

$$\frac{dU^2}{U^2}=\left(\frac{2}{M^2-1}\right)\frac{dA}{A} \tag{12}$$

Equation (12) shows the effect of stream wise area change on the speed of the flow. Using Eq. (12) to replace $\frac{\mathrm{dU}^2}{\mathrm{U}^2}$ in each of the relations in Eqs. (6),(7) and (8), we get

$$\frac{d\rho}{\rho}=\left(\frac{M^2}{M^2-1}\right)\frac{dA}{A} \tag{13}$$

$$\frac{dP}{P}=\left(\frac{\gamma M^2}{M^2-1}\right)\frac{dA}{A} \tag{14}$$

$$\frac{dT}{T}=-\frac{(\gamma-1)M^2}{2}\frac{dU^2}{U^2}\frac{dA}{A} \tag{15}$$

Eqs. (13), (14) and (15) describe the effects of area change on the thermodynamic state of the flow. Now use Eq. (12) in temperature Eq. (10).

We get

$$\left(\frac{2}{M^2-1}\right)\frac{dA}{A}=\left(\frac{(\gamma-1)M^2}{M^2-1}\right)\frac{dA}{A}$$
$$+\left(\frac{dM^2}{M^2}\,\frac{\gamma+1}{2(\gamma-1)}\right) \tag{16}$$

Rearranging Eq. (16). The effect of area change on the Mach number is

$$\frac{dA}{A}=\left(\frac{M^2-1}{2\left\{1+\left(\frac{\gamma-1}{2}\right)M^2\right\}}\right)\frac{dM^2}{M^2} \tag{17}$$

Integrate Eq. (17) from an initial Mach number M to one

$$\int_{M^2}^{1}\left(\frac{M^2-1}{2[1+\left(\frac{\gamma-1}{2}\right)M^2]}\right)\frac{dM^2}{M^2}=\int_{A}^{A^*}\frac{\mathrm{dA}}{\mathrm{A}} \tag{18}$$

We get

$$ln\left(\frac{A^*}{A}\right)=ln\left(\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\right)-\Bigg\{-ln(M)+$$
$$ln\left(\left(1+\left(\frac{\gamma-1}{2}\right)M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}\right)\Bigg\} \tag{19}$$

Evaluating equation at the Limits, we get the final equation as

$$\left(\frac{A^*}{A}\right)=\left\{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\frac{M}{\left(1+\left(\frac{\gamma-1}{2}\right)M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\right\} \tag{20}$$

In the above Eq., we referenced the integration process to M = 1. The area $A^*$ is a reference area at some point in the channel where M = 1 although such a point need not actually be present in a given problem. The area-Mach-number function is given by Eq. (20).

### 2.1 Nozzle Design

The Nozzle design for a Mach number 3.0 is shown in Fig. 2. It consists of a certain set of parameters that are used as standards for its design. It is important to note that, in the present study compressibility effects are pre-dominant in the flow as flow is generally of high speed. Thus the flow condition of the nozzles is blow down flow. The parameters involved are as follows:

1. Exit diameter of the Nozzle is (10 mm fixed).
2. Throat diameter which is different for all the nozzles has been determined from the Eq. (20)
3. The angle between the exit diameter and throat of the nozzle is maintained at an angle of approximately $5^0$ to $8^0$ ($6^0$ maintained in this study for all nozzles) as per Versteeg *et al.* (2009).
4. Similarly the angle between the inlet diameter and throat is maintained at an angle of approximately $18^0$ to $30^0$. Thus the angel

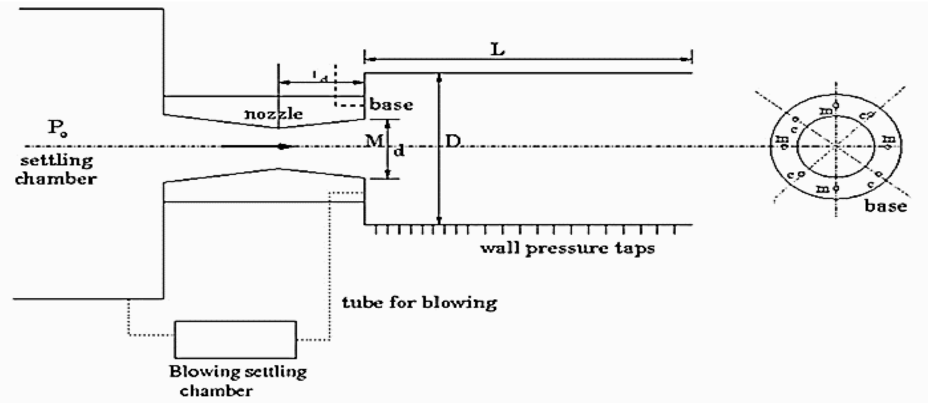


**Fig. 3. Experimental setup.**

between throat and exit is maintained at $6^0$ and angel between inlet and throat is maintained at $18^0$ for all nozzles viz. Mach 2.0, Mach 2.5 and Mach 3.0 respectively.

## 3. PLAN OF EXPERIMENTS

Taguchi design of experiments have been employed to conduct the set of experiments by using three factors namely Mach number (M), L/D ratio and area ratio (AR). The experiments were conducted by use of these factors at three levels i.e. Mach number (2.0, 2.5 and 3.0); L/D ratio (3, 5 and 8) and area ratio (3.24, 4.84 and 6.25) respectively. The corresponding factors and their levels are shown in Table 1. A standard $L_9$ orthogonal array (Basavarajappa *et al.* 2009) containing nine rows and four columns were chosen for conducting experiments as shown in Table 2. The experiments consisted of nine tests with Mach number assigned to column 1, L/D ratio to column 2 and area ratio to column 3.

**Table 1 Factors and their levels**

| Factors | Level 1 | Level 2 | Level 3 |
|---|---|---|---|
| Mach number | 2.0 | 2.5 | 3.0 |
| L/D ratio | 3 | 5 | 8 |
| Area ratio | 3.24 | 4.84 | 6.25 |

**Table 2 $L_9$ Orthogonal Array**

| $L_9$ Test | M | L/D | AR | $(P_b/P_a)$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 2 |
| 3 | 1 | 3 | 3 | 3 |
| 4 | 2 | 1 | 2 | 3 |
| 5 | 2 | 2 | 3 | 1 |
| 6 | 2 | 3 | 1 | 2 |
| 7 | 3 | 1 | 3 | 2 |
| 8 | 3 | 2 | 1 | 3 |
| 9 | 3 | 3 | 2 | 1 |

## 4. EXPERIMENTAL METHOD

A schematic representation of the experimental set up is as shown in Fig. 3. Eight holes in the form of jets have been used to control the dynamic flow of base pressure, out of which four have been used for measuring the base pressure and the remaining four for controlling the base pressure. The main settling chamber has been used to draw pressure energy by coupling it to the control chamber. This pressure from the settling chamber is used in the micro jet form by blowing through control holes at the nozzle exit known as Nozzle pressure ratio (NPR). The setup also consists of a convergent divergent axisymmetric nozzle possessing an exit diameter of (d) followed by an axisymmetric duct of larger diameter (D). The area ratios have been maintained at 3.24, 4.84 and 6.25 respectively for the present study. This was done by retaining the exit diameter of the nozzle at 10 mm and varying the diameter of the enlarged duct to 18, 22 and 25 mm respectively. The suddenly expanded ducts were fabricated of Brass material. The respective L/D values of 3, 5 and 8 for axisymmetric ducts have been employed for the present study. The lower L/Ds were achieved by cutting the length after testing for a particular L/D. The experiments have been conducted for L/D ratios from 10-1. However three selected values of L/Ds have been opted in order to facilitate the $L_9$ orthogonal array. The pressure in the control chamber, stagnation pressure in the settling chamber and pressure at the base was measured through a PSI 9010 model pressure transducer interfaced with a personal computer. It consisted of 16 channels with pressure readings ranging from 0-300 psi. It displays the reading by averaging 250 samples per second. The easy to use menu driven software gets information and showcases the pressure readings from all the 16 channels at the same time on the digital screen. The experiments are conducted twice and average values of base pressures have been documented in order to obtain the desired accuracy. All the non-dimensional base pressures presented in this paper are within an uncertainty band of ± 2.6 per cent. Further, all the results are band of ± 2.6 per cent. Further, all the results are repeatable within ±3 percent. The experimentation was conducted at the Supersonic Aerodynamic Laboratory, Bearys Institute of Technology, Mangalore, India.

**Table 3 Experimental Results.**

| S. I. No. | Mach Number (M) | Length to diameter ratio (L/D) | Area Ratio (AR) | $(P_b/P_a)$ Without Control | $(P_b/P_a)$ With Control |
|---|---|---|---|---|---|
| 1. | 2 | 3 | 3.24 | 0.22 | 0.226 |
| 2. | 2 | 5 | 4.84 | 0.479 | 0.466 |
| 3. | 2 | 8 | 6.25 | 0.499 | 0.497 |
| 4. | 2.5 | 3 | 4.84 | 0.699 | 0.714 |
| 5. | 2.5 | 5 | 6.25 | 0.679 | 0.682 |
| 6. | 2.5 | 8 | 3.24 | 0.399 | 0.397 |
| 7. | 3 | 3 | 6.25 | 0.862 | 0.862 |
| 8. | 3 | 5 | 3.24 | 0.695 | 0.69 |
| 9. | 3 | 8 | 4.84 | 0.753 | 0.754 |

## 5. RESULTS AND DISCUSSIONS

The data measured consists of Base pressure results $(P_b)$ computed for different lengths for the channel of the enlarged duct. The values are non-dimensionalized by dividing them by ambient pressure $(P_a)$ i.e. $(P_b/P_a)$ and are shown in Table 3. The studies are conducted at a constant nozzle pressure ratio (NPR) of 5 maintained for all Mach numbers and area ratios. NPR is defined the ratio of stagnation pressure/ settling chamber pressure $(P_0)$ to the ambient pressure $(P_a)$. The parameters of the present study are Mach number (M), L/D ratio and area ratio (AR). The non-dimensionalized base pressure $(P_b/P_a)$ is studied as a function of Mach number, area ratio and L/D ratio for cases of without control and with control respectively and are presented in Fig. 4. Fig. 4(a) presents the results for Mach 2.0. It is clearly observed that, for a high area ratio of 6.25, the base pressure decreases sharply from L/D=3 upto L/D = 6 and thereafter decreases marginally with increase in L/D. This is due to the fact that at high area ratios, suction at the base decreases and hence base pressure continuous to diminish with increasing L/D until it reaches a value of L/D = 6 where after the values experience marginal change. This duct value of L/D=6 is in good agreement the findings obtained by (Rehman *et al.* 2008). For the area ratio of 6.25, the influence of active control on base pressure plays a marginal influence. However for area ratio of 4.84, for L/D greater than 6, control results in increasing base pressure. To explain this case, it is known that the location of micro jets is fixed in the present study. Hence for an area ratio of 4.84, micro jets are very close to the base corner and for higher area ratio (6.25), they are away from the base corner. However once the reattachment length is reached, the micro jets for lower area ratio tend to counter the effects of shock tending to increase the base pressure when compared to those for higher area ratios (Khan *et al.* 2002). Moreover, this trend is mainly due to the fact that, for lower L/D ratios the influence of the atmospheric pressure influencing the flow development in the enlarged duct will be less when compared to those for higher L/D ratios (Baig *et al.* 2011).

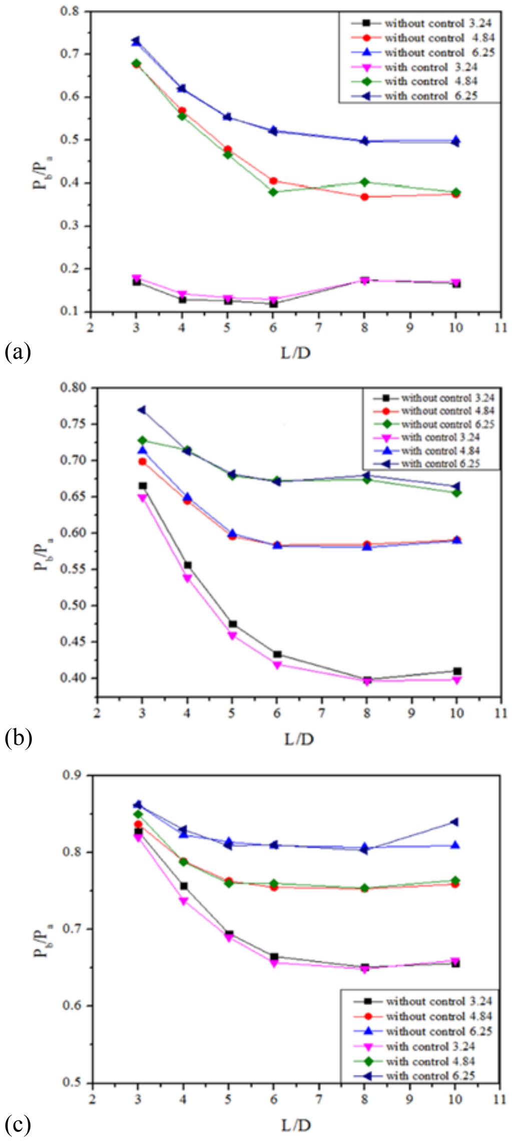


**Fig. 4. Variation of non-dimensional base pressure with respect to L/D ratios for a) Mach 2.0; b) Mach 2.5 and c) Mach 3.0.**

It has already been reported by (Khan *et al.* 2004b) that L/D=1 is the minimum value for the flow to

**Table 4 Analysis of Variance for non-dimensional base pressure without control**

| Source | DF | Adj. SS | Adj. MS | F- value | P-value | $^{a}$P % |
|---|---|---|---|---|---|---|
| M | 2 | 0.2255 | 0.1127 | 15.97 | 0.059 | 61.92 |
| L/D | 2 | 0.0068 | 0.0034 | 0.49 | 0.672 | 1.87 |
| AR | 2 | 0.1176 | 0.0588 | 8.33 | 0.107 | 32.29 |
| Error | 2 | 0.0141 | 0.0070 | | | 3.87 |
| Total | 8 | 0.3642 | 0.1127 | | | 100 |

attach for Mach 2.0 and area ratio 3.24. Thus the base pressure values for Mach 2.0 assume lower values when compared to the other Mach numbers. Therefore, once the flow is attached to the duct, the effect of back pressure tends to govern the flow which thereby tends to slightly increase base pressure beyond L/D=6. For the area ratio of 3.24, the influence of active control on base pressure plays a marginal influence. It is imperative to note that active control has played a relatively significant role by reducing base pressure for the case of area ratio 4.84 when compared to area ratios of 3.24 and 6.25. This is mainly due to the combined effect of relief enjoyed by the flow, Mach number and reattachment length which is responsible for this behavior (Khan *et al.* 2004a). Fig. 4(b) presents the results for Mach 2.5. Here, a slightly different behavior of base pressure variation is observed for few instances when compared to Mach 2.0. Here again the base pressure decreases for increasing L/D for the highest area ratio. Hence base pressure is significantly affected by area ratio as it decreases. The control is found to be marginal for the area ratios of 4.84 and 6.25. However for area ratio of 3.24, the base pressures are found to assume lower values for active control when compared to those for without control. The physical reason for this is the strong vortex that is created at the base for high Mach numbers. This in turn results in large suction at the base region. However at this point the use of active control (micro jets) entrains mass in the vicinity of the edge. However it must be noted that due to high Mach numbers, the nozzle exit and the base region is dominated by high strength of the shock due to which the use of control decreases the base pressure (Rehman *et al.* 2008). A similar trend is also observed for Mach 3.0 for a lower area ratio of 3.24 (Fig. 4c). This behavior is observed for L/D starting off from 3 upto 6 and for L/D=8 the base pressure for active control coincides with the base pressure value without control due to the effect of back pressure. Again post L/D = 6, control tends to again influence base pressure marginally. Additionally for Mach 2.5, area ratio of 6.25 and L/D=3, a considerable change was observed in the base pressure value for without and with control respectively. This is due overexpansion of the jets at Mach 2.5. At this stage, when the microjets are on, there is accumulation of mass in the vicinity of the edge. This mass along with shock strength at the nozzle exit and overexpansion might be the reason for this change in base pressure (Khan *et al.* 2002).This confirms the fact that active control is strongly affected by Mach number. In order to quantify the above discussion, base pressure results are subjected to multiple linear regression analysis and analysis of variance (ANOVA) for area ratios of 3.24, 4.84 and 6.25 which are discussed in the sections further below. Additionally, in comparison of Mach 2.5 with Mach 2.0, an L/D beyond 6 does not influence base pressure significantly except for a few cases. Similar results are illustrated for Mach 3.0 as shown in the Fig. 4(c) showing the effect of area ratio and L/D ratio on base pressure. Here again upto L/D=6, significant effect of base pressure has been observed where it decreases and then remains almost constant. However for one particular case of area ratio 6.25, the base pressure increases for L/D>8 with active control. The reason for this may be attributed to the effect of back pressure which is majorly dominant at the end of the enlarged duct i.e. (L/D>8). Therefore, from the above discussions it is imperative for one to distinguish the possible blend of parameters in order to accomplish base pressure control bringing about increment or decrement of it based upon the application desired. For example, for reducing base drag, one needs to consider expanding base pressure to the more extreme and similarly for the improvement of mixing; one needs to go for diminishing base pressure to the lowest possible value.

## 5.1 Regression Analysis and Analysis of Variance

The main aim for conducting the analysis of variance (ANOVA) was to quantify the parameter that greatly influences the base pressure. The analysis is conducted by use of commercial Minitab 17 software. The results for ANOVA are shown in Tables 4 and 5. The last columns ($^{a}$P%) depicts the percentage contribution of each factor on the total changeability of non-dimensional base pressure thereby indicating the impact of each of the factors employed in the present study. The F-value is the ratio of two variances. Variances are a measure of dispersion, and give an indication of how far is the data scattered from the mean. The P-value indicates the statistical significance of difference between the means. If the P-value is less than 0.05, then the difference between the means are statistically significant, otherwise they are not. Considering the ANOVA for base pressure variation without control, it is observed that, Mach number was the major factor of contribution with 61.92%, followed by area ratio (32.28%) and L/D ratio contributed comparatively less (1.87%), and this indicates that a small variation in base pressure will occur due to variation in L/D ratio when compared to area ratio and Mach number. The pooled error was

**Table 5 Analysis of Variance for non-dimensional base pressure with control.**

| Source | DF | Adj. SS | Adj. MS | F- value | P-value | $^{a}$P % |
|---|---|---|---|---|---|---|
| M | 2 | 0.2264 | 0.1132 | 16.43 | 0.057 | 62.28 |
| L/D | 2 | 0.0060 | 0.0030 | 0.44 | 0.695 | 1.66 |
| AR | 2 | 0.1172 | 0.0586 | 8.51 | 0.105 | 32.26 |
| Error | 2 | 0.0137 | 0.0068 | | | 3.78 |
| Total | 8 | 0.3635 | 0.1132 | | | 100 |

approximated to be 3.87%. In the case of active control, it was again observed that the major factor that contributed to variation in base pressure was Mach number with 62.28%, followed by area ratio (32.26%) and the contribution of L/D ratio was comparatively less (1.66%). The ANOVA had an associated pooled error of 3.78%. This approach gives the variation of means and variance to absolute values considered in the experiment and not the unit value of the variable. Here in both cases, Mach number influences base pressure significantly. This is because these Mach numbers experience a considerable decrease in base pressure for L/Ds between 3 and 6 for the Mach numbers 2.5 and 3 employed in the present study. This is due to high over expansion of the jets at Mach 2.5 and 3.0 where a stronger effect at the nozzle exit is experienced. However these shocks have larger shock angles and hence flow deflections are smaller. Thus these shocks will not dictate base pressure in the base region as this region is dominated by recirculating flow rather than shock flow thereby increasing base pressure (Baig *et al.* 2011). The regression equation for non-dimensionalized base pressure for suddenly expanded flow without active control and with active control are shown in Eqs. (21) and (22), respectively.

$$(P_b/P_a) = -0.767 + 0.3873(M) - 0.0067\,(L/D) + 0.0871\,(AR) \quad (21)$$

$$(P_b/P_a) = -0.758 + 0.3877\,(M) - 0.0083\,(L/D) + 0.0869\,(AR) \quad (22)$$

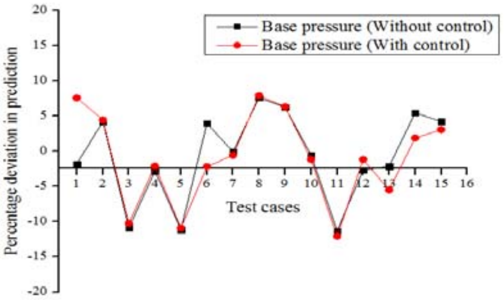


**Fig. 5. Standard deviation in prediction of base pressure.**

SS = Sum of squares; DF = degree of freedom; P=Percentage of contribution; $^{a}$95% confidence The coefficients of Mach number and area ratio are positive and coefficient of L/D is negative. This indicates that base pressure increases with increase in Mach number and area ratio and decreases with increase in L/D ratio. The non-dimensional base pressure calculated from the Eqs. (21) and (22) consist of positive coefficient values which suggest that base pressure results in increase with increased associated variables, whereas an opposite effect has been observed for negative coefficient values. The magnitudes of such variables indicate relative weight of each factor. The equations clearly suggest that Mach number has a greater effect on base pressure followed by area ratio and L/D. In order to test the linear regression models, a set of 15 random experimental test cases have been conducted the details of which have been elaborated in the sections below.

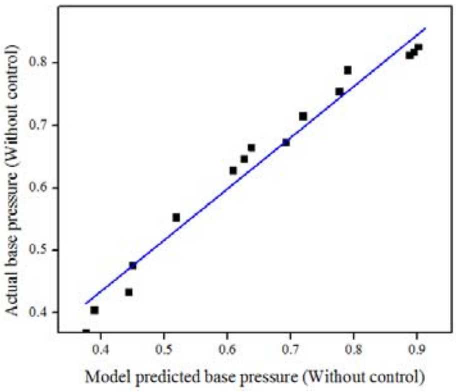


**Fig. 6. Comparison of model predicted base pressure (without control) with actual base pressure (without control).**

## 5.2 Testing of Linear Regression Models

It must be noted that, the regression models developed in the previous section must be tested for their accuracy and prediction capability. The performances of the regression models have been tested with the help of fifteen random test cases. The fifteen test cases have been conducted for the same level of nozzle pressure ratio i.e. 5 NPR. The test cases were performed randomly and real experiments were conducted to record the different responses of the above test cases. The experiments were conducted for selected values for Mach number; L/D ratio and area ratio falling in the respective range of their levels (refer Table 1) however were different from those conducted as per the $L_9$ orthogonal array and are shown in Table 6. The predicted values of non-dimensional base pressure obtained by using the regression models based on Taguchi design were compared with their respective target (i.e. experimental) values as shown in Figs. 6 and 7. In case of the regression model for

**Table 6 Random experimental test cases**

| Exp. No. | M | L/D | AR | $P_b/P_a$ (Without control) | $P_b/P_a$ (With control) |
|---|---|---|---|---|---|
| 1 | 2.0 | 8 | 3.24 | 0.369 | 0.402 |
| 2 | 3.0 | 8 | 3.24 | 0.651 | 0.649 |
| 3 | 3.0 | 4 | 6.25 | 0.823 | 0.83 |
| 4 | 3.0 | 6 | 4.84 | 0.755 | 0.76 |
| 5 | 3.0 | 6 | 6.25 | 0.809 | 0.81 |
| 6 | 2.0 | 5 | 3.24 | 0.405 | 0.38 |
| 7 | 3.0 | 4 | 4.84 | 0.789 | 0.788 |
| 8 | 2.5 | 4 | 4.84 | 0.645 | 0.65 |
| 9 | 2.0 | 5 | 6.25 | 0.553 | 0.554 |
| 10 | 2.5 | 4 | 6.25 | 0.715 | 0.713 |
| 11 | 3.0 | 5 | 6.25 | 0.814 | 0.809 |
| 12 | 2.5 | 8 | 6.25 | 0.674 | 0.68 |
| 13 | 2.5 | 6 | 3.24 | 0.434 | 0.42 |
| 14 | 2.5 | 5 | 3.24 | 0.476 | 0.46 |
| 15 | 3.0 | 6 | 3.24 | 0.665 | 0.657 |

base pressure without control shown in Fig. 6, the best fit line is used to make comparison i.e. actual measured values of the response are compared with the corresponding model predicted values. It can be clearly observed that, majority of data points lie closer to the ideal y=x line thereby indicating better prediction for the response of base pressure without control. The values of percentage deviation are found to lie in the range of -11.27% to +7.39% for the Taguchi based regression model of base pressure without control and are shown in Fig. 5.

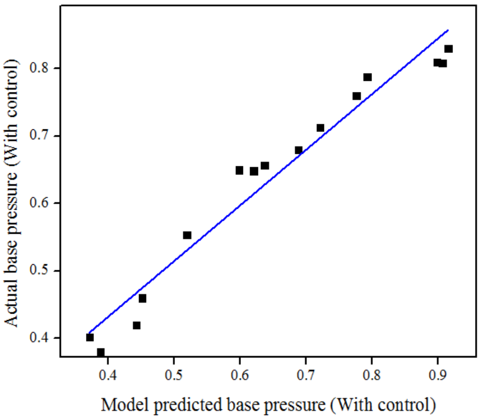


**Fig. 7. Comparison of model predicted base pressure (with control) with actual base pressure (with control).**

Figure. 7 shows the comparison of model predicted and actual values through a best fit line for base pressure with control. Here, again it is clearly observed that the data points are evenly distributed on either sides and close to the ideal y=x line thereby showing better prediction of the Taguchi based regression model for base pressure with control. Furthermore, the values of percentage deviation in prediction are found to lie in the range of -12.07% to +6.97% (refer Fig. 5). It is also imperative to note that, the regression model for base pressure without control has shown better prediction, in terms of average absolute percentage deviation when compared to that for base pressure with control (see Fig. 8).

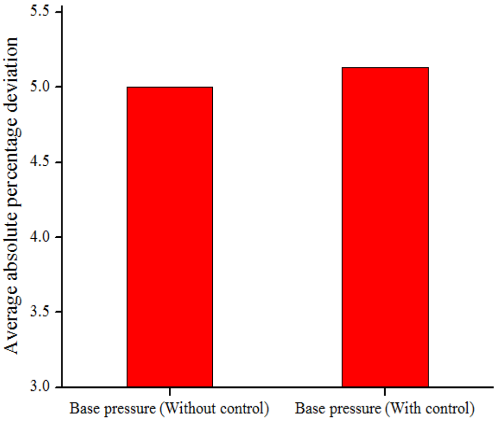


**Fig. 8. Comparison of average absolute percentage deviation for base pressure (without control) and base pressure (with control).**

## 5.3 Signal to Noise Ratio Analysis

Optimal parameter settings for a single characteristic performance could be established by conventional Taguchi method. This can be achieved by signal- to- noise ratio analysis. The process parameters that are set with the low or high value in the S/N ratio will always yield the optimum quality with minimum variance. Keeping in view the interest of its applications, the base pressure should be a minimum in case of combustion chamber in order to maximize mixing and maximum in case of the external flows like in case of rockets and projectiles to reduce base drag. For one of these particular cases, the base pressure quality characteristic selected was ‘Larger the better’ type

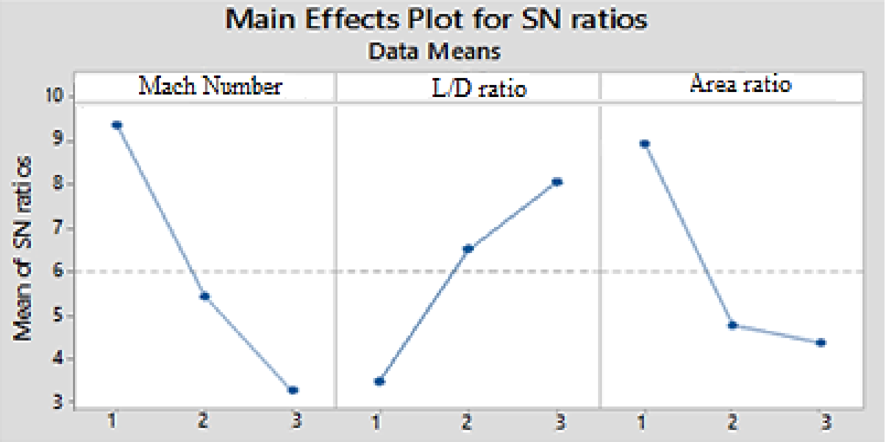
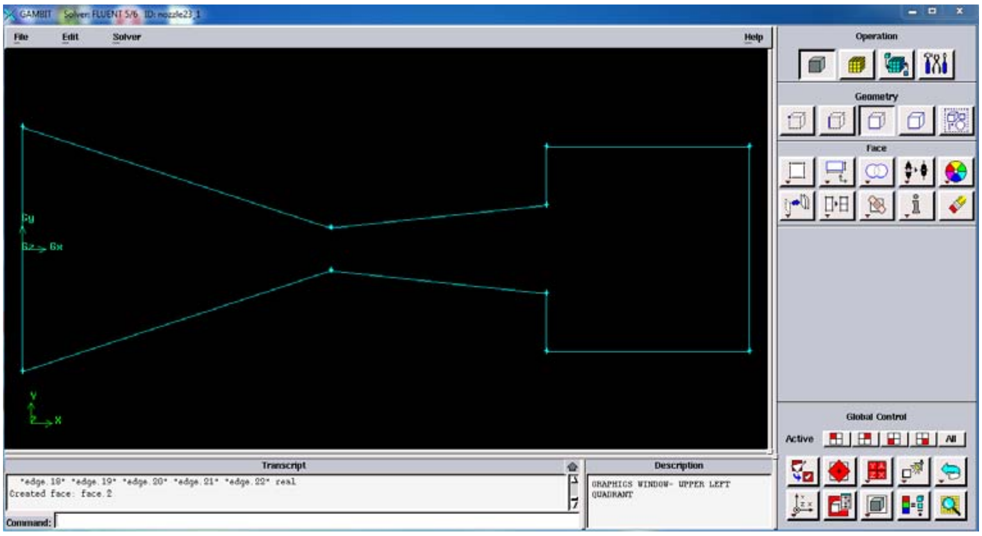


**Fig. 9. S/N analysis for base pressure without control.**



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  *edge.18* *edge.19* *edge.20* *edge.21* *edge.22* real
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**Fig. 10. General Gambit Model for analysis.**

in order to facilitate its applications for rockets, missiles and projectiles. The S/N ratio response has been analysed using Eq. (23) for the experimental results of without and with active control. Additionally, the control results are of marginal significance over base pressure variation. Hence S/N analysis has been conducted only for the case of without active control. From Fig. 9, it can be seen that the optimal parameter for maximum base pressure to be obtained without control is Mach number at level 3 i.e. (3.0 Mach), L/D ratio at level 1 (3 L/D) and area ratio at level 3 (6.25).

$$\eta = -10\ \log_{10}\left\{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right\} \qquad (23)$$

## 5.4 CFD Mesh and Flow Conditions

In order to create the geometry of the model, commercially licensed GAMBIT 16 software has been used. The analysis has been conducted for a nozzle of Mach number 2.5 and 3.0. The geometry is created using vertex, edge and face command (shown in Fig. 10 for Mach 3.0) as per the nozzle and enlarged duct dimensions as shown in Fig. 11. After this, meshing is generated over the edges and faces of the axisymmetric nozzle and the enlarged duct. A quadrilateral element of pave type with 0.75 mm spacing has been selected to get a fine face mesh as shown in Fig. 12. This generic model is then imported into the ANSYS Fluent to conduct the simulation of base pressure generated in the design module of the ANSYS Fluent (Bansal *et al*. 2014). The boundary growth ratio was used in two volume grid regions i.e. base region evaluation of airflow is generally performed by use of analytical method or CFD approach. For simple flows like laminar flow over a plate, analytical methods can be used. However, if the flow process is complex and turbulent, solving Navier-Stokes and continuity equations becomes a very tedious process. In order to overcome such a problem, a time averaged Navier-Stokes equation (Reynolds Averaged Navier-Stokes Equations (RANS) equations) together with turbulent models has been used. A $k$-$\varepsilon$ viscous model is selected for enhanced wall treatment. This $k$-$\varepsilon$ turbulence model is very robust and has a reasonable computational turnaround time, and is widely used by the auto industry. The model is defined by the type of solver used. In this

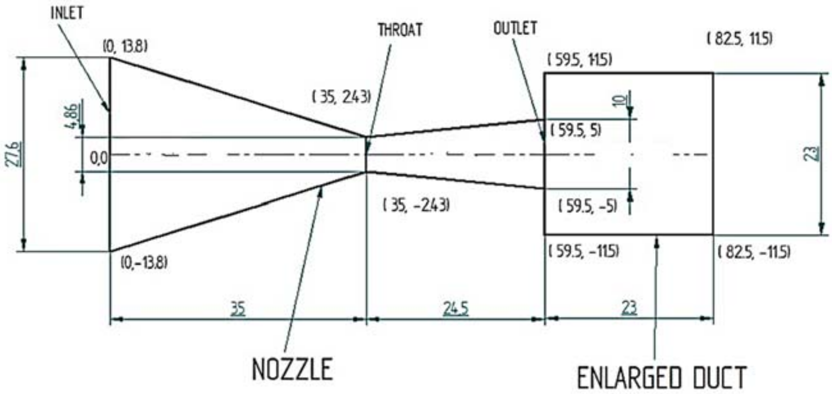
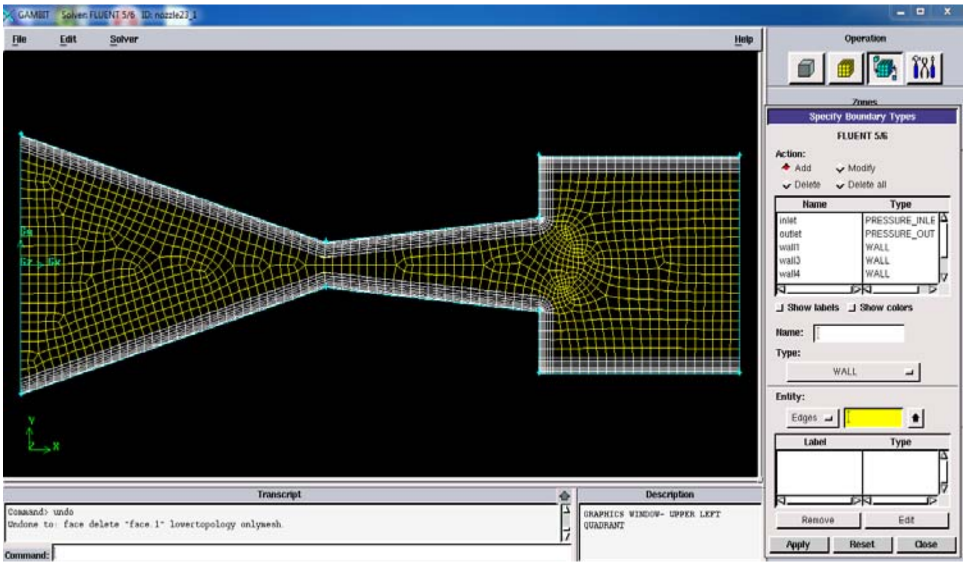


Fig. 11. Nozzle and enlarged duct dimensions.



| Name | Type |
|---|---|
| inlet | PRESSURE_INLE |
| outlet | PRESSURE_OUT |
| wall1 | WALL |
| wall3 | WALL |
| wall4 | WALL |

Type: WALL

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Fig. 12. Meshed model of the axisymmetric nozzle with enlarged duct.

particular case, in order to obtain accurate base pressure results, a density based solver is used since the flow is supersonic. A pressure based solver is generally used for subsonic flows. A time step of 0.000143 seconds was used, based on the Courant number of 0.5. For a CFD simulation, the Courant number gives information about how fluid is moving through the computational cells. If the courant number is <=1, fluid particles move from one cell to another within one time step. Similarly if it is >1, multiple cell movement of fluid particles takes place at each time steps owing to negative convergence. The convergence criterion of continuity, x and y velocities are set to $10^{-4}$, while energy convergence criterion is set to $10^{-6}$. Unsteady time accurate simulations are performed in a reference frame that is fixed since the flow process is unsteady and turbulent. The operating pressure is set to zero Pascal. In the discretization method, a second order upwind scheme is implemented for solving hyperbolic partial differential equations to get accurate results. Proper boundary conditions should be applied to get the accurate simulation results. Here inlet boundary condition is pressure inlet for nozzle inlet where stagnation pressure ($P_0$) is applied. The base pressures are measured at the nozzle exit periphery and hence outlet boundary condition is employed to the right hand side of enlarged duct. Boundary conditions are applied by selecting appropriate edges. The remaining edges are considered as wall. Iterations were carried out until convergence was reached. The computation was carried out using an Intel Core i7-5775C, a high-end quad-core desktop processor. The solution converged to a steady state and for each CFD computational solution; it took around 140 to 150 clock hours.

The computed CFD results are shown in Fig. 13. The CFD analysis was conducted in order to validate the experiment results obtained for different combinations of Mach number, area ratio and L/D ratio. The base pressures were computed without use of active control for the sake of computational ease and evasion of complicated design and large computational time to reach to the converging solution. The input values for nozzle pressure ratios were given in the form of $5\times10^5$ Pa as per experimentation. From Fig. 13(a) for Mach= 2.5, L/D = 3 and AR = 4.84, the value of pressure in

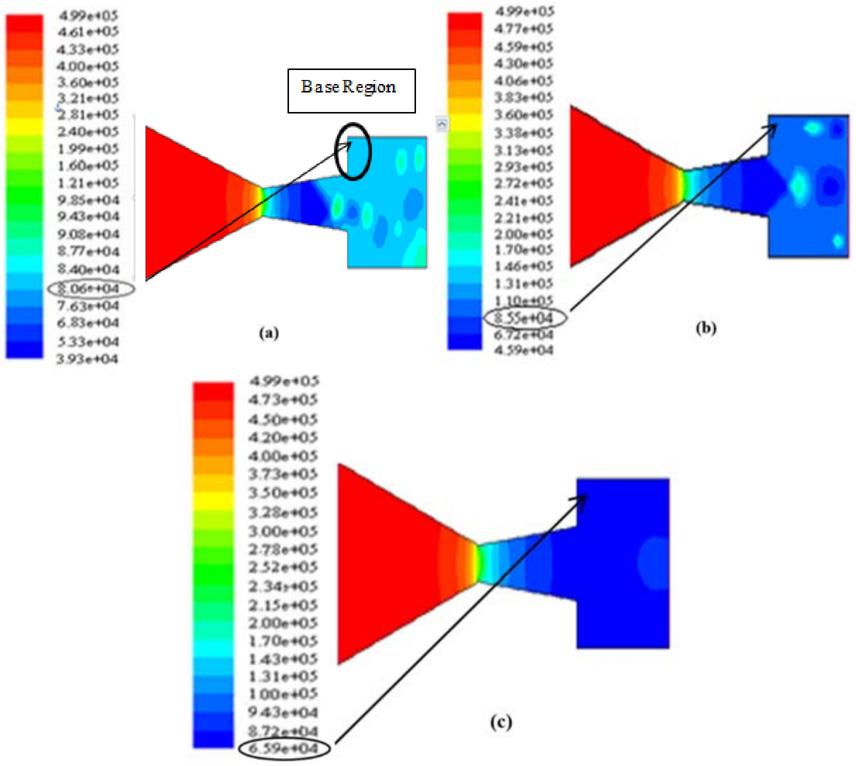
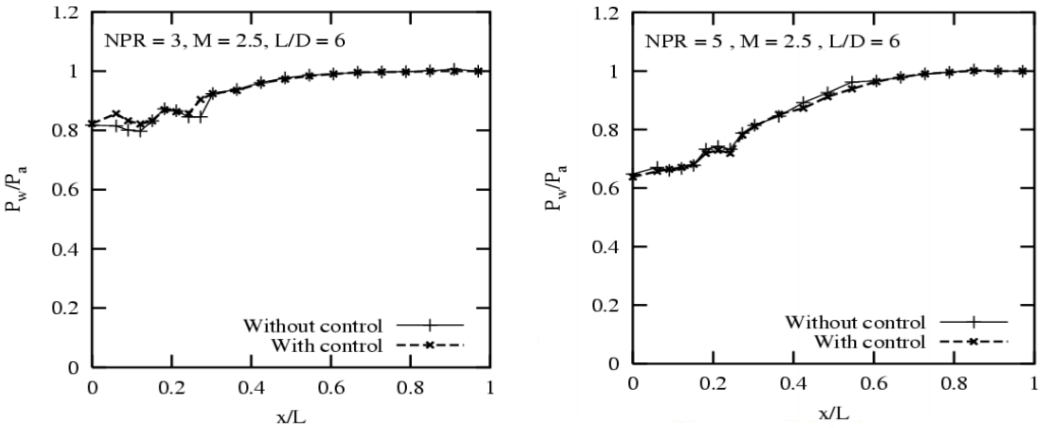


**Fig. 13. CFD results for (a) Mach = 2.5, L/D = 3 and AR = 4.84; (b) Mach = 3, L/D = 3 and AR = 6.25 And (c) Mach=2.5, L/D=5 and AR=4.84.**



(a) (b)

**Fig. 14. Wall Pressure distribution at a) NPR=3; b) NPR=5.**

the base region is 8.06×10$^4$ Pa. This value is non-dimensionalized for it to be compared with the experiment value by dividing it by atmospheric ambient pressure (1.013×10$^5$ Pa). Thus the non-dimensional base pressure value obtained from CFD analysis is 0.795, which is at a percentage deviation of 12.07% from the corresponding experimental value. Similarly from Fig. 13(b), for Mach = 3, L/D = 3 and AR = 6.25, it is evident that the value of pressure in the base region is 8.55×10$^4$ Pa. Hence the non- dimensional base pressure obtained from CFD analysis is 0.841 which is at a percentage deviation of 2.5% from the corresponding experimental value. Also Fig. 13(c) shows a base pressure value of 6.59 ×10$^4$ Pa which is at a deviation of 3% from the corresponding experimental value. Hence it is believed that the base pressure results predicted in the current work are reliable, at least for the relative merit comparisons of different configurations performed here. The respective deviation between the experimental and the CFD results may be due to measurement system errors or due to lack of surface finish at the base region thereby producing unwanted changes in base pressure readings. Also additionally, the experimental set up is subjected to huge amount of mechanical vibrations as the operations are carried out at high pressure and high velocity conditions, machine fluctuations and distinctness in the machining of nozzle and enlarged duct.

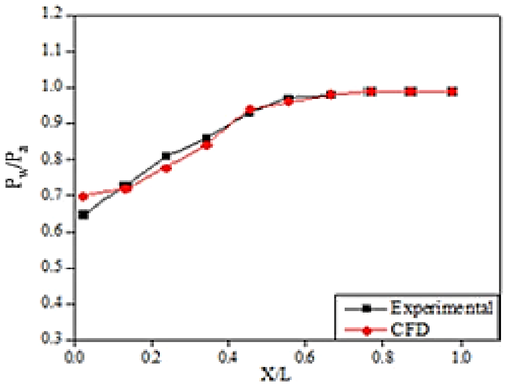


**Fig. 15. Comparison of experimental wall pressure distribution with CFD wall pressure along the enlarged duct length for M=2.5, NPR=5, L/D=6.**

## 5.5 Wall Pressure Distribution

It can be seen from Figs. 14 (a and b) that the location of Microjets does not augment the wall pressure. The wall pressure studies are verily required to understand the oscillatory nature of flow which is one of the major problems in active methods of controlling base flows. In other words, it is essential to make sure that the wall pressure field is not adversely influenced (i.e. made oscillatory) by the control for the current set of experiments conducted. To demonstrate this in the present investigation, the wall pressure distribution was measured for NPR 3 and NPR 5. The results of wall pressure distribution as a function jet Mach number and L/D show that the control does not influence the wall pressure adversely. Moreover the wall pressure distribution obtained from CFD is found to be in close proximity with the experimental wall pressure distribution for M=2.5, NPR=5, L/D=6 without control and is shown in Fig. 15.

## 6. CONCLUSIONS

Effectiveness of active control of micro jets has been experimentally examined through suddenly expanded axi-symmetric ducts for the purpose of controlling Base pressure. The experiments have been conducted for different flow parameters as per L9 orthogonal array for the cases of without active control and with active control. The main findings include:

- For Mach 2.0, for high area ratios of 4.84 and 6.25, the base pressure decreases drastically starting from L/D=3 to L/D = 6 and beyond the base pressure experiences marginal change with increase in L/D. This is mainly due to decrease in the suction at the base region However for the area ratio of 3.24, base pressure assumes a very low value decreasing marginally from L/D=3 to L/D=6 and then increase at L/D=8. This due to the effect of back pressure.
- For lower area ratios control results in increasing base pressure as the micro-jets are very close to the base corner when compared to the higher area ratios thereby tending to counter the effects of shock.
- Multiple linear regression analysis and analysis of variance is performed for the results obtained through experiments. The linear regression models have been tested for statistical adequacy by conducting experiments for random test cases. The non- dimensional base pressure predictions obtained through linear regression models developed have found to agree well with the experiments falling within a range of -11.27% to +7.39% for base pressure without control -12.07% to +6.97% for base pressure with control. Furthermore through analysis of variance, it has been observed that Mach number is the most significant factor influencing base pressure followed by area ratio and L/D ratio.
- Results obtained experimentally were assessed against CFD data for the nozzles of Mach 2.5 and Mach 3. A very good concurrence was found between the experimental and predicted results demonstrating how the proposed approach offers an economical and reliable design tool for industrial needs.
- The wall pressure studies indicate that the micro jets do not adversely affect wall pressure.

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