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Least Square Method for Porous Fin in the Presence of Uniform Magnetic Field

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ABSTRACT

In this study, the Least Square Method (LSM) is a powerful and easy to use analytic tool for predicting the temperature distribution in a porous fin which is exposed to uniform magnetic field. The heat transfer through porous media is simulated using passage velocity from the Darcy's model. It has been attempted to show the capabilities and wide-range applications of the LSM in comparison with a type of numerical analysis as Boundary Value Problem (BVP) in solving this problem. The results reveal that the present method is very effective and convenient, and it is suggested that LSM can be found widely applications in engineering and physics.

Keywords: Least Square Method; Porous fin; Magneto hydrodynamic.

NOMENCLATURE

A	cross-sectional or profile area,
d	diameter of fin,
h	convection heat transfer coefficient
h_b	convection heat transfer coefficient at the base
k	thermal conductivity
K	permeability of the porous fin
L	fin length,
P	fin perimeter,
q_{ideal}	ideal fin heat transfer rate,
q_f	fin heat transfer rate,
J	total current intensity
Jc	conduction current intensity
B_0	magnetic field intensity
Cp	specific heat
Ra	modified Rayleigh number
Rd	radiation–conduction parameter
V_w	average velocity of the fluid passing through the fin at any point
Nc	convection parameter
Nr	surface radiation
kr	thermal conductivity ratio,
H	Hartman number
T	temperature,
x	height coordinate,
δ	fin thickness,
m	mass flow rate
α	thermal diffusivity
β_R	Roseland extinction coefficient
$\tilde{\varepsilon}$	porosity
ε	emissivity
ε_a	emissivity of fin at the radiation sink temperature
η	fin efficiency
ρ	density of the fluid
ρ_ε	electrical density
θ	dimensionless temperature
θ_b	dimensionless radiation temperature
a	ambient conditions
b	base of fin
eff	porous properties
f	fluid properties
s	solid properties
u	axial velocity

1. INTRODUCTION

Fins are frequently used in many heat transfer applications to improve performance. In the other hand, for many years, High rate of heat transfer with reduced size and cost of fins are main targets for a number of engineering applications such as heat exchangers, economizers, super heaters, conventional furnaces, gas turbines, etc. Some engineering applications such as airplane and motorcycle also require lighter fin with higher rate of heat transfer. Increasing the heat transfer mainly depend on heat transfer coefficient (h), surface area available and the temperature difference between surface and surrounding fluid. However, this requirement is often justified by the high cost of the high-thermal-conductivity metals, that cost of high thermal conductivity metals is also high. Fin is porous to allow the flow of infiltrate through it. Extensive research has been done in this area and many references are available especially for heat transfer in porous fins. Described below are a few papers relevant to the study described herein. The theoretical study of MHD has been a subject of great interest due to its widespread applications, such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. For instance, MHD induced in rockets can improve heat transfer through porous fins, located on rocket surface. On the effect of MHD flow, although there are many studies regarding the free convection regime, there are only a few regarding the mixed convection regime. Chamkha *et al*. (2004) studied the effects of localized heating (cooling), suction (injection), buoyancy forces, and magnetic field for the mixed convection flow on a heated vertical plate. Aldoss *et al*. (1996) investigated the effect of MHD on heat transfer from a circular cylinder. For more information, some of studies in the MHD flow studies can be found in Refs. (Masood *et al*. 1996; Shehzad and Ali 2012; Aziz 2014). Nonlinear problems and phenomena play an important role in applied mathematics, physics, engineering and other branches of science specially some heat transfer equations. Except for a limited number of these problems, most of them do not have precise analytical solutions. Therefore, these nonlinear equations should be solved using approximation methods. Perturbation techniques are too strongly dependent upon the so-called ‘‘small parameters’’ (Nayfeh 2000). Other many different methods have introduced to solve nonlinear equation such as the δ-expansion method(Ganji and Hashemi Kachapi 2011a), Adomian’s decomposition method Ganji and Hashemi Kachapi 2011b), Homotopy Perturbation Method (He 2000, 2005a,b; Torabi *et al.* 2011; Esmaeilpour *et al*. 2009; Ganji *et al*. 2014; Ganji 2006), Variational Iteration Method (Ganji and Sadighi 2005; Singh *et al*. 2011; He 2006, 2007; Momani and Abuasad 2006; Ganji 2007a,b, 2009; He 1998, 1999), Homotopy analysis method(Ganji and Mohseni Languri 2011; Liao 2003 a, b, 2004) and Least Square Method (Aziz and Bouaziz 2011; Hatami *et al*. 2013).

In this work, we have applied LSM to find the approximate solution of nonlinear differential equations governing on porous fin is exposed to uniform magnetic field. Results demonstrate that LSM is simple and accuracy compared with the BVP as a numerical method.

2. General Guidelines

As shown in Fig. 1, a rectangular fin profile is considered. The dimensions of the fin are length *L*, width *W* and thickness *t*. The cross section area of the fin is constant. This fin is porous to allow the flow of infiltrate through it (Taklifi *et al*. 2011). For the sake of simplify of the solution. The following assumptions are made to solve this problem. The porous medium is homogeneous, isotropic, and saturated with a single-phase fluid. Both the fluid and the solid matrices have constant physical properties except the density in the buoyancy term where Boussenesq approximation is used. The temperature inside the fin is only a function of *x*. The interactions between the porous medium and the clear fluid can be simulated by the Darcy formulation. In order to reduce the complexity of the problem of radiative heat flux, the porous medium is assumed to behave as an optically thick gas. A uniform magnetic field is applied in y-direction as depicted in Fig. 1. It is assumed that the induced magnetic field, the imposed magnetic and electrical fields, and the induced electrical field due to polarization are negligible. Now applying energy balance equation at steady state condition (Taklifi *et al*. 2011; Khani *et al*. 2009) to the slice segment of the fin of thickness ΔX

$$\begin{aligned} q_{(X)} - q_{(X+\Delta X)} &= \dot{m} c_p \left(T_{(X)} - T_\infty\right) \\ &+ h P \Delta x (1-\tilde{\varepsilon})\left(T_{(X)} - T_\infty\right) + \frac{J_c \times J_c}{\sigma} \\ &+ P \Delta x \sigma_{St} \varepsilon \left(T_{(X)}^4 - \frac{\alpha}{\varepsilon} T_\infty^4\right) \end{aligned} \tag{1}$$

Where J_c is conduction current intensity that can be explained as:

$$J_c = \sigma\left(E + V \times B\right) \tag{2}$$

And J is total current intensity which can be stated as:

$$J = J_c + \rho_\varepsilon V \tag{3}$$

The mass flow rate of the fluid passing through the porous material can be written as:

$$\dot{m} = \rho \overline{\vartheta_w} \Delta x w \tag{4}$$

The value of $\overline{\vartheta}_w$ should be estimated from the consideration of the flow in the porous medium. From the Darcy’s model we have:

$$\overline{\vartheta_w} = \frac{g k \beta}{\upsilon}\left[T(x) - T_\infty\right] \tag{5}$$

The energy flux vector of combined radiation and conduction at the base of the fin can be expressed as

$$q_{fin\ base} = q_{conduction} + q_{radiation}\ , \tag{6}$$

Where the conduction term can be expressed, using Fourier’s law of conduction, as

$$q_{conduction} = -k_{eff}\ A_b \frac{dT}{dx}\ , \tag{7}$$

And the radiation heat flux term is expressed, based on the Rosseland diffusion approximation proposed, as

$$q_{radiation} = -\frac{4\sigma_{st}}{3\beta_R}\frac{dT^4}{dx} \tag{8}$$

Substitution of Eqs.(6) to (8) into Eq. (1) gives

$$\frac{d}{dx}\left[\frac{dT}{dx}+\frac{4\sigma}{3\beta_R k_{eff}}\frac{dT^4}{dx}\right]=\frac{\rho c_p g k\beta}{b\upsilon k_{eff}}\left(T_{(x)}-T_\infty\right)^2 + \frac{h\,p\,(1-\tilde{\varepsilon})}{k_{eff}}\left(T_{(x)}-T_\infty\right)+\frac{J_c\times J_c}{\sigma k_{eff}A_b}+\frac{\sigma_{st}\,\varepsilon}{k_{eff}}\frac{p}{A_b}\left(T_{(x)}^4-T_\infty^4\right) \tag{9}$$

Where

$$\frac{J_c\times J_c}{\sigma}=\sigma B_0^2 u^2 \tag{10}$$

In the situation where the temperature differences within the flow are assumed to be sufficiently small, then the term T^4 may be expressed as a linear function of temperature

$$T^4 = T_\infty^4 + 4T_\infty^3(T-T_\infty) + 6T_\infty^2(T-T_\infty)^2+...\cong 4T_\infty^3\,T-3T_\infty^4 \tag{11}$$

Using some simplifications and introducing the following dimensionless parameters:

$$\theta=\frac{T_{(x)}-T_\infty}{T_b-T_\infty},\quad X=\frac{x}{b},\quad \theta_b=\frac{T_b}{T_\infty} \tag{12}$$

By substituting them into Eq. (9) and using Eq. (11) yields

$$(1+4Rd)\theta''(X)-Ra^*\theta^2-Nc\,(1-\tilde{\varepsilon})\,\theta(X) - Nr\,\theta(X)-H\,\theta(X)=0 \tag{13}$$

Where

$$Ra=\frac{g\,k\,\beta(T_b-T_\infty)\,b}{\alpha\upsilon k_r},\quad Nc=\frac{p\,b\,h}{A_b\,k_{eff}},\quad Nr=\frac{4\sigma_{st}\,bT_\infty^3}{k_{eff}},\quad H=\frac{\sigma B_0^2 u^2}{k_{eff}A_b},\quad Rd=\frac{4\sigma_{st}\,T_\infty^3}{3\beta_R\,k_{eff}} \tag{14}$$

Where *Ra* is Modified Rayleigh number, *Nc* is a convection–conduction parameter, *Nr* is a Surface-ambient radiation parameter and *H* is a Hartman parameters and *Rd* is a Radiation–conduction parameter. In this research we study finite-length fin with insulated tip. For this case, the fin tip is insulated so that there will not be any heat transfer at the insulated tip and boundary condition will be,

$$\theta(0)=1\quad,\quad \theta'(1)=0 \tag{15}$$

2.1 Least Square Method (LSM)

There existed an approximation technique for solving differential equations called the Least Square Method (LSM). Suppose a differential operator D is acted on a function u to produce a function P (Hatami *et al*. 2013):

$$u\cong\tilde{u}=\sum_{i=1}^{n}C_i\varphi_i \tag{16}$$

It is considered that u is approximated by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is,

$$D(u(x))=p(x) \tag{17}$$

Now, when substituted into the differential operator D, the result of the operations is not, in general, p(x). Hence an error or residual will exist:

$$E(x)=R(x)=D(\tilde{u}(x))-p(x)\neq 0 \tag{18}$$

The notion in the collocation is to force the residual to zero in some average sense over the domain. That is:

$$\int_x R(x)\,W_i(x)=0\qquad i=1,2,...,n \tag{19}$$

Where the number of weight functions W_i is exactly equal the number of unknown constants C_i in $\tilde{u}$. The result is a set of n algebraic equations for the unknown constants C_i. If the continuous summation of all the squared residuals is minimized, the rationale behind the LSM's name can be seen. In other words, a minimum of

$$S=\int_X R(x)R(x)dx=\int_X R^2(x)dx \tag{20}$$

In order to achieve a minimum of this scalar function, the derivatives of S with respect to all the unknown parameters must be zero. That is,

$$\frac{\partial S}{\partial c_i}=2\int_X R(x)\frac{\partial R}{\partial c_i}dx=0 \tag{21}$$

Comparing with Eq. (21), the weight functions are seen to be

$$W_i=2\frac{\partial R}{\partial c_i} \tag{22}$$

However, the "2" coefficient can be dropped, since it cancels out in the equation. Therefore the weight functions for the Least Squares Method are just the derivatives of the residual with respect to the unknown constants

$$W_i=\frac{\partial R}{\partial c_i} \tag{23}$$

$$W_i=\frac{\partial R}{\partial c_i} \tag{24}$$

2.2 Application of LSM on problem

Because trial function must satisfy the boundary conditions in Eq. (15), each term in trial function (except the first term) should be in polynomial form $\left[x-\left(1/(n+1).\,x^{n+1}\right)\right]$ to satisfy the boundary condition. So it will be considered as,

$$\theta(X)=1+c_1\left(X-\frac{1}{2}X^2\right)+c_2\left(X-\frac{1}{3}X^3\right)+c_3\left(X-\frac{1}{4}X^4\right)+c_4\left(X-\frac{1}{5}X^5\right) \quad (25)$$

It is necessary to inform that other trial functions which satisfy the boundary condition of Eq. (15) can be acceptable for this problem; also its accuracy can be improved by increasing the number of its terms. By introducing this equation to the Eq. (13), residual function will be found:

$$\begin{aligned}
&R(c_1,c_2,c_3,c_4,X)=-Ra\,c_3^2\,X^2+\frac{1}{2}Ra\,c_3^2\,X^5\\
&-\frac{1}{16}Ra\,c_3^2\,X^8-Ra\,c_4^2\,X^2+\frac{2}{5}Ra\,c_4^2\,X^6\\
&-Ncc_1X+\frac{1}{2}Ncc_1X^2-Ncc_2X+\frac{1}{3}Ncc_2X^3\\
&-Ncc_3X+\frac{1}{4}Ncc_3X^4-Ncc_4X-Nrc_1X\\
&\frac{1}{2}Nrc_1X^2-Nrc_2X+\frac{1}{3}Nrc_2X^3-Nrc_3X\\
&+\frac{1}{4}Nr\,c_3X^4-Nr\,c_4X+\frac{1}{5}Nrc_4X^5-Hc_1X\\
&-Hc_2X+\frac{1}{3}Hc_2X^3-Hc_3x+\frac{1}{4}Hc_3X^4-Hc_4x\\
&+\frac{1}{5}Hc_4X^5+\frac{2}{5}Rac_2X^6c_4\frac{1}{2}Ra\,c_2\,X^5c_3\\
&-\frac{1}{4}Rac_1X^6c_3+\frac{2}{3}Rac_2X^4c_3-2Rac_1X^2c_2\\
&-2Rac_2X^2c_3-\frac{2}{15}Rac_2X^8c_4+\frac{2}{3}Rac_1X^4c_2\\
&-2Rac_1X^2c_3+\frac{2}{3}Rac_2X^4c_4+Rac_1X^3c_4-c_1\\
&+Ra\,c_1\,X^3\,c_2-\frac{1}{10}Ra\,c_3\,X^9c_4-2Ra\,c_1\,X^2c_4\\
&-\frac{1}{3}Rac_1X^5c_2+\frac{1}{2}Rac_3X^5c_4-2Rac_2\,X^2\,c_4\\
&+\frac{2}{5}Ra\,c_3X^6c_4-2Ra\,c_3x^2c_4+\frac{2}{5}Ra\,c_1x^6c_4\\
&-\frac{1}{5}Rac_1X^7c_4+Rac_1X^3c_3+Nc\varepsilon c_1X+Nc\varepsilon\\
&-\frac{1}{4}Nc\varepsilon c_3X^4+Nc\varepsilon c_4X-\frac{1}{5}Nc\varepsilon c_4X^5-4Rdc_1\\
&+\frac{1}{2}Rac_1X^5c_3-3c_3X^2-4c_4X^3-\frac{1}{6}Rac_2X^7c_3\\
&-\frac{1}{25}Rac_4^2X^{10}+\frac{1}{5}Ncc_4X^5+\frac{1}{2}Hc_1X^2-2c_2X
\end{aligned} \quad (26)$$

And via substituting the residual function into Eq. (17), a set of equation with four equations will appear and by solving this system of equations, coefficients $(c_i, i=1..4)$ will be determined. Using LSM, when $Ra=0.2, Rd=0.5, \varepsilon=0.2, Nc=0.3, Nr=0.8$ and $H=0.9$, following equations will be determined for temperature distribution.

$$\begin{aligned}
&\theta(X)=1-0.5738228590X+0.3565147734X^2\\
&-0.07356149367X^3+0.02257507249X^4\\
&-0.001764499350X^5
\end{aligned} \quad (27)$$

3. RESULT

In this manuscript, the Least Square Method such as analytical technique is employed to find an analytical solution of the temperature distribution in a porous fin. Figures 2 to 5 show comparison between the numerical solution and LSM solution for θ and θ' when M, G, Sh and εg are variables. It should be mention that, many advantages of LSM compared to other analytical and numerical methods make it more valuable and motivate researchers to use it for solving problems (Hatami *et al*. 2013; Hatami and Ganji 2014).

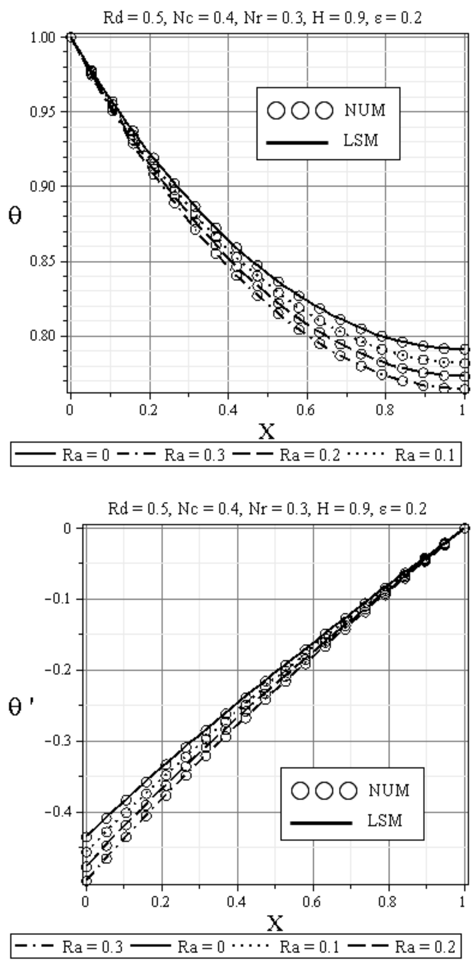

Fig. 2. The comparison between the Numerical and LSM solution for θ and θ' at different value of Ra.

It has the following benefits: firstly, unlike all previous analytic techniques, It solves the equations directly and no simplifications needs. Secondly, unlike all previous analytic techniques, it does not need to any perturbation, linearization or small parameter versus Homotopy Perturbation Method (HPM) and Parameter Perturbation Method (PPM). Thirdly, unlike homotopy asymptotic method, it does not need to determine the auxiliary function and parameter versus HAM. Also according previous publications this method is a simple and powerful technique for finding analytical solutions in science and engineering problems.

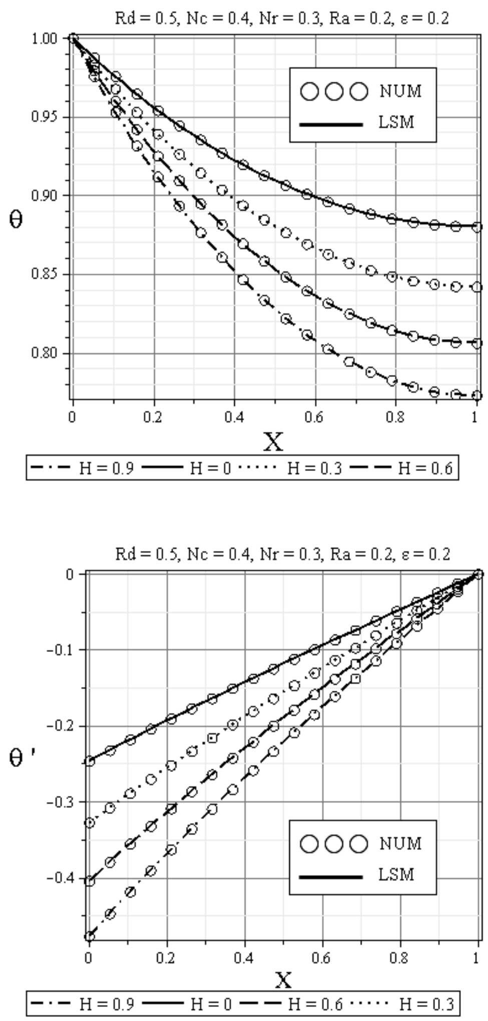

Fig. 3. The comparison between the Numerical and LSM solution for θ and θ' at different value of H .

As long as, the type of the current problem is boundary value problem (BVP) and the appropriate method need to be selected. The numerical solution is performed using the algebra package Maple 14.0, to solve the present problem. The package uses a uses a second-order difference scheme combined with an order bootstrap technique with mesh-refinement strategies: the difference scheme is based on either the trapezoid or midpoint rules; the order improvement/accuracy enhancement is either Richardson extrapolation or a method of deferred corrections. According to Tables 1 and Figs. 2 – 5, it is noticed that this comparison shows an excellent agreement, so that we are confident that the present results are accurate.

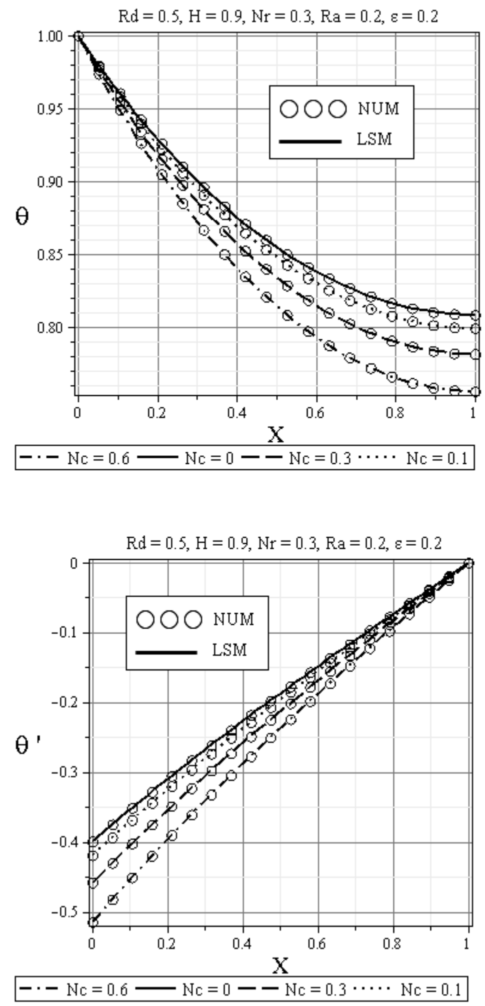

Fig. 4. The comparison between the Numerical and LSM solution for θ and θ' at different value of Nc .

4. CONCLUSION

In this work, Least Square Method (LSM) has been applied to achieve the solution of temperature distribution in a porous fin which is exposed to uniform magnetic field. The approximate solutions have been compared with the direct numerical solutions generated by the symbolic algebra package Maple 14. It has been shown that LSM provides very accurate for non-liner problems in comparison with numerical solutions. Finally, it has been attempted to show the capabilities and wide-range applications of the LSM in engineering.

Table 1 The results of LSM and Numerical methods for $\theta(X)$ for $Ra = 0.1,\ Rd = 0.5, \varepsilon = 0.2, Nc = 0.4\ , Nr = 0.3$ and $H = 0.9$.

X	$\theta(X)$		
	LSM	NUM	Error
0.00	1.000000000	1.000000000	0.0000000
0.10	0.956987665	0.956988020	3.554E-07
0.20	0.919132060	0.919132513	4.534E-07
0.30	0.886217828	0.886217964	1.366E-07
0.40	0.858057970	0.858057690	2.800E-07
0.50	0.834492969	0.834492509	4.597E-07
0.55	0.824390987	0.824390597	3.904E-07
0.60	0.815389906	0.815389668	2.385E-07
0.65	0.807477123	0.807477103	2.030E-08
0.70	0.800641589	0.800641806	2.166E-07
0.75	0.794873779	0.794874188	4.083E-07
0.80	0.790165668	0.790166187	5.195E-07
0.85	0.786510699	0.786511231	5.323E-07
0.90	0.78390376	0.783904154	3.947E-07
0.95	0.782341153	0.782341313	1.598E-07
1.00	0.781820569	0.781820729	1.597E-07

Table 2 The results of LSM and Numerical methods for $\theta'(X)$ for $Ra = 0.1,\ Rd = 0.5, \varepsilon = 0.2, Nc = 0.4,\ Nr = 0.3$ and $H = 0.9$.

X	$\theta'(X)$		
	LSM	NUM	Error
0.00	0.456697507	0.456697420	8.69000E-08
0.10	0.403956702	0.403953575	3.12730E-06
0.20	0.353514184	0.353515512	1.32790E-06
0.30	0.305081978	0.305086466	4.48820E-06
0.40	0.258380892	0.258384515	3.62290E-06
0.50	0.213140520	0.213140387	1.32700E-07
0.55	0.190986001	0.190983837	2.16360E-06
0.60	0.169099238	0.169095471	3.76710E-06
0.65	0.147448901	0.147444314	4.58660E-06
0.70	0.126004208	0.125999796	4.41250E-06
0.75	0.104734928	0.104731703	3.22490E-06
0.80	0.083611375	0.083610171	1.20427E-06
0.85	0.062604417	0.062605593	1.17589E-06
0.90	0.041685468	0.041688474	3.00629E-06
0.95	0.020826492	0.020829565	3.07318E-06
1.00	0.000000000	0.000000000	0.00000000

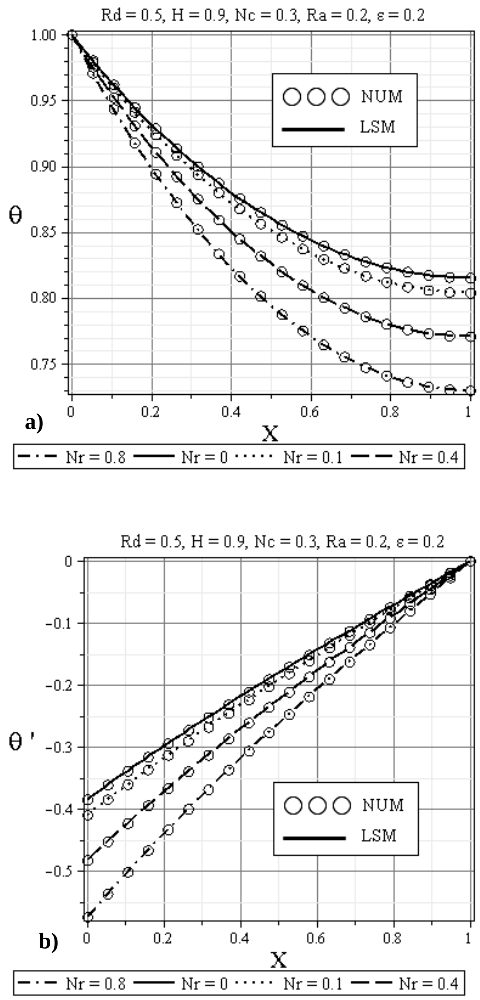

Fig. 5. Numerical and LSM solution for a) θ and b) θ' at different value of Nr .

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