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A Bio-Thermal Convection in Water-Based Nanofluid Containing Gyrotactic Microorganisms: Effect of Vertical Throughflow

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ABSTRACT

The effect of vertical throughflow on the onset of bio-thermal convection in a water-based nanofluid containing gyrotactic microorganisms is investigated using more realistic boundary conditions. The Galerkin weighted residual method is used to obtain numerical solutions of the mathematical model. The effects of bioconvection Rayleigh number, gyrotaxis number, bioconvection Péclet number, Lewis number, Péclet number, particle density increment number, modified diffusivity ratio, and nanoparticle Rayleigh number on thermal Rayleigh number are examined.The combined effect of Brownian motion and thermophoresis of nanoparticles, vertical throughflow, and gyrotactic microorganisms on the thermal Rayleigh number is found to be destabilizing and its value is decreased by first to third orders of magnitude as compared to regular fluids. Critical wave number is dependent on bioconvection parameters, nanofluid parameters as well as throughflow parameter. The results obtained using passive boundary conditions are compared with those of active boundary conditions. The present study may find applications in seawater convection at the ocean crust.

Keywords: Nanofluid; Vertical throughflow; Thermophoresis; Brownian motion; Bio-Thermal convection; Gyrotactic microorganism.

Nomenclature

a	wave number	t	time
a_c	critical wave number	W_0	vertical upward velocity
D_B	Brownian diffusion coefficient	W_c	microorganisms velocity
D_m	diffusivity of microorganism	(x, y, z)	space co-ordinates
D_T	thermophoresis diffusion coefficient	**Greek symbol**	
$\mathbf{g}$	gravity vector	α_0	measure of the cell eccentricity
H	dimensional layer depth	α_m	thermal diffusivity of the nanofluid
$\mathbf{j}$	flux of microorganisms	β_T	volumetric thermal expansion coefficient
$\hat{\mathbf{k}}$	vertically upward unit vector	$\Delta\rho = \rho_{cell} - \rho_f$	difference between cell density and a fluid density
k_m	thermal conductivity of nanofluid	μ	viscosity
L_b	bioconvection Lewis number	θ	average volume of microorganism
L_e	Lewis number	ϕ	nanoparticles volume fraction
n	microorganism concentration	ρ_p	density of nanoparticles
$\bar{n}$	average dimensionless concentration of microorganisms	$(\rho c)_p$	volumetric heat capacity for the nanoparticles
N_A	modified diffusivity ratio	$(\rho c)_f$	volumetric heat capacity for the nanofluid

N_B	particle density increment number	ρ_f	density of the nanofluid
$\hat{\mathbf{p}}$	average direction of microorganisms' swimming	∇_H^2	horizontal Laplacian operator
p	pressure	**Subscript**	
Q_v	Péclet number	b	basic state
Q_b	Bioconvection Péclet number	f	fluid
R_a	Thermal Rayleigh number	h	upper boundary
R_b	Bioconvection Rayleigh number	p	nanoparticles
R_m	Basic density Rayleigh number	0	lower boundary
R_n	Nanoparticle Rayleigh number	**Superscript**	
T	temperature	*	dimensional variable
T_h	reference temperature	'	perturbed state

1. INTRODUCTION

Bioconvection is a phenomenon that occurs when convective instability is induced by self-propelled up swimming microorganisms that are denser than cell fluid. Platt (1961) introduced the term bioconvection and studied the moving polygonal patterns in dense cultures of Tetrahymena. Plesset and Winet (1974) addressed the bioconvection in terms of Rayleigh-Taylor instability. Childress *et al*. (1975) studied the linear stability and pattern formation in stratified layers of negatively geotactic micro-organisms. Pedley *et al*. (1988) introduced the theoretical bio-convective model for the gyrotactic microorganisms. Later, Hill *et al*. (1989) studied the growth of bio-thermal convection patterns of microorganisms (Bacillus subtitles) in finite depth layer. Bioconvection models for different types of microorganisms were studied by Hillesden and Pedley (1996), Hill and Häder (1997), Bees and Hill (1997), Ghorai and Hill (1999). The effect of critical permeability on the bio-thermal convection was investigated by Kuznetsov and Avramenko (2002, 2003) and they found that for small permeability suspension of microorganisms is unstable, while for the larger permeability, suspension of microorganism is stable.

Due to vast range of applications, nanofluids have attracted the attention of many researchers in recent past. They are widely used in cooling, micro heat pipes, microchannel heat sinks, microreactors, cancer therapy, sterilization of medical suspensions, process industries, polymer coatings, aerospace tribology, microfluid delivery devices etc. (Ebrahimi et.al 2010; Fang et.al 2009). Choi (1995) defined a new class of fluid which consists of nano-sized particles and the base fluid, known as a nanofluid. Buongiorno (2006) developed a mathematical model for nanofluid and explored the various transport mechanisms in nanofluids. The research articles dedicated to the thermal instability problem in nanofluid using the Buongiorno model are well documented by Tzou (2008), Nield and Kuznetsov (2009, 2010).

Kuznetsov (2010, 2011) extended the work of Nield and Kuznetsov (2009) for the suspension containing both gyrotactic microorganisms and nanofluid. They observed that adding the microorganisms to a nanofluid increase the stability of a suspension. On the other hand, Tham, Nazar, and Pop (2013) studied the convection flow over a solid field and found that gyrotactic has a strong influence on the velocity of microorganisms transport rate.

Homsy and Sherwood (1976), Jones and Persichetti (1986) studied the effect of throughflow in layers of porous media and packed beds. Avramenko and Kuznetsov (2006) studied the bioconvection containing gyrotactic microorganisms in the porous layer with vertical throughflow and found that vertical throughflow stabilize the bio-thermal convection. The effect of vertical throughflow in a nanofluid was investigated by Nield and Kuznetsov (2011).

Baehr and Stephan (2006) were perhaps the first who gave the concept of physically realistic boundary conditions (zero nanoparticle flux on the boundaries). After the work of Baehr and Stephan (2006), Nield and Kuznetsov (2014, 2015) revised their work by using more realistic boundary conditions. Very recently, Saini and Sharma (2017) studied the effect of vertical throughflow in Rivlin-Ericksen nanofluid with the new set of boundary conditions.

The above review of the literature reveals that no work has been reported so far on throughflow with gyrotactic microorganisms in a nanofluid. The present study focuses on analytical and numerical investigations of the effect of vertical throughflow on the onset of bio-thermal convection by using more realistic boundary conditions. The effects of various non-dimensionless parameters are investigated and the results are illustrated in graphs and tables. The present study may find applications in seawater convection at the ocean crust.

2. MATHEMATICAL FORMULATION

A horizontal nanofluid layer with gyrotactic microorganisms confined between the two rigid permeable planes is presented in Fig. 1. We take temperatures T_0^* at $z^* = 0$ and T_h^* at $z^* = H$ $(T_0^* > T_h^*)$. Nanoparticles cause no effect on the velocity and direction of gyrotactic microorganisms. Suspension of nanoparticles is assumed to be dilute, stable and do not to agglomerate. The base fluid is water so that microorganisms can stay alive in it. The conservation equations for a water-based fluid containing gyrotactic microorganisms and nanoparticles are written below (see articles Pedley *et al*. 1988; Nield and Kuznetsov 2009, 2015).

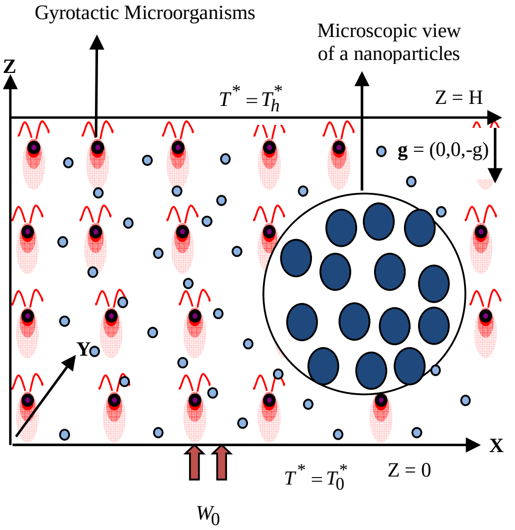


Fig. 1. Physical model and coordinate system

$$\nabla^*.\mathbf{V}^* = 0 \tag{1}$$

$$\rho_f\left(\frac{\partial}{\partial t^*}+\mathbf{V}^*.\nabla^*\right)\mathbf{V}^* = -\nabla^* p^* + \mu\nabla^{*2}\mathbf{V}^* + [\phi^*\rho_p + (1-\phi^*)\left[\rho_f\left\{1-\beta_T(T^*-T_c^*)\right\}\right] + n^*\theta\Delta\rho]\mathbf{g} \tag{2}$$

$$(\rho c)_f\left(\frac{\partial}{\partial t^*}+\mathbf{V}^*.\nabla^*\right)T^* = k_m\nabla^{*2}T^* + (\rho c)_p[D_B\nabla^*\phi^*.\nabla^*T^* + \frac{D_T}{T_c^*}\nabla^*T^*.\nabla^*T^* - (\phi^*-\phi_0^*)\mathbf{V}^*.\nabla^*T^*] \tag{3}$$

$$\left(\frac{\partial}{\partial t^*}+\mathbf{V}^*.\nabla^*\right)\phi = D_B\nabla^{*2}\phi^* + \frac{D_T}{T_C^*}\nabla^{*2}T^* \tag{4}$$

$$\frac{\partial n^*}{\partial t^*} = -\nabla^*.\left[n^*\mathbf{V}^* + n^*W_c\hat{\mathbf{p}} - D_m\nabla^*n^*\right] \tag{5}$$

Where $\mathbf{V}^*$ is the velocity, t^* is time, ρ_p is density of nanoparticles, μ is the viscosity, ρ_f is the density of the nanofluid, β_T is the volumetric thermal expansion coefficient, ϕ^* is the nanoparticles volume fraction, n^* is the microorganism concentration, θ is the average volume of microorganism, $\mathbf{g}$ is gravity vector, $\Delta\rho = \rho_{cell} - \rho_f$ is the difference between cell density and a fluid density, $(\rho c)_f$ is the heat capacity for the nanofluid, k_m is the thermal conductivity of nanofluid, D_T is the thermophoresis diffusion coefficient, $(\rho c)_p$ is the heat capacity for the nanoparticles, D_B is the Brownian diffusion coefficient. Temperature is assumed to be constant, and throughflow velocity has a constant value on the boundaries. The boundary conditions are

$$w^* = W_0,\ \frac{dw^*}{dz^*} = 0,\ T^* = T_0^*,\ \mathbf{j}.\hat{\mathbf{k}} = 0$$
$$D_B\frac{\partial\phi^*}{\partial z^*} + \frac{D_T}{T_h^*}\frac{\partial T^*}{\partial z^*} - W_0(\phi^*-\phi_0^*) = 0 \text{ at } z^* = 0 \tag{6a}$$

$$w^* = W_0,\ \frac{dw^*}{dz^*} = 0,\ T^* = T_h^*,\ \mathbf{j}.\hat{\mathbf{k}} = 0$$
$$D_B\frac{\partial\phi^*}{\partial z^*} + \frac{D_T}{T_c^*}\frac{\partial T^*}{\partial z^*} - W_0(\phi^*-\phi_0^*) = 0 \text{ at } z^* = H \tag{6b}$$

Where, $\mathbf{j} = (n^*\mathbf{V}^* + n^*W_c\hat{\mathbf{p}} - D_m\nabla^*n^*)$.

Dimensionless variables in the equations are as follows

$$t = \frac{t^*\alpha_m}{H^2}, \mathbf{V}(u,v,w) = \frac{\mathbf{V}(u^*,v^*,w^*)H}{\alpha_m}, (x,y,z) = \frac{(x^*,y^*,z^*)}{H},\ p = \frac{p^*H^2}{\mu\alpha_m}, \phi = \frac{\phi^*-\phi_0^*}{\phi_0^*}\ T = \frac{T^*-T_h^*}{T_0^*-T_h^*}, n = n^*\theta \text{ Where, } \alpha_m = \frac{k_m}{(\rho c)_f}. \tag{7}$$

Non-dimensional form of Eqs. (1)-(5) are as follows

$$\nabla.\mathbf{V} = 0 \tag{8}$$

$$\frac{1}{\Pr}\left(\frac{\partial}{\partial t}+\mathbf{V}.\nabla\right)\mathbf{V} = -\nabla p + \nabla^2\mathbf{V} - R_m\hat{\mathbf{k}} + R_aT\hat{\mathbf{k}} - R_n\phi\hat{\mathbf{k}} - \frac{R_b}{L_b}n\hat{\mathbf{k}} \tag{9}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{V}.\nabla\right)T = \nabla^2T - N_B\phi\mathbf{V}.\nabla T + \frac{N_B}{L_e}\nabla\phi.\nabla T + \frac{N_AN_B}{L_e}\nabla T.\nabla T \tag{10}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{V}.\nabla\right)\phi = \frac{1}{L_e}\nabla^2\phi + \frac{N_A}{L_e}\nabla^2T \tag{11}$$

$$\frac{\partial n}{\partial t} = -\nabla.\left(n\mathbf{V} + n\frac{Q_b}{L_b}\hat{\mathbf{p}} - \frac{1}{L_b}\nabla n\right) \tag{12}$$

With the non-dimensional boundary conditions as

$$w = Q_v,\ \frac{dw}{dz} = 0,\ T = 1, (Q_b + Q_vL_b)n = \frac{dn}{dz},$$
$$\frac{\partial\phi}{\partial z} + N_A\frac{\partial T}{\partial z} - QL_e\phi = 0 \quad at\ z = 0 \tag{13a}$$

$$w = Q_v,\ \frac{dw}{dz} = 0,\ T = 0, (Q_b + Q_vL_b)n = \frac{dn}{dz}$$
$$\frac{\partial\phi}{\partial z} + N_A\frac{\partial T}{\partial z} - QL_e\phi = 0, \quad at\ z = 1 \tag{13b}$$

The non-dimensional parameters in Eqs. (8)-(12) namely, Lewis number L_e, Prandtl number P_r, bioconvection Lewis number L_b, thermal Rayleigh number R_a, bioconvection Péclet number Q_b, Péclet number Q_v, basic density Rayleigh number R_m, particle density increment N_B, bioconvection Rayleigh number R_b, nanoparticle Rayleigh number R_n, and modified diffusivity ratio N_A are defined as

$$L_e=\frac{\alpha_m}{D_B},\ \Pr=\frac{\mu}{\rho_f\alpha_m},L_b=\frac{\alpha_m}{D_m},R_a=\frac{\rho g\beta_T H^3(T_0^*-T_h^*)}{\mu\alpha_m},$$
$$Q_b=\frac{W_c H}{D_m},Q_v=\frac{W_0 H}{\alpha_m}R_m=\frac{[\rho_p\phi_0^*+\rho_{f_0}(1-\phi_0^*)]gH^3}{\mu\alpha_m},$$
$$N_B=\frac{(\rho c)_p(\phi_0^*)}{(\rho c)_f},R_b=\frac{\Delta\rho g v H^3}{\mu D_m},R_n=\frac{\{(\rho_P-\rho)\phi_0^*\}gH^3}{\mu\alpha_m},$$
$$N_A=\frac{D_T(T_0^*-T_h^*)}{D_B T_h^*\phi_0^*} \tag{14}$$

3. BASIC SOLUTIONS

The basic state is described by

$$\mathbf{V}=\mathbf{V_b}=(0,0,Q_v),\ p=p_b(z),T=T_b(z),$$
$$\phi=\phi_b(z),\ n=n_b(z) \tag{15}$$

The Eqs. (8)-(12) are simplified as

$$\frac{-dp_b}{dz}-R_m+R_aT_b-R_n\phi_b-\frac{R_b}{L_b v}n_b=0 \tag{16}$$

$$\frac{d^2T_b}{dz^2}+\frac{N_B}{L_e}\frac{d\phi_b}{dz}\frac{dT_b}{dz}+\frac{N_AN_B}{L_e}\left(\frac{dT_b}{dz}\right)^2$$
$$-Q_v\frac{dT_b}{dz}-Q_vN_B\phi_b\frac{dT_b}{dz}=0 \tag{17}$$

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}-Q_vL_e\frac{d\phi_b}{dz}=0 \tag{18}$$

$$n(Q_vL_b+Q_b)-\frac{dn}{dz}=0 \tag{19}$$

Equation (19) is integrated, then the solution of n_b is obtained

$$n_b(z)=v\ \exp((Q_vL_b+Q_b)z) \tag{20}$$

Here v is the integration constant given by

$$v=\frac{\bar{n}(Q_vL_b+Q_b)}{\exp(Q_vL_b+Q_b)-1}$$

Where $\bar{n}=\int_0^1 n_b(z)dz$

On solving the Eqs. (17)–(18) with the help of boundary conditions (13a)-(13b), the solutions are

$$\phi_b=\frac{(1-L_e-N_A)(e^{Q_vL_ez}-1)}{(1-L_e)(e^{Q_vL_e}-1)}+\frac{N_A(e^{Q_vz}-1)}{(1-L_e)(e^{Q_v}-1)},$$
$$T_b=\frac{e^{Q_v}-e^{Q_vz}}{e^{Q_v}-1}. \tag{21}$$

4. PERTURBED SOLUTIONS

For small perturbations on the basic solution, we assume that

$$\mathbf{V}=\mathbf{V}_b+\mathbf{V}',\phi=\phi_b+\phi',n'=n_b+n,T=T_b+T',$$
$$p=p_b+p',\ \hat{\mathbf{p}}=\hat{\mathbf{k}}+p' \tag{22}$$

Substituting Eq. (22) in Eqs. (8)- (12), we get

$$\nabla.\mathbf{V}'=0 \tag{23}$$

$$\frac{1}{\Pr}\frac{\partial\mathbf{V}'}{\partial t}=-\nabla p'+\nabla^2\mathbf{V}'+R_aT'\hat{\mathbf{k}}-R_n\phi'\hat{k}-\frac{R_b}{L_bv}n'\hat{\mathbf{k}} \tag{24}$$

$$\frac{\partial T'}{\partial t}+\frac{dT_b}{dz}w'+Q_v\frac{\partial T'}{\partial z}=\nabla^2T'+\frac{N_B}{L_e}$$
$$\left(\frac{d\phi_b}{dz}\frac{\partial T'}{\partial z}+\frac{dT_b}{dz}\frac{\partial\phi'}{\partial z}\right)+\frac{2N_AN_B}{Le}\frac{dT_b}{dz}\frac{\partial T'}{\partial z}$$
$$-N_B(Q_v\frac{dT_b}{dz}\phi'+\phi_b\frac{dT'}{dz}w'+Q_v\phi_b\frac{\partial T'}{\partial z}) \tag{25}$$

$$\frac{\partial\phi'}{\partial t}+Q_v\frac{\partial\phi'}{\partial z}+\frac{d\phi_b}{dz}w'=\frac{1}{L_e}\nabla^2\phi'+\frac{N_A}{L_e}\nabla^2T' \tag{26}$$

$$\frac{\partial n'}{\partial t}=-\nabla.\left(n_b\left(\mathbf{V}+\frac{Q}{Lb}\hat{\mathbf{p}}\right)+n'\frac{Q}{Lb}\hat{\mathbf{k}}-\frac{1}{L_b}\nabla n'\right) \tag{27}$$

Applying the procedure outlined in Pedley and Hill (1988), Eq. (27) is written as

$$\frac{\partial n'}{\partial t}=-w'\frac{\partial n_b}{\partial z}-\frac{(Q_b+L_bQ_v)}{L_b}\frac{\partial n'}{\partial z}+GQ_bn_b((1-\alpha_0)$$
$$(\frac{\partial^2w'}{\partial x^2}+\frac{\partial^2w'}{\partial y^2})+(1+\alpha_0)\frac{\partial^2w'}{\partial z^2})+\frac{1}{L_b}\nabla^2n' \tag{28}$$

The pressure and horizontal component of velocity are eliminated from Eqs. (23)- (24), resulting to the following equation.

$$\frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^2w'-\nabla^4w'=R_a\nabla_H^2T'-R_n\nabla_H^2\varphi'-\frac{R_b}{L_bv}\nabla_H^2n' \tag{29}$$

In absence of oscillatory convection, perturbation quantity are taken in the following form

$$[w',T',\phi',n']=[W(z),\Theta(z),\Phi(z),N(z)]e^{(ik_xx+ik_yy)} \tag{30}$$

For spherical microorganisms, α_0 is set to be zero (value of α_0 lies between the values 0 to 0.5 Pedley et. al, 1988). Substituting Eq. (30) in Eqs. (25), (26)(28)-(29), we get

$$(-1+N_BQ_v)\frac{dT_b}{dz}W+(D^2+\frac{N_B}{L_e}\frac{d\phi_b}{dz}D+$$
$$\frac{2N_AN_B}{L_e}\frac{dT_b}{dz}D-N_BQ_v\phi_bD-Q_vD-a^2)\Theta$$
$$+(\frac{N_B}{L_e}\frac{dT_b}{dz}D-N_BQ_v\frac{dT_b}{dz})\Phi=0 \tag{31}$$

$$\frac{d\phi_b}{dz}W-\frac{N_A}{L_e}(D^2-a^2)\Theta-\frac{1}{L_e}((D^2-a^2)$$
$$-Q_vD)\Phi=0 \tag{32}$$

$$\frac{1}{L_b}(D^2-a^2)N-\frac{(Q_vL_b+Q_b)}{L_b}DN-e^{Q_bz}Q_b$$
$$v[1-G(D^2-a^2)]W=0 \tag{33}$$

$$(D^2-a^2)^2 W - R_a a^2 \Theta + R_n a^2 \Phi - \frac{R_b}{L_b v} a^2 N = 0 \tag{34}$$

With boundary conditions

$$\begin{aligned} & W=0, DW=0, \Theta=0, (Q_v L_b + Q_b N) = DN, \\ & D\Phi + N_A D\Theta - Q_v L_e \Phi = 0 \quad at\ z=0, z=1 \end{aligned} \tag{35}$$

Where $a=\sqrt{k_x^2+k_y^2}$ is the dimensionless horizontal wave number and $D=\frac{d}{dz}$.

5. METHOD OF SOLUTION

The above system of Eqs (31)-(34) subjected to boundary conditions given by Eq. (35) are solved by using single-term Galerkin's weighted residual method. Accordingly Θ, Φ, W and N are taken as

$$\begin{aligned} & \Theta_1 = z - z^2, W_s = z(1-z)^2, \\ & \Phi_1 = -N_A (Q_v L_e - 2 - 2Q_v L_e z) / Q_v L_e^2, \\ & N_1 = (2-(Q_v + QL_b)(1-2z) - (Q_v + QL_b)^2 (z-z^2)) \end{aligned} \tag{36}$$

To obtain an analytical formula for thermal Rayleigh number, we take $Q_v << 1$ so that first-order Galerkin approximations lead to a useful result. The above trial solutions Eq. (36) are substituted in Eqs. (31)-(34) to obtain residues and making these residues orthogonal (in inner product sense) to these trial functions, we get a system of linear simultaneous homogeneous equations which admits the non-trivial solution if its determinant vanishes (Finalayson, 1971) this results in the following Eigenvalue equation

$$\begin{aligned} & R_a = 7(20(10+a^2-\frac{N_A N_B Q_v}{L_e})(504+24a^2+a^4)-15(12+a^2) \\ & (1+N_B Q_v) N_A R_n - 14(10+a^2-\frac{N_A N_B Q_v}{L_e}) L_e R_n - 7200(28 \\ & -3(Q_b+Q_v L_b)^2) A_1 a^2 (10+a^2-\frac{N_A N_B Q_v}{L_e}) R_b) / (135a^2(1+N_B Q_v)) \end{aligned} \tag{37}$$

$$\begin{aligned} & +(72+Q_b^2) L_b^2 Q_v^2) - 2\mathrm{Sinh}(\frac{Q_b}{2})(4Q_b^2(12+Q_b^2)+2L_b Q_b (54 \\ & +5Q_b^2) Q_v + (72+7Q_b^2) L_b^2 Q_v^2)) / (Q_b^6 (10(Q_b+Q_v L_b)^4 \\ & +a^2(120-10(Q_b+Q_v L_b)^2 + (Q_b+Q_v L_b)^4))) \end{aligned}$$

In the absence of throughflow, gyrotactic microorganisms and nanofluid $(R_n=0, R_b=0, Q_v=0)$, the thermal Rayleigh number attains its minimum value of 1751.851 at a = 3.12. This value of R_a is two and half percent greater than the value obtained in Rayleigh-Bernard Problem Chandrasekhar (1961). Also, for the case when $R_n=0, R_a=0, Q_v=0,$ and $Q_b \to 0$ then $Q_b R_b \to 720$, which is same as given by Sparrow *et al*. (1964).

For the case $Q_v=0$, Eq. (37) becomes:

$$\begin{aligned} & R_a = (140(10+a^2)(504+24a^2+a^4) \\ & -105(12+a^2) N_A R_n - 98(10+a^2) L_e R_n \\ & -50400(28-3Q_b^2) A_1 a^2 (10+a^2) R_b) / (135a^2) \end{aligned} \tag{38}$$

From Eq. (38), when $Q_v=0$ thermal Rayleigh number is independent to N_B. To simplify the expression, we have fixed the values of $G=0.03$, $L_b=4$ (for alga Chlamydomonas nivails Hill et. al, 1989), and $Q_v=0.1$ (vertical through flow is small as compared to unity). Using these assumptions Eq. (37) is simplified as follows

$$\begin{aligned} & R_{a_c} = (1.16\times10^4 (19.73 - \frac{0.1\, N_A N_B}{L_e}) - 2.28\times10^2 \\ & (1+0.1N_B) N_A R_n - 9.5\times10^1 (19.73 - \frac{0.1 N_A N_B}{L_e}) L_e R_n - ((28 \\ & -3(Q_b+0.4)^2)(19.73 - \frac{0.1 N_A N_B}{L_e})(\mathrm{e}^{Q_b/2}(0.03\, Q_b^2 \\ & (Q_b(48Q_b^2 + 0.16\,(72 + Q_b^2) + 0.4Q_b(108 + Q_b^2)) \\ & \mathrm{Cosh}(Q_b/2) - 2\,(4\,Q_b^2(12 + Q_b^2) + 0.8\,Q_b(54 + 5Q_b^2) + \\ & 0.16\,(72 + 7\,Q_b^2))\,\mathrm{Sinh}(Q_b/2)) - 1.29(Q_b(0.48\,(60 + Q_b^2) \\ & + 2Q_b(60 + Q_b^2) + 2\,Q_b^2(66 + Q_b^2))\,\mathrm{Cosh}(Q_b/2) \\ & - 2\,(2.88\,(10 + Q_b^2) + 12Q_b(10 + Q_b^2) + Q_b^2(132 + 13\,Q_b^2)) \\ & \mathrm{Sinh}(Q_b/2))))/(Q_b^6\,(10\,(0.1 + 4\,Q_b)^4 \\ & + 9.7344\,(120 - 10\,(0.1 + 4\,Q_b)^2 \\ & + (0.1 + 4\,a)\text{^}4)))\}R_b) / (1.32\times10^2(1+0.01Q_v)) \end{aligned} \tag{39}$$

Where

$$\begin{aligned} & A_1 = (4e^{Q_b/2}((-1-a^2G)(-2\mathrm{Sinh}(\frac{Q_b}{2})(Q_b^2(132+13Q_b^2) \\ & +30Q_b L_b (10+Q_b^2) Q_v + 18L_b^2(10+Q_b^2) Q_v^2)) + \mathrm{Cosh}(\frac{Q_b}{2}) Q_b \\ & (2Q_b^2(66+Q_b^2) + 5L_b Q_b (60+Q_b^2) Q_v + 3L_b^2 (60+Q_b^2) Q_v^2)) \\ & +GQ_b^2(\mathrm{Cosh}(\frac{Q_b}{2}) Q_b (48Q_b^2 + L_b Q_b (108+Q_b^2) Q_v \end{aligned}$$

To simplify the above expression, we use three values of $Q_b = 0.1,\ 1$ and 10, which corresponds to slowly, intermediate, and faster swimming bacterial species. Then Eq. (39) becomes

$$\begin{aligned} & R_{a_c} = ((88.68(19.73\mathrm{L}_e - 0.1N_A N_B)/\mathrm{L}_e) - 1.73(1+0.1\mathrm{N}_B)\mathrm{N}_A R_n \\ & \quad -0.72(19.73L_e - 0.1N_A N_B) R_n - k R_b) / (1+0.1N_B) \end{aligned} \tag{40}$$

Where,

$k = 0.982(\text{for } Q_b = 0.1),\ 3.58(\text{for } Q_b = 1),\ 5.52\times10^3 (Q_b = 10).$ According to Buongiorno (2006), the value of Lewis number is of the order of (10^2-10^3), N_A is of order 1-10, and $N_B = 7.5\times10^{-4}$. For above stated ranges of these parameters, Eq. (40) shows that R_n has destabilizing effect on the system. The value of R_{a_c} decreases as bioconvection Rayleigh number increases. Thus for fixed values of $N_A, N_B,$ and L_e, R_b also has destabilizing effect.

6. Results and Discussion

To investigate the various effects of non-dimensional parameters on bio-thermal convection, we have used the MATHEMATICA 9.0 (software package) to obtain important numerical results. The present numerical scheme is also validated for regular fluid $(Q_v = 0, R_n = 0, R_b = 0)$ by comparing the present results with the result of Chandrasekhar (1961) (Fig. 2). It is clear that the present results are sufficiently in agreement to the results reported in it.

For alumina/water nanofluid with alga Chlamydomonas nivails (microorganisms), the ranges of bioconvection Péclet number ,modified diffusivity ratio,bioconvection Lewis number, nanoparticle Rayleigh number, are the order of $10^0 - 10^1$, Péclet number and gyrotaxis number are in the order $10^{-2} - 10^{-1}$, N_B is of the order $10^{-4} - 10^{-3}$, bioconvection Rayleigh number is of the order $10^0 - 10^2$, and the values of Lewis number in the range between $10^0 - 10^3$. The range of considered variables has been predicated from the data given by (Kessler 1986; Pedley 1988; Buongiorno 2006; Kuznetsov 2010). We have fixed the following values of dimensionless parameters:

$L_e = 500,\ L_b = 4,\ Q_b = 3.0,$
$Q_v = 0.05,\ R_b = 3, R_n = 0.1,$
$G = 0.03,\ N_b = 0.01, N_A = 5.$

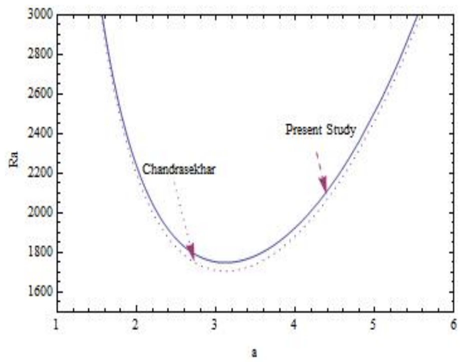
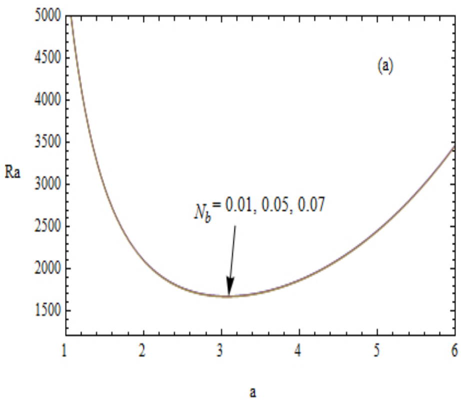


Fig. 2. Validation of the present code with the results of Chandrasekhar (1961)



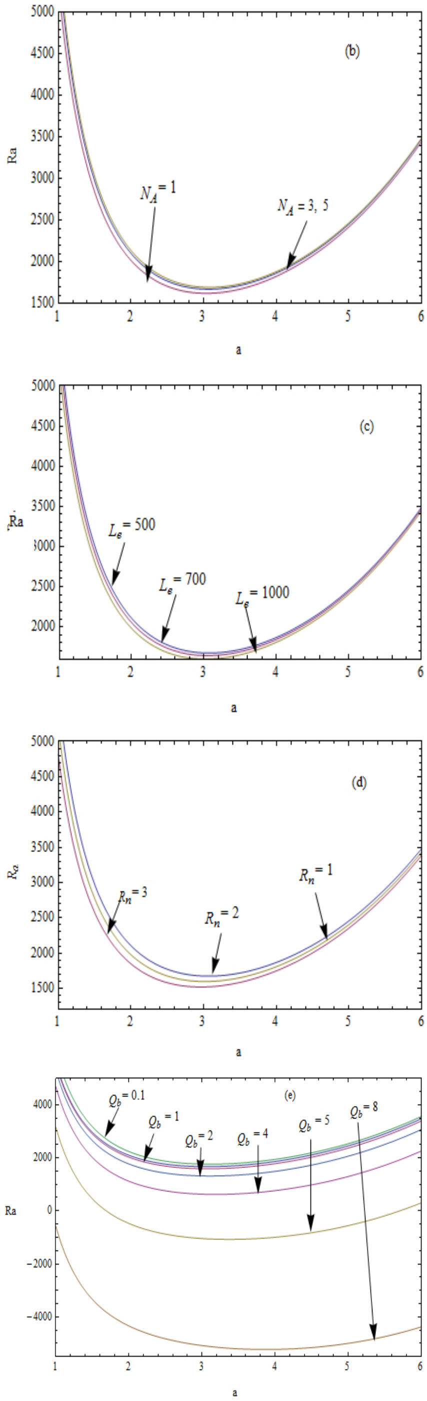


Fig. 3. Variation of thermal Rayleigh number with wave number for different values of (a) N_b, (b) N_A, (c) L_e, (d) R_n, and (e) Q_b.

Figures 3(a)-3(e) show the influence of different physical parameters on thermal Rayleigh number with respect to (a) modified particle density increment, (b) modified diffusivity ratio, (c) nanoparticle Lewis number, (d) nanoparticle Rayleigh number, (e) bioconvection Péclet number. The thermal Rayleigh number attains its minimum value at a = 3.069 in all cases expect the higher value of Q_b. The thermal Rayleigh number is insensitive to the variation of modified particle density increment as shown in Fig. 3 (a). Modified diffusivity ratio destabilized the suspension due to thermophoresis which pushes the lighter nanoparticles upwards and enhances the nanoparticles motion, as shown in Fig. 3 (b). Figure 3(c) shows that R_a decreases with increasing value of L_e. This may be physically interpreted as, the mass diffusivity of the nanofluid, increases the nanoparticle volume fraction and subsequently increase the amount of heat transfer. An increase in a volumetric fraction increases the Brownian motion of nanoparticles which produce a destabilizing effect, as shown in Fig. 3(d). From Fig. 3(e) it is found that a bioconvection Péclet number accelerates the onset of bio-thermal convection.

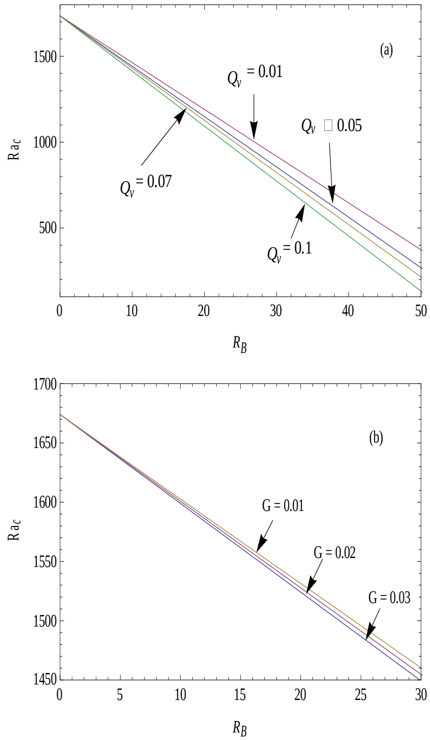


Fig. 4. Variation of thermal Rayleigh number with bioconvection Rayleigh number for different values of (a) Q_v, (b) *G*.

The value of R_a decreases with increase in the bioconvection Rayleigh number (R_b) as shown in Fig. (4). Therefore bioconvection Rayleigh number destabilizes the system. This result is expected from physical point of view also, because an increase in R_b enhances the concentration of gyrotactic microorganisms at the top layer and develops top-heavy density stratification. Vertical upward throughflow increases the upward speed of microorganism which helps to construct the bio-thermal convection pattern. Therefore, Péclet number accelerates the onset of bio-thermal convection for the small amount of throughflow, as shown in Fig. 4(a). From Fig. 4(b) it is noticed that as *G* increases, R_a decreases, it thus facilitates the development of bio-thermal convection.

To see the differences between the active (constant nanoparticle) and passive control (zero flux) of nanoflux on the onset of bio-thermal convection, the values of Ra_c, a_c for different values of non-dimensional parameters for both active and passive boundary conditions are compared in Table 1. It is observed that passive control of nanoflux has a more destabilizing effect than the active control. Higher concentration of microorganisms (higher R_b) implies increasing value of a_c, thus it reduces the size of cells.

Table 1 Numerical values of R_{ac}, a_c for different values of $L_e, L_b, N_A, N_B, R_n, R_b, G, Q_b$ and Q_v for the active and passive nanoflux

N_A	N_B	(Active Nanoflux)		(Passive Nanoflux)	
		$R_{a,c}$	α_c	$R_{a,c}$	α_c
3		1749.82	3.12	1674.52	3.07
5		1749.62	3.12	1674.17	3.07
7		1749.42	3.12	1673.81	3.07
	0.01	1749.62	3.12	1674.17	3.07
	0.05	1748.05	3.12	1669.25	3.07
	0.1	1748.05	3.12	1665.10	3.07
L_e	G				
500		1749.62	3.12	1674.17	3.07
700		1729.20	3.12	1643.51	3.05
1000		1699.62	3.12	1598.73	3.02
	0.01	1749.49	3.12	1673.83	3.07
	0.02	1749.52	3.12	1673.21	3.07
	0.03	1749.62	3.12	1674.17	3.07
Q_v	Q_B				
0.01		1750.70	3.12	1675.92	3.07
0.05		1749.62	3.12	1674.17	3.07
0.07		1749.02	3.12	1673.57	3.07
0.1		1748.05	3.12	1671.57	3.07
	1	1840.47	3.11	1665.94	3.06
	3	1749.62	3.12	1619.17	3.07
	4	1561.34	3.12	1486.07	3.07
R_n	R_b				
0.1		1749.62	3.12	1674.17	3.07
0.2		1647.56	3.12	1596.25	3.07
0.3		1597.06	3.12	1520.09	3.07
	1	1670.10	3.11	1684.71	3.06
	20	1455.64	3.12	1380.80	3.09
	30	1161.61	3.12	1087.38	3.10
	40	867.41	3.13	793.78	3.11

7. CONCLUSIONS

The main conclusions can be drawn as follows:

1. The combined behaviour of Brownian motion, thermophoresis of nanoparticles, vertical throughflow, and gyrotactic microorganisms is shown to have a strong destabilizing effect.
2. The highly promoted disturbance due to the presence of gyrotactic microorganisms enhances heat transfer in a nanofluid.
3. Faster swimmers produce stronger disturbance, it thus facilitates the development of bio-thermal convection resulting in a lower thermal Rayleigh number at a larger value of bioconvection Péclet number.
4. For larger values of R_b, the effect of vertical throughflow and gyrotaxis number are substantial

as compared to smaller values of R_b .

5. In the absence of throughflow, thermal Rayleigh number is insensitive to the variation of modified particle density increment.
6. As the swimming speed of gyrotactic microorganisms increases, the size of convection cells becomes narrower and concentrated at the upper boundary layer.
7. Active control of nanoflux has a less destabilizing effect than the passive control, for all considered range of non-dimensional numbers.

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