



Numerical Optimization of an Array of Triangular Microchannels using Constructal Theory

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ABSTRACT

In the present study, an arrangement of triangular microchannels with different contact angles is analyzed and optimized following the guidelines provided by the constructal theory to reach to the maximum heat removal rate. This investigation is performed analytically and numerically. Based on the obtained results, it is emerged that this optimization is independent of the number and the type of the arrangement of the microchannels. It is also observed that increasing the pressure drop through the triangular microchannels decreases the optimal hydraulic diameter. Numerical results recommend that the microchannel with contact angle of 60° possesses the highest heat transfer rate at a given pressure drop, and decreasing the contact angle of the triangular cross-section leads to lower heat transfer rates. Comparing the analytical and numerical results of the optimal hydraulic diameter of the microchannel heat sinks, a reasonable agreement is observed; however, due to some assumptions which are considered at the analytical method, the analytical predictions of the configurations having the highest heat transfer rate are inaccurate. Therefore, the numerical optimization should be used to choose the configuration with the highest cooling capacity.

Keywords: Microchannel; Geometrical optimization; Heat transfer; Constructal design.

NOMENCLATURE

A	channel cross-sectional area	\bar{U}	average velocity
Be	Bejan number	V	velocity
c_p	specific heat capacity	W	heat sink width
D_h	hydraulic diameter	ε	aspect ratio
H	height	Δ	difference
k	Thermal conductivity coefficient	μ	dynamic viscosity
L	axial length	ν	kinematic viscosity
Po	Poiseuille number	ρ	fluid density
P	pressure	φ	side angle
Pr	Prandtl number		

1. INTRODUCTION

Using microchannels for absorbing heat fluxes leads to high pressure drops in the fluid flow. Managing the high thermal volumetric densities generated by electronic equipment, requires advanced techniques of heat absorption, resulting in complicated designs.

Constructal design has been widely used in the engineering field to optimize the flow systems, Salimpour *et al.* (2013). A system with certain size should be able to provide easier access to the flows passing through it, Bejan (2000). Constructal theory is a view in which producing flow structures observed in the nature (rivers, lungs, atmospheric air

circulation, etc.) can be based on evolving principles through which access to these kinds of flows becomes easier. This principle is called constructal law. Constructal theory has gathered many researchers together to step in a new path including using constructal law for better engineering works, movements with more organization and relationship of human, goods and information as well. This path is called constructal design. Better structures and strategies are among purposes of this design path, Reis (2006). Typically, there are two types of problems in the realm of microchannel optimization by the help of constructal theory. Keeping the volume of microchannel fixed, at the first-type problems which are investigated in this study, the

wall temperature is considered invariant and optimization aims at finding the optimal microchannel dimensions to maximize the heat transfer rate; while at the second-type problems, the heat flux is set constant and the purpose of the optimization is minimizing the maximum temperature of the microchannel.

Bejan and Sciubba (1992) found the optimal spacing of parallel plates used for cooling electronic systems. Using the theoretical method provided by Bejan and Sciubba (1992), Muzychka (2005) obtained the optimal passage size to length ratios in terms of Bejan number for microchannels with different cross-sections. In this study, he investigated the heat transfer inside the circular and non-circular channels and showed that the optimal microchannel dimensions are independent of the channel arrangement.

Chen *et al.* (2009) modeled the fluid flow and heat transfer in non-circular microchannels. Comparing heat efficiencies of microchannels with different shapes (triangular, rectangular, and trapezoidal), they found that triangular microchannels have the highest heat efficiency. Xia *et al.* (2015) numerically studied the fluid flow and heat transfer in microchannel heat sinks with different cross-sectional shapes. A method that relies on the use of grooves parallel to the flow direction to increase the heat transfer, has been proposed by Moradi and Floryan (2013). They determined the optimal groove shape that maximizes the overall system performance. To improve the structures of the microchannel heat sinks for cooling the chips, Gong *et al.* (2015) investigated the flow and heat transfer in four types of heat sinks (traditional microchannel heat sink, rectangular column fin heat sink, single-hole jet-cooling heat sink and double-layer microchannel heat sink), numerically. The results showed that the arrangement optimization of the microchannel heat sinks can improve the thermal performance of the inlet header by boosting the flow distribution uniformity. Narvaez *et al.* (2014) analyzed the uniformity of the temperature profile inside the cold plate. Pressure, velocity, and temperature profiles were presented and compared with experimental results. The zone of high or low temperatures and critical details on the modeling were highlighted and discussed. Yilmaz *et al.* (2000) found that for a certain pressure drop of a microchannel, there is a position in which maximum heat transfer occurs. Salimpour *et al.* (2011) determined the optimal dimensions of microchannels with different cross-sections to achieve the highest heat transfer rate at constant volumes. Also, by evaluating the optimal hydraulic diameter and dimensionless heat transfer per unit volume of the microchannel, they found that the rectangular cross-section geometry is the best structure for a given pressure drop. Mardani and Salimpour (2016) studied the geometrical optimization of microchannel heat sinks with isosceles triangular cross-sections numerically and theoretically. For this purpose, the effect of pressure drop and volume fraction of the solid material on the maximum temperature of the microchannels has been

investigated. Considering the constraints and geometric parameters for the optimization, it was revealed that the microchannel heat sinks with side angle of 60° have the best performance compared to the other microchannels. Salimpour *et al.* (2019) invoked constructal theory to design circular multilayer microchannel heat sinks. They found that using three layers of microchannel can reduce the maximum temperature more than 10°C .

To the best knowledge of the authors, there is no study in the open literature on the investigation and optimization of an array of triangular microchannels. Therefore, in the present research, invoking constructal theory, this matter has been dealt with, analytically and numerically. For this purpose, an array of triangular microchannels with constant volume cooled via forced convection is considered. Using analytical and numerical optimization, optimal dimensions of triangular microchannels with three contact angles of 30° , 45° and 60° are determined so that the arrangement of these microchannels yields the maximum heat transfer in constant volume.

2. STATEMENT OF THE PROBLEM

The proposed system in this study is an array of triangular microchannels with fixed volume cooled by laminar flow convection, Fig. 1. Using theoretical analysis (intersection of asymptotes method) and numerical optimization, optimal dimensions of the microchannels with triangular cross-sections with three different contact angles of 30° , 45° and 60° are determined somehow the maximum heat transfer in a constant volume is achieved. Temperature and volume parameters are considered constant in this study and optimization aims at finding optimal microchannel dimensions to maximize heat transfer rate. Then, the results of numerical and theoretical analyses are compared, and the optimal geometry is ultimately selected out of the provided microchannels.

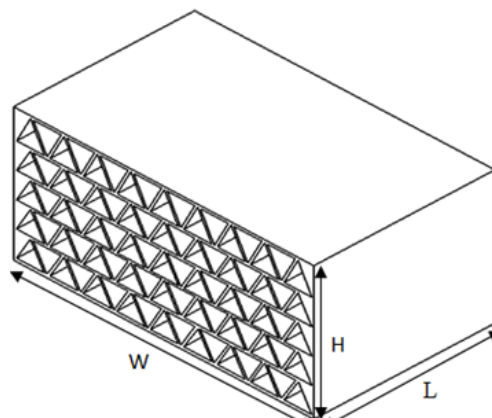


Fig. 1 An array of triangular microchannels.

3. THEORETICAL ANALYSIS

The theoretical analysis presented here is an application of the intersection of asymptotes method.

In this method, the relationship between the global thermal conductance and the geometry is found at two extreme modes: small channels ($D_h \rightarrow 0$) and large channels ($D_h \rightarrow \infty$). Assumptions used during the theoretical analysis are as follows, [Salimpour et al. \(2011\)](#):

1. Identical wall temperature of the channels (ignoring conductivity resistance of the arrangement),
2. Uniform flow distribution (equal flow rates in all channels),
3. Laminar flow,
4. Constant channel cross-sectional area,
5. Prandtl number larger than 0.1,
6. Fixed volume of the microchannels.

3.1 SMALL CHANNELS

If the arrangement includes small channels, the flow in the microchannels is fully-developed which is characterized as Hagen-Poiseuille flow, [Salimpour et al. \(2011\)](#); hence

$$Q_s = N \rho A_c \bar{U} c_p (\bar{T}_s - T_i) \quad (1)$$

\bar{U} is the average velocity which is defined as follows,

$$\bar{U} = \frac{A_c \Delta P D_h}{\mu P_c L P_o} \quad (2)$$

where, P_o is Poiseuille number which has been reported by [Shah and London \(1978\)](#). This number varies for different shapes and is mostly in the range of $6 < P_o D_h < 12$. Poiseuille number for the triangular microchannels is, [Mardani and Salimpour \(2016\)](#),

$$P_o D_h = \frac{6(\varepsilon^3 + 0.2595\varepsilon^2 - 0.2046\varepsilon + 0.0552)}{\varepsilon^3} \quad (3)$$

In Eq. (3), ε is the aspect ratio of the channel which is defined as the ratio of the channel height to its lower edge ($\varepsilon = H_c/d$).

Dimensionless heat transfer for small channels is defined as [Mardani and Salimpour \(2016\)](#),

$$Q_s^* \approx \frac{\rho c_p A_c^2 \Delta P}{k_f D_h \mu P_c P_o} \quad (4)$$

From Eq. (4), it is observed that,

$$Q_s^* \approx C_1 D_h^2 \quad (5)$$

3.2 Large Channels

When the arrangement consists of large channels, flow will be necessarily a boundary layer flow. In this extreme mode, heat transfer rate is determined from the following equation, [Salimpour et al. \(2011\)](#),

$$Q_l = \bar{h} N L P_c (\bar{T}_s - T_i) \quad (6)$$

where \bar{h} for the boundary layer flow on a flat surface is,

$$\frac{\bar{h} L}{k_f} = 0.664 \left(\frac{U_\infty L}{\nu} \right)^{1/2} Pr^{1/3} \quad (7)$$

In Eq. (7), U_∞ is the free flow velocity which is determined through a force balance over the arrangement as,

$$\bar{\tau}_w P_c L N = N A_c \Delta P \quad (8)$$

Average shear tension on the wall is obtained from the boundary layer solution,

$$\frac{\bar{\tau}_w}{\rho U_\infty^2 / 2} = 1.328 \left(\frac{U_\infty L}{\nu} \right)^{-1/2} \quad (9)$$

Combining Eqs. (8) and (9), U_∞ is calculated as follows,

$$U_\infty = 1.314 \left(\frac{\Delta P A_c}{P_c L^{1/2} \rho^{1/2} \nu^{1/2}} \right)^{2/3} \quad (10)$$

and from Eqs. (6), (7) and (10), dimensionless heat transfer is emerged as,

$$Q_l^* \approx \frac{0.7611 (\Delta P A_c P_c^2 L P_r)^{1/3} L}{D_h^2 (\rho^2)^{1/3}} \quad (11)$$

From Eq. (11), it can be shown that,

$$Q_l^* \approx C_2 D_h^{-2/3} \quad (12)$$

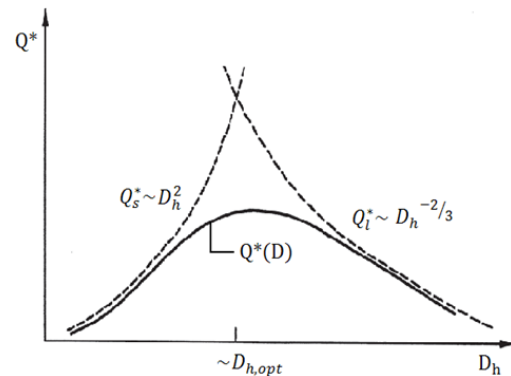


Fig. 2. The intersection of asymptotes method, Salimpour et al. (2013).

In the previous stages, two asymptotes for variations of Q^* versus D_h were achieved: Eqs. (5) and (12), as illustrated in Fig. 2. As is seen from this figure, there is a hydraulic diameter for which heat transfer is maximized. The optimal hydraulic diameter is obtained from intersecting the asymptotes, as

$$D_{h,opt} = 2.147 B e^{-1/4} L P_o^{3/8} \quad (13)$$

An interesting notion of Eq. (13) is that optimal diameter of the microchannels does not depend on

the number of channels, N . Therefore, it can be concluded that the optimal channel size provides the optimal overall arrangement, as well.

Introducing Eq. (13) into Eq. (5) or (12), maximum dimensionless heat transfer per unit volume, $Q_{max,th}^*$, is obtained. Generally, $Q_{max,th}^*$ is as follows,

$$Q_{max,th}^* = C_3 Be^{1/2} \quad (14)$$

C_3 is a constant determined from the channel geometry. It should be noted that Bejan number is the dimensionless pressure drop which is defined as $Be = \Delta PL^2 / \mu\alpha$.

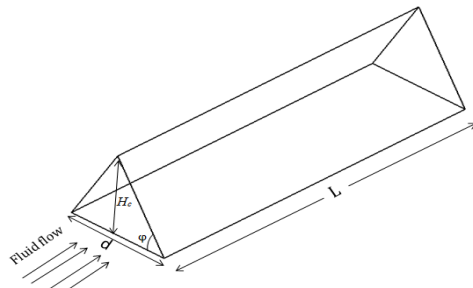


Fig. 3. Computational domain of a triangular microchannel.

4. MATHEMATICAL MODEL

Forced convection heat transfer is evaluated in the laminar flow regime in triangular microchannels. Considering the previous section equations, optimal channel size is independent of the arrangement; while it does depend on the length and pressure drop constraints.

Figure 3 presents the computational domain of the microchannel under consideration. Channel walls are kept in constant temperature. Cooling fluid with inlet temperature of T_{in} is derived into the channel through a constant pressure drop ($\Delta P = P(z = 0) - P(z = L)$).

To model the fluid flow and heat transfer, the following assumptions are used:

1. The continuity regime prevails.
2. Steady state conditions rule over the flow and heat transfer.
3. Flow is incompressible.
4. Properties of the fluid are constant.
5. Heat transfer due to radiation and free convection are negligible.

Taking into account these assumptions, continuity, momentum and energy equations for the cooling fluid flow are written as,

$$\vec{\nabla} \cdot (\rho \vec{U}) = 0 \quad (15)$$

$$\rho (\vec{U} \cdot \vec{\nabla} \vec{U}) = -\vec{\nabla} P + \mu \nabla^2 \vec{U} \quad (16)$$

$$\rho c_p (\vec{U} \cdot \vec{\nabla} T) = k_f \nabla^2 T \quad (17)$$

The hydraulic boundary condition on the walls is no-slip condition, while the inlet pressure is evaluated as,

$$P = \frac{\alpha \mu Be}{L^{2/3}} + P_{out} \quad (18)$$

The inlet temperature of the coolant is assumed as 20°C, while the outlet pressure of the channel is set at 1 atm. Also, the temperature of the channel walls is kept fixed at 70°C.

5. NUMERICAL ANALYSIS

In the present study, the control volume method, Patankar (1980), is used to resolve continuity, momentum and energy equations. SIMPLE algorithm is employed to solve the coupled pressure-velocity equations, while the second-order upwind scheme is invoked to discretize the equations. Convergence is obtained when the residuals for mass, momentum and energy equations are less than 10^{-5} , 10^{-5} and 10^{-9} , respectively.

To demonstrate the independency of the solution from the grid size, the number of grids is doubled until the differences in the maximum temperature and velocity obtained from the numerical solution are less than 1%.

Verification of the numerical scheme is performed through the comparison of the present study results with the results of Salimpour *et al.* (2011), as presented in Fig. 4. This figure shows the variations of the optimal hydraulic diameter of the microchannel with triangular cross-section with the contact angle of 45° versus the pressure drop. As is observed, there is a good agreement between the present numerical results with those reported by Salimpour *et al.* (2011). The deviation between the results is less than 2.2%. Moreover, the results of the present study pertaining to the maximum dimensionless heat transfer per unit volume of the microchannel are compared with those of Salimpour *et al.* (2011) in Fig. 5. These diagrams are drawn for $\Delta P = 2kPa$. As is observed, predictions of the present study are just slightly different from those existed in the literature (a deviation less than 0.4%) which shows the accuracy of the present numerical scheme.

6. RESULTS AND DISCUSSION

In this section, a series of numerical optimizations is performed to investigate the effect of pressure difference on the optimal hydraulic diameter and the dimensionless heat transfer per unit volume, Q^* . Air is considered as the coolant fluid flowing through a microchannel of 10mm length.

At the first step, the effect of hydraulic diameter of the triangular microchannel on the heat transfer is investigated. To this end, pressure difference along the unit cell is set at 2kPa. Figure 6 shows the variations in dimensionless heat transfer per unit

volume of the triangular microchannels with different contact angles with respect to the channel hydraulic diameter. These results are obtained numerically. This figure indicates that there is an optimal D_h for which heat transfer per unit volume is maximized. This optimal point had already been predicted by the theoretical method. As is seen in Fig. 6, for a given pressure drop, at a constant microchannel length, triangular microchannels transfer more heat at $D_{h,opt}$. When $D_h < D_{h,opt}$, the dimensionless heat transfer per unit volume experiences steeper gradients compared to the situations at $D_h > D_{h,opt}$. This means that at hydraulic diameters larger than the optimal value, the design is robust, and hence this region is favorable for design purposes. This matter is aligned with the results of Kandlikar and Upadhye (2005). Moreover, it is observed that the dimensionless heat transfer per unit volume increases by increasing the contact angle of triangular microchannels; thus the highest heat transfer belongs to the triangular cross section with the contact angle of 60° , and the lowest heat transfer belongs to that with the contact angle of 30° . As is seen in Fig. 6, at small hydraulic diameters, dimensionless heat transfer rates of all microchannels with different contact angles are close to each other; while at large diameters, the role of contact angle is more pronounced.

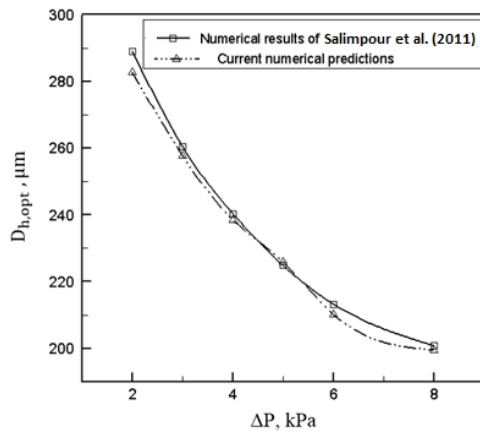


Fig. 4. Comparison of the numerically optimized hydraulic diameters of the present study with those of Salimpour *et al.* (2011).

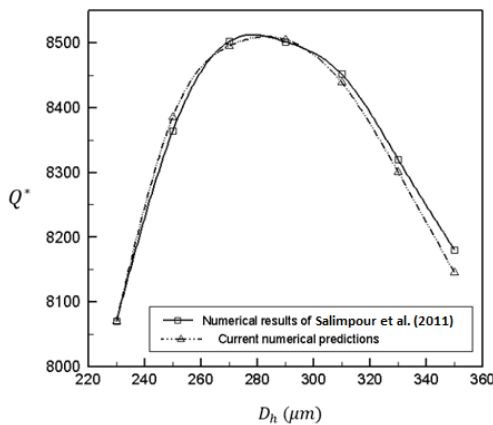


Fig. 5. Comparison of the numerically optimized Q^* of the present study with those of Salimpour *et al.* (2011).

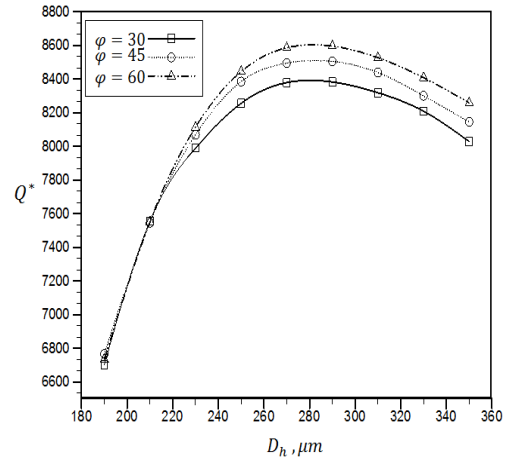


Fig. 6. The effect of hydraulic diameter on Q^* for triangular cross-sections with $\Delta P = 2 \text{ kPa}$.

Figure 7 presents the effect of pressure drop on the optimal hydraulic diameter of the triangular microchannels. Increasing pressure drop along the triangular microchannel decreases the optimal hydraulic diameter of the microchannel. This finding agrees with the results of the analytical section (Eq. (13)) where the optimal hydraulic diameter is inversely proportional to the Bejan number (dimensionless pressure drop).

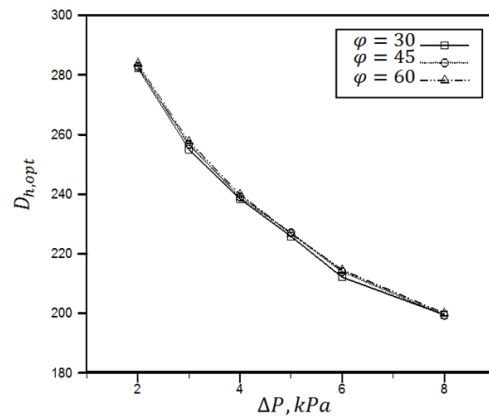


Fig. 7 The effect of pressure drop on the optimized hydraulic diameter of the microchannel.

Figures 8 and 9 illustrate the variations of the optimal diameter of the microchannels with different contact angles versus the Bejan number obtained from analytical method (Eq. (13)) and numerical method, respectively. It is clear that the trends of both curves are similar. Given in Fig. 10, is the comparison of the numerical and theoretical results of triangular microchannels with 60° contact angle. It is observed that the results of both methods are in good agreement with each other with a maximum difference less than 0.9%.

The optimal hydraulic diameters obtained from the analytical and numerical methods for the triangular microchannels with contact angles of 30° , 45° and 60° are correlated with Bejan number in a power law

form and presented in Table 1 to facilitate the prediction of the optimum triangular microchannel diameters for a specified pressure drop. Comparing these correlations discloses a good agreement between the analytical and numerical schemes in predicting the optimal hydraulic diameters. The maximum difference between these correlations is 1.5% which is for the triangular cross-section with the contact angle of 30°.

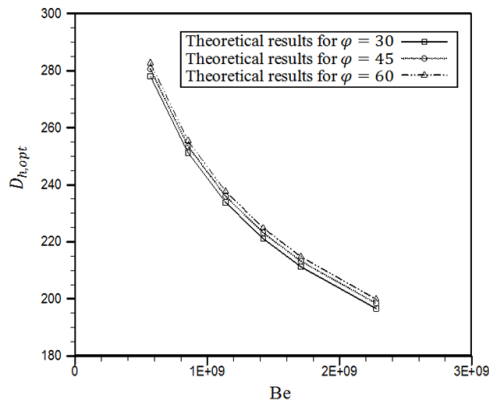


Fig. 8. Theoretical results for the optimal channel size for maximum Q^* .

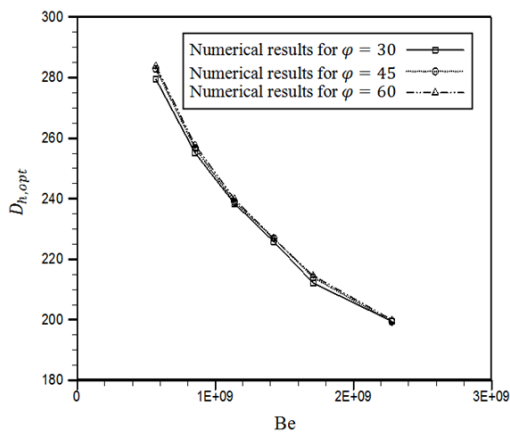


Fig. 9. Numerical results for the optimal channel size for maximum Q^* .

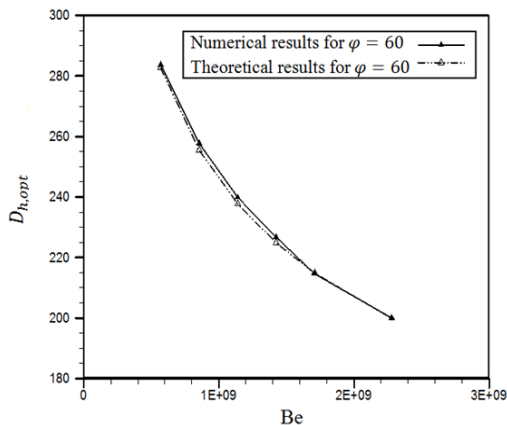


Fig. 10. Comparison of the numerical and theoretical optimal values of D_h for $\phi = 60^\circ$.

Table 1 Analytical and numerical correlations for predicting $D_{h,opt}/L$

Side angle of channel	Theoretical analysis	Numerical analysis
$\phi = 30^\circ$	$4.2954Be^{-0.25}$	$4.3594Be^{-0.25}$
$\phi = 45^\circ$	$4.3351Be^{-0.25}$	$4.3701Be^{-0.25}$
$\phi = 60^\circ$	$4.3673Be^{-0.25}$	$4.3848Be^{-0.25}$

Variations of numerically optimized Q^* with Bejan number are graphed in Fig. 11. The trend of these variations is similar to that of the analytical approach (Eq. (14)). From Fig. 11, it is observed that Q^*_{max} of a triangular cross-section with contact angle of 60° is larger than that of other cross-sections; and decreasing the contact angle of the microchannel cross-section reduces the dimensionless heat transfer. This contradicts the predictions of Eq. (14) which says that the lower contact angles yield higher heat transfer rates. This is due to the assumptions adopted in the analytical method.

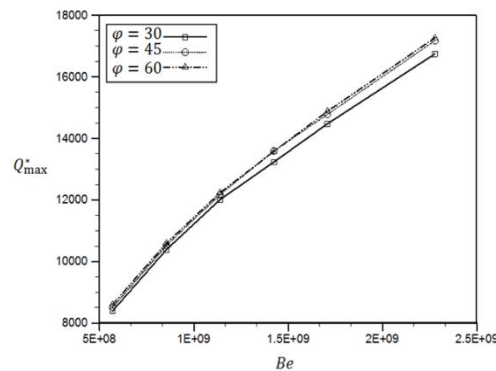


Fig. 11. The effect of Bejan number on the maximum value of Q^* .

The values of numerically optimized Q^* for different geometries of triangular microchannels, are correlated $Q^*_{max,num}$ with Bejan number in a power law form, and presented in Table 2. This table contains the analytically optimized Q^* as well for comparison purposes. From this table, it is seen that the dependencies of Q^*_{max} upon the Bejan number at both methods are similar; however, the values are different in some extent.

Table 2 Analytical and numerical correlations for predicting Q^*_{max}

Side angle of channel	Theoretical analysis	Numerical analysis
$\phi = 30^\circ$	$0.7258Be^{0.5}$	$0.3982Be^{0.4942}$
$\phi = 45^\circ$	$0.7214Be^{0.5}$	$0.3319Be^{0.5038}$
$\phi = 60^\circ$	$0.7178Be^{0.5}$	$0.3649Be^{0.4995}$

7. CONCLUSION

Constructal optimization of an array of triangular microchannels is carried out analytically and numerically. The contact angles of the triangular cross-sections are 30, 45 and 60°. The following results are drawn from this study.

- The optimal microchannel size decreases by increasing the pressure drop.
- The analytical and numerical correlations developed to predict the optimal hydraulic diameter of the microchannels are in good agreement.
- At hydraulic diameters larger than the optimal value, the design is robust, and hence this region is favorable for design purposes.
- For a given Bejan number, the microchannel with the contact angle of 60° at optimal conditions, has the highest dimensionless heat transfer per unit volume.
- Dependencies of Q_{max}^* upon the Bejan number at both analytical and numerical methods are similar.

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