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Interaction of Water Waves with Permeable Barrier using Galerkin Approximation

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ABSTRACT

In this paper we analyze the interaction of water waves with a permeable barrier which is slightly perturbed from its vertical position within the framework of linearised water wave theory. The barrier is placed in water of finite depth. Two different kinds of barriers are examined, namely, (I) a partially immersed barrier and (II) a submerged bottom standing barrier. The governing boundary value problem involving the velocity potential function is split into two boundary value problems involving the zeroth order as well as the first order velocity potential functions by using a simplified perturbation technique. The zeroth order reflection and transmission coefficients which are due to a vertical permeable barrier are evaluated by solving a Fredholm integral equation of second kind numerically by using a one term Galerkin approximation. Green's theorem is applied to evaluate the first order reflection and transmission coefficients. The first order transmission coefficient vanishes irrespective of the shape of the barrier. The numerical values for the first order reflection coefficient are determined by choosing some appropriate shape functions. The numerical results for the zeroth order reflection coefficient which stand for the case of a vertical barrier are validated against the known results for both the permeable and impermeable barriers. The first order reflection curves are also compared by making the porosity constant to be zero with those available in the literature for an impermeable nearly vertical barrier.

Keywords: Water waves; Permeable barrier; Perturbation technique; Galerkin approximation.

1. INTRODUCTION

The theoretical study of problems of scattering of surface water waves by barriers involves several important and interesting concepts of mathematical analysis which have been developed over the last few decades. These problems draw attention be-cause of its diverse applications in coastal engineering such as construction of breakwaters. Breakwaters play an important role to protect the coasts of the sea and harbors from the waves. The problem of scattering of water waves by various kinds of barriers has been studied well in literature by employing the assumption of linear theory of water waves (cf. Islam *et al*., 2018). Many researchers developed different mathematical methods in search for solutions to such scattering problems. However, it has been found that explicit solutions are possible to determine when a train of normally incident waves interact with thin vertical barriers present in deep water. Dean (1945) used complex variable technique to investigate the problem of water wave scattering by a submerged vertical barrier. Ursell (1947) considered the problem of scattering of normally incident water waves by a partially immersed barrier and utilized the Havelock's expansion of water wave potential to obtain the solution in closed form. Williams (1966) applied a simple reduction method to obtain the reflection and the transmission coefficients for the problem involving a surface piercing barrier. Evans (1970) studied the submerged plate problem and employed the complex variable technique to obtain the solution explicitly. Porter (1972) obtained the analytical solution for completely submerged vertical wall with a gap using complex variable method as well as an integral equation procedure based on the Green's integral theorem. Chakrabarti and Vijaya Bharathi (1992) used a modified Green's function technique to solve the water wave scattering problem by barriers (namely partially immersed barrier, submerged bottom standing barrier) by reducing it to uncoupled integral equations. They obtained the solution in the closed form by applying integral transform technique.

For a non-vertical barrier the governing boundary value problem does not possess any explicit solution. Parsons and Martin (1992, 1994) formulated the problem of scattering by flat or curved plate in terms of a first kind hypersingular integral equation for the discontinuity in the potential across the plate. McIver and Urka (1995) used two methods, one of them is based on the matched series expansions and the other involves a variational approximation procedure to obtain the numerical results for the reflection coefficient for a circular arc shaped plate submerged into the deep water. Parsons and McIver (1999) studied the problem of scattering of water waves by an inclined surface piercing plate by applying the method of matched asymptotic expansion. Zaghian *et al*. (2017) carried out an experimental study using the Particle Image Velocimetry (PIV) technique to investigate the interaction of solitary waves with an inclined plate.

All the works cited above are related with rigid or impermeable barriers. Failure of several rigid structures as wave barriers leads to the use of porous structures in coastal environment. Porous structures such as porous plates, rubble-mounds and concrete armors are often used in coastal engineering for dissipating wave energy from open sea. The topic of wave scattering by porous structures gained the importance since the analytical development of wave motion through porous media by Sollitt and Cross (1972) who derived a theoretical model based on the modification of Darcy's law to obtain the reflection and the transmission coefficients of normally incident ocean waves passing across a permeable structure. Macaskill (1979) employed Green's integral theorem to study the wave scattering problem related to a permeable thin barrier by converting it to integrodifferential equations. Chwang (1983) developed porous wavewmaker theory to analyze surface waves on water of finite depth, produced by horizontal oscillations of a porous vertical plate. Dalrymple *et al*. (1991) obtained eigenfunction solution to analyze the effect of wave past a porous structure of finite thickness. Yu (1995) employed an approximate method to solve the problem of scattering of surface water waves by a semi-infinite porous breakwater. McIver (1999) used WeinerHopf technique to obtain the asymptotic results for the problem of scattering by thin porous breakwater. Chwang and Chan (1998) published a review on the interaction between porous media and wave motion. Lee and Chwang (2000) used the method of eigenfunction expansion for the problem of scattering and radiation of small amplitude water waves by thin vertical porous barrier in the water of finite depth in two dimensions. Boundary value problems were converted to a dual series relation and applying least square method solutions were obtained. Huang *et al*. (2003) developed a numerical model to examine the effect of structural permeability on the interaction of solitary waves and porous sub-merged structure. Tsai and Young (2011) investigated the water wave diffraction by a semi-infinite thin porous breakwater using a combination of the method of fundamental solutions and the domain decomposition method. Gayen and Mondal (2014, 2015) employed the technique of hypersingular integral equation to study the reflective properties of water waves past an inclined porous plate and a pair of vertical porous plates respectively.

In this paper we consider a slightly curved permeable barrier. In the water wave literature this type of barriers is referred to as 'nearly vertical' barriers. Scattering of surface water waves by partially immersed nearly vertical barrier was first investigated by Shaw (1985) by using a perturbation technique that involved the solution of a singular integral equation. Later, Mandal and Chakrabarti (1989) employed a perturbation technique directly to the governing partial differential equation, together with the Green's integral theorem to deter-mine the first order correction to the reflection and transmission coefficients in deep water. Mandal and Banerjea (1992) employed a method based on Havelock's expansion of water wave potential to solve the boundary value problem for the first or-der correction to the velocity potential in the problem of diffraction of water waves by a partially immersed nearly vertical barrier. Chakrabarti and Sahoo (1996) employed the method of perturbation analysis with the application of Green's integral theorem to obtain the first order correction of the reflection and transmission coefficients for nearly vertical porous wall in deep water. Kaligatla and Manam (2014) extended the above work to the case when the surface of water is covered by a thin sheet of ice. They employed the perturbation expansion to the governing boundary value problem. The boundary value problem for the zeroth order potential function was solved explicitly by making use of the orthogonal mode-coupling relation. The first order reflection coefficient was determined analytically by two different methods, one was based on Green's second identity and the other was based on the first kind integral equation. Banerjea *et al*. (2017) considered the problem of scattering of flexural gravity waves by a rigid thin plate consists of two nearly vertical plates submerged in deep water.

The novelty of the present research lies in the geometry of the plates and the method of solution used here to tackle this problem. Apart from the papers of Chakrabarti and Sahoo (1996) and Kaligatla and Manam (2014) there does not exist any paper which discusses the influence of nearly vertical porous barriers on the wave propagation. Both of these papers deal with a vertical wall which ex-tend from free surface till the bottom of water. The objective of the present paper is to analyze the reflective properties of nearly vertical porous partial barriers present in finite depth water. The waves are incident on the barriers obliquely. We consider two types of nearly vertical barriers namely (I) partially immersed barrier, (II) submerged bottom standing barrier.

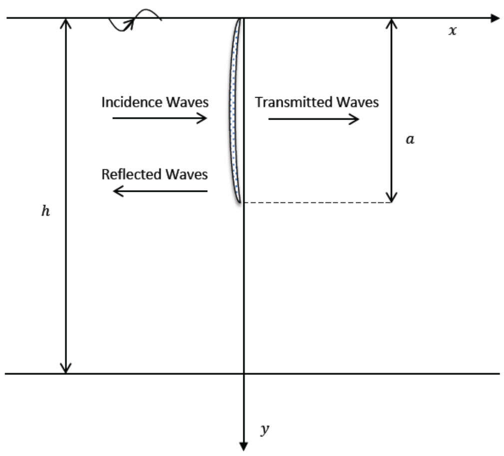
We apply Galerkin approximation to solve our present problem. The Galerkin approximation method is a useful method to obtain an approximate solution of integral equations. In this method, the unknown function is expanded in terms of a finite series involving real valued independent functions. The advantage of this method is that the set of functions need not to be an orthogonal set or not necessary to be complete. The Galerkin method converges faster than the collocation method (cf. Kanoria and Mandal, 2002). However, more computational time is needed for the Galerkin method compared to the collocation method because we have to compute a double integral at every stage in the former while a single integral in the latter.

The plan of the paper is as follows. The mathematical formulation of the problem is described in Section 2. In Section 3, a simplified perturbation expansion is employed in terms of a small parameter ε, for the velocity potential function together with the reflection and the transmission coefficients. Here ε is a dimensionless small parameter giving a measure of maximum deviation of the curved barrier from the vertical. The perturbation expansions of the velocity potential function and the reflection and the transmission coefficients are substituted into the governing partial differential equation, the boundary conditions, the edge condition as well as into the radiation condition. Equating the coefficients of the similar powers of the parameter ε, two independent boundary value problems (BVP) for the zeroth order and the first order potential functions are obtained. Martha and Bora (2007) and Panda (2016) used similar kind of perturbation involving a small parameter ε to investigate the oblique water wave diffraction by an ocean bed having small undulation. There the parameter ε is a measure of smallness of the bottom undulation. In Section 4.1, we solve the BVP for the zeroth order potential function (ϕ_0). It may be noted that this BVP actually stands for wave scattering by a vertical porous barrier. In order to solve this BVP we employ Havelock's expansion for ϕ_0 and Havelock's inversion theorem followed by the utilisation of porous plate condition. This produces a Fredholm integral equation of second kind in the potential difference across the barrier. A one term Galerkin approximation is employed to solve the integral equation. This requires representation of the unknown function to be presented as a product of an unknown constant and a known function to be chosen properly. We choose these functions to be the solutions of the scattering problems involving a thin vertical impermeable barrier present in deep water. Once the solution to the integral equation is determined, this is used to compute the zeroth order reflection coefficient. Although many scattering problems involving rigid vertical barriers are solved by employing Galerkin approximation (cf. Mandal and Dolai, 1994; Porter and Evans, 1995; Shivakumara *et al*., 2012), this approach is some-what new to solve any porous barrier problem. In Section 4.2, we determine the first order reflection and transmission coefficients by the application of Green's integral theorem. It is analytically shown that the first order transmission coefficient vanishes identically. The first order reflection coefficient is determined in terms of an integral involving the shape function. In Section 5, the numerical results are analyzed with the help of a number of graphs. Finally, conclusions are made in Section 6.

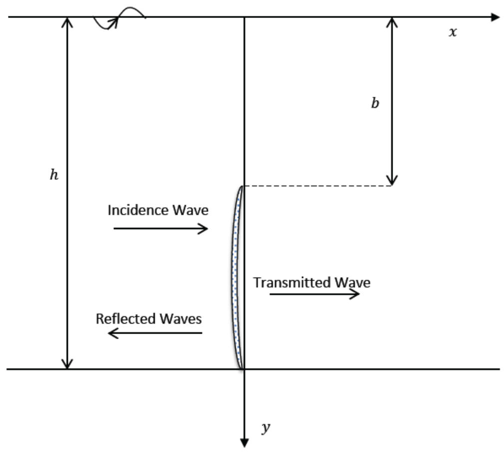
2. MATHEMATICAL FORMULATION

Cartesian coordinates are chosen in which the positive *y*-axis is directed vertically downwards inside the fluid region and the *xz* plane lies along the mean free surface. The fluid occupies the region $0 < y < h$, $-\infty < x, z < \infty$, where *h* is the uniform depth of water. The fluid is assumed to be inviscid, incompressible, homogeneous and the fluid motion is irrotational as well as time harmonic under the action of gravity only. The geometry of the problem is depicted in Fig. 1. If we denote the vertical barrier by L, then the position of the nearly vertical barrier can be represented as

$$S : x = \varepsilon c(y),\ \ y \in L \tag{1}$$



(a) Partially immersed barrier



(b) Submerged bottom standing barrier

Fig. 1. Schematic diagram of nearly vertical barrier.

where *c*(*y*) is a bounded continuous function and vanishes at the ends of the barrier. ε is a dimensionless small parameter representing a measure of maximum deviation from the vertical barrier *L*. Assuming the linearised water wave theory, let a train of incident surface waves be described by the potential function $\mathrm{Re}\{\phi^{inc}(x, y)e^{i\nu z - i\sigma t}\}$ where

$$\phi^{inc}(x, y) = \frac{\cosh k_0(h-y)}{\cosh k_0 h} e^{i\mu x}.$$

Here σ is the angular frequency, $\mu = k_0 \cos\alpha$, $\nu = k_0 \sin\alpha$; k_0 being the unique positive real root of the equation $k \tanh kh = K$ with $K = \sigma^2/g$. α is the angle of incidence of the waves, g is the acceleration due to gravity and t denotes the time.

Let the resulting motion in the fluid be described by the velocity potential $\mathrm{Re}\{\phi(x,y)e^{ivz-i\sigma t}\}$. Then $\phi(x,y)$ satisfies the modified Helmholtz equation (cf. Mandal and Chakrabarti, 2000)

$$(\nabla^2 - v^2)\phi = 0 \quad \text{in the fluid region,} \tag{2}$$

along with the free surface boundary condition

$$K\phi - \frac{\partial \phi}{\partial y} = 0 \quad \text{on } y = 0, \tag{3}$$

the bottom boundary condition

$$\frac{\partial \phi}{\partial y} = 0 \quad \text{on } y = h, \tag{4}$$

the boundary condition on the permeable barrier

$$\frac{\partial \phi}{\partial n} = -ik_0 G\left[\phi(\varepsilon c(y)^+, y) - \phi(\varepsilon c(y)^-, y)\right] \quad \text{on } S. \tag{5}$$

Here n is the outward normal direction and $G(G_r + iG_i)$ is the porous parameter defined by Chwang (1983).

The velocity potential ϕ satisfies the edge condition as given by

$$r^{1/2}\nabla\phi \quad \text{is bounded as } r \to 0 \tag{6}$$

where r is the distance of any fluid particle from either of the sharp edges of the nearly vertical barrier.

Let a train of regular, small-amplitude progressive waves propagate towards the barrier from the direction of $x = -\infty$. When it falls on the barrier, some part of it is transmitted above, below and through the pores of the barrier and rest of it is reflected back. If R and T denote the reflection coefficient and the transmission coefficient respectively, then the radiation conditions at infinity can be written as

$$\phi(x,y) \sim \begin{cases} T\phi^{inc}(x,y) & \text{as } x \to \infty, \\ \phi^{inc}(x,y) + R\phi^{inc}(-x,y) & \text{as } x \to -\infty. \end{cases} \tag{7}$$

Assuming that the parameter ε to be very small, and neglecting the $O(\varepsilon^2)$ terms, the boundary condition on the barrier S can be expressed, in approximate form, on $x = 0^{\pm}, y \in L$, as

$$\frac{\partial \phi}{\partial x}(0^{\pm}, y) - \varepsilon \frac{d}{dy}\{c(y)\frac{\partial \phi}{\partial y}\} = -ik_0 G[\phi(0^+, y) - \phi(0^-, y) + \varepsilon c(y)\frac{\partial \phi}{\partial x}(0^+, y) \tag{8}$$
$$\varepsilon c(y)\frac{\partial \phi}{\partial x}(0^+, y)] \quad \text{for } y \in L.$$

3. PERTURBATION FORMULATION

Due to involvement of the small parameter ε in the boundary condition (8) we take the perturbation expansions for ϕ, R and T in terms of the parameter ε, as given by

$$\left.\begin{aligned} \phi &= \phi_0 + \varepsilon\phi_1 + O(\varepsilon^2), \\ R &= R_0 + \varepsilon R_1 + O(\varepsilon^2), \\ T &= T_0 + \varepsilon T_1 + O(\varepsilon^2). \end{aligned}\right\} \tag{9}$$

After substituting these expansions into the governing Eq. (2), the boundary conditions (3) and (4), together with the edge condition (6) and the conditions at the infinity (7), two boundary value problems for the functions ϕ_0 and ϕ_1 are derived as given below.

3.1 Boundary Value Problem for the Zeroth Order Potential Function ϕ_0

The function $\phi_0(x,y)$ satisfies

$$(\nabla^2 - v^2)\phi_0 = 0 \quad \text{in } -\infty < x < \infty,\ 0 \le y \le h, \tag{10}$$

$$K\phi_0 - \frac{\partial \phi_0}{\partial y} = 0 \quad \text{on } y = 0, \tag{11}$$

$$\frac{\partial \phi_0}{\partial y} = 0 \quad \text{on } y = h, \tag{12}$$

$$\frac{\partial \phi_0}{\partial x}(0^{\pm}, y) = -ik_0 G[\phi_0(0^+, y) - \phi_0(0^-, y)] \text{ on } y \in L \tag{13}$$

$$r^{1/2}\nabla\phi_0 \quad \text{is bounded as } r \to 0. \tag{14}$$

The conditions at the infinity are given by

$$\phi_0(x,y) \sim \begin{cases} T_0\phi^{inc}(x,y) & \text{as } x \to \infty, \\ \phi^{inc}(x,y) + R_0\phi^{inc}(-x,y) & \text{as } x \to -\infty. \end{cases} \tag{15}$$

3.2 Boundary value problem for the first order potential function ϕ_1

The function $\phi_1(x,y)$ satisfies

$$(\nabla^2 - v^2)\phi_1 = 0 \quad \text{in } -\infty < x < \infty,\ 0 \le y \le h, \tag{16}$$

$$K\phi_1 - \frac{\partial \phi_1}{\partial y} = 0 \quad \text{on } y = 0, \tag{17}$$

$$\frac{\partial \phi_1}{\partial y} = 0 \quad \text{on } y = h, \tag{18}$$

$$\begin{aligned} &\frac{\partial \phi}{\partial x}(0^{\pm}, y) - \frac{d}{dy}\{c(y)\frac{\partial \phi_0}{\partial y}(0^{\pm}, y)\} \\ &= -ik_0 G[\phi_1(0^+, y) - \phi_1(0^-, y) \quad \text{on } y \in L, \end{aligned} \tag{19}$$

$$r^{1/2}\nabla\phi_1 \quad \text{is bounded as } r \to 0. \tag{20}$$

The conditions at the infinity are given by

$$\phi_1(x,y) \sim \begin{cases} T_1\phi^{inc}(x,y) & \text{as } x \to \infty, \\ R_1\phi^{inc}(-x,y) & \text{as } x \to -\infty. \end{cases} \tag{21}$$

Note that, if we put $G = 0$ in the Eq. (19), then the condition on an impermeable barrier is retrieved (cf. Mandal and Chakrabarti, 1989).

4. METHOD OF SOLUTION

4.1 Method of Solution for the Zeroth Order Potential Function ϕ_0

We employ the method of Havelock's expansion to obtain the solution for the zeroth order potential function $\phi_0(x,y)$. Thus, we express $\phi_0(x,y)$ satisfying the Eq. (10) and the conditions (11), (12) and (15), as given by

$$\phi_0(x,y)\sim\begin{cases} T_0\phi^{inc}(x,y) \\ +\sum_{n=1}^{\infty}A_n\cos k_n(h-y)e^{-s_n x} & x>0, \\ \phi^{inc}(x,y)+R_0\phi^{inc}(-x,y) \\ +\sum_{n=1}^{\infty}B_n\cos k_n(h-y)e^{s_n x} & x<0, \end{cases} \tag{22}$$

where $s_n^2=k_n^2+\nu^2$ and $k_n(n=1,2,...)$'s are positive real roots of $K+k\tan kh=0$. Let $f(y)$ and $g(y)$ denote the fluid velocity and potential difference across the barrier respectively. Thus

$$f(y)=\frac{\partial\phi_0}{\partial x}(0,y) \qquad 0<y<h \tag{23}$$

and

$$g(y)=\phi_0(0^+,y)-\phi_0(0^-,y) \qquad 0<y<h. \tag{24}$$

The normal velocity of the fluid passing through the pores of the barrier is linearly proportional to the potential difference between the two sides of the barrier (cf. Chwang, 1983) and since the pressure is continuous across the line below the barrier the functions $f(y)$ and $g(y)$ are connected as

$$f(y)=-\mathrm{i}k_0 G g(y) \qquad \text{on } y\in L \tag{25}$$

and

$$g(y)=0 \qquad \text{on } y\in\overline{L}=[0,h]-L. \tag{26}$$

Using (22) the expressions for $f(y)$ and $g(y)$ are obtained as

$$f(y)=\begin{cases} \frac{\partial\phi}{\partial x}(0^+,y)=\mathrm{i}\mu T_0\frac{\cosh k_0(h-y)}{\cosh k_0 h} \\ \qquad -\sum_{n=1}^{\infty}s_n A_n\cos k_n(h-y), \\ \frac{\partial\phi}{\partial x}(0^-,y)=\mathrm{i}\mu(1-R_0)\frac{\cosh k_0(h-y)}{\cosh k_0 h} \\ +\sum_{n=1}^{\infty}s_n B_n\cos k_n(h-y), \qquad 0<y<h \end{cases} \tag{27}$$

$$g(y)=(T_0-R_0-1)\frac{\cosh k_0(h-y)}{\cosh k_0 h} + \sum_{n=1}^{\infty}(A_n-B_n)\cos k_n(h-y), \quad 0<y<h. \tag{28}$$

Employing the Havelock's inversion formula the constants R_0 and T_0, the functions A_n and B_n and the relation between them are determined in terms of the function $f(y)$ as,

$$T_0=1-R_0=-\frac{4\mathrm{i}k_0\cosh k_0 h}{\mu(2k_0h+\sinh 2k_0 h)}\int_0^h f(y)\cosh k_0(h-y)dy, \tag{29}$$

$$A_n=-B_n=-\frac{4k_n}{s_n(2k_nh+\sin 2k_nh)}\int_0^h f(y)\cos k_n(h-y)dy. \tag{30}$$

In our present analysis, Eqs. (29) and (30) which represent the constants R_0 and T_0 and the functions A_n and B_n in terms of the function $f(y)$ do not play much significant role. But from these equations it is possible to obtain relations between the reflection coefficient R_0 and the transmission coefficient T_0, and between the functions A_n and B_n which we will use for further analysis.

Next, employing Havelock's inversion formula on (28), we obtain

$$T_0-R_0-1=\frac{4k_0\cosh k_0h}{2k_0h+\sinh 2k_0h}\int_0^h g(y)\cosh k_0(h-y)dy \tag{31}$$

$$A_n-B_n=\frac{4k_n}{2k_nh+\sin 2k_nh}\int_0^h g(y)\cos k_0(h-y)dy, \tag{32}$$

Now, using the relations between the constants R_0 and T_0 and the functions A_n and B_n obtained from (29) and (30) together with the Eq. (26) we get these constants and functions in terms of the potential difference $g(y)$ as

$$R_0=1-T_0=\frac{2k_0\cosh k_0h}{2k_0h+\sinh 2k_0h}\int_L g(y)\cosh k_0(h-y)dy, \tag{33}$$

$$A_n=-B_n=-\frac{2k_n}{2k_nh+\sin 2k_nh}\int_L g(y)\cos k_n(h-y)dy. \tag{34}$$

Substituting the expression of B_n from (34) into the Eq. (27) and then using the relation (25) an integral equation in $g(y)$ is obtained as

$$\frac{\cosh^2 k_0 h}{2k_0 h+\sinh 2k_0 h}\int_L g(u) M(y,u)du - \frac{ik_0 G}{2} g(y) = \frac{i\mu(1-R_0)}{2}\frac{\cosh k_0(h-y)}{\cosh k_0 h}, \quad y\in L \tag{35}$$

where the kernel $M(y,u)$ is given by

$$M(y,u) = \lim_{\xi\to 0}\frac{2k_0 h+\sinh 2k_0 h}{\cosh^2 k_0 h}\sum_{n=1}^{\infty}\frac{s_n k_n}{2k_n h+\sin 2k_n h}\cos k_n(h-u)\cos k_n(h-y)e^{-k_n\xi}; \tag{36}$$

the exponential term being introduced to ensure the convergence of the series. The above Eq. (35) is a non-homogeneous second kind integral equation for the potential difference across the barrier together with an unknown constant R_0. This equation contains two unknowns namely the potential difference function $g(y)$ and the zeroth order reflection coefficient R_0. To reduce the number of unknowns, we introduce a new function $\mathcal{G}(y)$ as given by

$$\mathcal{G}(y) = \frac{2\cosh^2 k_0 h}{i\mu(1-R_0)(2k_0 h+\sinh 2k_0 h)} g(y), \; y\in L. \tag{37}$$

By virtue of the above function, we rewrite the integral Eq. (35) as

$$\int_L \mathcal{G}(u) M(y,u)du - \frac{ik_0 G(2k_0 h+\sinh 2k_0 h)}{2\cosh^2 k_0 h} = \frac{\cosh k_0(h-y)}{\cosh k_0 h}, \quad y\in L. \tag{38}$$

In order to solve the above integral equation, we employ a single term Galerkin approximation (cf. Roy *et al.*, 2016). This requires $\mathcal{G}(y)$ to be ex-pressed as a product of an unknown constant and a known function. Thus we express $\mathcal{G}(y)$ as

$$\mathcal{G}(y) = a_0 g_0(y), \quad y\in L. \tag{39}$$

The choice of the function $g_0(y)$ depends on the configuration of the barrier and a_0 is the unknown constant to be determined. Substituting the expression for $\mathcal{G}(y)$ from the relation (39), into the integral Eq. (38), we find

$$\int_L a_0 g_0(u) M(y,u)du - \frac{ia_0 k_0 G(2k_0 h+\sinh 2k_0 h)}{2\cosh^2 k_0 h} g_0(y) = \frac{\cosh k_0(h-y)}{\cosh k_0 h}, \quad y\in L. \tag{40}$$

It may be noticed that the solution of the integral Eq. (38) can be derived only when the un-known constant a_0 is determined. To obtain the value of a_0, we multiply the Eq. (40) by $g_0(y)$ on both sides and integrate over L. This gives

$$a_0 = \frac{1}{P}\int_L g_0(y)\frac{\cosh k_0(h-y)}{\cosh k_0 h}dy. \tag{41}$$

where

$$P = \frac{2k_0 h+\sinh 2k_0 h}{\cosh^2 k_0 h}\left\{\left\{\sum_{n=1}^{\infty}\frac{s_n k_n}{2k_n h+\sin 2k_n h}\left[\int_L g_0(y)\cos k_n(h-y)dy\right]^2\right\} - \frac{ik_0 G}{2}\int_L [g_0(y)]^2 dy\right\} \tag{42}$$

We substitute the expression of a_0 from the above Eq. (41) into the Eq. (39) to obtain the expression of $\mathcal{G}(y)$ and then using the relations (37) and (33) we obtain the expression of R_0. in terms of the function $\mathcal{G}(y)$ as follows

$$\frac{iR_0}{(1-R_0)\cos\alpha} = k_0^2\int_L \mathcal{G}(y)\frac{\cosh k_0(h-y)}{\cosh k_0 h}dy. \tag{43}$$

Now, we introduce a constant A as given by

$$\frac{1}{A} = k_0^2\int_L \mathcal{G}(y)\frac{\cosh k_0(h-y)}{\cosh k_0 h}dy. \tag{44}$$

Using the relation (44) the expressions for the constants R_0 and T_0 are determined in terms of A. These are found as

$$R_0 = \frac{\cos\alpha}{\cos\alpha + iA}, \qquad T_0 = 1-R_0 = \frac{iA}{\cos\alpha+iA}. \tag{45}$$

Substituting the expressions for $\mathcal{G}(y)$ and a_0 from (39) and (41) into the Eq. (44), the constant A is found as

$$A = \frac{P}{k_0^2\left[\int_L g_0(y)\frac{\cosh k_0(h-y)}{\cosh k_0 h}dy\right]^2}. \tag{46}$$

To evaluate the value of A for any kind of barrier, we first need to choose an appropriate function $g_0(y)$. Then the expression of A given in (46) is substituted into (45) to obtain the values of R_0 and T_0 respectively.

4.2 Method of Solution for the First Order Potential Function $\phi 1$

In this section we determine the expressions for the first order corrections for the reflection and the transmission coefficients.

To obtain the expression for the first order reflection coefficient R_1, Green's integral theorem is applied to the functions ϕ_0 and ϕ_1 in the region bounded by the lines $y = 0$ $(0 < x \le X)$, $x = X$ $(0 \le y \le h)$, $y = h$ $(-X \le x \le X)$, $x = -X$ $(0 \le y \le h)$, $y = 0$ $(-X \le x < 0)$, $x = 0^-$ $(y \in L)$, $x = 0^+$ $(y \in L)$(with $X > 0$), and a circle of small radius δ with center at $(0,a)$. Denoting by l the contour which bounds the region, we find that

$$\int_l \left(\phi_0\frac{\partial\phi_1}{\partial n} - \phi_1\frac{\partial\phi_0}{\partial n}\right)dl = 0 \tag{47}$$

where n is the outward normal to the line element dl. The free surface condition and the bottom condition satisfied by ϕ_0 and ϕ_1 ensure that there is no contribution to the integral on the left side of the Eq. (47) from the lines y = 0 ($0 < x \le X$ and $-X \le x < 0$) and $y = h$ ($-X \le x \le X$). As both ϕ_0 and ϕ_1 describe outgoing waves as $x \to \infty$, there is no contribution to the integral from the line $x = X(0 \le y \le h)$ as $X \to \infty$. The contribution from the circle with center at (0,a) for partially immersed barrier or (0,b) for bottom standing barrier becomes negligible as its radius $\delta \to 0$. The only contribution arises from the line integral around the lines x = 0 (y ∈ L) and $x = -X(0 \le y \le h)$ as $X \to \infty$. Thus we obtain

$$\frac{\mathrm{i}}{2}\cos\alpha R_1\left[\frac{2k_0h+\sinh 2k_0h}{\cosh^2 k_0h}\right] = \int_L\left[\phi_0(0^+,y)\frac{d}{dy}\left\{c(y)\frac{\partial\phi_0}{\partial y}(0^+,y)\right\} -\phi_0(0^-,y)\frac{d}{dy}\left\{c(y)\frac{\partial\phi_0}{\partial y}(0^-,y)\right\}\right]dy. \quad (48)$$

Using the expression of ϕ_0 $(0^{\pm},y)$ as given in Eq.(22), finally R_1 is determined as

$$R_1 = -\frac{2\mathrm{i}}{\cos\alpha}\left[\frac{\cosh^2 k_0h}{2k_0h+\sinh 2k_0h}\right]\times \left[4k_0^2R_0\int_L c(y)\frac{\sinh^2 k_0(h-y)}{\cosh k_0h}dy + \sum_{n=1}^{\infty}4k_0k_nA_n\int_L c(y)\frac{\sinh k_0(h-y)\sin k_n(h-y)}{\cosh k_0h}dy\right]. \quad (49)$$

Again to derive the expression for the first order correction of the transmission coefficient T_1, Green's integral theorem is applied on the functions $\psi_0(x,y)$ and ϕ_1 (x,y) in the same region mentioned above, where $\psi_0(x,y)$ is the velocity potential function for the problem of scattering of water waves by a thin vertical barrier when a train of surface waves travel from the positive infinity. Thus

$$\frac{\mathrm{i}}{2}\cos\alpha T_1\left[\frac{2k_0h+\sinh 2k_0h}{\cosh^2 k_0h}\right] = \int_L\left[\phi_0(0^-,y)\frac{d}{dy}\left\{c(y)\frac{\partial\phi_0}{\partial y}(0^+,y)\right\} -\phi_0(0^+,y)\frac{d}{dy}\left\{c(y)\frac{\partial\phi_0}{\partial y}(0^-,y)\right\}\right]dy. \quad (50)$$

By employing a similar argument employed above, we find T_1 as

$$\frac{\mathrm{i}}{2}\cos\alpha T_1\left[\frac{2k_0h+\sinh 2k_0h}{\cosh^2 k_0h}\right] = \int_L\left[\phi_0(0^-,y)\frac{d}{dy}\left\{c(y)\frac{\partial\phi_0}{\partial y}(0^+,y)\right\} -\phi_0(0^+,y)\frac{d}{dy}\left\{c(y)\frac{\partial\phi_0}{\partial y}(0^-,y)\right\}\right]dy. \quad (51)$$

For determination of the above expression we use the relation $\psi_0(0^{\pm},y) = \phi_0(0^{\mp},y)$. Using integration by parts, the Eq. (51) simplifies to

$$\frac{\mathrm{i}}{2}\cos\alpha T_1\left[\frac{2k_0h+\sinh 2k_0h}{\cosh^2 k_0h}\right] = \left[c(y)\left\{\phi_0(0^-,y)\frac{\partial\phi_0}{\partial y}(0^+,y) -\phi_0(0^+,y)\frac{\partial\phi_0}{\partial y}(0^-,y)\right\}\right]_L. \quad (52)$$

The right hand side of the Eq. (52) vanishes identically. Thus

$$T_1 \equiv 0. \quad (53)$$

5. NUMERICAL RESULTS AND DISCUSSIONS

Since T_1 vanishes identically and T_0 is connected with R_0 by the relation $T_0 = 1 - R_0$, in this section we only discuss the numerical results for the zeroth order reflection coefficient and for the first order reflection coefficient. A graph for total reflection for a permeable nearly vertical surface piercing barrier with varying porosity is presented at the end of this section.

5.1 The Zeroth Order Reflection Coefficient

To obtain the numerical results for the zeroth order reflection coefficient, first we choose a proper form of the function $g_0(y)$.

Partially Immersed Barrier

Here we choose the function $g_0(y)$ as the explicit solution for the difference of the velocity potential across the barrier for the problem of scattering of water waves by a surface piercing barrier in deep water as given by Ursell (1947) which is of the form

$$g_0(y) = \mathrm{e}^{-Ky}\int_a^y \frac{ue^{Ku}}{\sqrt{(a^2-u^2)}}\mathrm{d}u, \qquad 0 < y < a. \quad (54)$$

We substitute the above expression for $g_0(y)$ into the Eqs. (41) and (46) to obtain the constants a_0 and A numerically. Then substituting the value of A in (45) the numerical values of the zeroth order reflection and transmission coefficients are obtained.

In order to verify the correctness of the numerical results based on the present analysis, we com-pare our results for a surface piercing barrier with those in Porter and Evans (1995). They solved the problem of scattering of water waves by four configurations of thin vertical barrier using multi-term Galerkin approximation. In Fig. 2, the numerical values of $|R_0|$ are plotted against the dimension-less wavenumber k_0a for $G = 0$ and $\alpha = 0$ and for three different values for a/h (= 0.1,0.5,0.9). The present results (denoted by solid lines) show good agreement with those of Porter and Evans (1995) (denoted by '∗'). This gives a partial check on the correctness of the numerical results derived in this paper.

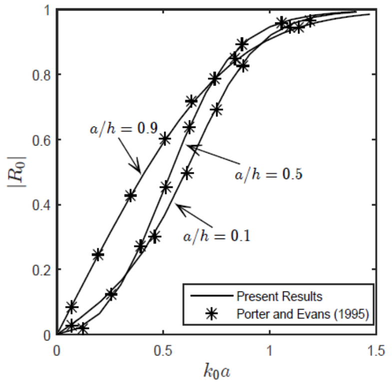


Fig. 2. $|R_0|$ vs. dimensionless wavenumber k_0a for a surface piercing barrier with a/h = 0.1,0.5,0.9, α = 0 and G = 0.

In Fig. 3, we compare the results of our paper with those in Lee and Chwang (2000) for a partially immersed permeable barrier present in water of uniform finite depth h. The figure is plotted for constant values of a/h (= 0.5) and α(= 0). The graphs are drawn against the dimensionless wave number k_0a for several values of G(= 0,0.25,0.5,1). The solid lines and the stars represent the values for the present results and the results obtained by Lee and Chwang (2000) respectively. The stars almost coincide with the solid lines, proving that our results obtained by single term Galerkin approximation are matching with those in Lee and Chwang (2000) who obtained the reflection curves using an eigenfunction matching method. Figure 3 depicts that as the value of G is increased, the curves for $|R_0|$ subside, since the energy is dissipated through the pores of the barrier.

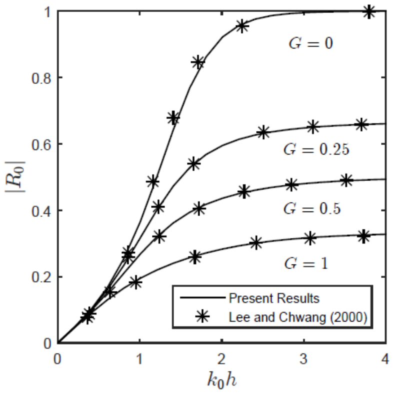


Fig. 3. $|R_0|$ vs. dimensionless wavenumber k_0a for a surface piercing barrier with G = 0,0.25,0.5,1, a/h = 0.5 and α = 0.

In Fig. 4, the absolute value of the zeroth order reflection coefficient against the angle of incidence is depicted for a/h = 0.5 and k_0a = 1.0. The general shape of the curves for $|R_0|$ for different values of G(= 0,0.5,1) is of similar nature, but compressed downwards. This figure depicts that with the increase in the incidence angle of the waves the value of $|R_0|$ decreases. The maximum value for $|R_0|$ is attained for the normal incidence of waves. It is also clear from this figure that the amount of the reflection reduces with the increasing values of G. This happens due to the fact that the increase in the permeability of the barrier causes more energy dissipation by it and as a result amount of reflection decreases.

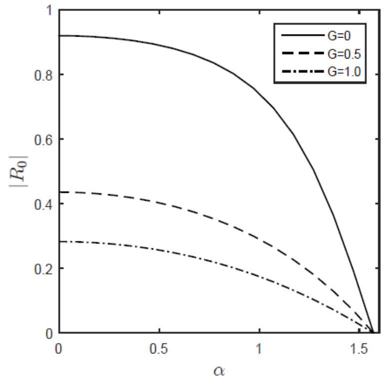


Fig. 4. $|R_0|$ vs. angle of incidence α for a surface piercing barrier with k_0a = 1.0, G = 0,0.5,1 and a/h = 0.5.

Submerged Bottom Standing Barrier

In this case $g_0(y)$ is chosen as the explicit solution for the problem of water wave scattering by a submerged bottom standing barrier in deep water in the case of the normal incidence, and is given by Ursell (1947) as,

$$g_0(y) = \mathrm{e}^{-Ky}\int_b^y \frac{e^{Ku}}{\sqrt{(u^2 - b^2)}}\mathrm{d}u, \quad b < y < h. \quad (55)$$

We substitute the above expression for $g_0(y)$ into the relations (41) and (46) to compute a_0 and A respectively.

In Fig. 5, the variation of $|R_0|$ against the dimensionless wavenumber k_0b for the different values of b/h(= 0.1,0.2,0.4) and for the fixed values of α (= 0) and G (= 0) is displayed. The figure reveals that our results (represented by solid lines) exactly coincide with the corresponding results in Porter and Evans (1995) (represented by '∗') whose analysis was based on a multi-term Galerkin approximation.

In Fig. 6, we analyze the effect of the permeability of the barrier on the reflection coefficient $|R_0|$

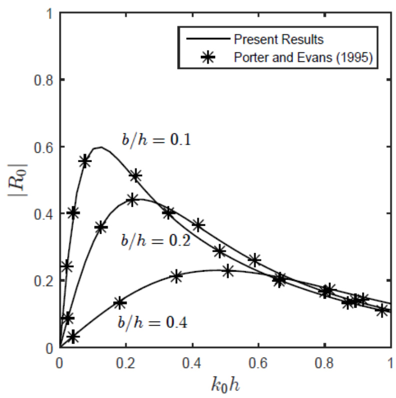


Fig. 5. $|R_0|$ vs. dimensionless wavenumber k_0b for a bottom standing barrier with b/h = 0.1,0.2,0.4, α = 0 and G = 0.

for a vertical bottom standing barrier. The graphs for $|R_0|$ are plotted against the dimensionless wave number k_0h for different values of $G(= 0,0.5,1)$ with $b/h = 0.25$ and $\alpha = 0$. The stars in this figure represent the results of Lee and Chwang (2000) for $|R_0|$. It can be observed that the graphs for |R0|show good agreement with the results of Lee and Chwang (2000). As observed in Figs. 3 and 4, here also the reflection reduces with the increase in the value of porosity.

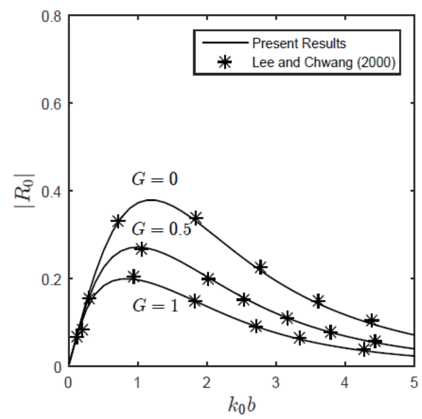


Fig. 6. $|R_0|$ vs. dimensionless wavenumber k_0h for a bottom standing barrier with G = 0,0.25,1, b/h = 0.5 and α = 0.

In Fig. 7, the reflection coefficient $|R_0|$ versus the angle of incidence α is plotted for different values of G with fixed values of b/h (= 0.1) and k_0b (= 0.1). It is clear from these graphs that the values of $|R_0|$ decrease with the increase in angle of incidence as in the case of partially immersed barrier. The common feature of less reflection for higher values of the porosity is also visible.

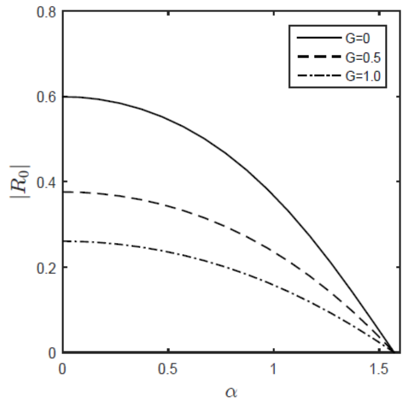


Fig. 7. $|R_0|$ vs. angle of incidence α for a bot-tom standing barrier with different values of G and b/h = 0.1 and k_0b = 0.1.

5.2 The First Order Reflection Coefficient

It may be noted from the Eq. (53) that the first order transmission coefficient vanishes identically for any kind of the barrier. To obtain the numerical results for the first order reflection coefficient an appropriate shape function is required to be chosen.

In this section we discuss the numerical results for the first order reflection coefficient R_1. For computing R_1, we need to choose appropriate shape functions for the two configurations viz. partially immersed and completely submerged barriers. Also, from the Eq. (49), it is evident that to compute R_1 , we need to evaluate an infinite series. We have chosen the truncation size N of this infinite series to be equal to 50, because after N = 50, the values of $|R_1|$ converge up to fourth decimal places.

Partially Immersed Barrier

In order to compute the numerical results for the first order reflection coefficient for a partially immersed barrier, we choose the shape function $c(y)$ as given below:

$$c(y) = y(1-e^{-\lambda(a-y)}), \qquad 0 \le y \le a. \qquad (56)$$

The above form of $c(y)$ ensures that it vanishes at the end points of the barrier and λ is a constant. Now, the expression for $c(y)$ is substituted in the Eq. (48) to evaluate the first order reflection coefficient R_1.

In Fig. 8, the variation of the first order reflection coefficient $|R_1|$ with the different values of a/h(= 0.1,0.5,0.9) is described. The curves of $|R_1|$ are plotted against the dimensionless wavenumber Ka for G = 0, α = 0 and λh = 1, where h is the uniform depth of the water. If a barrier is situated in the water region whose uniform depth is ten times the length of the barrier, then the barrier can be effectively regarded as being submerged in deep water. Thus, we take a/h = 0.1 to compare our results with those in Mandal and Chakrabarti (1989) in which the barriers were placed in deep water. Substituting $c(y)$ as given in (56) in Eq. (3.14) appearing in Mandal and Chakrabarti (1989), we take the data for $|R_1|$ and represent these by '$*$'s. We also compute $|R_1|$ from our present analysis and represent it by a continuous line in Fig. 8. This figure evidences that the stars almost coincide with the continuous line, proving the excellent matching of our results with those in Mandal and Chakrabarti (1989). It is also evident from this figure that as the value of a/h increases, the barrier curbs the water wave propagation as a result of which the total amount of reflection increases.

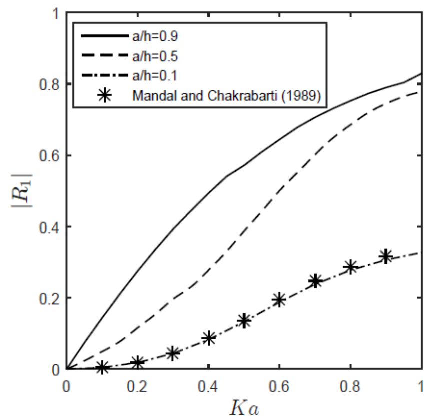


Fig. 8. $|R_1|$ vs. dimensionless wavenumber Ka for a surface piercing barrier with a/h = 0.1,0.5,0.9, G = 0, α = 0 and λh = 1.

In Fig. 9, the effect of the porosity of the barrier on the first order reflection coefficient $|R_1|$ is portrayed for different values of $G(= 0,0.25,0.5,1)$ when $a/h = 0.4$, $\alpha = 0$ and $\lambda h = 1$. The curves show that the values of the first order reflection co-efficient decrease with the increase in G as in the case of zeroth order reflection coefficient.

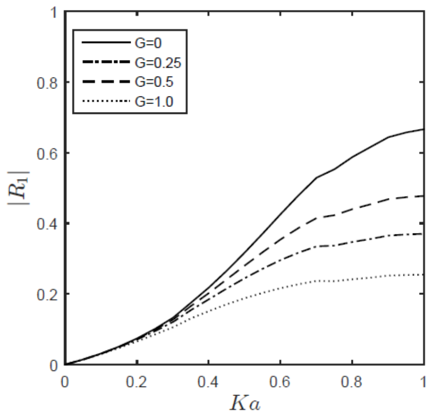


Fig. 9. $|R_1|$ vs. dimensionless wavenumber Ka for a surface piercing barrier with $G = 0,0.25,.5,1$, $a/h = 0.4$, $\alpha = 0$ and $\lambda h = 1$.

In Fig. 10 the graph for the first order reflection coefficient |R1| is plotted as a function of the angle of incidence α and with $Kh = 0.1$, $a/h = 0.1$, $\lambda h = 1$, $G = 0.25$. It is noticed from this figure that the value of $|R_1|$ decreases with the increase in the angle of incidence α.

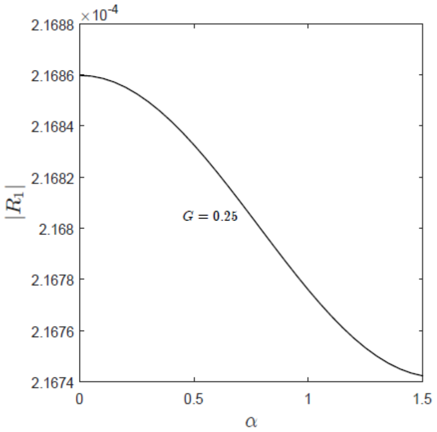


Fig. 10. $|R_1|$ vs. angle of incidence for a surface piercing barrier with $G = 0.25$, $a/h = 0.1$, $Kh = 0.1$ and $\lambda h = 1$.

Submerged Bottom Standing Barrier

We choose the shape function corresponding to the submerged bottom standing barrier, satisfying $c(b) = 0$ and $c(h) = 0$ as

$$c(y) = (h - y)\ (1 - e^{-\lambda(y-b)}), \qquad b \le y \le h. \quad (57)$$

Substituting the above form of $c(y)$ into the Eq. (48), we compute the value of R_1 numerically.

Figure 11 describes the variation of the first or-der reflection coefficient $|R_1|$ for different values of the depth of the submergence of the barrier for normal incidence in the water of uniform depth h. The graphs for $|R_1|$ are depicted against the dimensionless wavenumber Kb varying the dimension-less depth of submergence $b/h(= 0.1,0.2,0.4)$ and keeping G (= 0.1 + i), α (= 0) and λh (= 1) fixed. From this figure, it is clearly noticed that the nature of the curves for $|R_1|$ is the same as that for the zeroth order reflection coefficient $|R_0|$ in Fig. 5.

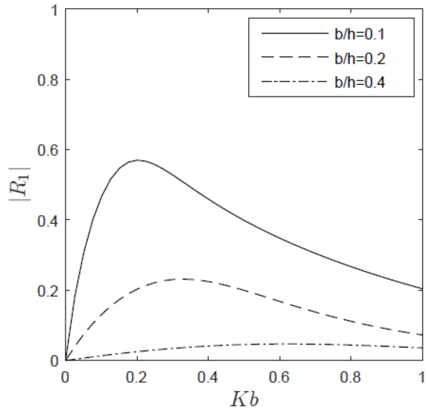


Fig. 11. $|R_1|$ vs. dimensionless wavenumber Kb for a bottom standing barrier with $b/h = 0.1,0.2,0.4$, $G = 0.1 + i$, $\alpha = 0$ and $\lambda h = 1$.

Figure 12 shows the results for $|R_1|$ against the dimensionless wavenumber Kb for various values of $G(= 0,0.25,0.5)$ with $b/h = 0.2$, $\alpha = 0$ and $\lambda h = 1$. This figure reveals that as the porosity of the barrier increases, the value of $|R_1|$ decreases. This phenomenon was also observed earlier in the case of partially immersed barrier in Fig. 9.

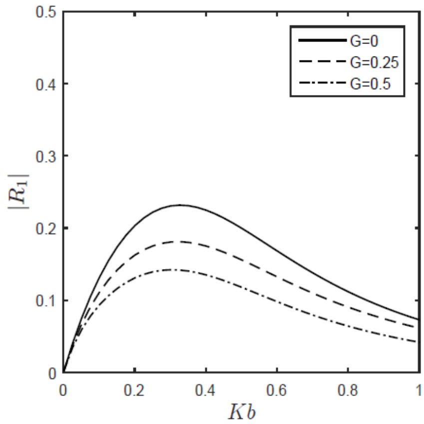


Fig. 12. $|R_1|$ vs. dimensionless wavenumber Kb for a bottom standing barrier with $G = 0,0.25,0.5$, $b/h = 0.2$, $\alpha = 0$ and $\lambda h = 1$.

In Fig. 13, the graph for the first order reflection coefficient $|R_1|$ versus the angle of incidence α is plotted for the fixed values of $Kh = 0.1$, $b/h = 0.9$, $G = 0.25$ and $\lambda h = 1$. It is evident from this figure that the amount of reflection decreases as angle of incidence increases as in the case of partially immersed barrier.

Finally in Fig. 14, we plot the graphs for $|R_0 + \varepsilon R_1|$ for a nearly vertical permeable surface piercing barrier against dimensionless wavenumber Ka for various values of the porous effect parameter $G(= 0, 0.5, 1.0)$. The graphs are drawn considering $a/h = 0.3$, $\alpha = \pi/6$, $\lambda h = 1$ for $\varepsilon = 0.001$. The amount of total reflection is observed to be decreased as the permeability of the barrier increases. This phenomenon is already noticed in cases of the zeroth and first order reflection coefficients. An in-crease in the permeability of the barriers results in dissipation of more wave energy. As the amount of dissipation of wave energy increases, the amount of the reflection decreases.

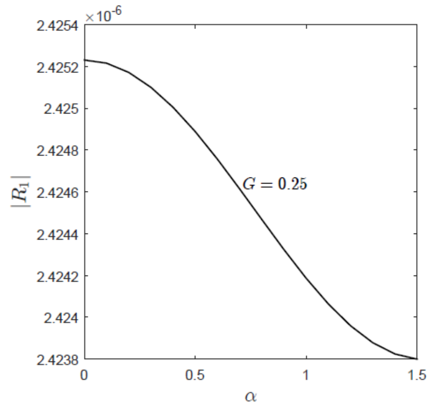


Fig. 13. $|R_1|$ vs. angle of incidence α for bottom standing barrier with $G = 0.25$, $Kh = 0.1$, $b/h = 0.9$ and $\lambda h = 1$.

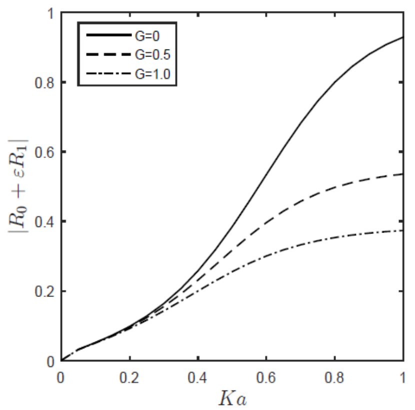


Fig. 14. $|R_0 + \varepsilon R_1|$ vs. dimensionless wavenumber Ka for a surface piercing barrier with $G = 0,0.5,1$, $a/h = 0.3$, $\alpha = \pi/6$, $\lambda h = 1$ for $\varepsilon = 0.001$.

6. CONCLUSION

The problem of scattering of obliquely incident water waves by permeable barriers present in the water of finite depth has been solved in this paper within the framework of the linearised theory of water waves. The barriers are curved having a very small curvature. The governing boundary value problem is solved by employing a perturbation technique. In order to obtain the numerical estimates of the zeroth order reflection and transmission coefficients (R_0 and T_0), a single term Galerkin approximation is applied to the solution of the second kind Fredholm integral equation for the difference in the velocity potential across the barrier. The accuracy of the numerical results for R_0 obtained here has been validated against those obtained by Porter and Evans (1995) for vertical rigid barriers for obliquely incident waves and Lee and Chwang (2000) for vertical permeable barriers for normally incident waves. Green's integral theorem has been employed to obtain the expressions for the first or-der reflection and transmission coefficients (R_1 and T_1). As in the case of impermeable nearly vertical barriers, it has been proved analytically that T_1 also vanishes when the barrier is permeable. Appropriate shape functions are chosen to compute the numerical estimates for R_1 for surface piercing as well as submerged bottom standing barriers. The results for R_1 corresponding to a surface piercing barrier have been compared with those in Mandal and Chakrabarti (1989). Some new results are obtained here for surface piercing and submerged bottom standing porous barriers by varying the permeability of the barriers. The permeability reduces the amplitude of the zeroth order as well as the first order reflection coefficient. The method employed in the present analysis is straight forward and provides an alternative technique to tackle vertical as well as nearly vertical porous plate problems. This method may be employed to study the reflective properties of a nearly vertical porous submerged plate and a porous wall with gap.

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REFERENCES

Banerjea, S., P. Maiti and D. Mondal (2017). Scattering of fexural gravity waves by a two-dimensional thin plate. *Journal of Applied Fluid Mechanics* 10(1), 199-208.

Chakrabarti, A. and L. Vijaya Bharathi (1992). A new approach to the problem of scattering of water waves by vertical barriers. ZAMM-*Journal of Applied Mathematics and Mechanics* 72(9), 415-423.

Chakrabarti, A. and T. Sahoo (1996). Reflection of water waves by a nearly vertical porous wall. *Journal of the Australian Mathematical Society* 37(3), 417-429.

Chwang, A. T. (1983). A porous-wavemaker theory. *Journal of Fluid Mechanics* 132, 395-406.

Chwang, A. T. and A. T. Chan (1998). Interaction between porous media and wave motion. *Annual Review of Fluid Mechanics* 30(1), 53-84.

Dalrymple, R. A., M. A. Losada and P. Martin

(1991). Reflection and transmission from porous structures under oblique wave attack. *Journal of Fluid Mechanics* 224, 625-644.

Dean, W. R. (1945). On the reflexion of surface waves by a submerged plane barrier. *Mathematical Proceedings of the Cambridge Philosophical Society* 41(03), 231-238.

Evans, D. V. (1970). Diffraction of water waves by a submerged vertical plate. *Journal of Fluid Mechanics* 40(03), 433-451.

Gayen, R. and A. Mondal (2014). A hypersingular integral equation approach to the porous plate problem. *Applied Ocean Research* 46, 70-78.

Gayen, R. and A. Mondal (2015). Scattering of water waves by a pair of vertical porous plates. *Geophysical & Astrophysical Fluid Dynamics* 109(5), 480–496.

Huang, C. J., H. H. Chang and H. H. Hwung (2003). Structural permeability effects on the interaction of a solitary wave and a sub-merged breakwater. *Coastal Engineering* 49(1-2), 1-24.

Islam, N., R. Gayen and B. N. Mandal (2018). Wave motion due to a ring source in two superposed fluids covered by a thin elastic plate. *Journal of Applied Fluid Mechanics* 11(4), 1047-1057.

Kaligatla, R. B. and S. R. Manam (2014). Flexural gravity wave scattering by a nearly vertical porous wall. *Journal of Engineering Mathematics* 88(1), 49-66.

Kanoria, M. and B. N. Mandal (2002). Water wave scattering by a submerged circular-arc-shaped plate. *Fluid Dynamics Research* 31(5), 317-331.

Lee, M. M. and A. T. Chwang (2000). Scattering and radiation of water waves by permeable barriers. *Physics of Fluids* 12(1), 54-65.

Macaskill, C. (1979). Reflexion of water waves by a permeable barrier. *Journal of Fluid Mechanics* 95(1), 141-157.

Mandal, B. N. and A. Chakrabarti (1989). A note on diffraction of water waves by a nearly vertical barrier. *IMA Journal of Applied Mathematics* 43(2), 157–165.

Mandal, B. N. and A. Chakrabarti (2000). *Water wave scattering by barriers*. Wit Pr/Computational Mechanics.

Mandal, B. N. and D. P. Dolai (1994). Oblique water wave diffraction by thin vertical barriers in water of uniform finite depth. *Applied Ocean Research* 16(4), 195-203.

Mandal, B. N. and S. Banerjea (1992). Solution of a boundary value problem associated with diffraction of water by a partially immersed nearly vertical barrier. ZAMM- *Journal of Applied Mathematics and Mechanics* 72(10), 517-519.

Martha, S. C. and S. N. Bora (2007). Reflection and transmission coefficients for water wave scattering by a sea-bed with small undulation. ZAMM- *Journal of Applied Mathematics and Mechanics* 87(4), 314-321.

McIver, M. and U. Urka (1995). Wave scattering by circular are shaped plates. *Journal of Engineering Mathematics* 29(6), 575-589.

McIver, P. (1999). Water-wave diffraction by thin porous breakwater. *Journal of Waterway, Port, Coastal, and Ocean Engineering* 125(2), 66-70.

Panda, S. (2016). A study on inviscid flow with a free surface over an undulating bottom. *Journal of Applied Fluid Mechanics* 9(3), 1089-1096.

Parsons, N. F. and P. A. Martin (1992). Scattering of water waves by submerged plates using hypersingular integral equations. *Applied Ocean Research* 14(5), 313-321.

Parsons, N. F. and P. A. Martin (1994). Scattering of water waves by submerged curved plates and by surface-piercing flat plates. *Applied Ocean Research* 16(3), 129-139.

Parsons, N. F. and P. McIver (1999). Scattering of water waves by an inclined surface-piercing plate. *The Quarterly Journal of Mechanics and Applied Mathematics* 52(4), 513-524.

Porter, D. (1972). The transmission of surface waves through a gap in a vertical barrier. *Mathematical Proceedings of the Cambridge Philosophical Society* 71(02), 411-421.

Porter, R. and D. V. Evans (1995). Complementary approximations to wave scattering by vertical barriers. *Journal of Fluid Mechanics* 294, 155-180.

Roy, R., U. Basu, and B. N. Mandal (2016). Oblique water wave scattering by two un-equal vertical barriers. *Journal of Engineering Mathematics* 97(1), 119-133.

Shaw, D. C. (1985). Perturbational results for diffraction of water-waves by nearly-vertical barriers. *IMA Journal of Applied Mathematics* 34(1), 99-117.

Shivakumara, I. S., S. Sureshkumar and N. Devaraju (2012). Effect of non-uniform temperature gradients on the onset of convection in a couple-stress fluid-saturated porous medium. *Journal of Applied Fluid Mechanics* 5(1), 49-55.

Sollitt, C. K. and R. H. Cross (1972). Wave transmission through permeable breakwaters. *Coastal Engineering Proceedings* 1(13).

Tsai, C. H. and D. L. Young (2011). The method of fundamental solutions for water-wave diffraction by thin porous breakwater. *Journal of Mechanics* 27(1), 149-155.

Ursell, F. (1947). The effect of a fixed vertical barrier on surface waves in deep water. *Mathematical Proceedings of the Cambridge Philosophical Society* 43(03), 374-382.

Williams, W. E. (1966). Note on the scattering of water waves by a vertical barrier. *Mathematical*

Proceedings of the Cambridge Philosophical Society 62(03), 507-509.

Yu, X. (1995). Diffraction of water waves by porous breakwaters. *Journal of Waterway, Port, Coastal, and Ocean Engineering* 121(6), 275-282.

Zaghian, R., M. R. Tavakoli, M. Karbasipour and M. N. Ahmadabadi (2017). Experimental study of flow structures of a solitary wave propagating over a submerged thin plate in different angles using PIV technique. *International Journal of Heat and Fluid Flow* 66, 18-26.