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Stability of Vertical Throughflow of a Power Law Fluid in Double Diffusive Convection in a Porous Channel

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ABSTRACT

The instability of non-Newtonian power law fluid in double diffusive convection in a porous medium with vertical throughflow is investigated. The lower and upper boundaries are taken to be permeable, isothermal and isosolutal. For vertical throughflow the linear stability of flow is determined by the power law index (n), non-Newtonian Rayleigh number (Ra), Buoyancy ratio (N), Péclet number (Pe) and Lewis number (Le). The eigenvalue problem is solved by two-term Galerkin approximation to obtain the critical value of Rayleigh number and neutral stability curves. It is observed that the neutral stability curves, as well as the critical wave number and Rayleigh number, are affected by the parameters such as Péclet number, buoyancy ratio and Lewis number. The neutral stability curves indicate that power law index n has destabilizing nature when it takes values for dilatant fluid at low Péclet numbers while for the pseudoplastic fluids it shows stabilizing effect. In the absence of buoyancy ratio and vertical throughflow, the present numerical results coincide with the solution of standard Horton-Rogers-Lapwood Problem. The numerical analysis of linear stability for the limiting case of absolute pseudoplasticity is also done by using Galerkin method.

Keywords: Porous medium; Non-newtonian fluid; Buoyancy ratio; Rayleigh number; Lewis number.

NOMENCLATURE

a	dimensionless wave number
D	diffusivity
g	gravitational acceleration
K^*	generalised permeability
K	permeability
Le	Lewis number
N	buoyancy ratio
n	power law index
P	dynamic pressure
Pe	Peclet number
Ra	Rayleigh number
χ	thermal diffusivity
μ^*	effective consistency factor
β_C	concentration expansion coefficient
β_T	thermal expansion coefficient
σ	heat capacity ratio
ϕ	porosity

1. INTRODUCTION

The study of heat and mass transfer in natural convection for Newtonian fluid flow in a porous medium is an interesting topic of re-search due to its applications in science and engineering, such as geophysics, biology, meteorology and chemical engineering etc. Vast literature can be found on the linear stability of convection in a porous medium saturated by the Newtonian fluid. Nield and Bejan (2017) and Drazin and Reid (2004) gives comprehensive details related to the instability of Newtonian fluid.

For non-Newtonian fluid, the study of hydro-dynamic and thermoconvective instability has many applications, such as petroleum production, chemical engineering and in liquid food, etc. Most frequently used models for non-Newtonian fluids are Ostwald-de Waele power law, Carreau-Yasuda, Bingham Herschel-Bulkly, Maxwell, Oldroyd-B, etc. Shenoy (1994) focused on heat transfer in different models of non-Newtonian fluid flow in a porous medium. For Ostwald-de Waele power law model, Barletta and Nield (2011) explained the linear instability of a power law fluid saturated porous layer for horizontal throughflow where boundary planes were considered as impermeable and isothermal. For more general temperature boundary conditions in a porous medium, it was extended by Alves and Barletta (2013). The convective instability of vertical throughflow of a Newtonian fluid saturated horizontal porous medium was developed by Sutton (1970) and by Homsy and Sherwood (1976). In a non-Newtonian power law fluid saturated porous layer, the convective instability of vertical throughflow was investigated by Barletta and Storesletten (2016).

The combined effect of heat and mass transfer (double diffusive convection) in porous media received the considerable attention of the researchers due to its physical importance in real life applications such as, in seawater flow, chemical processes, geology, food processing, etc. In natural convection, the impact of double diffusive convection in a cavity occupied by Newtonian fluid was numerically investigated by the Nikbakhti and Khodakhah (2016). The problem on double diffusive fingering convection, where flow is assumed to be periodic and two dimensional in the horizontal direction was studied by Chen and Chen (1993). It was numerically solved by Galerkin and finite difference methods. The linear stability of non-Newtonian Maxwell fluid for double diffusive convection was examined by Wang and Tan (2008). They explained that the effect of double diffusion and relaxation time on critical Rayleigh number for Maxwell fluid. For inclined thermal gradient, the convective instability in a horizontal porous medium was examined by Nield (1991). The numerical computation was carried out using Galerkin method. The combined effect of inclined thermal and solutal gradient in porous layers was studied by Nield *et al*. (1993) and for Soret effect it was extended by Narayana *et al*. (2008). They explained the various modes of instability and determined the critical Rayleigh number using two-terms Galerkin approximation.

The purpose of the present investigation is to examine the effect of double diffusive convection on linear stability of non-Newtonian power law fluid saturated porous medium with vertical throughflow. From this analysis, it is noticed that the linear stability of vertical through flow for a power law fluid is influenced by large parameter space. For numerical computation, we used two-terms Galerkin approximation. The numerical analysis of the neutral stability is also done for the limiting case of absolute pseudoplastic fluid. In the absence of solute concentration and vertical throughflow, the result obtained agree with the conclusion drawn by Barletta and Nield (2011).

2. MATHEMATICAL MODEL

Consider, a horizontal porous medium saturated by Ostwald-de-Waele power law fluid of thickness H, with permeable, isothermal and isosolutal boundary planes at z = 0 and z = H. The x-axis is along the horizontal direction and gravitational acceleration **g** is acting in the opposite direction of vertical z-axis. The vertical temperature and concentration difference across the boundaries is ΔT and ΔC respectively and **u** is a velocity vector with Cartesian components (u,v,w).

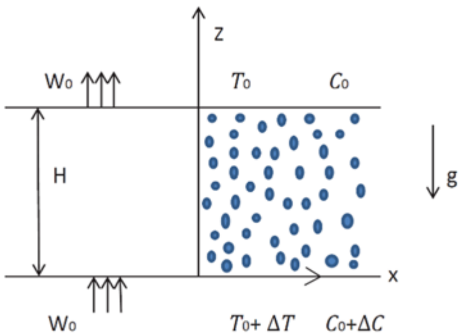

Fig. 1. Schematic of the power law fluid saturated porous layer with vertical throughflow and isothermal and isosolutal boundary conditions.

The governing equation for non-Newtonian power law fluid flow in a porous medium with thermal and solutal buoyancy forces which are modeled by Oberbeck-Boussinesq approximation, the generalized Darcy's law as

$$\frac{\mu^*}{K^*}|\boldsymbol{u}|^{n-1}\boldsymbol{u} = -\nabla P - \rho_0 \boldsymbol{g}\left[\beta_T(T-T_0)+\beta_C(C-C_0)\right]. \tag{1}$$

In the above K^* is the generalized permeability for the non-Newtonian power law fluid and ρ_0 is the fluid density at some reference temperature T_0 and reference concentration C_0. For Newtonian fluid (n = 1) $\frac{\mu^*}{K^*}$ coincides with $\frac{\mu}{K}$.

2.1 Governing Equations

Under the extended form of Boussinesq approximation for concentration, the governing equations for the generalised Darcy's Law of power law fluid with double diffusive transport in a porous medium may be written as

$$\nabla.\boldsymbol{u} = 0, \tag{2}$$

$$\frac{\mu^*}{K^*}|\boldsymbol{u}|^{n-1}\boldsymbol{u} = -\nabla P - \rho_0 \boldsymbol{g}\left[\beta_T(T-T_0)+\beta_C(C-C_0)\right], \tag{3}$$

$$\sigma\frac{\partial T}{\partial t} + \boldsymbol{u}.\nabla T = \chi \nabla^2 T, \tag{4}$$

$$\phi\frac{\partial C}{\partial t} + \boldsymbol{u}.\nabla C = D \nabla^2 C, \tag{5}$$

For permeable (constant vertical throughflow W_0), isothermal and isosolutal boundary planes in a porous medium, the boundary conditions are,

$$z = 0, w = W_0, T = T_0 + \Delta T, \quad C = C_0 + \Delta C,$$
$$z = H,\ w = W_0,\ T = T_0,\ C = C_0. \tag{6}$$

2.2 Dimensional Analysis

Non dimensionalizing the governing Eqs. (2) to (6) by using the following nondimensional quantities as,

$$\frac{1}{H}(x,y,z)\to(x',y',z');\frac{H}{x}\boldsymbol{u}=\frac{H}{x}(u,v,w)\to\boldsymbol{u}';$$
$$\frac{x}{\sigma H^2}t\to t';\frac{T-T_0}{\Delta T}\to T';H\nabla\to\nabla'; \tag{7}$$
$$H^2\nabla^2\to\nabla'^2;\frac{C-C_0}{\Delta C}\to C'.$$

By applying the curl operator on both side of Eq. (3), and removing (') from all parameters we obtain the pressure eliminated form of non dimensional governing equations and the boundary conditions as

$$\nabla.u=0, \tag{8}$$

$$\nabla\times|\boldsymbol{u}|^{n-1}\boldsymbol{u}=Ra\left[\nabla\times(T+NC)\hat{e}_z\right], \tag{9}$$

$$\frac{\partial T}{\partial t}+\boldsymbol{u}.\nabla T=\nabla^2 T, \tag{10}$$

$$\wedge\frac{\partial C}{\partial t}+\boldsymbol{u}.\nabla C=\frac{1}{Le}\nabla^2 C, \tag{11}$$

$$z=0,\quad w=Pe,\quad T=1,\quad C=1,$$
$$z=1,\ w=Pe,\ T=0,\ C=0 \tag{12}$$

with $\hat{e}_z$ is the unit vector in z direction. The dimensionless parameters $Pe=\frac{w_0 H}{\chi}$ is Péclet number, $Ra=\frac{\rho_0 g\beta_T\Delta TK^* H^n}{\mu^*\chi^n}$ is the non-Newtonian form of a Rayleigh number and $\Lambda=\frac{\phi}{\sigma}$. The Lewis number $Le=\frac{\chi}{D}$ is defined as a ratio of thermal diffusivity and solutal diffusivity and $N=\frac{\beta_C\Delta C}{\beta_T\Delta T}$ is the buoyancy ratio.

2.3 Basic Solution

The basic steady state solution of governing Eqs. (8) to (11) subject to boundary conditions (12) is given as

$$u_b=0,v_b=0,w_b=Pe,T_b=\frac{e^{Pe}-e^{Pez}}{e^{Pe}-1},$$
$$C_b=\frac{e^{LePe}-e^{LePez}}{e^{LePe}-1} \tag{13}$$

For the case of $Pe\to 0$, the present basic solution for temperature is linear, which is coincides with the one that was given in Barletta and Nield (2011).

3. LINEAR STABILITY ANALYSIS

3.1 Disturbance Equations

The linear stability of the basic steady state solution is investigated by introducing small disturbance in velocity, temperature and concentration as given below

$$\boldsymbol{u}=\boldsymbol{u}_b+\varepsilon\hat{\boldsymbol{u}},T=T_b+\varepsilon\hat{T},C=C_b+\varepsilon\hat{C}, \tag{14}$$

where ε is a small perturbation parameter. After substituting Eq. (14) into Eqs. (8) to (12) and neglecting $O(\varepsilon^2)$ and beyond, we obtain the linearized equations and the corresponding boundary conditions as

$$\frac{\partial\hat{u}}{\partial x}+\frac{\partial\hat{v}}{\partial y}+\frac{\partial\hat{w}}{\partial z}=0, \tag{15}$$

$$n\frac{\partial\hat{w}}{\partial y}-\frac{\partial\hat{v}}{\partial z}=\lambda\left(\frac{\partial\hat{T}}{\partial y}+N\frac{\partial\hat{C}}{\partial y}\right), \tag{16}$$

$$n\frac{\partial\hat{w}}{\partial x}-\frac{\partial\hat{u}}{\partial z}=\lambda\left(\frac{\partial\hat{T}}{\partial x}+N\frac{\partial\hat{C}}{\partial x}\right), \tag{17}$$

$$\frac{\partial\hat{T}}{\partial t}+Pe\frac{\partial\hat{T}}{\partial z}+\hat{w}\frac{dT_b}{dz}=\frac{\partial^2\hat{T}}{\partial x^2}+\frac{\partial^2\hat{T}}{\partial y^2}+\frac{\partial^2\hat{T}}{\partial z^2}, \tag{18}$$

$$\Lambda\frac{\partial\hat{C}}{\partial t}+Pe\frac{\partial\hat{C}}{\partial z}+\hat{w}\frac{dC_b}{dz}=\frac{1}{Le}\left(\frac{\partial^2\hat{C}}{\partial x^2}+\frac{\partial^2\hat{C}}{\partial y^2}+\frac{\partial^2\hat{C}}{\partial z^2}\right), \tag{19}$$

$$z=0,1:\ \hat{w}=0,\ \hat{T}=0,\ \hat{C}=0 \tag{20}$$

Where

$$\lambda=\frac{Ra}{|Pe|^{n-1}}. \tag{21}$$

Differentiating Eq. (16) with respect to y and (17) with respect to x, then by adding the resulting equations and making use of the continuity Eq. (15), we obtain $(\hat{w},\hat{T},\hat{C})$ formulation of the linear stability problem as

$$n\left(\frac{\partial^2\hat{w}}{\partial x^2}+\frac{\partial^2\hat{w}}{\partial y^2}\right)+\frac{\partial^2\hat{w}}{\partial z^2}=\lambda\left(\frac{\partial^2\hat{T}}{\partial x^2}+\frac{\partial^2\hat{T}}{\partial y^2}\right)$$
$$+N\lambda\left(\frac{\partial^2\hat{C}}{\partial x^2}+\frac{\partial^2\hat{C}}{\partial y^2}\right), \tag{22}$$

$$\frac{\partial\hat{T}}{\partial t}+Pe\frac{\partial\hat{T}}{\partial z}+\hat{w}\frac{dT_b}{dz}=\frac{\partial^2\hat{T}}{\partial x^2}+\frac{\partial^2\hat{T}}{\partial y^2}+\frac{\partial^2\hat{T}}{\partial z^2}, \tag{23}$$

$$\Lambda\frac{\partial\hat{C}}{\partial t}+Pe\frac{\partial\hat{C}}{\partial z}+\hat{w}\frac{dC_b}{dz}$$
$$=\frac{1}{Le}\left(\frac{\partial^2\hat{C}}{\partial x^2}+\frac{\partial^2\hat{C}}{\partial y^2}+\frac{\partial^2\hat{C}}{\partial z^2}\right), \tag{24}$$

$$z=0,1:\ \hat{w}=0,\ \hat{T}=0,\ \hat{C}=0 \tag{25}$$

Consider an arbitrary disturbance in normal modes form,

$$\hat{w} = W(z)e^{\eta t} f(x,y), \hat{T} = \theta(z)e^{\eta t} f(x,y),$$
$$\hat{C} = \psi(z)e^{\eta t} f(x,y), \tag{26}$$

where η is a complex parameter whose real part describes growth rate of the disturbance, while imaginary part is angular frequency and *f (x,y)* is a solution of the two dimensional Helmholtz equation given by

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + a^2 f = 0, \tag{27}$$

with *a > 0* is the wave number. By substituting Eqs. (26) and (27) into Eqs. (22) to (25) and by setting η = 0, we obtained the eigenvalue problem for the neutrally stable modes as :

$$W'' - a^2(nW - \lambda\theta - \lambda N\psi) = 0, \tag{28}$$

$$\theta'' - Pe\theta' - a^2\theta + WPeF(z) = 0, \tag{29}$$

$$\frac{1}{Le}\psi'' - Pe\psi' - \frac{a^2}{Le}\psi + WLePeG(z) = 0, \tag{30}$$

$$Z = 0,1: \quad W = 0, \ \theta = 0, \ \Psi = 0 \tag{31}$$

Where

$$F(z) = \frac{e^{Pez}}{e^{Pe} - 1}, G(z) = \frac{e^{LePez}}{e^{LePe} - 1}. \tag{32}$$

3.2 Numerical Solution

The two-term Galerkin approximation is employed to solve the system of ordinary differential Eqs. (28)-(30) along with the boundary conditions (31) to find the eigenvalue Ra; detail descriptions are as given in Finlayson (2013). The trial solution for unknown variables W , θ and ψ, which satisfies the boundary conditions are taken as $W_i = \sin \pi iz, \theta_i = \sin \pi iz, \psi_i = \sin \pi iz$ for i=1,2… Now the two-term approximations for W , θ and ψ are written in terms of series of trial functions as:

$$W = \sum_{i=1}^{2} A_i W_i, \theta = \sum_{i=1}^{2} B_i \theta_i, \psi = \sum_{i=1}^{2} C_i \psi_i \tag{33}$$

where A_1, A_2, B_1, B_2, C_1 and C_2 are constants. We substitute Eq. (33) into Eqs. (28)-(31) and multiply first, second and third equation by W_1, θ_1 and ψ_1 respectively. For a two-term approximation, the same process is repeated with W_2, θ_2 and ψ_2, then integrate each term with respect to z from $z = 0$ to 1. After that, using integration by parts with boundary conditions, the system of ordinary differential equation is converted into six homogeneous equations in A_1, A_2, B_1, B_2, C_1 and C_2. The nontrivial solution of six homogeneous equations is obtained, when det(a_{ij}) = 0. The determinant of a 6 × 6 matrix gives a polynomial in *Ra* whose coefficients are functions of remaining parameters such as *N, Le, Pe, n* and *a*. For a given set of input parameters (*N, Le, Pe, n*), the critical value of Rayleigh number Ra_c corresponding to a wave number *a* is determined by using a FindMinimum command in Mathematica 9.

4. RESULT AND DISCUSSION

In the present analysis, the convective instability of the vertical throughflow with double diffusive convection in a non-Newtonian power law fluid saturated porous medium is investigated. The critical value of Rayleigh number is described for various value of *n, Pe, N* and *Le.* The results are presented in the form of neutral stability curves on the *(a,Ra)* plane. The region above and below the neutral stability curve represents the unstable and stable state of the basic solution respectively. The numerical scheme is validated for two special cases.

For the case of $|Pe| \to 0$ (no vertical through flow) with pure thermal convection *(N = 0)* the present results agree well with the conclusion given by Barletta and Nield (2011). From present investigation, it is observed that for $Pe = 10^{-10}$($|Pe| \to 0$) the critical Rayleigh number is 5.75261 × 10^{-9} at the critical wave number $a_c = 2.62649$ for a dilatant fluid (n = 2) and $Ra_c = 2.87621 \times 10^6$ at a_c = 3.736 for a pseudoplastic fluid *(n = 0.5*). For Newtonian fluid *(n = 1)*, the critical value of Rayleigh number is Ra_c = 39.4784 with a_c = 3.14159 and it coincides with the data given in Barletta and Nield (2011) (Eq. 43). The results obtained for pure thermal instability *(N = 0)* with vertical throughflow also agrees with the results given in Barletta and Storesletten (2016).

What follows is the discussion on linear stability of power law fluid for vertical throughflow in double diffusive convection. Unlike in the case of pure thermal convection *(N = 0)* induced convective instability when one considers the double diffusive convection induced flow of power law fluid, the instability is governed by two crucial parameters namely the buoyancy ratio and the diffusivity ratio in addition to the other parameters. For numerical computation, some realistic range of parameters is considered as $0 < n \leq 2$, $0 \leq Pe \leq 2$ (Barletta and Storesletten (2016)), $-1 < N \leq 1$ and $0 < Le \leq 10$ (Charrier-Mojtabi *et al.* (2007)). The effect of Lewis number *Le* and the buoyancy ratio N on the linear stability of fluid flow for varying values of power law index *n* has been investigated for some selected range of *N* and *Le.*

The effect of buoyancy ratio and diffusivity ratio parameters on convective instability, the critical Rayleigh number Ra_c for different values of power law index *n* and Péclet number *Pe* is shown in Table 1 for the aiding buoyancy and in Table 2 for the opposing buoyancy. From Table 1, it is observed that, for moderate vertical throughflow *(Pe = 0.1)*, the critical Rayleigh number Ra_c where the instability of this flow occurs decreased consistently with increasing value of Lewis number *Le* (in the range considered) for aiding buoyancy. This behavior is observed for both non-Newtonian dilatant and pseudoplastic fluids. As the intensity of this vertical throughflow is increased (which is signified by the increase in value of *Pe*), the critical Rayleigh number Ra_c is seen to decrease upto certain values of *Le* and beyond which this

Table 1 Critical Rayleigh number for Pseudoplastic, Newtonian and Dilatant fluid for aiding buoyancy

		Le	*Le=0.1*	*Le=1*	*Le=5*	*Le=10*
			Ra_c	Ra_c	Ra_c	Ra_c
	n=0.5	*N=0.1*	90.0824	82.7123	60.8288	46.2852
		N=0.4	87.4832	64.9882	30.4935	18.6825
		N=1	82.7102	45.4917	15.2651	8.51779
Pe=0.1	*n*=1	*N*=0.1	39.101	35.902	26.4083	20.1131
		N=0.4	37.9728	28.2087	13.2408	8.12368
		N=1	35.901	19.7461	6.62892	3.70463
	n=2	*N*=0.1	5.69755	5.2314	3.84868	2.93378
		N=0.4	5.53315	4.11039	1.92999	1.1856
		N=1	5.23126	2.87727	0.966309	0.540768
	n=0.5	*N*=0.1	29.3991	27.0007	23.8507	27.2444
		N=0.4	28.5296	21.2148	14.5664	18.2026
		N=1	26.9356	14.8504	8.00404	9.34225
Pe=1	*n*=1	*N*=0.1	40.4412	37.1427	33.1509	378.2444
		N=0.4	39.2429	29.1835	20.5647	26.9894
		N=1	37.0463	20.4285	11.3961	13.8243
	n=2	*N*=0.1	59.0402	54.2255	48.3314	56.6399
		N=0.4	57.2881	42.6058	30.7058	42.1701
		N=1	54.0768	29.824	17.1415	21.6075
	n=0.5	*N*=0.1	22.755	20.9139	21.7294	23.4853
		N=0.4	22.0349	16.4323	17.538	25.4525
		N=1	20.7204	11.5026	11.007	42.6844
Pe=2	*n*=1	*N*=0.1	44.5503	40.9491	43.0284	46.0496
		N=0.4	43.1302	32.1743	36.0919	50.2197
		N=1	40.5393	22.522	22.9936	72.1946
	n=2	*N*=0.1	130.801	120.235	127.36	135.209
		N=0.4	126.608	94.4704	110.267	147.311
		N=1	118.961	66.1293	71.638	236.422

Ra_c increased. This behavior is significantly affected by the buoyancy ratio parameter *N*. This phenomenon is tabulated for three different values of *Pe*. Thus, from this we conclude that the dual nature of $Ra_c$with *Le* depends on the intensity of the initial vertical through flow and the buoyancy ratio parameter. From Table 1 it is also noticed that the critical Rayleigh number Ra_c for low Péclet number *(Pe = 0.1)* is higher for pseudoplastic fluid than the dilatant fluid, but this gets reversed as the value of the Péclet number becomes large.

The destabilizing effect of the slow vertical through flow due to the (aiding) buoyancy parameter *N* for various values of the diffusivity ratio *(Le)* is clearly seen from the Figs. 2 and 3. When *N = 1* the shift in neutral stability curve towards downward direction indicates that for *Pe = 0.1* destabilizing nature of fluid will increase with increasing value of $0 < Le \leq 5$. Thus the vertical throughflow ceases to be stable in the double diffusive convection even for small values of buoyancy ratio *N* and the Lewis number *Le*. From these neutral stability curves it is also evident that as the power law index *n* increases, the flow gets destabilized, which means, pseudo-plastic fluids are more stable compared to the dilatant fluid for small value of *Pe*.

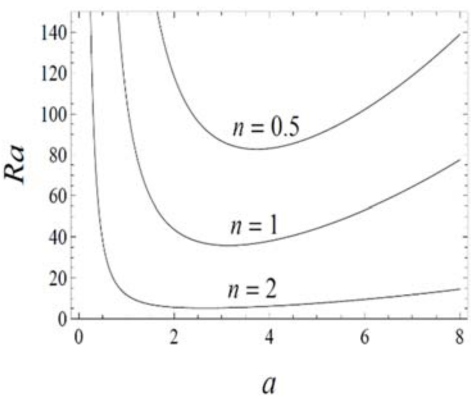

Fig. 2. Variation of Rayleigh number *Ra* with wavenumber a in aiding buoyancy *(N = 1)* for *Pe* = 0.1 and *Le = 0.1*.

In opposing buoyancy situation, where thermal buoyancy and solutal buoyancy forces are oppositely directed, the flow instability phenomenon becomes more complicated and different from the aiding buoyancy case. The convective instability of a vertical through flow and Ra_c is highly influenced by the buoyancy ratio *N*, diffusivity ratio *Le* and power law index *n* which is shown in Table 2. From this table, it can be seen

Table 2 Critical Rayleigh number for Pseudoplastic, Newtonian and Dilatant fluid for opposing buoyancy

		Le	*Le=0.1*	*Le=1*	*Le=5*	*Le=10*
			Ra_c	Ra_c	Ra_c	Ra_c
Pe=0.1	*n=0.5*	*N=-0.05*	91.4408	95.7721	120.936	174.562
		N=-0.02	91.1659	92.8403	100.991	112.644
	n=1	*N*=-0.05	39.6907	41.5707	52.4832	75.5943
		N=-0.02	39.5713	40.2981	48.8333	48.8661
	n=2	*N*=-0.05	5.78347	6.05741	7.6461	10.9893
		N=-0.02	5.76608	5.87198	6.38672	7.11647
Pe=1	*n*=0.5	*N*=-0.05	29.8539	31.2639	33.6033	30.8319
		N=-0.02	29.7618	30.3069	31.1676	30.1593
	n=1	*N*=-0.05	41.068	43.0073	45.9156	40.072
		N=-0.02	40.9411	41.6908	42.7655	41.3485
	n=2	*N*=-0.05	59.9567	62.7874	66.6464	61.0646
		N=-0.02	59.7711	60.8654	62.2974	60.2199
Pe=2	*n*=0.5	*N*=-0.05	23.1324	24.216	23.622	22.7859
		N=-0.02	23.0559	23.4747	23.2535	22.916
	n=1	*N*=-0.05	45.2949	47.4147	46.0177	44.5865
		N=-0.02	45.144	45.9633	45.4358	44.8577
	n=2	*N*=-0.05	132.999	139.22	134.629	130.915
		N=-0.02	132.554	134.958	133.212	131.711

that for a small value of Péclet number *(Pe = 0.1)*, the critical value of Rayleigh number Ra_c increased consistently with increasing value of Lewis number Le. As Pe becomes large *(Pe = 1)*, Ra_c increased in the range $0 < Le \leq 5$ for $N < 0$, beyond which Ra_c decreased.

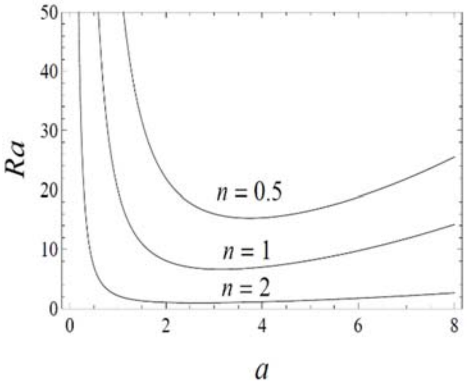

Fig. 3. Variation of Rayleigh number *Ra* with wavenumber a in aiding buoyancy *(N = 1)* for *Pe=0.1* and *Le = 5*.

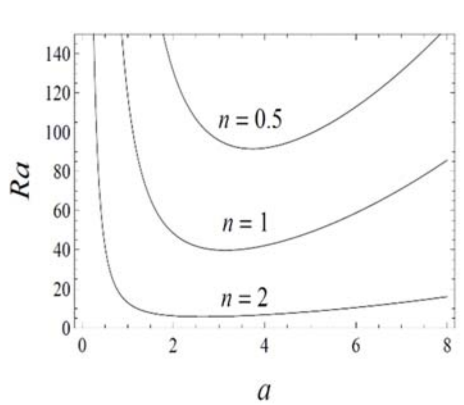

Fig. 4. Variation of Rayleigh number *Ra* with wavenumber a in opposing buoyancy *(N = −0.05)* for *Pe = 0.1* and *Le = 0.1.*

Further increase in the Péclet number *(Pe = 2)*, this dual nature for Ra_c is seen to be starting even for smaller values of *Le*. This behavior is presented in Table 2. Similar to the case of aiding buoyancy, from this table, it is evident that in the case of opposing buoyancy also, the critical Rayleigh number Ra_c for low Péclet number (*Pe = 0.1)* is higher for pseudoplastic fluid than the dilatant fluid, but this gets reversed as the value of the Péclet number becomes very large.

The neutral stability curves for the opposing buoyancy case show the stabilizing effect of the vertical through flow, which is contrary to the case of aiding buoyancy. In Figs. 4 and 5, neutral stability curves are displayed for two values of *Le* when $N = -0.05$ for pseudoplastic and dilatant fluids. In the opposing buoyancy case, the fluid flow in a vertical direction is stabilized by the solutal buoyancy component while the thermal buoyancy which is oppositely directed to the solutal buoyancy, tries to destabilize this action. In this scenario, the value of the Lewis number plays an important role in mostly stabilization of the flow. From this figure, it is also seen that, for a low Péclet number, such as *Pe = 0.1*, the neutral stability curves shifted towards upward direction with increasing values of Lewis number. It means that increasing value of Lewis number will stabilize the basic flow. Neutral stability curves presented in Figs. 6 and 7 for *Pe = 2, Le = 0.1* shows a shift towards the downward direction with increasing value of *N* right from the opposing buoyancy to the aiding buoyancy. It means that for a large value of *Pe*, the basic flow of dilatant fluid is more stable than the pseudoplastic fluid. Increasing value of buoyancy ratio *N* will increase the destabilizing effect of fluid. From all these graphs we can conclude that the presence of solute concentration has a significant effect on the linear stability of different kinds of non-Newtonian fluids.

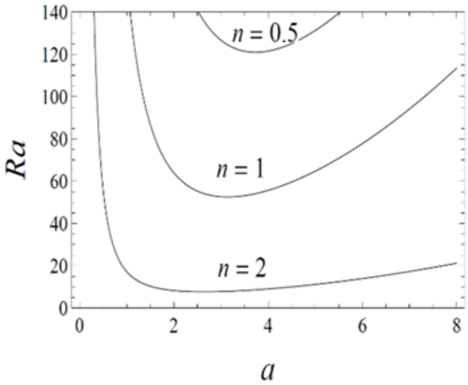

Fig. 5. Variation of Rayleigh number *Ra* with wavenumber a in opposing buoyancy *(N = −0.05)* for *Pe = 0.1* and *Le = 5.*

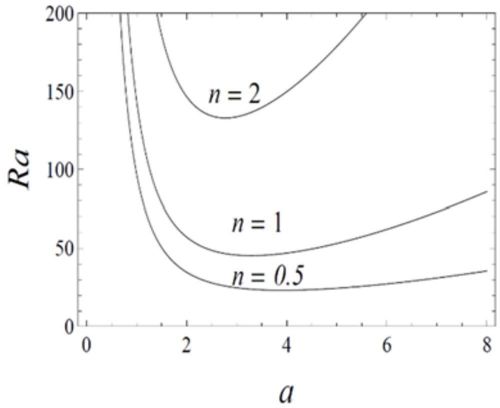

Fig. 6. Variation of Rayleigh number *Ra* with wavenumber a for *Pe = 2, Le = 0.1* and *N=−0.05.*

4.1 The Limiting Cases of Absolute Pseudoplasticity, *n* → *0* and Absolute Dilatancy, *n*→∞

We observe the effect of vanishingly small value of power law index n (which is referred to as the case of absolute pseudoplasticity) on neutral stability curve mathematically for the double diffusive convective instability. For pure thermal convection case, Barletta and Storesletten (2016) presented a detailed discussion for the limiting case of $n \to 0$ and $n \to \infty$, in terms of the Bessel functions. At n → 0, the system of coupled ordinary differential equation is given by

$$W'' + a^2\lambda(\theta + N\psi) = 0 \tag{34}$$

$$\theta'' - Pe\theta' - a^2\theta + WPeF(z) = 0 \tag{35}$$

$$\frac{1}{Le}\psi'' - Pe\psi' - \frac{a^2}{Le}\psi + WLePeG(z) = 0 \tag{36}$$

$$Z = 0,1: \quad W = 0, \ \theta = 0, \ \Psi = 0\,. \tag{37}$$

The eigenvalue problem is solved numerically by using two term Galerkin approximation. The in-stability of pseudoplastic fluid in the limiting case is shown in the Figs. 8 and 9 by neutral stability curves.

In the aiding buoyancy, Ra is seen to be monotonically decreasing with a for all value of Le, which means that due to the presence of solute concentration, the destabilizing character of pseudoplastic fluid is further increased with increasing Pe and this is shown in the Fig. 8, the results are shown here for $N = 1$. When Pe increases, the curves shift towards the downward direction, increasing the instability region. Increasing Pe will enhance the shear rate of the fluid which is instrumental in reducing the value of apparent viscosity, the aiding buoyancy promotes early onset of convective instability of pseudoplastic fluid in this limiting case also. In the opposing buoyancy case also, Ra is monotonically decreasing with a for all value of Le, but with increase in Pe, the curves are seen to shift towards the upward direction leading to, increase in the stability region and this is shown in the Fig. 9, the results are presented for $N = -0.05$. The case of extremely large value of $n(n \to \infty)$ is referred to as the case of absolute dilatancy, which is physically unrealistic. The mathematical significance of this physically unrealistic phenomenon for the case of thermal convection was discussed at length by Barletta and Storesletten (2016), but in the present investigation we ignore presenting this case.

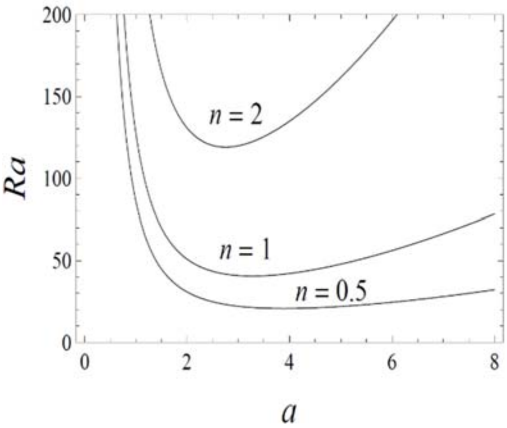

Fig. 7. Variation of Rayleigh number *Ra* with wavenumber *a* for *Pe = 2, Le = 0.1* and *N = 1.*

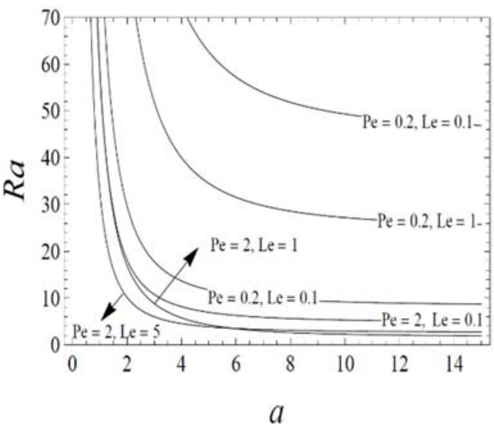

Fig. 8. Asymptotic case *n* → *0*: neutral stability curve in the plane *(a,Ra)* with different value of *Le* and Pe for *N = 1.*

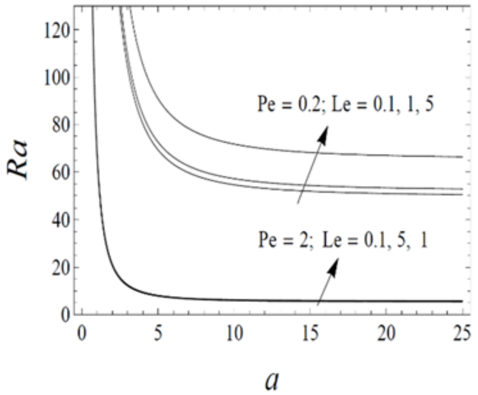

Fig. 9. Asymptotic case *n* → *0*: neutral stability curve in the plane *(a,Ra)* with different value of *Le, Pe* for *N* = −*0.05*.

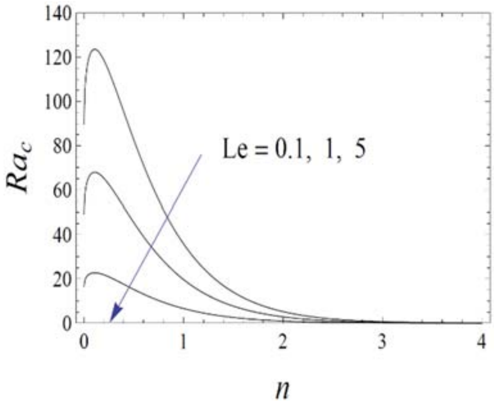

Fig. 10. Variation of critical Rayleigh number Ra_c against *n* with different *Le* for *N* = 1 and *Pe=0.1*.

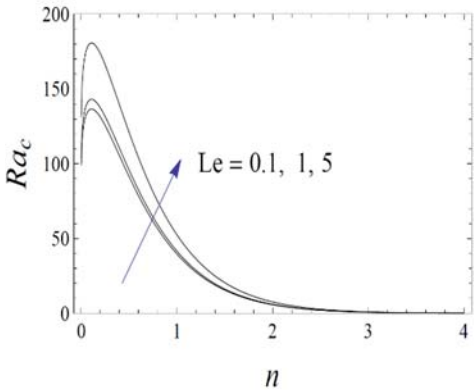

Fig. 11. Variation of critical Rayleigh number Ra_c against n with different *Le* for *N* = −*0.05* and *Pe* = *0.1*.

4.2 Critical Case

The effect of buoyancy ratio *N* on the convective instability of the vertical through flow for varying values of Lewis number *Le* and the power law index *n*, has been investigated for varying Péclet number *Pe* and it is presented in Figs. 10 and 11 for *Pe* = *0.1* and in Figs. 12 and 13 for *Pe* = *2* respectively. It is evident from the Figs. 10 and 11 that the critical Rayleigh number Ra_c is more for the pseudoplastic fluids compared to the dilatant fluid. With increase in *Le*, the Ra_c decreased in the aiding buoyancy while the reverse nature is seen for the opposing buoyancy. For large values of the aiding buoyancy parameter, this critical value is the least. The physical explanation of this is that the effective viscosity of a dilatant fluid is zero as shear rate tends to zero (i.e., for small *Pe*) while it becomes infinite for pseudoplastic fluid at low shear rate. The onset of convective instability of dilatant fluid is represented by the vanishing value of Ra_c while the flow instability for pseudoplastic fluid is represented by the large value of Ra_c. In both aiding and opposing buoyancy situation, with large shear rate (higher value of *Pe*) the critical value of Rayleigh number is monotonically increasing with increasing value of *n*, which is shown in Figs. 12 and 13. The higher value of *Pe* will enhance the shear rate of the fluid, which increase the effective viscosity of the dilatant fluids and decreases that for the pseudoplastic fluids. The instability phenomena of the fluid for dilatant fluid is shown by the higher value of Ra_c and for pseudoplastic fluid, it is explained by the lower value of Ra_c. But in case of large *Pe* no regular trend is seen for varying value of Lewis number *Le*.

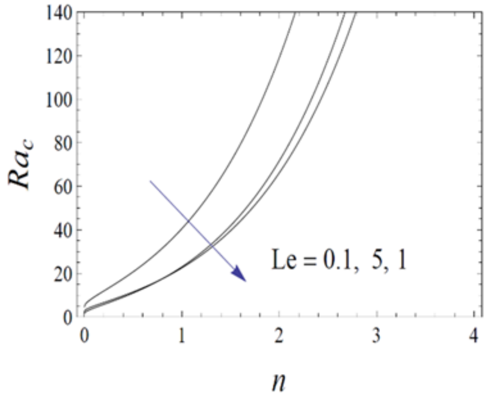

Fig. 12. Variation of critical Rayleigh number Ra_c against n with different *Le* for *N* = *1* and *Pe=2*.

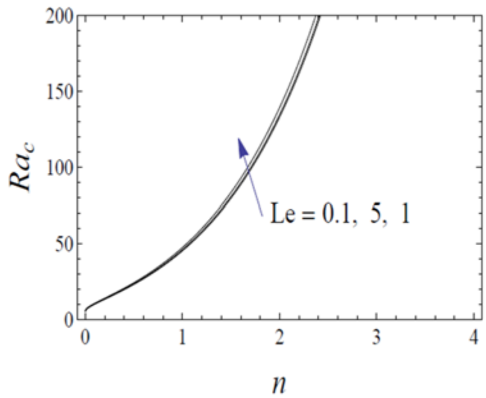

Fig. 13. Variation of critical Rayleigh number Ra_c against *n* with different *Le* for *N* = −*0.05* and *Pe* = *2*.

5. CONCLUSION

Unlike in the case of pure thermal convection induced instability, when one considers the double diffusive convection induced flow of power law fluid, the instability is governed by two crucial parameters namely the buoyancy ratio *N* and the diffusivity ratio Le in addition to the other parameters. For moderate vertical throughflow (*Pe* = 0.1), the critical Rayleigh number Ra_c decreased consistently with increasing value of Le in the aiding buoyancy, for both dilatant and pseudoplastic flu-ids. As the intensity of this vertical throughflow is increased, Ra_c is seen to be decreasing upto certain Le, beyond which there is a raise in this Ra_c. This behavior is pronounced with increasing values of *N*. The vertical throughflow ceases to be stable in the double diffusive convection even for small values of *N* and *Le*. It is also noticed that the value of Ra_c is higher for low Péclet number for pseudoplastic fluid than the dilatant fluid, but this gets reversed as the value of the Péclet number becomes large. The critical Rayleigh number Ra_c increased consistently with increasing value of Le in the opposing buoyancy case for small Péclet numbers. As *Pe* becomes large Ra_c increased up to certain Le, beyond which Ra_c decreased. Further increase in the Péclet number *(Pe = 2)*, this dual nature for Ra_c is seen to be starting even for smaller values of *Le*. In the case of opposing buoyancy also, the critical Rayleigh number Ra_c for low Péclet number is higher for pseudoplastic fluid than the dilatant fluid, but this gets reversed as the value of the Péclet number becomes large. The neutral stability curves presented for various values of these crucial parameters clearly indicate these phenomenon in both aiding and opposing buoyancy cases. Large value of *Pe*, the dilatant basic flow is more stable than the pseudoplastic fluid flow. Increasing value of Buoyancy Ratio *N* has the tendency of more destabilizing effect of the basic flow. The presence of solute concentration has significant influence on the linear stability of different kinds of non Newtonian power law fluids.

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