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A Semi-Analytical Solution for a Porous Channel Flow of a Non-Newtonian Fluid

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ABSTRACT

A theoretical study of steady laminar two-dimensional flow of a non-Newtonian fluid in a parallel porous channel with variable permeable walls is carried out. Solution by Differential Transform Method (DTM) is obtained and the flow behavior is studied. The non-Newtonian fluid considered for the study is couple stress fluid. Thus, in addition to the effects of inertia and permeabilities on the flow, the couple stress effects are also analyzed. Results are presented and comparisons are made between the behaviour of Newtonian and non-Newtonian fluids.

Keywords: Laminar flow; Couple stress fluid; Porous channel; Variable permeability; Differential transform method.

NOMENCLATURE

f, g	generic functions
h	height of the channel
l^2	dimensionless couple stress parameter
$\bar{q}$	velocity vector
R_1	Reynolds number in case (i)
R_2	Reynolds number in case (ii)
R^*	entrance Reynolds number
$U(0)$	entrance velocity
u	axial velocity
V_1	permeability velocity at the lower plate
V_2	permeability velocity at the upper plate
v	normal velocity
α_1	suction/injection parameter for case (i)
α_2	suction/injection parameter for case (ii)
λ	dimensionless y coordinate
ρ	fluid density
μ	fluid viscosity
γ_1	ratio of injection/suction Reynolds number to entrance Reynolds number-case
γ_2	ratio of injection/suction Reynolds number to entrance Reynolds number-case
η	couple stress parameter

1. INTRODUCTION

Laminar flow through porous channels (or) ducts have gained considerable importance because of their applications in industries and biophysical laboratories, such as binary gas diffusion, filtration, ablation cooling, transpiration cooling, paper manufacturing, oil production, blood dialysis in artificial kidneys and blood flow in capillaries.

Several researchers have studied the wall porosity effects on the two-dimensional steady laminar flow of an incompressible Newtonian fluid through uniform porous walls. Berman (1953) has investigated the effect of wall porosity on the two-dimensional steady laminar flow of an incompressible Newtonian fluid in a rectangular channel and obtained solution by perturbation. Yuan (1956) has analyzed the two-dimensional steady laminar flow of an incompressible Newtonian fluid in channels with porous walls at moderate Reynolds numbers. Terrill (1964) has solved the flow in a two-dimensional channel with permeable walls by a numerical technique with uniform injection or suction. Terrill and Shrestha (1965) have studied the flow through parallel porous walls of different permeabilities for small Reynolds number and solved using perturbation technique and numerical method.

Many studies on fluid flows are confined to the use of Newtonian fluids owing to its simple nature of the linear constitutive equation. However, in many practical applications, the fluids used are non-Newtonian. In the classical continuum theory, various non-Newtonian models are used to describe the non-linear relation between stress and rate of strain. In particular, power-law model, cubic equation model, Oldroyd B model, Rivlin-Ericksen model, Hershel-Bulkley model, Bingham plastic model and Maxwell model have received remarkable attention among fluid dynamists. Further, analytical solution of fluid flow problems using Newtonian fluid is mostly possible and available in literature whereas it is rare in flow problems using non-Newtonian fluids. Therefore, it is reasonable to attempt at determining an analytical or a semi-analytical solution procedure for a non-Newtonian fluid flow problem.

The theory of couple stress fluids proposed by Stokes (1966) shows all the important features and effects of couple stresses in fluid medium. The basic equations are similar to Navier Stokes equations. The importance of couple stress effects in flow between parallel porous plates have been analyzed by several researchers. Kabadi (1987) has studied the flow of couple stress fluid between two parallel horizontal stationary plates with fluid injection through the lower porous plate. Ariel (2002) has provided an exact solution for flow of second grade fluid through two parallel porous flat walls. Kamisili (2006) has analyzed the laminar flow of a non-Newtonian fluid in channels with the upper plate stationary, while the lower plate is uniformly porous and moving in *x*-direction with constant velocity. Srinivacharya *et al.* (2009) have analyzed the flow and heat transfer of couple stress fluid in a porous channel with expanding and contracting walls. Srinivacharya *et al.* (2010) have investigated the flow of couple stress fluid between two porous plates for suction at both plates with different permeabilities.

A powerful semi-analytical technique namely Differential Transform Method (DTM) proposed by Zhou (1986) has been used to solve both linear and non-linear initial value problems in electric circuit theory. Several researchers like Chen and Ho (1999), Bert (2002), Kurnaz *et al.* (2005), Erturk *et al.* (2007), Rashidi and Sadri (2010), Rashidi *et al.* (2010), Rashidi and Mohimanian Pour (2010), Rashidi and Erfani (2011), Vimala and Blessie (2013) have used DTM to solve partial differential equations, system of algebraic equations and fluid flow problems with highly non-linear terms. In this paper, a steady laminar two-dimensional flow of couple stress fluid in a porous channel with variable permeablities is considered. The effects of inertia, porosity and couple stresses on the flow behavior are studied. At every stage of the formulation and solution, the present problem in the Newtonian case is compared with existing literature and the results agree well, which validate the present results.

2. MATHEMATICAL FORMULATION

The problem of two-dimensional steady laminar flow of an incompressible viscous couple stress fluid in a parallel porous channel is considered. Various types of flow occur and they fall under two major categories: (i) $|V_1| \geq |V_2|$ and (ii) $|V_2| \geq |V_1|$.

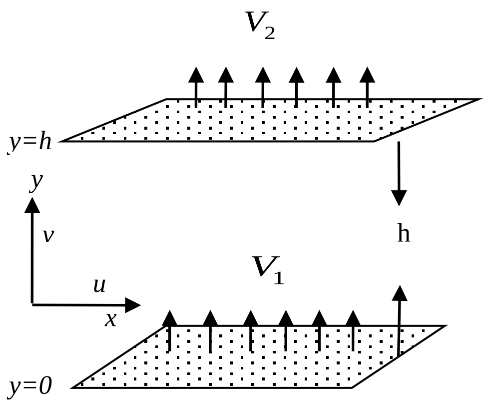

Fig. 1. Flow Geometry.

Based on the Stokes microcontinuum theory, the governing equations of the flow in the absence of body forces and body moments are the continuity equation and the momentum equations (Stokes 1966) given by

$$\nabla.\bar{q} = 0 \ , \tag{1}$$

$$\rho\left(\bar{q}.\nabla\right)\bar{q} = -\nabla p - \mu\nabla\times(\nabla\times\bar{q}) - \eta\nabla\times(\nabla\times(\nabla\times(\nabla\times\bar{q}))) \ . \tag{2}$$

The boundary conditions are

$$\begin{aligned} u(x,0) &= 0, \qquad v(x,0) = V_1 \\ u(x,h) &= 0, \qquad v(x,h) = V_2 \end{aligned} \ , \tag{3}$$

and the no-couple stress conditions at the boundary are

$$\begin{aligned} \frac{\partial^2 u}{\partial y^2} &= 0 \qquad \text{at} \qquad y = 0, \\ \frac{\partial^2 u}{\partial y^2} &= 0 \qquad \text{at} \qquad y = h. \end{aligned} \ . \tag{4}$$

Taking $\lambda = \dfrac{y}{h}$, the above equations become

$$\frac{\partial u}{\partial x} + \frac{1}{h}\frac{\partial v}{\partial \lambda} = 0 \ , \tag{5}$$

$$u\frac{\partial u}{\partial x} + \frac{v}{h}\frac{\partial u}{\partial \lambda} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 u}{\partial \lambda^2}\right),$$

$$-\frac{\eta}{\rho}\left(\frac{\partial^4 u}{\partial x^4}+\frac{1}{h^4}\frac{\partial^4 u}{\partial \lambda^4}+\frac{2}{h^2}\frac{\partial^4 u}{\partial x^2 \partial \lambda^2}\right) \tag{6}$$

$$u\frac{\partial v}{\partial x}+\frac{v}{h}\frac{\partial v}{\partial \lambda}=\frac{-1}{h\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \lambda^2}\right),$$
$$-\frac{\eta}{\rho}\left(\frac{\partial^4 v}{\partial x^4}+\frac{1}{h^4}\frac{\partial^4 v}{\partial \lambda^4}+\frac{2}{h^2}\frac{\partial^4 v}{\partial x^2 \partial \lambda^2}\right) \tag{7}$$

where p is the pressure and ν is the kinematic viscosity.

The boundary conditions are given by

$$u(x,0)=0, \quad v(x,0)=V_1, \quad u(x,1)=0, \quad v(x,1)=V_2. \tag{8}$$

and

$$\frac{\partial^2 u}{\partial \lambda^2}=0 \quad \text{at} \quad \lambda=0, \quad \frac{\partial^2 u}{\partial \lambda^2}=0 \quad \text{at} \quad \lambda=1. \tag{9}$$

Using a stream function (Berman, 1953) of the form

$$\psi(x,\lambda)=\left[\frac{hU(0)}{s(1)-s(0)}-\frac{V_1-V_2}{s(1)-s(0)}x\right]s(\lambda), \tag{10}$$

where $U(0)=\int_0^1 u(0,\lambda)\,d\lambda$ is the entrance velocity, the partial differential Eqs. (5)-(7) reduce to an ordinary differential equation.

Without loss of generality, it is assumed that $|V_1|\ge|V_2|$ in the first case and $|V_2|\ge|V_1|$ in the second case.

2.1 Case(i) $|V_1|\ge|V_2|$

The stream function of Eq.(10) in this case becomes

$$\psi(x,\lambda)=\left[\frac{hU(0)}{\alpha_1}-V_1 x\right]f(\lambda), \tag{11}$$

where $f(\lambda)=\dfrac{s(\lambda)}{s(0)}, \quad \alpha_1=\dfrac{V_2}{V_1}-1.$ (12)

Further, Eq. (6) and Eq. (7) respectively become

$$\left(\frac{U(0)}{\alpha_1}-\frac{V_1 x}{h}\right)\left[\frac{V_1}{h}\left(ff''-f'^2\right)-\frac{\nu}{h^2}f'''+\frac{\eta}{\rho h^4}f^{v}\right]=\frac{-1}{\rho}\frac{\partial p}{\partial x}, \tag{13}$$

$$\frac{\mu V_1}{h}f''-\rho V_1^2 ff'-\frac{\eta V_1}{h^3}f^{iv}=\frac{\partial p}{\partial \lambda}, \tag{14}$$

Eq.(14) is a function of λ only and therefore

$$\frac{\partial^2 p}{\partial x \partial \lambda}=0 \quad . \tag{15}$$

Using Eq.(15) in Eq.(13), a fifth order differential equation is obtained as

$$l^2 f^{v}-f'''-R_1(f'^2-ff'')=A_1 \quad , \tag{16}$$

where A_1 is an arbitrary constant, $l^2=\dfrac{\eta}{\mu h^2}$ is the dimensionless couple stress parameter and $R_1=V_1 h/\nu$ is the Reynolds number. It may be noted that Eq.(16) reduces to its Newtonian counterpart when l^2 is made to vanish .

The boundary conditions from Eq. (8) and Eq. (9) become

$$f(0)=1, \quad f'(0)=0, \quad f(1)=1+\alpha_1, \quad f'(1)=0, \quad f'''(0)=0, \quad f'''(1)=0. \tag{17}$$

For the case of suction at the upper wall and injection at the lower wall, α_1 takes the range $-1<\alpha_1\le 0$ and $R_1>0$. For injection at the upper wall and suction at the lower wall, the range is $-1<\alpha_1\le 0$ and $R_1<0$. For suction at both walls, it is required that $R_1<0,\ -2\le\alpha_1<-1$, and for injection at both walls, it is $R_1>0,\ -2\le\alpha_1<-1$.

2.2 Case (ii) $|V_2|\ge|V_1|$

In this case, the stream function becomes

$$\psi(x,\lambda)=\left[\frac{hU(0)}{\alpha_2}-V_2 x\right]g(\lambda), \tag{18}$$

where $\alpha_2=1-\dfrac{V_1}{V_2}$ and $g(\lambda)=\dfrac{s(\lambda)}{s(1)}$.

Further Eq. (6) and Eq. (7) in this case reduce to

$$\left(\frac{U(0)}{\alpha_2}-\frac{V_2 x}{h}\right)\left[\frac{V_2}{h}\left(gg''-g'^2\right)-\frac{\nu}{h^2}g'''+\frac{\eta}{\rho h^4}g^{v}\right]=\frac{-1}{\rho}\frac{\partial p}{\partial x}, \tag{19}$$

$$\frac{\mu V_2}{h}g''-\rho V_2^2 gg'-\frac{\eta V_2}{h^3}g^{iv}=\frac{\partial p}{\partial \lambda} \quad . \tag{20}$$

Eq.(20) is independent of x and therefore Eq.(15) holds good in this case also. Using Eq. (15) in Eq. (19) as in case (i), a fifth order differential equation in $g(\lambda)$ is obtained as

$$l^2 g^{v}-g'''-R_2(g'^2-gg'')=A_2, \tag{21}$$

where A_2 is an arbitrary constant and $R_2=V_2 h/\nu$ is the Reynolds number in this case. Here again Eq.(21) reduces to the Newtonian case when l^2 is made equal to zero. The boundary conditions from Eq. (8) and Eq.(9) become

$$g(0)=1-\alpha_2,\quad g'(0)=0,\quad g(1)=1,$$
$$g'(1)=0,\ g'''(0)=0,\quad g'''(1)=0. \qquad (22)$$

For the case of suction at the upper wall and injection at the lower wall, α_2 takes the range $0\le\alpha_2<1$ and $R_2>0$. For injection at the upper wall and suction at the lower wall, the range of values is $0\le\alpha_2<1$ and $R_2<0$. For suction at both the walls, it is required that $R_2>0,\ 1<\alpha_2\le 2$ and for injection at both the walls, it is $R_2<0, 1<\alpha_2\le 2$.

3. Solution Of The Problem

3.1 Solution by DTM for Case (i)

Using differential transform about $\lambda=0$, Eq. (16) can be transformed into

$$l^2(k+1)(k+2)(k+3)(k+4)(k+5)F(k+5)$$
$$-(k+1)(k+2)(k+3)F(k+3)$$
$$-R_1\left(\begin{array}{l}\sum_{k_1=0}^{k}(k_1+1)(k-k_1+1)F(k_1+1)F(k-k_1+1)\\ -\sum_{k_1=0}^{k}(k-k_1+1)(k-k_1+2)F(k_1)F(k-k_1+2)\end{array}\right),(23)$$
$$=A_1\delta(k)$$

and the first two boundary conditions and fourth condition from Eq. (17) are

$$F(0)=1,\qquad F(1)=0,\qquad F(3)=0. \qquad (24)$$

It is assumed that $F(2)=b_1,\ F(4)=d_1$, where $b_1\ \&\ d_1$ are undetermined constants. Using these and Eq. (24) in Eq. (23), the values of *F(k)* are obtained iteratively and the other three boundary conditions are written as

$$f(1)=1+\alpha_1 \quad\text{or}\quad \sum_{k=0}^{N}F(k)=1+\alpha_1 \qquad (25)$$

$$f'(1)=0 \quad\text{or}\quad \sum_{k=0}^{N-1}(k+1)F(k+1)=0 \qquad (26)$$

$$f'''(1)=0 \text{ or} \sum_{k=0}^{N-3}(k+1)(k+2)(k+3)F(k+3)=0 \ (27)$$

For N=17, solving the three Eqs. (25), (26) and (27), the values of b_1, d_2 and A_1 are obtained.

Using Eqs. (13) and (14), the general expression for pressure distribution is obtained as

$$p(x,\lambda)=p(0,0)$$
$$+\frac{\mu V_1}{h}f'(\lambda)-\frac{\rho V_1^2}{2}\left[f^2(\lambda)-f^2(0)\right] ,(28)$$
$$-\frac{\eta V_1}{h^3}f'''(\lambda)-\frac{\mu A_1}{h^2}\left(\frac{U(0)}{\alpha_1}x-\frac{V_1}{2}\frac{x^2}{h}\right)$$

From this general expression the pressure distribution for *x* and λ directions are deduced. The non- dimensional pressure drop in the *x* and λ directions are respectively given by

$$p_x=\frac{p(0,\lambda)-p(x,\lambda)}{\rho U^2(0)/2}$$
$$=\frac{1}{R^*}\frac{x}{h}\left(\frac{1}{\alpha_1}-2\gamma_1\right)(8A_1) , \qquad (29)$$

$$p_\lambda=\frac{p(x,0)-p(x,\lambda)}{\rho U^2(0)/2}$$
$$=\left(\frac{4R_1}{R^*}\right)^2\left[f^2(\lambda)-(1)^2\right]-2R_1\left(\frac{4}{R^*}\right)^2 f'(\lambda). \ (30)$$
$$-\left(\frac{4}{R^*}\right)^2 R_1 l^2\left[f'''(\lambda)-f'''(0)\right]$$

where $\gamma_1=\frac{R_1}{R^*}\frac{x}{h}$ is a non-dimensional variable and $R^*=4hU(0)/\nu$ is the entrance Reynolds number.

The non-dimensional stream function is given by

$$\Psi(x,\lambda)=\frac{\psi(x,\lambda)}{hU(0)}=\left(\frac{1}{\alpha_1}-4\gamma_1\right)f(\lambda). \qquad (31)$$

The skin friction is defined as

$$c_f=\frac{\tau_{wall}}{\rho U(0)^2/2}=\frac{2\mu(\partial u/\partial\lambda)_{wall}}{\rho\, \mathrm{h}\mathrm{U}(0)^2}, \qquad (31)$$

where τ_{wall} is the shear stress at the wall and the skin friction here becomes

$$c_f=\frac{8h}{R_1 x}\gamma_1\left(\frac{1}{\alpha_1}-4\gamma_1\right)[f''(\lambda)]_{wall}. \qquad (32)$$

3.2 Solution by DTM for Case (ii)

In this case, Eq. (21) is transformed to

$$l^2(k+1)(k+2)(k+3)(k+4)(k+5)G(k+5)$$
$$-(k+1)(k+2)(k+3)G(k+3)$$
$$-R_2\left(\begin{array}{l}\sum_{k_1=0}^{k}(k_1+1)(k-k_1+1)G(k_1+1)G(k-k_1+1)\\ -\sum_{k_1=0}^{k}(k-k_1+1)(k-k_1+2)G(k_1)G(k-k_1+2)\end{array}\right),(33)$$
$$=A_2\delta(k)$$

and the first two boundary conditions and fourth condition from Eq. (22) are

$$G(0)=1-\alpha_2,\quad G(1)=0,\quad G(3)=0. \qquad (34)$$

It is assumed that $G(2)=b_2,\quad G(4)=d_2$, where b_2, d_2 are undetermined constants. Using these and Eq. (34) in Eq. (33), the values of *G(k)* are obtained iteratively and the other three boundary conditions are written as

$$g(1)=1 \quad \text{or} \quad \sum_{k=0}^{N} G(k)=1 \tag{35}$$

$$g'(1)=0 \quad \text{or} \quad \sum_{k=0}^{N-1}(k+1)G(k+1)=0 \tag{36}$$

$$g'''(1)=0 \,\text{or} \sum_{k=0}^{N-3}(k+1)(k+2)(k+3)G(k+3)=0 \tag{37}$$

For N=17, solving the three equations (35),(36) and (37), the values of b_2, d_2 and A_2 are obtained.

Using (19) and (20), the general expression for pressure distribution in this case is obtained as

$$\begin{aligned} p(x,\lambda) &= p(0,0) \\ &+ \frac{\mu V_2}{h} g'(\lambda) - \frac{\rho V_2^{\,2}}{2}\left[g^2(\lambda)-g^2(0)\right] \quad , \\ &- \frac{\eta V_1}{h^3} g'''(\lambda) - \frac{\mu A_2}{h^2}\left(\frac{U(0)}{\alpha_2}x - \frac{V_2}{2}\frac{x^2}{h}\right) \end{aligned} \tag{38}$$

From this general expression, the pressure distribution for x and λ directions are deduced. The non-dimensional pressure drop in the x and λ directions are respectively given by

$$p_x = \frac{1}{R^*}\frac{x}{h}\left(\frac{1}{\alpha_2} - 2\gamma_2\right)(8A_2)\,, \tag{39}$$

$$\begin{aligned} p_\lambda &= \left(\frac{4R_2}{R^*}\right)^2\left[g^2(\lambda)-(1-\alpha_2)^2\right] \\ &-2R_2\left(\frac{4}{R^*}\right)^2 g'(\lambda) - \left(\frac{4}{R^*}\right)^2 R_1 l^2\left[g'''(\lambda)-g'''(0)\right] \end{aligned} \quad .(40)$$

where $\gamma_2 = \frac{R_2}{R^*}\frac{x}{h}$ is a non-dimensional variable.

In this case, the non-dimensional stream function and the skin friction are given by

$$\Psi(x,\lambda) = \frac{\psi(x,\lambda)}{hU(0)} = \left(\frac{1}{\alpha_2} - 4\gamma_2\right) g(\lambda) \quad \& \tag{41}$$

$$c_f = \frac{8h}{R_2 x}\gamma_2\left(\frac{1}{\alpha_2} - 4\gamma_2\right)\left[g''(\lambda)\right]_{wall} \;. \tag{42}$$

4. RESULTS AND DISCUSSION

The problem of a steady, two-dimensional laminar flow of couple stress fluid in a porous channel has been solved using DTM under two cases (i) $|V_1| \ge |V_2|$ and (ii) $|V_2| \ge |V_1|$.

In case (i), R_1 is taken to be positive and the values of wall permeability $\alpha_1 = -0.2, -0.6$ correspond to suction at the upper wall and injection at the lower wall, that of $\alpha_1 = -1$ correspond to a solid upper wall and injection at lower wall and those of $\alpha_1 = -1.4, -1.8$ correspond to injection at both the walls. Other cases of suction at both walls, injection at upper wall and suction at lower wall correspond to negative values of R_1. In particular, $V_2 = 0$ in case (i) reduces to the problem of Kabadi (1987).

Figures 2-8 correspond to the results of case (i). Fig. 2 shows the effects of α_1 on normal velocity for the Reynolds number $R_1 = 10$, couple stress parameter $l^2 = 0.0 \,\&\, 0.5$ and for different values of the wall permeability parameter α_1.

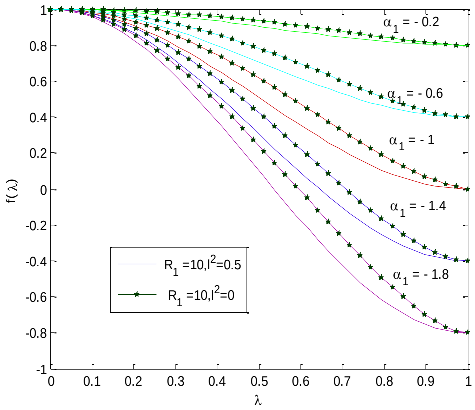

Fig. 2. Effects of wall permeability on normal velocity (case (i)).

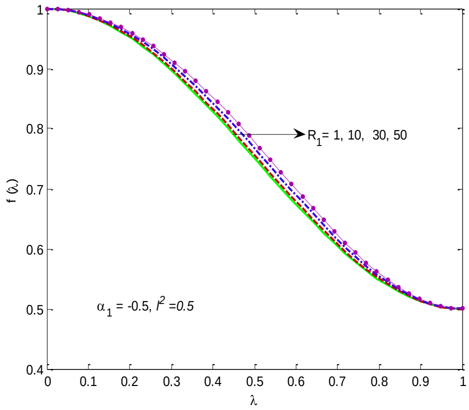

Fig. 3. Effects of inertia on normal velocity (case (i)).

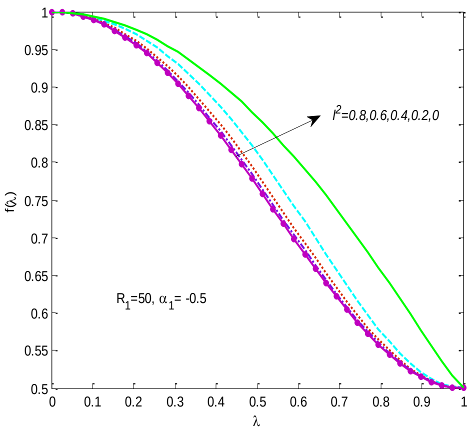

Fig. 4. Effects of couple stresses on normal velocity (case (i)).

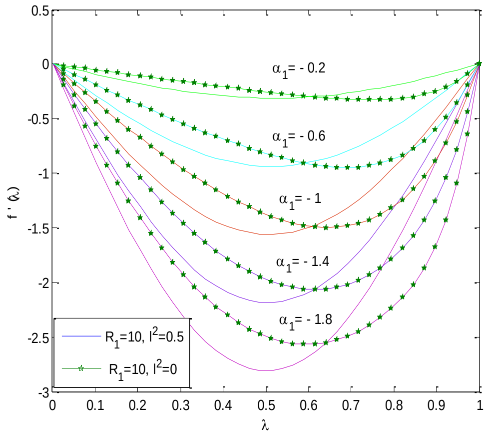

Fig. 5. Effects of wall permeability on axial velocity (case(i)).

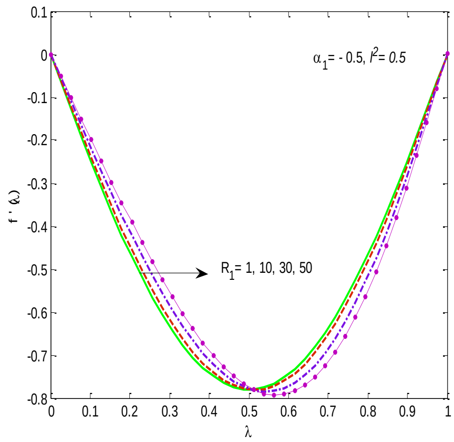

Fig. 6. Effects of inertia on axial velocity (case (i)).

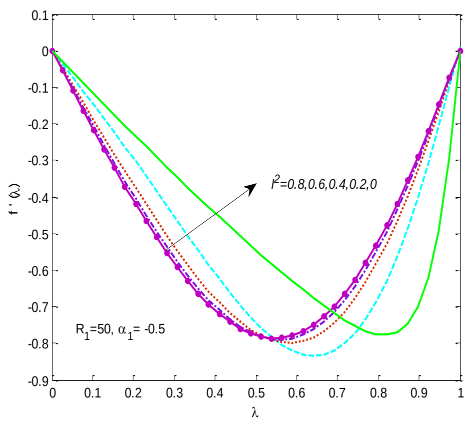

Fig. 7. Effects of couple stresses on axial velocity (case (i)).

Here $l^2 = 0.0$ correspond to the Newtonian case. The normal velocity decreases with increase in magnitude of α_1. Also, the effect of couple stresses $l^2 = 0.5$ on the flow behavior is to further decrease the normal velocity. Fig. 3 shows the inertia effects on normal velocity for $\alpha_1 = -0.5, l^2 = 0.5$ and different values of R_1. It is observed that the normal velocity increases with increasing Reynolds number.

Figure 4 shows the effects of couple stresses on the normal velocity. The normal velocity component decreases throughout the domain for increasing values of the couple stress parameter as seen in Fig. 2. Fig. 5 gives the effects of α_1 on axial velocity for $R_1 = 10, l^2 = 0.0 \,\&\, 0.5$ and different values of α_1. $l^2 = 0.0$ corresponds to the Newtonian case where the axial velocity profiles are skewed close to the upper wall. For $l^2 = 0.5$, increase in magnitude of α_1 increases the magnitude of axial velocity, with minimum magnitudes at both walls and maximum magnitudes near the middle of the wall. Thus couple stresses are seen to permit a smooth flow. Fig. 6 shows the inertia effects on axial velocity for $\alpha_1 = -0.5, l^2 = 0.5$ and for different values of R_1. It is observed that the axial velocity decreases in magnitude as the value of R_1 is increased up to a certain value of λ. Above this value, the velocity increases in magnitude due to increase in lower wall velocity V_1 which is greater than upper wall velocity V_2.

The effects of couple stresses on axial velocity are shown in Fig. 7. It can be seen that the axial velocity decreases for increasing couple stress parameter near the lower wall and the trend is reversed near the upper wall. Here, it is clearly seen that the flow becomes smoother as the couple stress parameter increases from $l^2 = 0.0$ to $l^2 = 0.8$. Fig. 8 shows the pressure drop in axial direction for various values of Reynolds number and for various values of couple stress parameter. The pressure drop decreases with decreasing Reynolds number and with decreasing couple stress parameter.

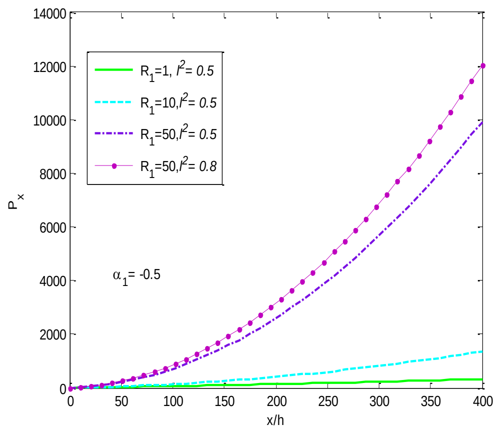

Fig. 8. Axial pressure drop (case (i)).

Similar graphs are plotted for case (ii) in Figs. 9-15. In case (ii), R_2 is taken to be positive and the values of $\alpha_2 = 0.2, 0.6$ correspond to suction at the upper wall and injection at the lower wall, that of $\alpha_2 = 1$ correspond to a solid lower wall and suction at upper wall and those of $\alpha_2 = 1.4, 1.8$ correspond to suction at both walls. Other cases of injection at both walls, injection at upper wall and suction at lower wall correspond to negative values of R_2. Fig. 9 shows the effect of α_2 on normal velocity for the Reynolds number $R_2 = 10$, couple stress parameter $l^2 = 0.0$ & 0.5 and for different values of the wall permeability parameter α_2. The normal velocity decreases as the value of α_2 decreases for $l^2 = 0.5$. Also, the effect of couple stresses on the flow behavior is to further decrease the normal velocity. Fig. 10 shows the inertia effects on normal velocity for $\alpha_2 = 0.5, l^2 = 0.5$ and different values of R_2. It is observed that the normal velocity decreases with increasing Reynolds number.

Figure 11 shows the effects of couple stresses on the normal velocity. The normal velocity decreases with decrease in couple stress parameter. Fig. 12 gives the effects of α_2 on axial velocity for $R_2 = 10, l^2 = 0.0$ & 0.5 and different values of α_2. $l^2 = 0.0$ corresponds to the Newtonian case where the axial velocity profiles are skewed close to the upper wall. For $l^2 = 0.5$, increase in α_2 increases the axial velocity, with minimum values at both walls and maximum values near the middle of the wall. Thus couple stresses are seen to admit a smooth flow. Fig. 13 shows the effects of inertia on axial velocity profile for $\alpha_2 = 0.5, l^2 = 0.5$ and different values of R_2. It is observed that the axial velocity decreases as the value of R_2 increases up to a certain value of λ. Above this value, the velocity increases due to increase in V_2 which is greater than V_1. The effects of couple stresses on the axial velocity for fixed values of $R_2 = 10, l^2 = 0.5$ are shown in Fig. 14. The axial velocity is increasing with increasing couple stress parameter for certain values of λ and after that it is decreasing. Here, it is clearly seen that the flow becomes smoother as the couple stress parameter increases from $l^2 = 0.0$ to $l^2 = 0.8$. In Figs. 11 and 14, when the couple stress parameter is equal to zero, it represents the Newtonian fluid case. Fig. 15 shows the pressure drop in axial direction for various values of Reynolds number and for various values of couple stress parameter. The pressure drop decreases with increasing Reynolds number and with increasing couple stress parameter.

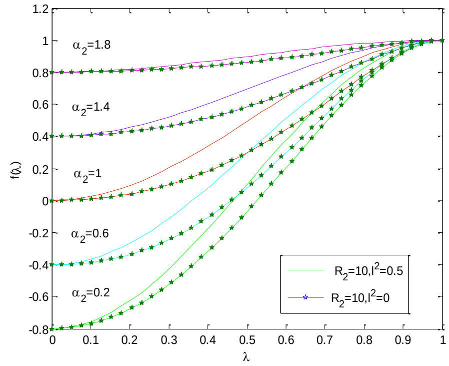

Fig. 9. Effects of wall permeability on normal velocity (case (i)).

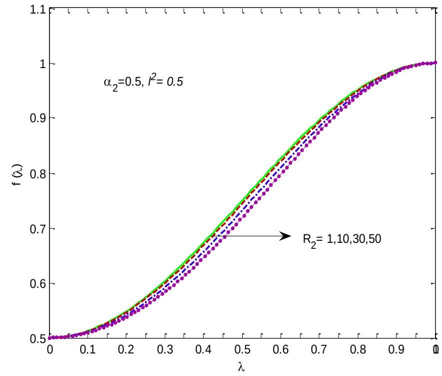

Fig. 10. Effects of inertia on normal velocity (case (ii)).

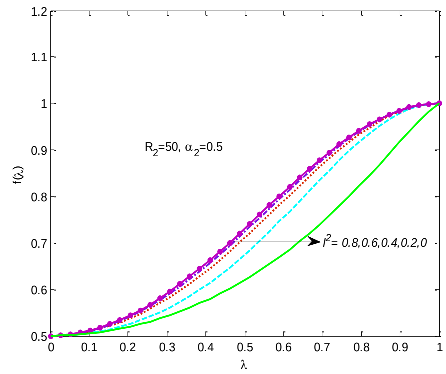

Fig. 11. Effects of couple stresses on normal velocity (case (ii)).

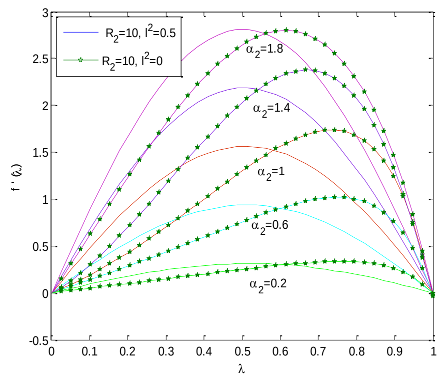

Fig. 12. Effects of wall permeability on axial velocity (case (ii)).

Table 1 Effects of couple stresses on skin friction by DTM – case (i)

l^2	$R_1 = 2$, α_1 = -0.5		$R_1 = 10$, α_1 = -0.5		$R_1 = 10$, α_1 = -1	
	$f''(0)$	$f''(1)$	$f''(0)$	$f''(1)$	$f''(0)$	$f''(1)$
0	-2.3730	3.7915	-1.5070	7.3454	-3.6484	10.0952
0.2	-2.4552	2.5739	-2.2353	2.8128	-4.6619	5.4084
0.4	-2.4759	2.5388	-2.3551	2.6645	-4.8128	5.2195
0.6	-2.4835	2.5263	-2.4002	2.6117	-4.8707	5.1500
0.8	-2.4875	2.5199	-2.4239	2.5846	-4.9013	5.1139

Table 2 Effects of couple stresses on skin friction by DTM – case (ii).

l^2	$R_2 = 2$, $\alpha_2 = 0.5$		$R_2 = 10$, $\alpha_2 = 0.5$		$R_2 = 10$, $\alpha_2 = 1$	
	$g''(0)$	$g''(1)$	$g''(0)$	$g''(1)$	$g''(0)$	$g''(1)$
0	2.3466	-3.9354	1.1486	-10.388	1.6085	-19.770
0.2	2.4550	-2.5740	2.2211	-2.8274	4.6159	-5.4668
0.4	2.4758	-2.5389	2.3509	-2.6692	4.8009	-5.2350
0.6	2.4835	-2.5264	2.3983	-2.6141	4.8655	-5.1572
0.8	2.4875	-2.5200	2.4228	-2.5860	4.8997	-5.1157

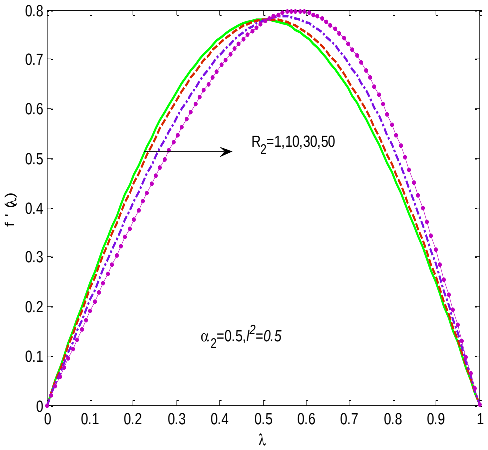

Fig. 13. Effects of inertia on axial velocity (case (ii)).

Tables 1 and 2 present comparisons between Newtonian ($l^2 = 0$) and non-Newtonian results ($l^2 \neq 0$), revealing the effects of couple stresses on the skin friction in case (i) and case (ii) respectively. From Table 1 of case (i), it is observed that the magnitude of skin friction increases at the lower wall and decreases at the upper wall with an increase in couple stress parameter for all values of α_1 & R_1. Here, α_1 = -0.5 corresponds to the case of one wall injection and other wall suction, whereas α_1 = -1 corresponds to the case of solid upper wall and injection at the lower wall. Further, the magnitude of the skin friction decreases at the lower wall whereas it increases at the upper wall with an increase in Reynolds number for fixed values of α_1 and l^2 .

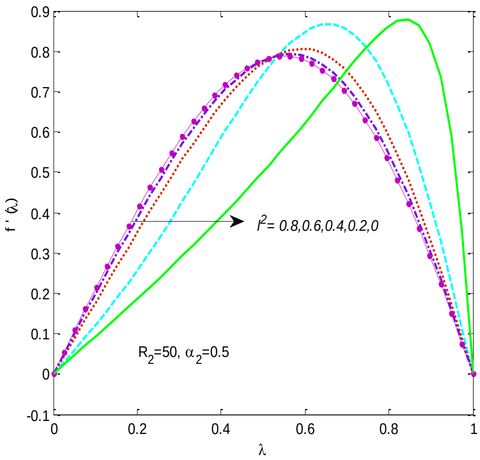

Fig. 14. Effects of couple stresses on axial velocity (case (ii)).

From Table 2 of case (ii), it is observed that the magnitude of skin friction increases at the lower wall and decreases at the upper wall with an increase in couple stress parameter for all values of α_2 & R_2 as in case (i). In Table 2 $\alpha_2 = 0.5$ corresponds to the case of one wall injection and other wall suction, whereas $\alpha_2 = 1$ corresponds to the case of solid lower wall and suction at the upper wall. Also, the magnitude of the skin friction decreases at the lower wall whereas it increases at the upper wall with an increase in Reynolds number for fixed values of α_2 and l^2. Thus the effect of couple stresses on the magnitude of skin friction in both cases gets enhanced at one wall and diminished at the other wall.

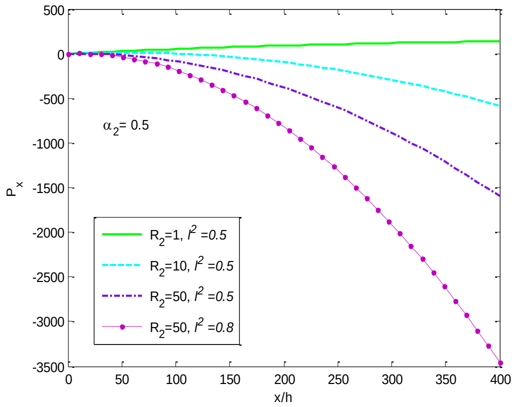

Fig. 15. Axial pressure drop (case(ii)).

5. Conclusion

In this paper, differential transform method is used successfully for finding the solution of two dimensional steady laminar flow of an incompressible couple stress fluid in a porous channel. It is seen that this method is simple and easy to use and solves the problem without any discretization and does not require exertion of any flow parameter as in perturbation method. Although the non-Newtonian fluid flow problem deals with more complicated governing equations than the Newtonian case, the method does not get disturbed and works well. Besides this, the reliability of the method and reduction in size of computational domain gives this method a wider applicability. Hence, the Differential Transform Method is an effective and reliable tool in finding the semi- analytical solution to many non-linear systems. This paper uses DTM for the flow problem with the couple stress model of the non-Newtonian fluid. However, there is no limitation for applying the DTM for such problems in similar geometry with any other non-Newtonian fluid model.

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