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The Influence of Surface Tension on Oblique Wave Scattering by a Rectangular Trench

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ABSTRACT

The diffraction of obliquely incident wave by a symmetric rectangular submarine trench with the effect of surface tension at the free surface is investigated using two dimensional linearized potential theory. The reflection and transmission coefficients are computed numerically using appropriate multiterm Galerkin approximations involving ultraspherical Gegenbauer polynomials. These coefficients are represented graphically against the wave number in a number of figures. The theoretical observations are validated computationally. The derived result will coincide analytically and graphically with the results already present in the literature neglecting the effect of surface tension, which confirms the correctness of the result presented here. We observed the zero reflection phenomenon in the graphical representation. It is also noted that the values of reflection coefficient decreases as the surface tension increases. We conclude that realistic changes in surface tension on the free surface have a significant effect on the present study.

Keywords: Water wave scattering; Galerkin approximation; Surface tension; Submarine trench; Reflection and transmission coefficients.

NOMENCLATURE

a	depth of the fluid	τ	coefficient of surface tension
d	depth of the trench	$2l$	width of the trench
g	acceleration due to gravity	θ	incident angle
i, j, n	space index	ω	angular frequency
M	surface tension	$\phi(x, y)$	potential function
N	truncation size	λ_1	incident wave number
R	reflection coefficient	$\phi_1(x, y)$	symmetric part of potential function
T	transmission coefficient	ρ	density of fluid
$\phi_2(x, y)$	antisymmetric part of potential function		

1. INTRODUCTION

For over half a century, there has been considerable interest of the problem of the diffraction of obliquely incident wave by obstacles of various geometrical shapes of infinite depth water and finite depth water in the literature. The study of scattering of water wave over various depth geometries has been investigated using different method for a long time (Kreisel (1949), Mei and Black (1969), Lassiter (1972), Lee and Ayer (1981), Miles (1982), Kirby and Dalrymple (1983)). Recently, problem of obliquely incidence water wave scattering by rectangular trench is investigated by Chakraborty and Mandal (2015) applying the multiterm Galerkin approximations involving ultraspherical Gegenbauer polynomials for solving the integral equations arising in the mathematical analysis.

None of the results documented above accounts for the effect of surface tension. Very few attempts have been made to include the effect of surface tension in water wave problems involving obstacles in the free surface. But the problems of water wave scattering by obstacles in presence of surface tension are investigated by some of the researchers such as Evans (1968a), Evans (1968b), Rhodes-Robinson (1970), Rhodes-Robinson (1971), Rhodes-Robinson (1982), Chakrabarti and Sahoo (1998). Indeed, to the authour's knowledge no previous investigation has been made on the existence of water wave scattering by rectangular submarine trench in presence of surface tension. The amplitude and the frequency of the wave depend on both the surface tension and gravity. For this reason it may not be possible to neglect the effect of surface tension while doing experimental study. As mentioned by Hocking and Mahdmina (1991), another important reason for including surface tension is that in the absence of surface tension the transient motion initiated by an impulsive start is singular, but when the effect of surface tension is taken into account this singularity is removed. The uniqueness of the solution of the problem depends on the behavior of a special combination of the derivatives of the velocity potential at the edge because of the effect of surface tension as mentioned by Chakrabarti and Sahoo (1998). This is also an important reason for including surface tension.

Our aim in the present study is to enhance understanding of whether it is physically realistic to include the effect of surface tension of the problem of oblique wave scattering by rectangular trench of finite depth. Problem is split into two separate problems involving the symmetric and antisymmetric potential functions describing the resultant motion in the fluid region because of the geometrical symmetry of the rectangular trench as was done by Kanoria *et al*. (1999). To solve the problem the potential function is reduced to linear integral equations using the eigenfunction expansions along with the Havelock inversion formula followed by a matching process. Using the multiterm Galerkin approximations the integral equations are approximated involving ultraspherical Gegenbauer polynomials. We have shown the effect of surface tension of the problem of normal incident wave as a special case. It has been seen that when surface tension effect is excluded the result is analytically equal to the result done by Chakraborty and Mandal (2015) and Chakraborty and Mandal (2014). The transmission coefficient is represented graphically against wave number neglecting effect of surface tension for oblique incident wave in a number of figure which are coincide with the results obtained by Kirby and Dalrymple (1983). For normal incidence wave, the numerical estimate of reflection coefficient is depicted graphically against wave number neglecting effect of surface tension which agree with the result obtained by Lee and Ayer (1981). Numerical estimates for the reflection coefficient are obtained for various values of different parameters involved in the problem. We have seen that surface tension affects the reflection coefficient significantly. It is also found that in presence of surface tension width of the trench affects the reflection and transmission coefficient significantly. The zero reflection phenomenon and the multiple reflections are also observed here.

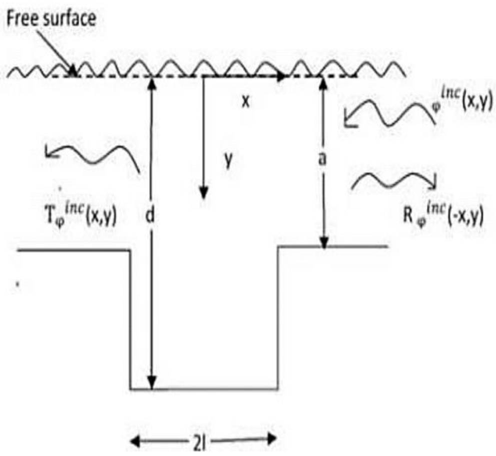

Fig. 1. Definition diagram of the trench.

2. FORMULATION OF THE PROBLEM

Under the assumption of irrotational motion of a homogeneous, inviscid and incompressible fluid, the problem is studied in 3D cartesian coordinate system in which y axis is taken vertically downwards along the line of symmetry of the rectangular trench of width $2l$. The depth of the trench from the mean free surface is d and the bottom of an ocean of uniform finite depth a (see Fig. 1). The fluid occupies the region $-\infty < x, z < \infty$, except for the trench in the fluid region. Assuming linear theory, a train of progressive waves represented By $\Re\{\frac{2\cosh\alpha_0(a-y)}{\cosh\alpha_0 a} e^{i\{\alpha_0(1-x)\cos\theta+\alpha_0 z\sin\theta-\omega t\}}\}$ is obliquely incident from very large distance on the right side of the trench with θ $(0 \le \theta \le \frac{\pi}{2})$ being the angle of incident of the wave with positive x axis and ω the angular frequency of the wave. The wave number α_0 $(\alpha_0 = 2\pi / \lambda_1)$ satisfies the dispersion equation $\alpha(1+M\alpha^2)\tanh\alpha a = K$, where $K = \frac{\omega^2}{g}$ with g is acceleration due to gravity and $M = \frac{\tau}{\rho g}$, τ is the coefficient of surface tension at the free surface of the ocean, ρ is the density of fluid and λ_1 λ1 the wavelength. Due to the geometrical symmetry of the problem, the z dependence can be eliminated by assuming the velocity potential to be of the form $\Re\{\phi(x,y)e^{i\{\alpha_0 z\sin\theta-\omega t\}}\}$. Then $\phi(x,y)$ satisfies the following boundary value problem

$$(\nabla^2 - \nu^2)\phi = 0 \text{ in the fluid region} \tag{1}$$

Where $\nu = \alpha_0 \sin\theta$

$$K\phi + \frac{\partial\phi}{\partial y} + M\frac{\partial^3\phi}{\partial y^3} = 0 \quad on \quad y = 0, \tag{2}$$

$$\frac{\partial\phi}{\partial x} = 0 \quad on\, x = \pm l, \quad y \in (a,d), \quad (d > a) \tag{3}$$

$$\frac{\partial\phi}{\partial y}=0 \quad on \;\; x=a, \quad |x|>l \tag{4}$$

$$\frac{\partial\phi}{\partial y}=0, \;\; on \;\; x=d, \quad |x|<l \tag{5}$$

$$r^{1/3}\nabla\phi \;\; \text{is bounded as} \;\; r\to 0 \tag{6}$$

where r is the distance from a submerged edge of the trench

$$\phi(x,y)\sim\begin{cases}\frac{\cosh\alpha_0(a-y)}{\cosh\alpha_0 a}\{e^{-i\mu(x-l)}+\mathrm{R}e^{i\mu(x-l)}\}, \\ \qquad as\, x\to\infty \\ \frac{\cosh\alpha_0(a-y)}{\cosh\alpha_0 a}Te^{-i\mu(x-l)}, \;\; as\;\; x\to-\infty\end{cases} \tag{7}$$

where R and T are unknown reflection and transmission coefficients to be determined, respectively; and $\mu=\alpha_0\cos\theta$.

3. SOLUTION PROCEDURE

To solve the problem we split the velocity potential function $\phi(x,y)$ into a symmetric and antisymmetric parts $\phi_1(x,y)$ and $\phi_2(x,y)$ respectively due to the geometrical symmetry of the trench about y axis. Thus

$$\phi(x,y)=\phi_1(x,y)+\phi_2(x,y) \tag{8}$$

Where

$$\phi_1(-x,y)=\phi_1(x,y), \;\; \phi_2(-x,y)=-\phi_2(x,y) \tag{9}$$

We may analyze only the region x ≥ 0. Now $\phi_1(x,y)$ and $\phi_2(x,y)$ satisfy Eqs. (1) to (6) together with

$$\frac{\partial\phi_1(0,y)}{\partial x}=\phi_2(o,y)=0, \quad 0<y<d \tag{10}$$

Let for large x the behavior of $\phi_{1,2}(x,y)$ be represented by

$$\phi_{1,2}(x,y)\sim\frac{\cosh\alpha_0(a-y)}{\cosh\alpha_0 a}\{e^{-i\mu(x-l)}+R_{1,2}e^{i\mu(x-l)}\} \quad as\;\; x\to\infty \tag{11}$$

where R_1 and R_2 unknown constants which are to be determined by using the Eq. (7). These constants are related to R and T by the equations

$$R=\frac{(R_1+R_2)}{2}e^{-2i\mu l}, \;\; T=\frac{(R_1-R_2)}{2}e^{-2i\mu l} \tag{12}$$

Now the eigenfunction expansions of $\phi_{1,2}(x,y)$ satisfying Eqs. (1).− (3), (5), (6), (10) and (11) are given below.

$$\phi_1(x,y)=\begin{cases}\frac{\cosh\alpha_0(a-y)}{\cosh\alpha_0 a} \\ \times\{e^{-i\mu(x-l)}+R_1e^{i\mu(x-l)}\} \\ +\sum_{n=1,}^{\infty}A_n\cos\alpha_n(a-y)e^{-p_n(x-l)} \\ \quad for\,(x>l, 0<y<a) \\ \frac{\cosh\beta_0(d-y)}{\cosh\beta_0 d}B_0\cos(sx) \\ +\sum_{n=1'}^{\infty}B_n\cosh(t_n x)\cos\beta_n(d-y) \\ \quad for\,(0<x<l, 0<y<d)\end{cases} \tag{13}$$

$$\phi_2(x,y)=\begin{cases}\frac{\cosh\alpha_0(a-y)}{\cosh\alpha_0 a} \\ \times\{e^{-i\mu(x-l)}+R_2e^{i\mu(x-l)}\} \\ +\sum_{n=1,}^{\infty}C_n\cos\alpha_n(a-y)e^{-p_n(x-l)} \\ \quad for\,(x>l, 0<y<a) \\ \frac{\cosh\beta_0(d-y)}{\cosh\beta_0 d}D_0\cos(sx) \\ +\sum_{n=1}^{\infty}D_n\cosh(t_n x)\cos\beta_n(d-y), \\ \quad for\,(0<x<l, 0<y<d)\end{cases} \tag{14}$$

Where

$p_n=\sqrt{\alpha_n^2+\nu^2}, \;\; s=\sqrt{\beta_n^2-\nu^2}, \;\; t_n=\sqrt{\beta_n^2+\nu^2},$

$\alpha_n, \;\; \beta_n \;\; (n=1,2,........)$ are the real positive roots of the dispersion equation $x(1-Mx^2)\tan(xa)=-K$ and β_0 is the root of the dispersion equation $\beta(1+M\beta^2)\tanh(\beta d)=K$.

Now we define a function as follows

$$\frac{\partial\phi_{1,2}(l+0,y)}{\partial x}=f_{1,2}(y), \;\; 0<y<a \tag{15}$$

Using the matching conditions $\frac{\partial\phi_{1,2}(l+0,y)}{\partial x}=\frac{\partial\phi_{1,2}(l+0,y)}{\partial x}, \;\; 0<y<a$ in Eq. (15) we get

$$\frac{\partial\phi_{1,2}(l-0,y)}{\partial x}=\begin{cases}f_{1,2}(y), & 0<y<a \\ 0, & a<y<d\end{cases} \tag{16}$$

Also, due to the edge condition described by Eq. (6), we find that

$$f_{1,2}(y)=O(|y-a|^{-1/3}), \;\; as \;\; y\to a \tag{17}$$

Using the expansion (13) for $\phi_{1,2}(x,y)$ in Eq. (15) followed by Havelock's inversion formula, we obtain

$$1-R_{1,2}=\frac{4i\,\alpha_0\cosh(\alpha_0 a)}{\gamma_0\mu}\int_0^a f_{1,2}(y)\cosh\alpha_0(a-y)\,dy \tag{18}$$

With

$$\gamma_0=\frac{2\alpha_0 a(1+M\,\alpha_0^2)+(1+3M\,\alpha_0^2)\sinh(2\alpha_0 a)}{(1+M\,\alpha_0^2)}$$

And

$$A_n=\frac{4\alpha_n}{\gamma_n p_n}\int_0^a f_1(y)\cos\alpha_n(a-y)\,dy, \tag{19}$$

$$C_n=\frac{4\alpha_n}{\gamma_n p_n}\int_0^a f_2(y)\cos\alpha_n(a-y)\,dy, \tag{20}$$

With

$$\gamma_n=\frac{2\alpha_n a(1+M\,\alpha_n^2)+(1+3M\,\alpha_n^2)\sinh(2\alpha_n a)}{(1+M\,\alpha_n^2)}$$

Using the expansion (14) for $\phi_{1,2}(x,y)$ in Eq. (16) followed by Havelock's inversion formula, we obtain

$$(B_0,D_0)=\frac{4\beta_0\cosh(\beta_0 d)}{\delta_0 s\,(-\sin(sl),\cos(sl))}\times\int_0^a f_{1,2}(y)\cosh\beta_0(d-y)\,dy, \tag{21}$$

With

$$\delta_0=\frac{2\beta_0 d\,(1+M\,\beta_0^2)+(1+3M\,\beta_0^2)\sinh(2\beta_0 d)}{(1+M\,\beta_0^2)}$$

And

$$(B_n,D_n)=\frac{4\beta_n}{\delta_n t\;(\sinh(t_n l),\cosh(t_n l))}\times\int_0^a f_{1,2}(y)\cos\beta_n(d-y)\,dy, \tag{22}$$

with

$$\delta_n=\frac{2\beta_n d\,(1-M\,\beta_n^2)+(1-3M\,\beta_n^2)\sin(2\beta_n d)}{(1-M\,\beta_n^2)}$$

Now matching of $\phi_{1,2}(x,y)$ across the line $x=l$ through the right corner points of the gap, gives the relation

$$\phi_{1,2}(l+0,y)=\phi_{1,2}(l-0,y),\ 0<y<a \tag{23}$$

which ultimately produce the integral equations

$$\int_0^a g_{1,2}(u)F_{1,2}(y,u)\,du=\frac{\cosh\alpha_0(a-y)}{\cosh\alpha_0 a},\quad 0<y<a \tag{24}$$

Where

$$g_{1,2}(y)=\frac{4\alpha_0\cosh^2(\alpha_0 a)}{\gamma_0\mu(1+R_{1,2})}f_{1,2}(y),\ 0<y<a \tag{25}$$

And

$$F_{1,2}(y,u),\ (0<y,u<a) \tag{26}$$

The function $F_{1,2}(y,u),\ (0<y,u<a)$ is real and symmetric in y and u.

We define the constants k1 and k2 by

$$k_1=-i\,\frac{1-R_1}{1+R_1},\ k_2=-i\,\frac{1-R_2}{1+R_2} \tag{27}$$

Now using relations (18) and (24), we find that

$$\int_0^a g_{1,2}(y)\frac{\cosh\alpha_0(a-y)}{\cosh\alpha_0 a}dy=k_{1,2} \tag{28}$$

It is important to note that $g_{1,2}(y)$ and k_1 , k_2 are real quantities. To evaluate k_1 , k_2 we can used the solution of the integral equation in the relation (24). Then using the value of k_1 , k_2 we produce the actual reflection and transmission coefficients |R| and |T |respectively, as follows

$$|R|=\frac{|1+k_1k_2|}{\sqrt{1+k_1^{\,2}+k_2^{\,2}+(k_1k_2)^2}}$$
$$and\ |T|=\frac{|k_1-k_2|}{\sqrt{1+k_1^{\,2}+k_2^{\,2}+(k_1k_2)^2}} \tag{29}$$

which are obtained from Eqs. (27) and (12). Now from the Eq. (29), we can easily show the energy identity as follows

$$|R|^2+|T|^2=1$$

4. MULTI-TERM GALERKIN APPROXIMATION

In this section, we solve the integral Eq. in (24) using Multi-Term Galerkin Approximation method. The unknown functions $g_{1,2}(y)$ are approximated as $g_{1,2}(y)\approx F_{1,2}(y)$. An appropriate multi-term form of the functions $F_1(y)$ and $F_2(y)$ are chosen in terms of suitable basis functions as follows

$$F_1(y)=\sum_{i=0}^N c_i b_i(y),\ F_2(y)=\sum_{i=0}^N e_i h_i(y),\quad 0<y<a \tag{30}$$

with c_i and e_i are unknown constants. Since the horizontal velocity of the fluid near the corner points $(\pm l,a)$ of the trench has a cube root of singularity, as mention by Chakraborty and Mandal (2014), the basis functions $b_i(y)$ and $h_i(y)$ can be found as

$$b_i(y)=-\frac{d}{dy}\left[e^{-Ky}\int_y^a e^{Kt}b_i(t)\,dt\right]\quad 0<y<a \tag{31}$$

$$h_i(y) = -\frac{d}{dy}\left[e^{-Ky}\int_y^a e^{Kt} b_i(t)dt\right] \quad 0 < y < a \tag{32}$$

where $\hat{b}_i(t)$ is chosen in terms of ultraspherical Gegenbauer polynomials of order 1/6 as

$$b_i(t) = \frac{2^{7/6}\Gamma(\frac{1}{6})(2i)!}{\pi\Gamma(2i+\frac{1}{3})a^{1/3}(a^2-y^2)^{1/3}} C_{2i}^{1/6}(\tfrac{y}{a}), \quad 0 < y < a \tag{33}$$

Now we substitute the expansion (33) in the Eq. (32) and putting in the Eq. (31), we get the approximate forms of $F_1(y)$ and $F_2(y)$. Multiplying both side of the Eq. (24) by $b_i(y)$ and $h_i(y)$, using the approximation form of $F_1(y)$ and $F_2(y)$ and integrating over $(0 < y < a)$, we determined the linear systems

$$\sum_{i=0}^{N} c_i \mathcal{K}_{ij} = \mathcal{H}_i, \ \sum_{i=0}^{N} e_i \mathcal{G}_{ij} = \mathcal{L}_i, \ j = 0,1,2,........,N \tag{34}$$

Where

$$\mathcal{K}_{ij} = \int_0^a \int_a^a \mathcal{F}_1(y,u) b_i(u) b_j(y) du dy \quad i,j = 0,1,2,........,N \tag{35}$$

$$\mathcal{G}_{ij} = \int_0^a \int_a^a \mathcal{F}_2(y,u) h_i(u) h_j(y) du dy \quad i,j = 0,1,2,........,N \tag{36}$$

and

$$\mathcal{H}_i = \int_0^a \frac{\cosh\alpha_0(a-y)}{\cosh\alpha_0 a} b_i(y) dy, i = 0,1,2,........,N \tag{37}$$

$$\mathcal{L}_i = \int_0^a \frac{\cosh\alpha_0(a-y)}{\cosh\alpha_0 a} h_i(y) dy, i = 0,1,2,........,N \tag{38}$$

Using the different properties and standard results on Gegenbauer polynomials, integrating Eqs. (35), (36), (37) and (38) we can evaluated explicitly, as done by Kanoria *et al*. (1999). Thus we obtained

$$\begin{aligned}(\mathcal{K}_{ij}, \mathcal{G}_{ij}) = &\frac{\gamma_0 \mu}{\alpha_0 \cosh^2(\alpha_0 a)} \times [4(-1)^{i+j} \sum_{r=1}^{\infty} \\ &\times \{\frac{\alpha_r \cos^2(\alpha_r a)}{\gamma_r p_r (\alpha_r a)^{1/3}} J_{2i+1/6}(\alpha_r a) J_{2i+1/6}(\alpha_r a) \\ &+ \frac{\beta_r(\coth(t_r l), \tanh(t_r l))}{\delta_r t_r (\beta_r a)^{1/3}} \\ &\times \cos^2(\beta_r d) J_{2i+1/6}(\beta_r a) J_{2i+1/6}(\beta_r a)\} \\ &+ \frac{\beta_0(-\cot(sl), \tan(sl))}{\delta_0 s(\beta_0 a)^{1/3}} \\ &\times \cosh^2(\beta_0 d) I_{2i+1/6}(\beta_0 a) I_{2i+1/6}(\beta_0 a)], \\ &i,j = 0,1,2,........,N\end{aligned} \tag{39}$$

and

$$\mathcal{L}_i = \mathcal{H}_i = \frac{I_{2i+1/6}(\alpha_0 a)}{(\alpha_0 a)^{1/6}}, \ i = 0,1,2,.....,N \tag{40}$$

Now the constants $c_i, e_i \ (i = 0,1,2,.....,N)$ are deter-mined by solving the linear systems (34), and then relation (28) produce

$$k_1 = \sum_{i=0}^{N} c_i \mathcal{H}_i, \ k_2 = \sum_{i=0}^{N} e_i \mathcal{G}_i \tag{41}$$

Now using the values of k_1 and k_2 we evaluated the reflection and transmission coefficient from the relation (29).

5. NUMERICAL RESULTS AND DISCUSSION

In this section, to construct a solution, it is first necessary to truncate infinite series in the equation (34) involving $\mathcal{K}_{ij}, \mathcal{G}_{ij}$ to a finite number of terms given by N. We have seen that accuracy of the solution depends on the number of terms. It is observed that the accuracy can be further increased by taking more terms in the series in (39) as mentioned by Chakraborty and Mandal (2014). It is also seen that the computed results for reflection coefficients in presence of surface tension converges very rapidly with N. The reflection coefficients are computed numerically for various values of different dimensionless parameters and they are presented graphically.

In Fig. 2 the reflection coefficients are plotted against wave number with fixed value of $d/a = 2, \ l/a = 5, M/a^2 = 1$ for different values of angle $(\theta = 30^o, \ 45^o, 60^o, 89^o)$. It is observed that the peak value of reflection coefficient increases as the angle of incident increases. Also we have seen that number of zeros of reflection coefficient increases as the angle of incident decreases. From the figure, it is clear that present result satisfy the geometry of the problem. It is also noted that when value of incident angle tends to 90^o reflection coefficient be-comes almost unity.

Table 1 Values of $|R|$, $|T|$ and $|R|^2+|T|^2$ for fixed values of $M/a^2=1.5, \theta=45^o, l/a=4, d/a=2$

$a\alpha_0$	$\|R\|$	$\|T\|$	$\|R\|^2+\|T\|^2$
0.1	0.137817	0.990458	1.0000
0.3	0.301439	0.953485	1.0000
0.5	0.243715	0.969847	1.0000
0.7	0.0514609	0.998675	1.0000
0.9	0.0420428	0.999116	1.0000

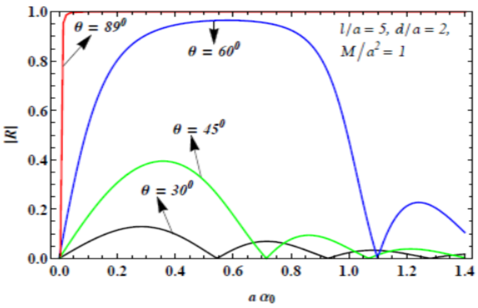

Fig. 2. Reflection coefficient against $a\alpha_0$ for different values of angles.

The effects of surface tension on reflection and transmission coefficients are shown in Fig. 3 and Fig. 4, respectively . In these figures the graphs are plotted for three different values of $M/a^2\,(=1,1.5,\ 2)$ and for $\theta=45^o, l/a=4, d/a=2$. From Fig. 3, it is clear that the peak value of reflection coefficients decreases as the surface tension increases. Its happens because of the fact that the cohesive force between fluid molecules at the free surface. The inter molecular force increases with the surface tension. Due to this fact the free surface of the fluid becomes stretched. Thus in presence of surface tension, more energy will be transmitted and less energy will be reflected. From Fig. 4, it is observed that for large wave number, transmission coefficient is almost equal to unity. The waves of short wavelength are confined near the free surface and these do not feel the existence of submerged trench and as such these propagate without any hindrance. The results using the present method are validated through the calculated values of $|R|$, $|T|$ and $|R|^2+|T|^2$ and given in Table 1. This table shows that in presence of surface tension the value of $|R|^2+|T|^2$ is exactly equal to unity. This implies that the energy identity is always satisfied numerically. Thus, the present procedure for the numerical computation of $|R|$ (and $|T|$) is quite efficient.

The reflection coefficients are plotted against wave number in Fig. 5 with fixed value of $d/a=2, \theta=$

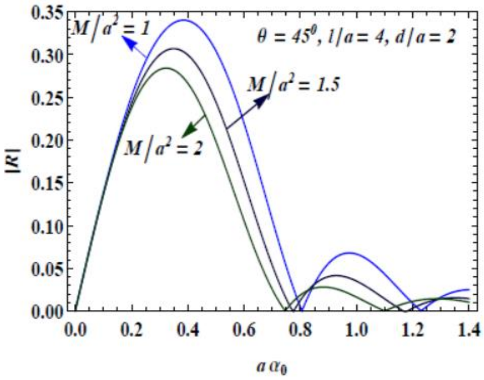

Fig. 3. Reflection coefficients against $a\alpha_0$ for different values of surface tension.

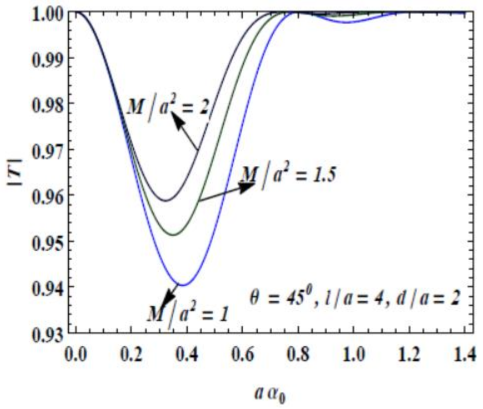

Fig. 4. Transmission coefficients against $a\alpha_0$ for different values of surface tension.

$45^o, M/a^2=1$ for different values of $l/a\,(=2,3,5)$. It is observed that the peak value of reflection coefficient increases as the width of the trench increases. Also we have seen that number of zeros of reflection coefficient increases as the width of the trench increases.

The graphs depicted in Fig. 6 are the reflection coefficients against wave number with fixed value of $l/a=4, \theta=45^o, M/a^2=1$ for different values of depth of the trench $d/a\,(=2,2.5,3)$. It is seen that the peak value of reflection coefficient increases as the depth of the trench increases.

Results for normal incidence of surface waves on a rectangular trench are calculated and analyzed numerically in Figs. 7-9 as a function of $a\alpha_0$. Mathematically, this can be achieved by considering $\theta=0^o$ in the problem. In Fig. 7 $|R|$ are depicted for $M/a^2=0.1, 0.5, 0.9$ with $\theta=0^o, d/a=2, l/a=5$. It is observed from the figure that the reflection coefficients $|R|$ decreases as the surface tension increases.

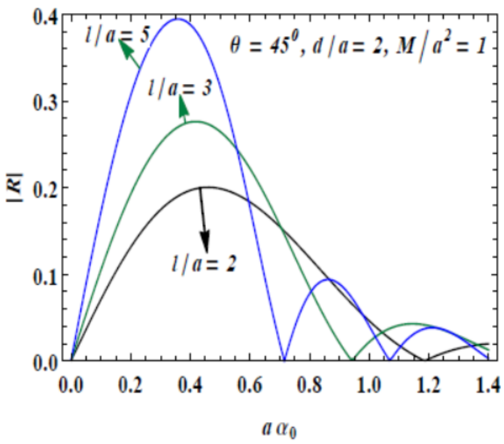

Fig. 5. Reflection coefficients against $a\alpha_0$ for different values of width of the trench.

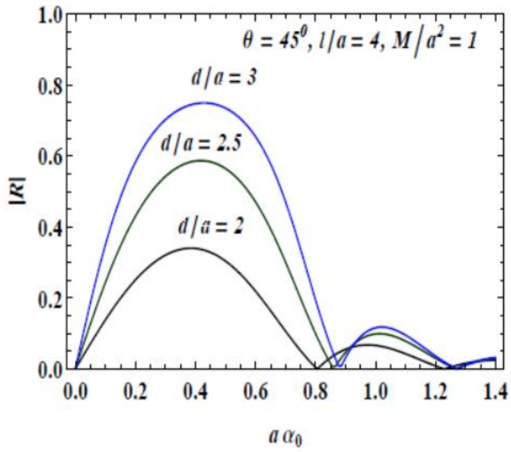

Fig. 6. Reflection coefficient against $a\alpha_0$ for different values of depth of the trench.

The reflection coefficients $|R|$ are plotted against the wave number for different values of depth of trench in Fig. 8. In this figure we consider $\theta = 0^o,\ M/a^2 = 1,\ l/a = 5$ and for $d/a = 2, 2.25, 2.5$. It is observed that the reflection coefficients $|R|$ increases as the depth of the trench increases.

The graphs for $|R|$ are plotted against the wave number aα0 in Fig. 9 for three different values of $l/a (= 2,3,5)$ with $\theta = 0^o,\ M/a^2 = 1,,\ d/a = 2$. It is clear that the pick values of $|R|$ are decreases as the width of the trench decreases.

All these figures show the existence of zeros of reflection coefficient for discrete values of wavenumber. This occurs when the two sides of the trench are equal and the number of zeros increases with the depth and width of the trench increases. Zero re-flection phenomenon is also observed by Xie *et al*. (2011) and Liu *et al*. (2013) for symmetric trench. It is also noted that the curve for reflection and transmission coefficients are oscillatory in nature. The oscillatory nature is attributed to multiple re-

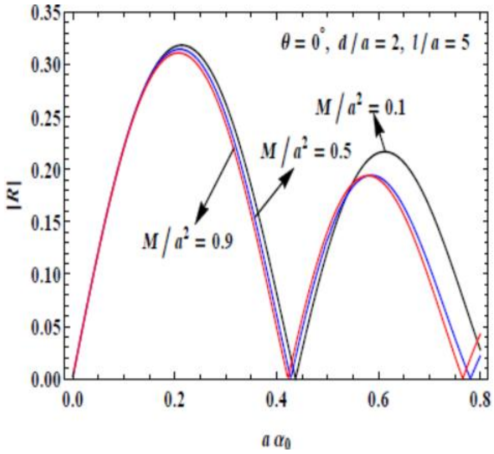

Fig. 7. Reflection coefficients versus $a\alpha_0$ for different values of surface tension.

flections of the incident wave train by two sides of the symmetric trench.

5.1 Accuracy of the Results

In this section we consider the motion of the fluid be under the action of gravity only. So we take $M = 0$ in Eq. (2). The corresponding problem matches exactly with the one considered by Chakraborty and Mandal (2015) and Lee and Ayer (1981). A numerical study of the results in absence of surface tension have been done in Figs. 10-13 and compared with some previously published works in the literature. We plotted transmission coefficients versus $a\alpha_0$ in the Figs. 10-12 for the different values of non-dimensional parameters and angle of incidence, such as taken by Kirby and Dalrymple (1983). It is observed that the Fig.10 from present study almost coincide with the Fig. (4) plotted by Kirby and Dalrymple (1983). In Fig. 11 and Fig. 12 transmission coefficients are plotted for two different depth of the trench at defferent angles, namely, $\theta = 0^o$ and $\theta = 45^o$ ignoring the surface tension.

These graphs matches closely with the Fig. (4) and Fig. 5 by Kirby and Dalrymple (1983).

For normally incident waves (i.e. when $\theta = 0^o$) and neglecting the effect of surface tension (i.e. when $M = 0$), the numerical values of $|R|$ are calculated. Corresponding graph is depicted as a function of incident wave number $\frac{a\alpha_0}{2\pi}$ for $M/a^2 = 0,\ d/a = 2,\ l/a = 2.5,\ \theta = 0^o$ in Fig. 13. From this figure, it is clear that present results agree closely with the result plotted in Fig. 2 by Lee and Ayer (1981) and Fig. 4 by Chakraborty and Mandal (2015).

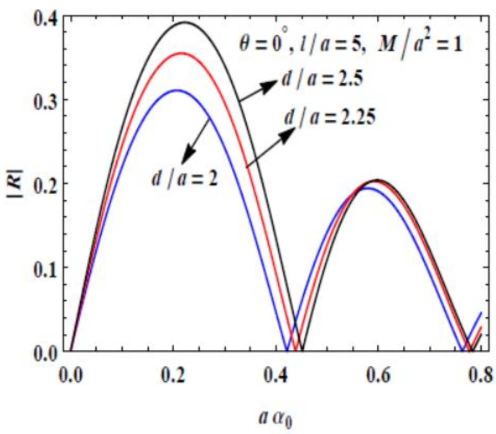

Fig. 8. Reflection coefficients versus $a\alpha_0$ for different values of trench depth.

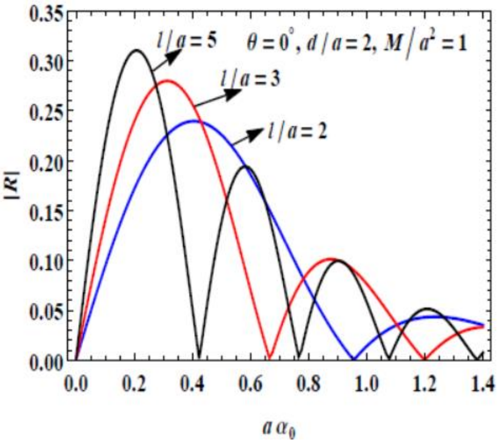

Fig. 9. Reflection coefficients versus $a\alpha_0$ for different values of trench width.

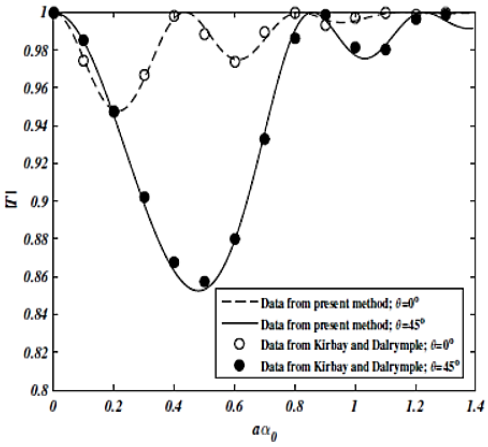

Fig. 10. Transmission coefficient for $M/a^2 = 0$ with fixed values of $d/a = 2, l/a = 5$ and different values of q $\theta = 0^o$ and $\theta = 45^o$.

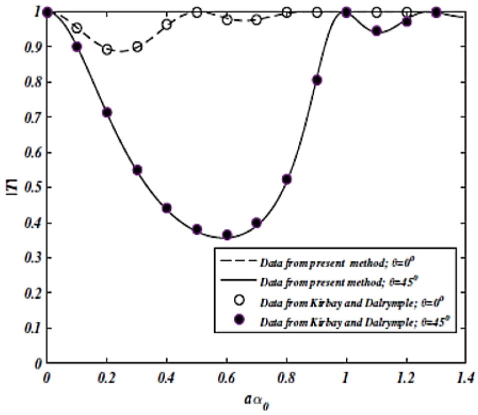

Fig. 11. Transmission coefficient for $M/a^2 = 0$ with fixed values of $d/a = 3, l/a = 5$ and different values of $\theta = 0^o$ and $\theta = 45^o$.

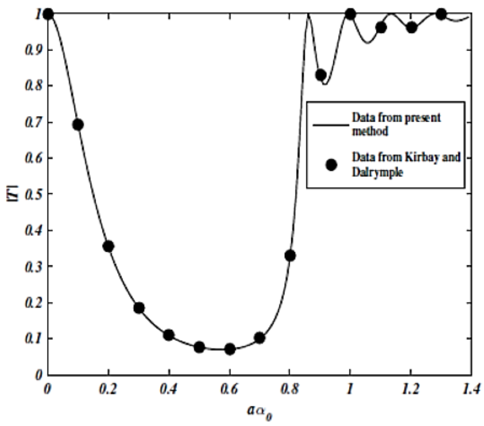

Fig. 12. Transmission coefficient for $M/a^2 = 0$ with fixed values of $d/a = 3, l/a = 10$ and different values of $\theta = 45^o$.

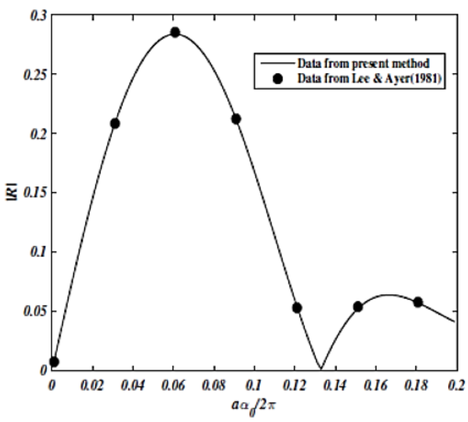

Fig. 13. Reflection coefficient against $\frac{a\alpha_0}{2\pi}$ for $\theta = 0^o$, $M/a^2 = 0, d/a = 2, l/a = 2.5$.

6. CONCLUSION

In the present study, the scattering of oblique incident waves by a rectangular symmetric trench in presence of surface tension at the free surface is investigated by employing multiterm Galerkin approximation method. The numerical results are illustrated graphically. It is seen that the derived result will coincide analytically and graphically with the results already present in the literature. From the computational results it is clear that for some fixed wave number the peak value of reflection coefficient decreases when the value of surface tension increases. In presence of surface tension the length and depth of the trench play an important role to the scattering behavior of the surface waves by a rectangular trench. The zero reflection and multiple reflections phenomenon are also observed here. It is also noted that the values of reflection are negligible for small wave number. This is due to fact that for long wave, corresponding to smaller wavenumbers, the potential behaves like a uniform horizontal flow far from the trench and free surface is stretched with the effect of surface tension, so more wave energy transmitted.

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