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Linear Instability of Throughflow in a Rectangular Box Saturated by Nanofluid

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ABSTRACT

An analysis is made for the effect of throughflow on the onset of convection in a rectangular box under the assumption that total flux (sum of diffusive, thermophoretic, and convective) is zero on the boundaries. A linear stability analysis and Galerkin weighted residual method are used to obtain the Rayleigh number and stability curves for the onset of convection. Three dominating combination of parameters are extracted from the non-dimensional analysis. All rescaled parameters promote the convection. Aspect ratios, throughflow, and nanoparticles play an important role in the formulation of cell distribution and development of convection. Oscillatory convection is possible for permissible range of nanofluid parameters. It is also found that the size of a cellular mode is altered by throughflow and nanoparticles.

Keywords: Aspect ratio; Convective nanoparticle flux; Lateral walls; Rectangular box.

1. INTRODUCTION

Beck (1972) was the first who examined the instability in rectangular box using linear stability and energy method and found that lateral walls have less influence on Rayleigh number. Tewari and Torrance (1981) studied the thermal instability in a rectangular box when the top of the box is permeable. Assuming that the aspect ratio is one; Yamaguchi *et al*. (1999) explored the effect of the magnetic field in the rectangular box. Later, Wang (1999) proposed the model for thermo-convective instability when the bottom of the box is heated by constant flux. In the same context, Davis (2006) investigated the linear stability theory of three-dimensional rectangular box. Very recently, Saini and Sharma (2018a) examined the thermal instability of rectangular box saturated by a nanofluid.

Choi (1995) defined a new class of fluid which consistent with nano-sized particles, known as a nanofluid. Nanofluid has many applications such as cooling, microchannel heat sinks, microheat pipes, microreactors, polymer coatings, process industries, aerospace tribology, biomedical such as cancer therapy, microfluid delivery devices etc. Incorporating the effect of thermophoresis and Brownian motion, Buongiorno (2006) developed a mathematical model for nanofluid. Pioneering work on convection has been analyzed by Tzou (2008) and Nield and Kuznetsov (2009, 2010). They observed that nanoparticles enhance the thermal conductivity of the fluid. Nield and Kuznetsov (2014a, 2014b) suggested the more realistic flow on the boundaries. Saini and Sharma (2018b) examined the onset of double diffusive convection using the revised boundary conditions.

The readers are referred to the work of Nield and Kuznetsov (2011a, 2015) to study the throughflow effect on nanofluid. Recently, Saini and Sharma 2018c, 2018d) examined the instability of nanofluid bioconvection with the effect of throughflow.

In this paper, we study the onset of convection in a rectangular box using the modified mass flux conditions (nanoparticle flux is zero on boundaries) with the effect of throughflow. Nield and Kuznetsov (2011b) studied the similar analysis for the regular fluid and they observed that cellular mode is not altered by throughflow. In this article, we found that throughflow and nanoparticles play an important part in the formulation of the cell and the development of convection. It is also found that oscillatory convection is possible which is completely ruled out in previous studies (Nield and Kuznetsov 2014a, 2015). These are the benchmarks of this study. In this article, we derive the Rayleigh number in terms of throughflow and nanofluid parameters. The effects of various physical parameters are graphically presented.

2. MATHEMATICAL FORMULATION

We assume a three-dimensional rectangular box $0 \le z^* \le H, 0 \le y^* \le L_y, 0 \le x^* \le L_x$, and the direction of

Z-axis is taken upward. Here L_y, L_x are the lengths of the domain in the Y- and X- direction, and H is depth of the box. We take temperatures T_h^* and T_c^* at the bottom and the top wall of the box and another walls (side walls) are taken as insulated. Suspension of nanoparticles is assumed to be dilute, stable and do not to agglomerate. Oberbeck–Boussinesq approximations are used.

Following (Buongiorno 2006; Nield and Kuznetsov 2011a, 2014a, 2015), the conservation equations in dimensional form are as

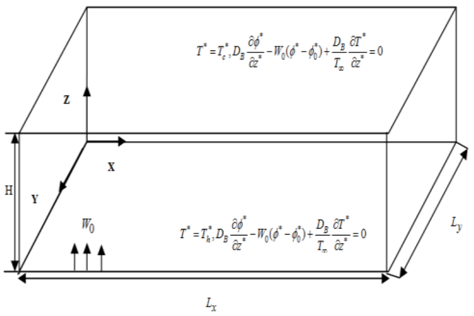

Fig. 1. Geometric configuration of the problem.

$$\nabla^*.\mathbf{V}^*=0 \tag{1}$$

Here $\mathbf{V}^*=(u^*,v^*,w^*)$ is the nanofluid velocity.

$$\mu\mathbf{V}^*=-\nabla^*p^*+\left[\phi^*\rho_p+(1-\phi^*)\left\{\rho_f\,(1-\beta_T\,(T^*-T_c^*))\right\}\right]\mathbf{g} \tag{2}$$

In Eq. (2), μ is the viscosity, ρ_p is the density of nanoparticles, ϕ^* is the nanoparticles volume fraction, p^* is the pressure, ρ_f is the density of the nanofluid, β_T is the volumetric thermal expansion coefficient, $\mathbf{g}$ is the gravity vector, T^* is the nanofluid temperature, and T_c^* is the reference temperature.

$$(\rho c)_f\left(\frac{\partial}{\partial t^*}+\mathbf{V}^*.\nabla^*\right)T^*=k_m\nabla^{*2}\,\mathrm{T}^*+(\rho c)_p\times$$
$$\left[D_B\nabla^*\phi^*.\nabla^*T^*+\frac{D_T}{T_c^*}\nabla^*T^*.\nabla^*T^*-(\phi^*-\phi_0^*)\mathbf{V}^*.\nabla^*T^*\right] \tag{3}$$

Equation (3) is the thermal energy equation. First-term on R.H.S represents the heat transfer by conduction and the expression inside the square bracket represents the total nanoparticle flux. Inside the square bracket, the Ist and IInd terms represent the heat transfer by nanoparticles that have a relative velocity to the base fluid (due to thermophoresis and Brownian motion) and IIIrd term represents the convective contribution of nanoparticle flux.

$$\left(\frac{\partial}{\partial t^*}+\mathbf{V}^*.\nabla^*\right)\phi^*=\frac{D_T}{T_C^*}\nabla^{*2}T^*+D_B\nabla^{*2}\phi^* \tag{4}$$

Equation (4) is the nanoparticle conservation equation that accounts for nanoparticle transport by thermophoresis (Ist term) and Brownian diffusion (IInd term) of R.H.S.

In Eqs. (3) - (4), t^* is the time, $(\rho c)_f$ is the volumetric heat capacity for the nanofluid, D_T is the thermophoresis diffusion coefficient, ϕ_0^* is the reference volume fraction, and D_B is the Brownian diffusion coefficient.

To non-dimensionalise the equations, variables are introduced as

$$u=\frac{u^*L_x}{\alpha_m}, v=\frac{u^*L_y}{\alpha_m} w=\frac{w^*H}{\alpha_m}, x=\frac{x^*}{L_x}, y=\frac{y^*}{L_y}, z=\frac{z^*}{H},$$
$$t=\frac{t^*\alpha_m}{H^2}, p=\frac{p^*H^2}{\mu\alpha_m}, \phi=\frac{\phi^*-\phi_0^*}{\phi_0^*}, T=\frac{T^*-T_c^*}{T_h^*-T_c^*}$$

Equations (1)- (4) take the form as

$$M_x^2\frac{\partial u}{\partial x}+M_y^2\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{5}$$

$$\mathbf{V}=-\nabla p-R_m\hat{\mathbf{k}}+R_aT\hat{\mathbf{k}}-R_n\phi\hat{\mathbf{k}} \tag{6}$$

$$\frac{\partial T}{\partial t}+(1+N_B\phi)\left(M_x^2u\frac{\partial T}{\partial x}+M_y^2v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=M_x^2\frac{\partial^2T}{\partial x^2}+$$
$$M_y^2\frac{\partial^2T}{\partial y^2}+\frac{\partial^2T}{\partial z^2}+\frac{N_B}{L_e}\left(M_x^2\frac{\partial\phi}{\partial x}\frac{\partial T}{\partial x}+M_y^2\frac{\partial\phi}{\partial y}\frac{\partial T}{\partial y}+\frac{\partial\phi}{\partial z}\frac{\partial T}{\partial z}\right)$$
$$+\frac{N_AN_B}{L_e}\left(M_x^2\left(\frac{\partial T}{\partial x}\right)^2+M_y^2\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right) \tag{7}$$

$$\frac{\partial\phi}{\partial t}+M_x^2u\frac{\partial\phi}{\partial x}+M_y^2v\frac{\partial\phi}{\partial y}+w\frac{\partial\phi}{\partial z}=\frac{1}{L_e}\left(M_x^2\frac{\partial^2\phi}{\partial x^2}\right.$$
$$\left.+M_y^2\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)+\frac{N_A}{L_e}\left(M_x^2\frac{\partial^2T}{\partial x^2}+M_y^2\frac{\partial^2T}{\partial y^2}+\frac{\partial^2T}{\partial z^2}\right) \tag{8}$$

Now applying the operator curl on both sides of Eq. (6) and write the resulting equation in component form, we get

$$\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}=R_a\frac{\partial T}{\partial y}-R_n\frac{\partial\phi}{\partial y} \tag{9(a)}$$

$$\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=-R_a\frac{\partial T}{\partial x}+R_n\frac{\partial\phi}{\partial x} \tag{9(b)}$$

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=0 \tag{9(c)}$$

The dimensionless parameters in Eqs. (5)-9(c) namely aspect ratios M_x, M_y, the Rayleigh number R_a, basic density Rayleigh number R_m, Lewis number L_e, nanoparticle Rayleigh number R_n, Péclet number Q, modified particle density increment N_B, and modified diffusivity Ratio N_A are defined as

$$M_x = \frac{H}{L_x}, M_y = \frac{H}{L_y}, R_m = \frac{[\rho_p \phi_0^* + \rho_f (1-\phi_0^*)]\mathbf{g}H^3}{\mu \alpha_m},$$

$$N_B = \frac{(\rho c)_p (\phi_0^*)}{(\rho c)_f}, N_A = \frac{D_T (T_h^* - T_c^*)}{D_B T_0^* \phi_0^*},$$

$$R_n = \frac{\left\{(\rho_P - \rho_f)\phi_0^*\right\}\mathbf{g}H^3}{\mu \alpha_m},$$

$$R_a = \frac{\rho_f \mathbf{g} \beta_T H^3 (T_h^* - T_c^*)}{\mu \alpha_m}, Q = \frac{W_0 H}{\alpha_m}, L_e = \frac{\alpha_m}{D_B}$$

where $\alpha_m = k_m / (\rho c)_f$

In horizontal boundaries, we have taken the temperature to be constant, through flow velocity has uniform value and in addition the nanoparticle flux (thermophoretic, convective, and diffusive) is supposed to be zero. The horizontal boundary conditions are taken as

$$w = Q, T = 1, \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} - QL_e \phi = 0 \text{ at } z = 0 \qquad 10(a)$$

$$w = Q, T = 0, \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} - QL_e \phi = 0 \text{ at } z = 1 \qquad 10(b)$$

Vertical boundaries are assumed to be impermeable and adiabatic. Then we have vertical boundary conditions are

$$u = 0, \frac{\partial T}{\partial x} = 0, \frac{\partial \phi}{\partial x} + N_A \frac{\partial T}{\partial x} - QL_e \phi = 0 \text{ at } x = 0 \text{ and } 1 \qquad (11)$$

$$v = 0, \frac{\partial T}{\partial y} = 0, \frac{\partial \phi}{\partial y} + N_A \frac{\partial T}{\partial y} - QL_e \phi = 0 \text{ at } y = 0 \text{ and } 1 \qquad (12)$$

We seek a time-independent solution of Eqs.(7)-(8) with $u = 0,\ v = 0,\ w = Q$, then Eqs. (7)-(9c) become

$$0 = R_a \frac{\partial T}{\partial y} - R_n \frac{\partial \phi}{\partial y} \qquad 13(a)$$

$$0 = -R_a \frac{\partial T}{\partial x} + R_n \frac{\partial \phi}{\partial x} \qquad 13(b)$$

$$(1 + N_B \phi) Q \frac{\partial T}{\partial z} = M_x^2 \frac{\partial^2 T}{\partial x^2} + M_y^2 \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{N_B}{L_e}\left(M_x^2 \frac{\partial \phi}{\partial x}\frac{\partial T}{\partial x} + M_y^2 \frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y} + \frac{\partial \phi}{\partial z}\frac{\partial T}{\partial z}\right) + \frac{N_A N_B}{L_e}\left(M_x^2 \left(\frac{\partial T}{\partial x}\right)^2 + M_y^2 \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 \right) \qquad (14)$$

$$Q \frac{\partial \phi}{\partial z} = \frac{1}{L_e}\left(M_x^2 \frac{\partial^2 \phi}{\partial x^2} + M_y^2 \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) + \frac{N_A}{L_e}\left(M_x^2 \frac{\partial^2 T}{\partial x^2} + M_y^2 \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \qquad (15)$$

By using Eqs. 13(a)-13(b), Eqs.(14)-(15) permit basic solutions of temperature and nanoparticle concentration of the form $T = T_b(z), \phi = \phi_b(z)$, then Eqs.(14)-(15) become

$$Q \frac{dT_b}{dz} = \frac{d^2 T_b}{dz^2} - N_B Q \phi_b \frac{dT_b}{dz} + \frac{N_B}{L_e}\frac{d\phi_b}{dz}\frac{dT_b}{dz} + \frac{N_A N_B}{L_e}\left(\frac{dT_b}{dz}\right)^2 \qquad (16)$$

$$Q \frac{d\phi_b}{dz} = \frac{1}{L_e}\frac{d^2 \phi_b}{dz^2} + \frac{N_A}{L_e}\frac{d^2 T_b}{dz^2} \qquad (17)$$

Using the boundary conditions 10(a)-10(b), we get

$$T_b = \frac{e^Q - e^{Qz}}{e^Q - 1}, \phi_b = e^{QL_e z} + \frac{N_A e^{Qz}}{(1 - L_e)(e^Q - 1)}$$

3. LINEAR INSTABILITY ANALYSIS

The perturbed state is assumed as $(u, v, w) = (u', v', Q + w'), T = T_b + T', \phi = \phi_b + \phi'$.
Substituting perturbed solution in Eqs. (7)-9(c), we get

$$M_x^2 \frac{\partial u'}{\partial x} + M_y^2 \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \qquad (18)$$

x

$$\frac{\partial T'}{\partial t} + \frac{dT_b}{dz} w' + Q \frac{\partial T'}{\partial z} = M_x^2 \frac{\partial^2 T'}{\partial x^2} + M_y^2 \frac{\partial^2 T'}{\partial y^2} + \frac{\partial^2 T'}{\partial z^2} - N_B \left(Q \frac{dT_b}{dz}\phi' + \phi_b \left(\frac{dT_b}{dz} w' + Q \frac{\partial T'}{\partial z}\right)\right) + \frac{N_B}{L_e}\left(\frac{d\phi_b}{dz}\frac{\partial T'}{\partial z} + \frac{dT_b}{dz}\frac{\partial \phi'}{\partial z}\right) + \frac{2 N_A N_B}{L_e}\frac{dT_b}{dz}\frac{\partial T'}{\partial z} \qquad (19)$$

$$\frac{\partial \phi'}{\partial t} + Q \frac{\partial \phi'}{\partial z} + \frac{d\phi_b}{dz} w' = \frac{1}{L_e}\left(M_x^2 \frac{\partial^2 \phi'}{\partial x^2} + M_y^2 \frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial^2 \phi'}{\partial z^2}\right) + \frac{N_A}{L_e}\left(M_x^2 \frac{\partial^2 T'}{\partial x^2} + M_y^2 \frac{\partial^2 T'}{\partial y^2} + \frac{\partial^2 T'}{\partial z^2}\right) \qquad (20)$$

$$\frac{\partial w'}{\partial y} - \frac{\partial v'}{\partial z} = R_a \frac{\partial T'}{\partial y} - R_n \frac{\partial \phi'}{\partial y} \qquad 21(a)$$

$$\frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x} = -R_a \frac{\partial T'}{\partial x} + R_n \frac{\partial \phi'}{\partial x} \qquad 21(b)$$

$$\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} = 0 \qquad 21(c)$$

Applying the operator $M_y^2 \frac{\partial}{\partial y}$ to the both sides of Eq. 21(a) and $M_y^2 \frac{\partial}{\partial y}$ to the both sides of Eq. 21(b), then subtracting and by using Eq. (18), we get

$$M_x^2\frac{\partial^2 w'}{\partial x^2}+M_y^2\frac{\partial^2 w'}{\partial y^2}+\frac{\partial^2 w'}{\partial z^2}= R_a\left(M_x^2\frac{\partial^2 T'}{\partial x^2}+M_y^2\frac{\partial^2 T'}{\partial y^2}\right) -R_n\left(M_x^2\frac{\partial^2 \phi'}{\partial x^2}+M_y^2\frac{\partial^2 \phi'}{\partial y^2}\right) \quad (22)$$

with

$$w'=0,\ T'=0,\ \frac{\partial\phi'}{\partial z}+N_A\frac{\partial T'}{\partial z}-QL_e\phi'=0 \quad \text{at } z=0 \text{ and } z=1 \quad 23(a)$$

$$u'=0,\ \frac{\partial T'}{\partial x}=0,\ \frac{\partial\phi'}{\partial x}+N_A\frac{\partial T'}{\partial x}-QL_e\phi'=0 \quad \text{at } x=0 \text{ and } 1 \quad 23(b)$$

$$v'=0,\ \frac{\partial T'}{\partial y}=0,\ N_A\frac{\partial T'}{\partial y}+\frac{\partial\phi'}{\partial y}-Q_vL_e\phi'=0 \quad \text{at } y=0 \text{ and } 1 \quad 23(c)$$

Let us assume solutions of Eqs. (19)-(20),(22) are of the form

$$\left.\begin{aligned} u' &= \sin m\pi x \cos n\pi y\ U(z)e^{st},\\ v' &= \cos m\pi x \sin n\pi y V(z)e^{st},\\ w' &= \cos m\pi x \cos n\pi y W(z)e^{st},\\ T' &= \cos m\pi x \cos n\pi y\ \Theta(z)e^{st},\\ \phi' &= \cos m\pi x \cos n\pi y\ \Phi(z)e^{st} \end{aligned}\right\} \quad (24)$$

where $m,\ n=1,\ 2,\ 3...$

These solutions are substituted in Eqs. (19),(20), (22) with boundary condition Eq. 23(a) which results a system of ordinary differential equations (ODE), as follows

$$(D^2-\alpha^2)W+R_a\alpha^2\Theta-R_n\alpha^2\Phi=0 \quad (25)$$

$$-(QN_B+1)\frac{dT_b}{dz}W+\Big(D^2+\frac{N_B}{L_e}\frac{d\phi_b}{dz}D+\frac{2N_AN_B}{L_e}\frac{dT_b}{dz}D -QN_B\phi_bD-QD-\alpha^2-s\Big)\Theta+\frac{dT_b}{dz}\frac{N_B}{L_e}(D-QN_B)\Phi=0 \quad (26)$$

$$\frac{d\phi_b}{dz}W-\frac{N_A}{L_e}(D^2-\alpha^2)\Theta -\left(\frac{1}{L_e}(D^2-\alpha^2)-QD-s\right)\Phi=0 \quad (27)$$

with

$$W=0, \Theta=0,\ N_AD\Theta+D\Phi-QL_e\Phi=0 \quad \text{at } z=0,1 \quad (28)$$

where $D=\frac{d}{dz}, \alpha=\left(m^2M_x^2+n^2M_y^2\right)^{1/2}\pi$

4. METHOD OF SOLUTION

Equations (25)-(27) with Eq.(28) are solved numerically using the single-term Galerkin weighted residual method. Accordingly W, Θ , and Φ are taken as

$$W=\sum_{i=1}^{N}P_iw_i, \Theta=\sum_{i=1}^{N}Q_i\theta_i, \Phi=\sum_{i=1}^{N}R_i\phi_i, \quad (29)$$

Here P_i, Q_i , and R_i are constants. The trial functions are chosen as

$$W_i=\mathrm{Sin}\, i\pi z,\ \Theta_i=\mathrm{Sin}\, i\pi z,\ \Phi_i=-N_A\,\mathrm{Sin}\, i\pi z \quad (30)$$

Equations (29)-(30) are substituted in Eqs. (25)-(27) to obtain residues and making these residues orthogonal (in inner product sense) to these trial functions, we get a system of linear simultaneous homogeneous equations. The vanishing of the determinant of coefficients produces the eigenvalue for the system (Finlayson, 1972). For a first-order Galerkin approximation, we take N = 1 and follow the above procedure, we get the following eigenvalue equation

$$\begin{vmatrix} B_{11} & B_{12} & B_{13}\\ B_{21} & B_{22} & B_{23}\\ B_{31} & B_{32} & B_{33}\end{vmatrix}=0 \quad (31)$$

where

$$B_{11}=\langle W(D^2-\alpha^2)W\rangle=-\frac{\delta}{2},$$

$$B_{12}=R_a\alpha^2\langle W\Theta\rangle=\frac{R_a\alpha^2}{2},$$

$$B_{13}=-R_n\alpha^2\langle W\Phi\rangle=\frac{R_nN_A\alpha^2}{2},$$

$$B_{21}=-(1+N_BQ)\langle\Theta\frac{dT_b}{dz}W\rangle =\frac{\left(16\pi^4-4\pi^2Q^2+Q^4\right)(1+QN_B)}{32\pi^4}+O[Q]^6$$

$$B_{22}=\Big\langle\Theta\Big(D^2+\frac{N_B}{L_e}\frac{d\phi_b}{dz}D+\frac{2N_AN_B}{L_e}\frac{dT_b}{dz}D -QN_B\phi_bD-QD-\alpha^2-s\Big)\Theta\Big\rangle$$

$$=(2Q\left(16\pi^4-4\pi^2Q^2+Q^4\right)(-1+L_e)L_eN_AN_B +L_e(-32\pi^5\left(\alpha^2+\pi^2+s\right)(-1+L_e)L_e+Q(2(-1+L_e) L_e\left(16\pi^4-4\pi^2Q^2L_e^2+Q^4L_e^4\right)+(16\pi^4(-1+3L_e) -4\pi^2Q^2\left(-1+L_e+2L_e^3\right)+Q^4(-1+L_e+2L_e^5)) N_A)N_B))/64\pi^5(-1+L_e)L_e^2+O[Q^6]$$

$$B_{23}=\frac{N_B}{L_e}\left\langle \Theta(\frac{dT_b}{dz}(D-QN_B))\Theta\right\rangle =$$
$$-\frac{QN_AN_B\left(16\pi^4-4\pi^2Q^2+Q^4\right)\left(1+2\pi L_e\right)}{64L_e\pi^5}+O[Q]^6$$

$$B_{31}=\left\langle \Phi\frac{d\phi_b}{dz}W\right\rangle =$$
$$\frac{-N_A}{2}-\frac{N_AQ^2\left(-4\pi^2L_e^2-4\pi^2N_A-4\pi^2L_eN_A\right)}{32\pi^4}$$
$$-\frac{N_AQ^4\left(L_e^4+N_A+N_AL_e+N_AL_e^2+N_AL_e^3\right)}{32\pi^4}+O[Q]^6$$

$$B_{32}=-\frac{N_A}{L_e}\left\langle \Phi(D^2-\alpha^2)\Theta\right\rangle =-\frac{\delta N_A^2}{2L_e},$$

$$B_{33}=\frac{1}{L_e}\left\langle \Phi((D^2-\alpha^2)-QD-s)\Phi\right\rangle =\frac{N_A^2(\delta+sL_e)}{2L_e},$$

Here

$$\left\langle G(z)\right\rangle \equiv\int_0^1 G(z)dz,$$

$$\delta=\alpha^2+\pi^2=\pi^2(1+m^2M_x^2+n^2M_y^2)$$

To obtain an approximate solution, we take $Q<<1$, so that Galerkin method leads to a useful result. This produces the following expression for Rayleigh number

$$R_a=\frac{\delta\left(2\delta\left(s+\delta\right)+2s\left(s+\delta\right)L_e-sN_AQ_v\right)}{a^2\left(2\delta+\left(-1+2\delta\right)Q_v+2sL_e\left(1+Q_v\right)\right)}-\frac{\left(2\left(s+\delta\right)L_e+N_A\left(2\delta+\left(-1+2\delta\right)Q_v\right)\right)R_n}{2\delta+\left(-1+2\delta\right)Q_v+2sL_e\left(1+Q_v\right)} \quad (32)$$

where $Q_v=N_BQ$ is rescaled Péclet number. Setting, $s=i\omega$, here ω is real, we get

$$R_a=\frac{\begin{array}{l}\left(2\delta+\left(-1+2\delta\right)Q_v\right)(2\delta^3+2\delta\left(-a^2+\omega^2\right)L_e\\ +a^2N_A\left(-2\delta+Q_v-2\delta Q_v\right)R_n)+2\omega^2L_e\\ \left(1+Q_v\right)(\delta(2\delta-N_AQ_v)+2L_e(\delta^2-a^2R_n))\end{array}}{4\omega^2L_e^2\left(1+Q_v\right)^2+\left(2\delta+\left(-1+2\delta\right)Q_v\right)^2}+i\omega P_1 \quad (33)$$

where

$$P_1=\frac{\begin{array}{l}-2L_e\left(1+Q_v\right)(2\delta^3+2\delta\left(-a^2+\omega^2\right)L_e+a^2N_A\\ (-2\delta+Q_v-2\delta Q_v)R_n)+\left(2\delta+\left(-1+2\delta\right)Q_v\right)\\ (\delta\left(2\delta-N_AQ_v\right)+2L_e(\delta^2-a^2R_n))\end{array}}{4\omega^2L_e^2\left(1+Q_v\right)^2+\left(2\delta+\left(-1+2\delta\right)Q_v\right)^2} \quad (34)$$

The Rayleigh number is a physical parameter and it must be real. Then from Eq. (33) either $P_1=0$ or $\omega=0$. We first consider the case of direct bifurcation ($\omega=0$), then Eq. (33) becomes

$$R_a=\frac{\delta^3}{\alpha^2((1+Q_v)\delta-(Q_v/2))}-\tilde{R}_n(\tilde{N}_A+\frac{\delta}{(1+Q_v)\delta-(Q_v/2)}) \quad (35)$$

where $\tilde{N}_A=\frac{N_A}{L_e}$ is rescaled diffusivity ratio and $\tilde{R}_n=L_eR_n$ is rescaled nanoparticle Rayleigh number.

In the absence of throughflow and nanoparticles, Eq. (35) becomes

$$R_a=\frac{\delta^3}{\alpha^2} \quad (36)$$

This is the similar expression for R_a was gotten by Beck (1972).

To study the behavior of rescaled Péclet number Q_v, we examine $\frac{\partial R_a}{\partial Q_v}$ analytically. Then from Eq. (35) we get

$$\frac{\partial R_a}{\partial Q_v}=-\frac{\begin{array}{c}2(\pi^2+\alpha^2)(2(\pi^2+\alpha^2)-1)\\ ((\pi^2+\alpha^2)^2-\alpha^2\tilde{R}_n)\end{array}}{\alpha^2(Q_v-2(\pi^2+\alpha^2)(1+Q_v))^2} \quad (37)$$

From Eq. (37), it is found that the effect of throughflow on convection depends on rescaled nanoparticle Rayleigh number as well as the aspect ratios of a rectangular box. To simplify the above expression, the value of α is assumed to be π. Under these assumptions Eq. (37) becomes as follows

$$\frac{\partial R_a}{\partial Q_v}=-\frac{4\pi^2(4\pi^2-1)(4\pi^2-\tilde{R}_n)}{(Q_v-4\pi^2(1+Q_v))^2} \quad (38)$$

This shows that the rescaled Péclet number has a destabilized effect if $4\pi^2>\tilde{R}_n$.

Rayleigh number attains its minimum over the positive integers m,n. The corresponding positive integers are

$$m=m(Q_v,\tilde{R}_n,M_x) \quad 39(a)$$

$$n=n(Q_v,\tilde{R}_n,M_y) \quad 39(b)$$

Full mathematical expressions of m,n are too lengthy to present here. From Eqs.39 (a) -39(b), it is observed that positive integers (m,n) are dependent on the rescaled Péclet number, rescaled nanoparticle Rayleigh number and aspect ratios, respectively.

From Table 1, it is observed that an increase in the values of rescaled Péclet number tends to increase α. Thus, Q_v reduces the size of a convection cell. The value of α decreases with increasing values of rescaled nanoparticle Rayleigh number thus its effects is to make higher the size of the cell.

Now, we investigate the possibility of oscillatory convection. When $P_1=0,\omega\neq0$ (Hopf bifurcation), Eq. (34) providing the frequency of oscillation

$$\omega^2 = \frac{\begin{array}{l}4a^2\delta L_e^2\left(1+Q_v\right)+\delta\left(2\delta+\left(-1+2\delta\right)Q_v\right)\\ \left(2\delta-N_AQ_v\right)+2L_e(-\delta^2Q_v\\ +a^2\left(2\delta+\left(-1+2\delta\right)Q_v\right)\left(-1+N_A\left(1+Q_v\right)\right)R_n)\end{array}}{4\delta^2L_e^2\left(1+Q_v\right)} \quad (40)$$

It is found that for the modest value of R_n and for small values of Péclet number, Lewis number, and diffusivity ratio, ω^2 is positive and real. Therefore, oscillatory convection is possible for an admissible range of parameters. This result is in marked contrast with the corresponding result obtained by Kuznetsov (2014a, 2015) who completely ruled out the possibility of oscillatory convection.

Table 1 Numerical values of α for different values of R_n and Q_v

Q_v	$\tilde{R}_n$	α
0.01	1	3.14038
0.02	1	3.14076
0.03	1	3.14113
0.01	2	3.14110
0.01	3	3.14107
0.01	4	3.14035

5. RESULTS AND DISCUSSION

Using the data given by Buongiorno (2006), Nield and Kuznetsov (2009) the following parameters value for alumina/water nanofluid of 1-100 nm are utilized:

$$L_e = 10, R_n = 0.1, N_A = 2, Q = 0.1, N_B = 0.1, \alpha = 3.14$$
$$(m = 2, n = 1, L_x = 2.82, L_y = 1.41, H = 1)$$

The above parameters are fixed except when the variation is considered with respect to that particular parameter. The trends of R_a verses Q_v for various values of $\tilde{R}_n$ are shown in Fig. 2. It is observed that R_a decreases with increasing value of $\tilde{R}_n$. Therefore, the rescaled nanoparticle Rayleigh number destabilizes the system. This may be physically interpreted as; an increase in a volumetric fraction increases the Brownian motion of nanoparticles which produce a destabilizing effect. From figure, it is also observed that the value of R_a decreases with increase in rescaled Péclet number. Thus, throughflow destabilized the convection for the small amount of throughflow. This may be physically interpreted as, increasing the rescaled Péclet number (Q_v), increases the convective contribution of nanoparticle flux in the thermal energy equation, which enhances the destabilizing effect. Thus, the rescaled Péclet number helps to construct the convection pattern.

The variation of the square of oscillation frequency with nanoparticle Rayleigh number is shown in Figs. 3(a)-3(b). From Fig. 3(a), it is found that for small values of L_e $(1\sim10)$, ω^2 is positive when the value of R_n is lies between in range 0 to 0.02. For larger values of $(50\sim100)$, ω^2 is positive when the value of R_n is lies between in range 0 to 0.002. From Fig. 3(b), it is found that for small values of $N_A(1\sim10)$, ω^2 is positive when the value of R_n is lies between in range 0 to 0.002. Thus, oscillatory convection is possible for the small amount of throughflow, small values of N_A and L_e, and the modest value of R_n.

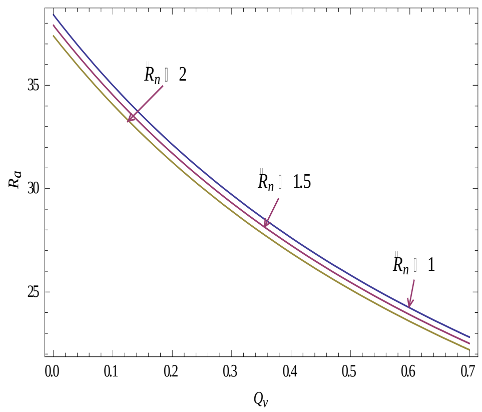

Fig. 2. Plot of Rayleigh number versus rescaled Péclet number for various values of $\tilde{R}_n$.

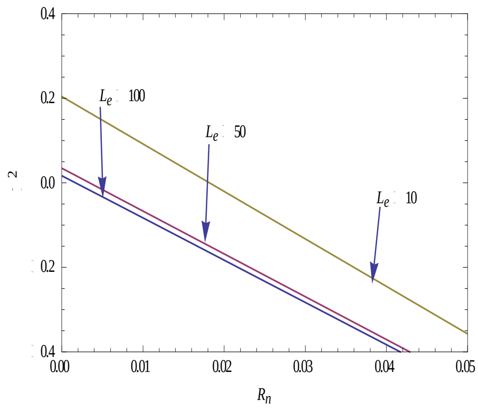

Fig. 3(a) Plot of square of oscillation frequency (ω^2) versus R_n for various values of Lewis number.

6. CONCLUSIONS

We have taken a fresh look to study linear instability in a rectangular box saturated by nanofluid with the effect of throughflow using the modified mass flux conditions (nanoparticle flux is zero on boundaries). Stationary Rayleigh number depends on three combinations of parameters such as N_A/L_e, QN_B, R_nL_e .We rescale these combinations of parameters and found that all three rescaled parameters ($\tilde{N}_A, Q_v, \tilde{R}_n$) prompt the convection. By using the modified flux conditions, the minimization process over the integers m,n is affected by nanofluid and throughflow parameters. It is found that aspect ratios, throughflow, and nanoparticles play a significant role in the formulation of cell distribution. The effect of throughflow is dependent on aspect ratios as well as nanofluid parameters. The convective component of nanoparticle flux enhances the destabilizing effect, this result that modified boundary conditions produce a more destabilizing effect as compared to previous boundary conditions. Oscillatory convection is also possible for a permissible range of nanofluid and throughflow parameters.

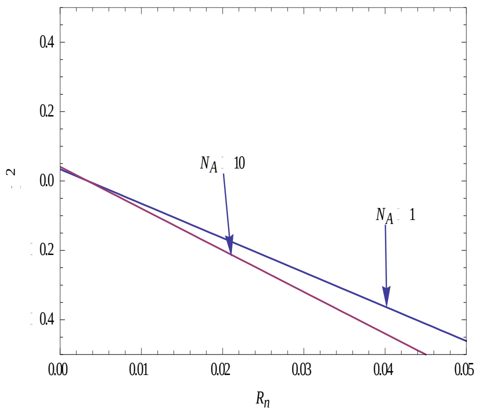

Fig. 3(b) Plot of square of oscillation frequency (ω^2) versus R_n for various values of modified diffusivity ratio.

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