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Diffusion of Chemically Reactive Species in Stagnation-Point Flow of a Third Grade Fluid: a Hybrid Numerical Method

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ABSTRACT

The boundary layer flow of a third grade fluid and mass transfer near a stagnation-point with diffusion of chemically reacting species on a porous plate is investigated. Due to a porous plate the suction is taken into an account. Using suitable transformations, the momentum and concentration equations are first transformed into nonlinear ordinary ones and then solved using a hybrid numerical method. This method combines the features of finite difference and shooting methods. The effects of various controlling parameters on the flow velocity, concentration profile, skin friction and rate of mass transfer on surface are analyzed graphically and in tabular form. Comparison of the present results with the previous reported results has been found in excellent agreement.

Keywords: Third grade fluid, Chemical reaction; Stagnation point flow; Porous plate; Numerical solution.

1. INTRODUCTION

Many liquids in industry and technology do not obey the linear relationship between the shear stress and rate of strain and are classified as non-Newtonian in nature. Boundary layer theory has been applied successfully to various non-Newtonian fluids and has gained considerable attention of the researchers in the last few decades. The most common examples of these fluids are polymer solutions, polymer melts, paints, certain oils, blood etc. Furthermore, the equations of motion of non-Newtonain fluids are highly non-linear and one order higher than the Navier-Stokes equations. Due to complexity of these equations, finding an accurate solution is not easy and this poses challenges to scientist, mathematicians and engineers alike.

The stagnation point flow against a flat surface is a classic problem in the field of fluid dynamics whose study can be traced back to the seminal work of Hiemenz (1911) and has been investigated by many researchers. The problem of two-dimensional stagnation point flow for non-Newtonian fluids is an interesting problem both from physical and mathematical point of view. Rajeshwari and Rathna (1962) first analyzed the stagnation-point flow for a viscoelastic second-order fluid and found the solution using Karman-Polhausen method. Beard and Walters (1964) discussed the boundary layer flow of an elastico-viscous fluid near a stagnation point. They reduced the governing PDEs to a single nonlinear ODE using similarity transformations and used the perturbation approach to obtain results upto order one. Rajagopal *et al.* (1984) have presented the Falkner-Skan flows of an incompressible second grade fluid. Later, numerous attempts for stagnation point flow in a viscoelastic fluid using various techniques were made by Teipel (1988), Garg and Rajagopal (1990, 1991), and Pakdemirli and Suhubi (1992). Ariel (1995) studied the stagnation point flow of a second grade fluid with and without suction velocity at the wall numerically using a hybrid method. Later, Ariel (2002) discussed the stagnation-point flow of a Walter's B fluid using an accurate hybrid method (combining the features of the finite difference method and the shooting method). He has augmented boundary conditions upto four in order to solve the fourth-order ODE by imposing an extra condition at the wall based on the governing equation itself. Chamkha (1998) discussed the analysis for the plane and axisymmetric MHD flow near a stagnation point with heat generation.

El-Kabeir (2005) investigated the two-dimensional Hiemenz flow of a micropolar viscoelastic fluid with a transverse magnetic field. Sadeghy *et al.* (2006) gave the numerical analysis for a stagnation point flows of upper-convected Maxwell fluid using both Runge-Kutta method in a shooting scheme and spectral method. Recently, Labropulu and Li (2008) examined the stagnation-point flows of a second grade fluid with slip condition using a quasi-linearization technique. Ishak *et al.* (2008) analyzed the MHD mixed convection stagnation point flow towards a vertical surface in an incompressible micropolar fluid using finite difference method. Li *et al.* (2009) discussed the two-dimensional forced convection stagnation-point flow and heat transfer of a viscoelastic second grade fluid obliquely on an infinite plane wall. Hayat *et al.* (2010) presented the unsteady flow with heat and mass transfer of a third grade fluid bounded by a stretching surface in the presence of chemical reaction. Mukhopadhyay and Bhattacharyya (2012) discussed the numerical analysis of unsteady two-dimensional flow of a Maxwell fluid on a stretching sheet with first order constructive/destructive chemical reaction. Abbas *et al.* (2013) studied the mass transfer in two MHD viscoelastic fluids over a shrinking sheet in porous medium with chemical reaction analytically using homotopy analysis method. Loganathan and Stepha (2013) investigated the effects of chemical reaction and mass transfer on flow of micropolar fluid past a continuously moving porous plate with variable viscosity. Mukhopadhyay and Vajravelu (2013) discussed the diffusion of chemically reactive species in Casson fluid past an unsteady permeable stretching sheet. Choudhary and Das (2014) presented the analysis of visco-elastic MHD free convective flow through porous medium in presence of radiation and chemical reaction with heat and mass transfer.

A literature survey indicates that very few studies describing the boundary layer flow of third grade fluids has been analyzed. Pakdemirli (1992) discussed the boundary layer flow of third grade fluid and the flow equations derived using the special coordinates system. Sajid and Hayat (2007) gave the non-similar series solution for the boundary layer flow of a third order fluid over a stretching sheet using homotopy analysis method (HAM). In other attempts Sajid *et al.* (2006, 2007) investigated two-dimensional and axisymmetric flows of third grade fluids past a stretching surface analytically by employing homotopy analysis method. Recently, Sahoo (2009) examined the Hiemenz flow and heat transfer of an electrically conducting third grade fluid numerically using second-order numerical technique. Very recently, Sahoo (2010) discussed the flow and heat transfer of a third grade fluid past a stretching sheet with partial slip using a hybrid numerical method.

The main objective of this paper is to study the boundary layer flow of a third grade fluid near a stagnation point with suction at the wall and the nth-order chemically reactive species. We transformed the governing momentum and concentration equations into a system of ordinary differential equations by means of suitable transformations and then numerically solved these equations for some values of involving parameters using a hybrid numerical method.

2. FLOW EQUATONS

We consider steady, two-dimensional and incompressible flow of a third grade fluid near a stagnation point over a permeable plate situated at $y = 0$. The fluid is being removed from the plate at a velocity $v_w = -v_0, (v_0 > 0)$ and the velocity of the flow external to the boundary layer is $U(x) = ax,$ where $a(> 0)$ a constant. The mass transfer is the flow along a plate that contains a species slightly soluble in a fluid $B.$ The concentration of the reactant is considered at a constant value C_w at the surface and solubility of A in $B,$ and C_∞ is the concentration of A which is assumed to vanish far away from the surface. Under these assumptions along with boundary layer approximations, the flow is governed by Pakdemirli (1992)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\begin{pmatrix} v\dfrac{\partial^3 u}{\partial y^3} + \dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2} + \\ u\dfrac{\partial^3 u}{\partial x \partial y^2} + \dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial y^2} \end{pmatrix} + \frac{6(\beta_3)}{\rho}\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial \hat{p}}{\partial y} = 0, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_n C^n, \tag{4}$$

where u and v are the velocity components in the $x-$ and $y-$directions, respectively, ρ is the fluid density, ν is the kinematic viscosity, $\alpha_1 \geq 0, \beta_3 \geq 0$ are material constants, C is the concentration of the species of the fluid, D is the diffusion coefficient of the diffusing species in the fluid and k_n the nth order homogeneous and irreversible reaction, n is the reaction-order parameter and $\hat{p}$ is the modified pressure defined as

$$\hat{p} = p - (2\alpha_1 + \alpha_2)\left(\frac{\partial u}{\partial y}\right)^2, \tag{5}$$

and α_2 is a material constant.

The corresponding boundary conditions for this problem are

$$u = 0,\ \ v = -v_0,\ C = C_w\ at\ \ y = 0, \tag{6}$$

$$u \to U(x),\ \ C \to 0\ \ as\ \ y \to \infty. \tag{7}$$

To simplify the flow equations, we define the following nondimensional variables as

$$\eta = \sqrt{\frac{U}{\nu x}}y, \quad \psi = \sqrt{U\nu x}\, f(\eta), \quad C = C_w \Phi(\eta), \tag{8}$$

where ψ is the stream function satisfying $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x.$ Using Eq. (9), the continuity Eq. (1) is satisfied automatically and Eqs. (2) and (4) reduce to

$$f''' + ff'' - f'^2 + 1 + \alpha(2ff''' - ff^{iv} - f''^2) + 6\beta \mathrm{Re}_x f''^2 f''' = 0, \tag{9}$$

$$\Phi'' + Scf\Phi' = Sc\gamma\Phi^n, \tag{10}$$

subject to the boundary conditions

$$f(0) = f_w, \quad f'(0) = 0, \quad f'(\infty) = 1, \quad f''(\infty) = 0, \tag{11}$$

$$\Phi(0) = 1, \quad \Phi(\infty) = 0, \tag{12}$$

where prime denotes differentiation with respect to η . In Eqs. (9)-(11), $\alpha = a\alpha_1^* / \mu$ and $\beta = a^2\beta_3 / \mu$ are the non-dimensional fluid parameters, $\mathrm{Re}_x = ax_2 / \nu$ is the local Reynolds number, $f_w = v_0 / (a\nu)^{1/2}$ is the suction velocity (> 0), $Sc = \nu / D$ is the Schmidt number and $\gamma = k_n C_w^{n-1} / a$ is the chemical reaction rate parameter.

It is worthmentioning (see Cortell (2007) and Hayat *et al.* (2008)) to note that the parameters α, β, Sc, n are positive and γ can be a real number ($\gamma > 0$ indicates destructive chemical reaction; $\gamma < 0$ denotes generative chemical reaction and $\gamma = 0$ for a non-reactive species). In general for the case $n > 1$, the non-linearity in Eq. (10) prevents us from obtaining exact solutions for all values of physical parameters involved and one has to use numerical technique. It may be noted that for Eq. (9) $\beta = 0$, reduces to those of by Ariel (1995) without concentration field.

3. NUMERICAL SOLUTION

In order to solve this system of Eqs. (9, 10), subject to the boundary condition (11, 12), we convert the infinite domain problem to a finite domain [0, 1] by means of the following transformation

$$\zeta = e^{-c\eta}, \tag{13}$$

where c is an adjustable parameter to incorporate the effect of boundary layer thickness for different values of the parameters involved in the equation. The dependent variables are also transformed as

$$F = \eta - f\ \ and\ \ \phi = \Phi, \tag{14}$$

which give rise to the following boundary value problem

$$F''' + (\eta - F)F'' + F'^2 - 2F' + \alpha\left\{\begin{matrix} 2(1 - F')F''' - \\ (\eta - F)F^{iv} + F''^2 \end{matrix}\right\} + 6\beta \mathrm{Re}_x F''^2 F''' = 0, \tag{15}$$

$$\phi'' + Sc(\eta - F)\phi' = Sc\gamma\phi^n, \tag{16}$$

with the boundary conditions

$$F'(0) = 0, \quad F''(0) = 0, \quad F(1) = -f_w, \quad F'(1) = 1, \tag{17}$$

$$\Phi(0) = 0, \quad \Phi(1) = 1. \tag{18}$$

Following Ariel (2008), we convert Eqs. (15)-(16) into system of equations in the following way

$$y_1 = F, \quad y_2 = F', \quad y_3 = F'', \quad z = \phi, \tag{19}$$

and by using the transformation (13), the system of differential equation takes the following form

$$-c\zeta \frac{dy_1}{d\zeta} = y_2, \tag{20}$$

$$-c\zeta \frac{dy_2}{d\zeta} = y_3, \tag{21}$$

$$c\zeta \frac{dy_3}{d\zeta} + \left(c^{-1}\ln\zeta + y_1\right)y_3 + 2y_2 - y_2^2 - \alpha\left\{\begin{matrix} \left(c^{-1}\ln\zeta + y_1\right)c^2\zeta \frac{d}{d\zeta}\left(\frac{dy_3}{d\zeta}\right) \\ -2(1 - y_2)c\zeta \frac{dy_3}{d\zeta} + y_3^2 \end{matrix}\right\} + 6\beta \mathrm{Re}_x y_3^2 c\zeta \frac{dy_3}{d\zeta} = 0, \tag{22}$$

$$c^2\zeta \frac{d}{d\zeta}\left(\zeta \frac{dz}{d\zeta}\right) + Sc\left(c^{-1}\ln\zeta + y_1\right)c\zeta \frac{dz}{d\zeta} = Sc\gamma z^n. \tag{23}$$

The boundary conditions are thus transformed to

$$y_2(0) = 0, \quad y_3(0) = 0, \quad y_1(1) = -f_w, \quad y_2(1) = 1, \tag{24}$$

$$z(0) = 0, \quad z(1) = 1. \tag{25}$$

It may be pointed out that the boundary conditions at $\zeta = 0$ are automatically satisfied in Eqs. (21) and (22). We take mesh on the ζ -axis as

$$\zeta_i = ih\ :\ i = 0, 1, 2, ...N, \tag{26}$$

where N is an integer. By using the following central difference approximation to the derivatives in the Eqs. (22) and (23) which will be written at the i th node as

$$\left\{\zeta \frac{dy_3}{d\zeta}\right\}_i = \zeta_i \frac{y_3^{i+1} - y_3^{i-1}}{2h}, \tag{27}$$

$$\left\{\zeta \frac{d}{d\zeta}\left(\frac{dy_3}{d\zeta}\right)\right\}_i = \frac{\zeta_i}{h}\left[\begin{matrix} \left\{\zeta \frac{dy_3}{d\zeta}\right\}_{i+1/2} \\ -\left\{\zeta \frac{dy_3}{d\zeta}\right\}_{i-1/2} \end{matrix}\right],$$

$$= \zeta_i \frac{\begin{pmatrix} \zeta_{i+1/2}\left(y_3^{i+1} - y_3^{i}\right) \\ -\zeta_{i-1/2}\left(y_3^{i} - y_3^{i-1}\right) \end{pmatrix}}{h^2}. \quad (28)$$

Equations (20) and (21) are obtained at $(i+1/2)$ th node

$$\left\{\frac{dy_1}{d\zeta}\right\}_i = \zeta_{i+1/2}\frac{y_1^{i+1} - y_1^{i}}{h} \quad (29)$$

and thus system of Eqs. (20)-(23) discretized to

$$y_1^{i+1} = y_1^{i} - \frac{y_2^{i} - y_2^{i+1}}{c(2i+1)}, \quad (30)$$

$$y_2^{i+1} = y_2^{i} - \frac{y_3^{i} - y_3^{i+1}}{c(2i+1)}, \quad (31)$$

$$y_3^{i+1} = \begin{bmatrix} \frac{1}{2}ci - \alpha_1 \begin{Bmatrix} c^2 i\left(c^{-1}\ln\zeta_i + y_1^{i}\right) \\ (i+1/2) - \\ ci\left(1 - y_2^{i}\right) \end{Bmatrix} + \\ 3ci\beta \mathrm{Re}_x \left(y_3^{i}\right)^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \frac{1}{2}ciy_3^{i-1} - \left(c^{-1}\ln\zeta_i + y_1^{i}\right)y_3^{i} - 2y_2^{i} + \left(y_2^{i}\right)^2 + \\ \alpha \begin{Bmatrix} c^2 i \begin{pmatrix} c^{-1}\ln\zeta_i \\ +y_1^{i} \end{pmatrix}\begin{pmatrix} -2iy_3^{i} + \\ (i-1/2)y_3^{i-1} \end{pmatrix} + \\ ci\left(1 - y_2^{i}\right)y_3^{i-1} + \left(y_3^{i}\right)^2 \end{Bmatrix} + \\ +3ci\beta \mathrm{Re}_x \left(y_3^{i}\right)^2 y_3^{i-1} \end{bmatrix}, \quad (32)$$

$$z^{i+1} = \left[c^2 i(i+1/2) + \frac{1}{2}Sc\left(c^{-1}\ln\zeta + y_1\right)ci\right]^{-1} \begin{bmatrix} c^2 i\left\{2iz^{i} - (i-1/2)z^{i-1}\right\} + \\ \frac{1}{2}Sc\left(c^{-1}\ln\zeta + y_1\right)ciz^{i-1} + Sc\gamma(z^{i})^n \end{bmatrix} \quad (33)$$

and the boundary conditions are to

$$y_2^0 = 0, \quad y_3^0 = 0, \\ y_1^N = -f_w, \quad y_2^N = 1, \quad (34)$$

$$z^0 = 0, \quad z^N = 1. \quad (35)$$

The algorithm given by Eqs. (30)-(33) subject to the boundary conditions given in Eqs. (34) and (35) is used to calculate the values of $y_1^j,\ y_2^j,\ y_3^j$ and z^j at each $j = 0,1,2,...,N$ stage. The computation order in which these values are calculated is given as: Since it is given in Eq. (34) that $y_2^0 = 0$ and $y_3^0 = 0,$ the value of y_2^1 can be obtained from Eq. (31) if y_3^1 is known. In the same way, the value of y_1^1 can be obtained from Eq. (30) after using the value of y_2^1 and calculated from the previous step, if y_1^0 is also known. Thus $y_2,\ y_1$ and y_3 can be calculated at level $j = 0$ and 1. From this point onward, we can cycle through Eqs. (32)-(30) to calculate the values of $y_2,\ y_1$ and y_3 at all the subsequent levels. Hence if y_3^1 and y_1^0 are known, the values of $y_1,\ y_2$ and y_3 can be computed at all the mesh points by using the above algorithm. Our main objective just narrows down to finding the appropriate values of y_3^1 and y_1^0 such that the terminal boundary conditions $y_1^N = -f_w$, $y_2^N = 1$ are satisfied. For this purpose, any zero finding algorithms in two dimensions may be chosen and we have used analog of secant method in one dimension due to its rapid convergence rate and other advantages. On the other hand, to compute the value at each level of z^j from Eq. (33), only z^1 is missing as $z^0 = 0$ is given. In this case, our objective is again to find the appropriate value of z^1 such that the terminal boundary conditions $z^N = 1$ are satisfied and for this purpose, we have used secant method. The results of the computations through this algorithm are given in next section.

4. RESULTS AND DISCUSSION

The system of nonlinear boundary values problems consisting of Eqs. (9) and (10) along with the boundary conditions (11) and (12) is solved using a hybrid numerical method (Ariel (2002, 2008) and Sahoo (2009)). The obtained numerical results are shown graphically in Figs. 1-8 to see the variations in velocity and concentration profile for various values of the controlling fluid parameters. To show the validity and accuracy of the present method, a comparison of the obtained results with existing numerical results is given and found in good agreement.

Figure 1 shows the effect of mass suction parameter f_w on the dimensionless fluid velocity $f'(\eta)$ in the presence of second and third grade fluids with $\mathrm{Re}_x = 1.0$.

From this Fig. one can see that in case of second grade fluid $(\alpha = 0.2,\ \beta = 0)$ there is not much of change in the fluid velocity $f'(\eta)$ as the value of suction parameter is increased form zero. This is because for this value of second grade fluid parameter $\alpha = 0.2,$ the suction boundary layer is a nonstarter, and it never develops even we increase the value of f_w beyond the values used in this Fig. It is also noted that this change in the fluid velocity $f'(\eta)$ is larger in case of third grade fluid $(=1)$ as we increase the values of the suction parameter f_w (see Ariel (1995), and the present results are agreed with those reported by Ariel (1995, 2002) with $\beta = 0.$ Figure 2 illustrates the variations of the fluid velocity $f'(\eta)$ for several values of viscoelastic fluid (second grade fluid) parameter α with $f_w = 10$

and $\beta = 0.$ Form this Fig., it is evident that the momentum boundary layer effects due to suction velocity are quite obvious in the absence of viscoelasticity $(\alpha = 0)$ and as we increase the viscoelasticity the velocity profiles become flat. It is further noted from this Fig. that as we take the value of $\alpha > 1,$ the effects of suction at the wall can hardly be recognized, the present results agree with the results reported by Ariel (1995) with $\beta = 0.$

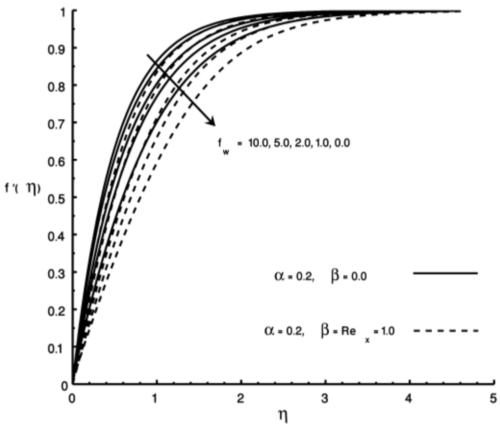

Fig. 1. Effects of suction parameter f_w on the velocity $f'(\eta)$ verses η : solid lines for second grade fluid $(\beta = 0)$ and: dashed lines for third grade fluid $(\beta \neq 0)$.

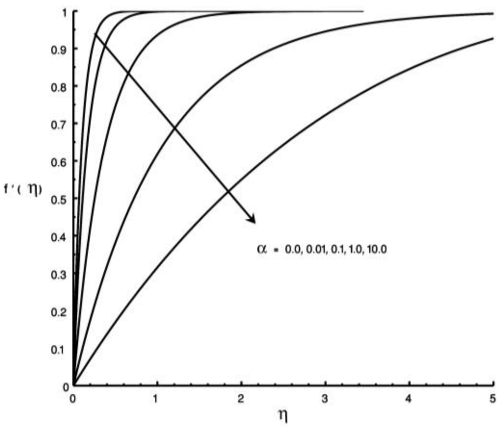

Fig. 2. Effects of second grade parameter α on the velocity $f'(\eta)$ verses η : for case of second grade fluid $(\beta = 0)$ and $f_w = 10.$

Figure 3 gives the change of the fluid velocity $f'(\eta)$ for different values of second grade (viscoelastic) parameter α by keeping $f_w = 10$ and $\beta = 0.$ It is noticed from this Fig. that as increase in the value of viscoelasticity is to decrease the fluid velocity whereas the momentum boundary thickness increases as α increases. It is also found that there is not much of variation in the velocity profiles for very large values of α .

Figure 4 presents the effects of third grade fluid parameter β on the fluid velocity $f'(\eta)$ with $f_w = 1 = \mathrm{Re}_x$ and $\alpha = 2$ are fixed. From this Fig., it is clear that the fluid velocity $f'(\eta)$ decreases with an increasing value of third grade parameter $\beta.$ This is due to the fact that the shear thickening effects which are increased by increasing value of third grade parameter. It is also noted that the effects of third grade parameter β is to increase the momentum boundary layer thickness.

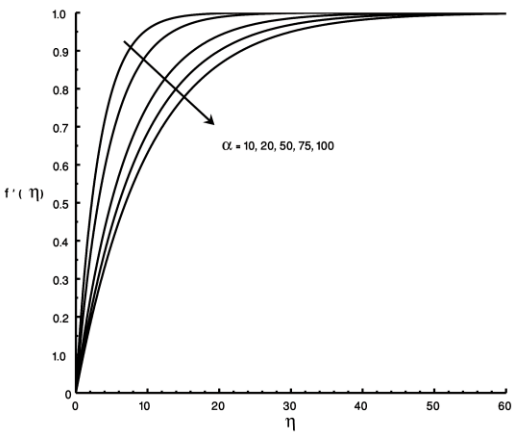

Fig. 3. Effects of viscoelastic or second grade parameter α on the velocity $f'(\eta)$ verses η : for second grade fluid $(\beta = 0)$ and $f_w = 10$.

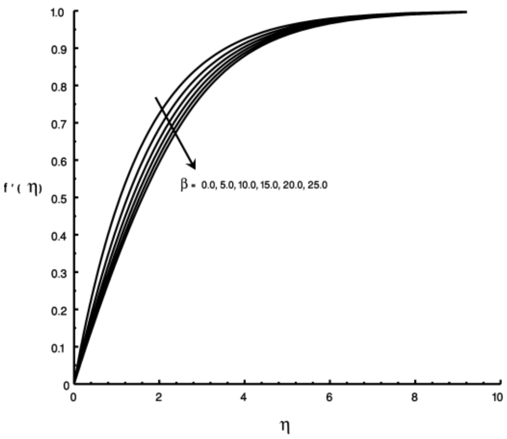

Fig. 4. Effects of third grade parameter β on the velocity $f'(\eta)$ verses η when $f_w = 1 = \mathrm{Re}_x$ and $\alpha = 2.$

The effect of the Schmidt number Sc on the species concentration profile $\Phi(\eta)$ for destructive $(\gamma > 0)$ and generative $(\gamma < 0)$ chemical reaction parameter is shown in Fig. 5 with all other fluid parameters are fixed. As the Schmidt number Sc is the ratio between a viscous diffusion rate and a molecular diffusion rate, so the species concentration of reactants depends on the Schmidt number Sc From this Fig., it is evident that the effect of Schmidt number is to decrease species concentration profile and this change is very slowly for higher values of Sc. It is also noticed that an increase in the Schmidt number Sc produces decrease in the concentration boundary larger thickness, associated with the reduction of the species concentration profile.

Figure 6 gives the variation of the species concentration field $\Phi(\eta)$ for various value of destructive /generative chemical reaction parameter γ with all other involved parameters fixed. One can see from this Fig. that the increase of chemical reaction parameter γ is to decrease the species concentration profile $\Phi(\eta)$ in the boundary layer. It is further noted that the concentration of the species value of 1 is decreased to reach the minimum value of zero at the end of the boundary layer and this situation is noted for all value of γ.

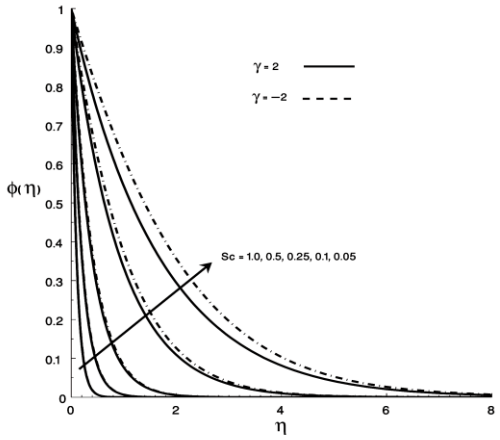

Fig. 5. Effects of the Schmidt number Sc on the concentration field $\phi(\eta)$ verses η : solid lines for parameter $\gamma > 0$, and dashed lines for generative parameter $\gamma < 0$: when $\alpha = 2, \beta = \mathrm{Re}_x = 1, n = 2$ and $f_w = 10$.

Figure 7 illustrates the effect of mass suction parameter f_w on the species concentration field $\Phi(\eta)$ in case of destructive chemical reaction parameter $(\gamma = 2.0)$. From this Fig., we can see that the concentration boundary layer thickness decreases with an increase of suction parameter f_w. This is because of the fact that for external free stream velocity, the velocity of the fluid is increased due to suction, and this leads to the decrease in species concentration profile.

Figure 8 shows the effect of the reaction-order parameter n on the species concentration field $\Phi(\eta)$ for both destructive and generative chemical reaction parameter γ with all other parameters fixed. It is noticed from this Fig. that the increase in the reaction-order parameter n is to increase the fluid concentration in case of destructive chemical reaction $(\gamma > 0)$, while an opposite trend is noted for the case of generative chemical reaction $(\gamma < 0)$ which occurs with the results reported by Cortell (2007).

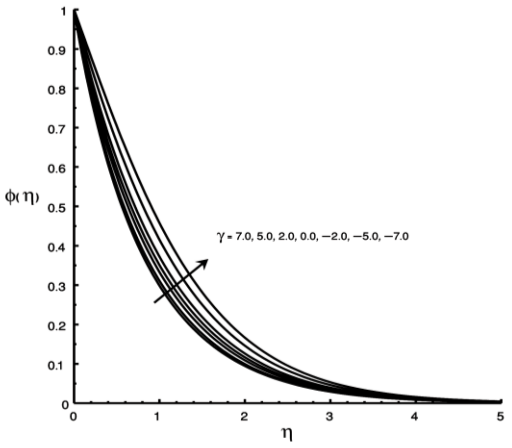

Fig. 6. Effects of the destructive/generative chemical reaction parameter γ on the concentration field $\phi(\eta)$ verses η : for case of third grade fluid, when $\beta = \mathrm{Re}_x = 1, \alpha = 2,$ $f_w = 10,\ Sc = 0.1$ and $n = 2.$

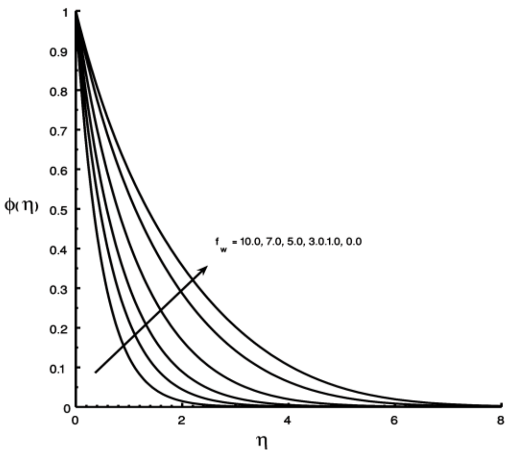

Fig. 7. Effects of the suction parameter f_w on the concentration field $\phi(\eta)$ verses η : for case of destructive chemical reaction $\gamma > 0$, when $\alpha = 2,\ \beta = \mathrm{Re}_x = 1,\ n = 2,\ Sc = 0.2$ and $\gamma = 2.$

Table 1 shows the numerical values of $f''(0)$ for various values of viscoelastic/second grade parameter α and two values of f_w when $\beta = 0.$

It is noticed from this table that by increasing the viscoelastic parameter α is to decrease the wall shear stress, where as the value of $f''(0)$ decrease with an increase of f_w.

It is also worthwhile to note that the present results are to be found in a good agreement with those reported by Ariel (1995, 2002) and Labropulu and Li (2008).

Table 1 Numerical values of $f''(0)$ for second grade parameter α and two values of f_w when $\beta = 0$.

α	$f_w = 0$			$f_w = 10$	
	Ariel (2002)	Labropulu and Li (2008)	Present results	Ariel (1995)	Present results
0	1.232588	1.23259	1.23259	10.193554	10.1941
0.001				9.330114	9.330234
0.002				8.693450	8.693451
0.005				7.442309	7.442481
0.01				6.276552	6.276557
0.02				5.072516	5.072580
0.05				3.629950	3.629960
0.1	1.134114	1.13425	1.134121	2.734932	2.734951
0.2	1.058131	1.05818	1.058141	2.023021	2.023042
0.5	0.902500	0.90248	0.902531	1.331220	1.331286
1	0.752766	0.75276	0.752780	0.960444	0.960419
2	0.596769	0.59677	0.596780	0.688459	0.688449
5	0.412885	0.41288	0.412890	0.440519	0.440517
10	0.302828	0.30283	0.302829	0.313218	0.313217
20	0.218554	0.21857	0.218550	0.222283	0.222282
50	0.140077	0.14008	0.140080	0.140989	0.140989
100	0.099515	0.09952	0.099515	0.099819	0.099819
500	0.044677	0.04469	0.044677		
1000	0.031607		0.031607		

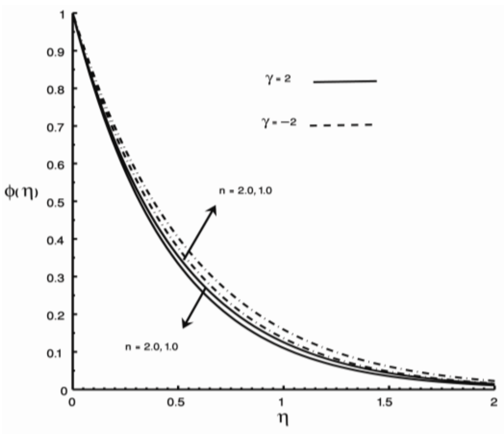

Fig. 8. Effects of the reactor order parameter n on the concentration field $\phi(\eta)$ verses η : solid lines for parameter $\gamma > 0$, and dashed lines for parameter $\gamma < 0$: when $\beta = \mathrm{Re}_x = 1, \alpha = 2, f_w = 10, Sc = 0.2.$

Table 2 gives the numerical values of $f(\eta), f'(\eta)$ and $f''(\eta)$ versus η when $\alpha = 0$ and $\beta = 0.$ It is evident from this table that the present numerical results are found to be in excellent agreement with those discussed by Wu *et al.* (2005). Table 3 shows the numerical values of concentration gradient at the wall $\Phi'(0)$ for several values of $\alpha, \beta, Sc, \gamma$ and three values of reaction-order parameter n when $f_w = 5.$ It is found from this table that rate of mass transfer at the wall $\Phi'(0)$ decreases by increasing α and $\beta,$ whereas it is increased with an increase of $Sc,$ γ and $n.$

5. CONCLUSIONS

In the present analysis, the two-dimensional stagnation point flow of a third grade fluid with diffusion of chemically reacting species and uniform suction is considered. The governing nonlinear boundary layer equations are solved numerically using a hybrid numerical method. The influences of several involving fluid parameters of interest on the flow velocity and species of concentration profile are analyzed and shown graphically. We found that the fluid velocity in the boundary layer thickness decreases by increasing of n in case of destructive chemical reaction parameter $(\gamma > 0).$ From this investigation; we hope that the numerical results obtained will not only give useful information for application, but also provide a compliment to the previous studies.

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Table 2 Numerical values of $f(\eta), f'(\eta)$ and $f''(\eta)$ with η in the case of Newtonian fluid when $\alpha = f_w = \beta = 0$.

η	Wu *et al.* (2005)			Present results		
	f	f'	f''	f	f'	f''
0	0.000	0.000	1.1233	0.000	0.000	1.1233
0.2	0.023	0.227	1.034	0.023	0.227	1.034
0.4	0.088	0.414	0.846	0.088	0.414	0.846
0.6	0.187	0.566	0.675	0.187	0.566	0.675
0.8	0.312	0.686	0.525	0.312	0.686	0.525
1	0.459	0.778	0.398	0.459	0.778	0.398
1.2	0.622	0.847	0.294	0.622	0.847	0.294
1.4	0.797	0.897	0.211	0.797	0.897	0.211
1.6	0.980	0.932	0.147	0.980	0.932	0.147
1.8	1.169	0.957	0.100	1.169	0.957	0.100
2	1.362	0.973	0.066	1.362	0.973	0.066
2.2	1.558	0.984	0.042	1.558	0.984	0.042
2.4	1.755	0.991	0.026	1.755	0.991	0.026
2.6	1.954	0.995	0.016	1.954	0.995	0.016
3.8	2.153	0.997	0.009	2.153	0.997	0.009
3.6	2.952	1.000	0.001	2.952	1.000	0.001
4.4	3.75	1.000	0.000	3.75	1.000	0.000

Table 3 Numerical values of concentration gradient at the wall $\phi'(0)$ for $\alpha, \beta, Sc, \gamma$ and three values of n when $f_w = 5$.

α	β	Sc	γ	$n=1$	$n=2$	$n=3$
0.1	1	1	1	5.2260	5.1365	5.1049
0.2				5.2243	5.1347	5.1030
0.5				5.2196	5.1297	5.0979
1.0				5.2145	5.1237	5.0918
2.0				5.2092	5.1179	5.0858
5.0				5.2031	5.1111	5.0789
1	0.1			5.2156	5.1249	5.0932
	0.2			5.2154	5.1248	5.0931
	0.5			5.2151	5.1244	5.0927
	1.0			5.2145	5.1237	5.0920
	2.0			5.2134	5.1228	5.0908
	5.0			5.2111	5.1201	5.0881
2	1	0.2		1.2162	1.1544	1.1295
		0.5		2.7208	2.6396	2.6097
		1.0		5.2145	5.1237	5.0918
		1.5		7.7094	7.6153	7.5828
		2.0		10.205	10.1098	10.077
		1	0.0	5.0203	5.0203	5.0202
			0.5	5.1164	5.0694	5.0335
			1.0	5.2092	5.1179	5.0858
			2.0	5.3862	5.2130	5.1503
			5.0	5.8628	5.4858	5.3384
			10.0	6.5354	5.9045	5.6355

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