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Variational Approach of Marangoni Mixed Convection Boundary Layer Flow with Pade Technique

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ABSTRACT

In this paper, Variational Iteration method with combining Pade approximation (Modified Variational Iteration Method-MVIM) is performed to Marangoni convection flow over the surface with buoyancy effects which is occurred gravity and external pressure. After the appropriate transformation of equations, we get the dimensionless form to solve numerically with modified variational iteration method. We compare the our results with well-known asymptotic expansion method used by Zhang Yan and Zheng Liancun and also compare with Fourth order Runge Kutta solution which are presented in tables. Very efficient and accurate results are obtained with presented method.

Keywords: Variational iteration method; Marangoni convection; Pade approximation; Boundary layer flow.

NOMENCLATURE

f	velocity function	y	Cartesian coordinates normal to S
g	gravitational acceleration	x	Cartesian coordinate along
Pr	Prandtl number		
S	interface	λ	Maranon mixed convection parameter
T	temperature	η	similarity variable
ΔT	positive Temp. increment along the interface	Γ	buoyancy forces parameter
		σ	surface tension
X	component of velocity	θ	temperature function
$u_e(x)$	velocity outside boundary layer	φ	Lagrange multiplier
y	component of velocity	Ψ	stream function

1. INTRODUCTION

Marangoni convection flow is stimulated by variations of surface tension throughout liquid-liquid or liquid-gas surfaces and it is important in many fields of nature and engineering. Fundamental treatment of Marangoni flow has been analyzed by (Gelles 1978; Napolitano 1982, 1979, 1978; Okano *et al*. 1989). Some of numerical studies depend on the Marangoni convection in various geometries have been presented by (Golia and Viviani 1985, 1986), (Cristopher and Wang 2001), (Pop *et al*. 2001), (Chamkha *et al*. 2006), (Arafune and Hirata 1999), (Magyari and Chamkha 2008), (Aini *et al*. 2012), (Hamid and Arifin 2014), (Remeli *et al*. 2013), (Zhang and Zheng 2012), (Chen 2007) and (Al-Mudhaf and Chamkha 2005).

In real applications, mathematical problems are usually modeled by nonlinear ordinary and partial differential equations such as physical and engineering applications. In generally nonlinear models may not be an exact solutions. Therefore, we try to find approximate or numerical solutions of these models as seen in references (Freidoonimehr and Rashidi 2015; Jhankal 2015). One of the most popular technique is variational iteration method that is very powerful.

In this context, we will consider to extend the work studied by (Zhang and Zheng 2014) in order to find analytical solutions by using variational iteration method with combining Pade approximation called modified variational iteration method (MVIM). Also, we will give comparison between MVIM and asymptotic expansion method (Zhang and Zheng 2014). Additionally, unknown parameters of velocity gradient $f'(0)=a$ and temperature gradient $\theta'(0)=b$ which obtained by MVIM is compared with Runge-Kutta method as shown in tables. The effects and variations of $\Pr, \lambda$ and Γ on velocity and temperature profiles are presented graphically. All the calculations for solutions are provided by only one or two step iterations. Thus, it is found that the present results are in very good agreement with other known result as presented in Table 1, 2 and 3.

2. MATHEMATICAL DESCRIPTION OF PROBLEMS

We will consider two-dimensional Marangoni boundary layer flow with buoyancy effects due to external pressure gradient and gravity. It's occurs along an interface S between two fluids as in Fig. 1.

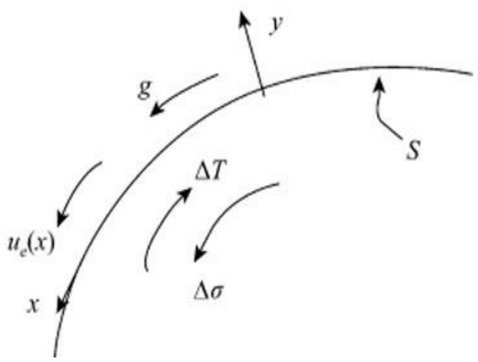

Fig. 1. Schematic of the problem.

Gravity g occurs throughout on interface S, the surface tension changes with temperature. Viscous dissipation and interface tortuosity are negligible. Also the flow fields for two interfacing fluids are independent (Golia and Viviani 1985, 1986; Zhang and Zheng 2014).

Considering these information, we can write the governing equations for the Marangoni boundary layer with water based fluid as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\frac{\partial^2 u}{\partial y^2}-\Gamma\lambda T \qquad (1)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}$$

Also boundary conditions are given as

$$if\ \ y=0,\ \frac{\partial u}{\partial y}=-\frac{\partial T}{\partial x}, v=0, T=T_0 x^5$$

$$if\ \ y\to\infty, u\to u_e(x)=u_0 x^3, T\to 0 \qquad (2)$$

where u and v are the velocity components corresponding to x and y axis respectively. $u_e(x)$ is external velocity, T is fluid temperature, λ is Marangoni mixed convection parameter. Also, if $\Gamma=-1$, then buoyancy force is available and if $\Gamma=1$, then buoyancy force is not available. Additionally, variation of surface tension as

$$\sigma_T=-\frac{d\sigma}{dT},\ \sigma=\sigma_m-\sigma_T\left(T-T_m\right). \qquad (3)$$

By using these facts and boundary conditions, we can write the transform variables as

$$\eta=\sqrt[4]{T_0}xy,\ T=-T_0x^5\theta(\eta)$$
$$\Psi(x,y)=\sqrt[4]{T_0}x^2 f(\eta) \qquad (4)$$
$$\frac{u_0}{\sqrt{T_0}}=r,\ \sqrt[4]{T_0}=k$$

Combining (1)-(4) and considering the literature (Golia and Viviani 1985, 1986; Chamkha *et al* 2006; Al-Mudhaf and Chamkha 2005) we obtain the main ordinary differential system which is the reduced form of (1) as

$$f'''-3\left(f'\right)^2+2ff''+3r^2+\Gamma\lambda\theta=0$$
$$\theta''+\Pr\left(2f\theta'-5f'\theta\right)=0 \qquad (5)$$

and boundary conditions turn into

$$f(0)=0, f'(\infty)=r=1,$$
$$f''(0)=-5k=-1, \qquad (6)$$
$$\theta(0)=1,\ \theta(\infty)=0$$

Here, prime denotes the derivatives with respect to η.

3. MODIFIED VARIATIONAL ITERATION METHOD (MVIM)

Variational iteration method (VIM) is one of the powerful mathematical tool to solve various kinds of linear and nonlinear problems as shown some of in ref. (He 2007, 1999, 1997; He and Hong 2007). In order to basic definition of VIM, we consider the following general nonlinear problem (He 2007, 1999, 1997; He and Hong 2007)

$$L\left[u(x)\right]+R\left[u(x)\right]+N\left[u(x)\right]=g(x) \qquad (7)$$

where $L=\frac{d^m}{dx^m}$, $m\in\mathbb{N}$ and R are linear operators, N is a nonlinear operator and g is given continuous function. According to the originally VIM, we construct the correction functional as

$$u_{n+1}(x) = u_n(x) + \int_0^x \varphi(s) \begin{bmatrix} L(u_n(s)) \\ +R(u_n(s)) \\ +N(\tilde{u}_n(s)) \\ -g(s) \end{bmatrix} ds \quad (8)$$

Here, $n \geq 0$, φ is a Lagrange multiplier (Inokuti *et al*. 1978), $\tilde{u}_n$ is considered as a restricted variation, i.e. $\delta\tilde{u}_n = 0$. If we apply the variation to correction functional (8) by using variational analysis, then we write down as

$$\delta u_{n+1}(x) = \delta u_n(x) + \delta \int_0^x \varphi(x,s) \begin{bmatrix} L(u_n(s)) \\ +R(u_n(s)) \\ +N(\tilde{u}_n(s)) \\ -g(s) \end{bmatrix} ds \quad (9)$$

$$= \delta u_n(x) + \int_0^x \delta\varphi(x,s) \begin{bmatrix} K(u_n(s)) \\ -g(s) \end{bmatrix} ds = 0 \quad (10)$$

From solution of Euler-Lagrange problem shown in (10), we determine the Lagrange multiplier and successive iterations $u_n(x)$, $n \geq 0$ are obtained by using Lagrange multiplier and initial approximation u_0 that satisfy, at least, the initial and boundary conditions with possible unknowns. Consequently the exact solution of (7) can be obtained by using (He 2007, 1999, 1997; He and Hong 2007) as

$$u(x) = \lim_{n \to \infty} u_n(x) \quad (11)$$

3.1 Pade Technic

Some techniques exist to increase the convergence of a given series. Among them so-called Pade Technique is widely applied (Baker and Morris 1981).

Supposed that a function $y(x)$ is presented by a power series

$$\sum_{n=0}^{\infty} c_n x^n \quad (12)$$

$[L,M]$ Pade approximant is

$$\frac{g_0 + g_1 x + \cdots + g_L x^L}{h_0 + h_1 x + \cdots + h_M x^M} + O\left(x^{L+M+1}\right) \quad (13)$$

which agree with (12) as far as possible. Here there are $L+1$ independent numerator coefficients and M independent denominator coefficients, so making $L+M+1$ unknown coefficient in all. This is suggested that normally $[L,M]$ ought to fit the power series (12) namely

$$\sum_{n=0}^{\infty} c_n x^n = \frac{g_0 + g_1 x + \cdots + g_L x^L}{h_0 + h_1 x + \cdots + h_M x^M} + O\left(x^{L+M+1}\right) \quad (14)$$

If the equations equate with respect to $x^{L+1}, x^{L+2}, ..., x^{L+M}$, we write down

$$\begin{aligned} & h_M c_{L-M+1} + h_{M-1} c_{L-M+2} + \cdots + h_0 c_{L+1} = 0 \\ & h_M c_{L-M+2} + h_{M-1} c_{L-M+3} + \cdots + h_0 c_{L+2} = 0 \\ & \vdots \qquad\qquad \vdots \\ & h_M c_L + h_{M-1} c_{L+1} + \cdots + h_0 c_{L+M} = 0 \end{aligned} \quad (15)$$

If $n<0$, $c_n = 0$ for consistency. Since $h_0 = 1$, (15) become a set of M linear equations for M unknown denominator coefficients and also the numerator coefficients $g_1, g_2, ..., g_L$ follow immediately from (14) by equating the coefficients of $1, x, x^2, ..., x^{L+M}$ as

$$\begin{aligned} & g_0 = c_0, \\ & g_1 = c_1 + c_0 h_1, \\ & g_2 = c_2 + h_1 c_1 + h_2 c_0, \\ & \vdots \qquad \vdots \\ & g_L = c_L + \sum_{n=0}^{\min(L,M)} h_n c_{L-n} \end{aligned} \quad (16)$$

Thus, (16) normally determine the Pade numerator and denominator. The $[L,M]$ Pade approximant is constructed which agrees with $\sum_{n=0}^{\infty} c_n x^n$, through order x^{L+M}.

In order to find the infinite boundary conditions in (6) and increase convergence and efficiency of the series solution (11), we apply the Pade approximation technic to (11). Therefore, we combine the variational iteration method and pade technic so called modified variational iteration method (MVIM).

4. SOLUTION PROCEDURE OF PROBLEMS

Now, we will apply our proposed method MVIM to eqs. (5)-(6) to obtain analytic solutions.

Let, assumed that $f'(0)=a$ and $\theta'(0)=b$ for the boundary conditions (6). By using these cases, the initial approximations $f_0(\eta)$ and $\theta_0(\eta)$ which provided boundary conditions (6) are considered as

$$f_0(\eta) = a\eta - \frac{\eta^2}{2} \text{ and}$$
$$\theta_0(\eta) = b\eta + 1 \qquad (17)$$

where a,b are unknown coefficients that will be obtained by applying boundary conditions (6). By using the variations theory (9)-(10), Lagrange multipliers are found as follow respectively

$$\varphi_f = -\frac{1}{2}(s-\eta)^2 \text{ and } \varphi_\theta = (s-\eta) \qquad (18)$$

Thus, Lagrange multipliers (18) put into (8), then successive iteration equations are written as

$$f_{n+1}(\eta) = f_n(\eta) - \frac{1}{2}\int_0^\eta (s-\eta)^2 \begin{pmatrix} f_n'''(s) - 3(f_n'(s))^2 \\ +2f_n(s) f_n''(s) + 3r^2 \\ +\Gamma\lambda\theta_n(s) \end{pmatrix} ds \qquad (19)$$

and

$$\theta_{n+1}(\eta) = \theta_n(\eta) + \int_0^\eta (s-\eta) \begin{pmatrix} \theta_n''(s) \\ +\Pr\begin{pmatrix} 2f_n(s)\theta_n'(s) \\ -5f_n'(s)\theta_n(s) \end{pmatrix} \end{pmatrix} ds \qquad (20)$$

By applying (17) to (19)-(20), we obtain the solutions of (5) respect to boundary conditions (6) as follow

$$f(\eta) = a\eta - \frac{1}{2}\eta^2 + \begin{pmatrix} -\frac{1}{6}\Gamma\lambda \\ +\frac{1}{2}a^2 - \frac{1}{2} \end{pmatrix}\eta^3 + \left(-\frac{1}{6}a - \frac{1}{24}\Gamma\lambda b\right)\eta^4 + \left(-\frac{\Gamma\lambda\Pr a}{24} + \frac{a^3-a}{20} + \frac{2-\Gamma\lambda a}{60}\right)\eta^5 + \begin{pmatrix} \frac{\Gamma\lambda\Pr + 2\Gamma\lambda}{144} + \frac{1-a^2}{24} \\ -\frac{1}{240}\Gamma\lambda\Pr ba \end{pmatrix}\eta^6 + \cdots \qquad (21)$$

$$\theta(\eta) = 1 + b\eta + \frac{5}{2}\Pr a\eta^2 + \left(-\frac{5\Pr}{6} + \frac{ab\Pr}{2}\right)\eta^3 + \begin{pmatrix} \frac{5(a^2\Pr^2 - \Gamma\lambda\Pr)}{24} \\ +\frac{5\Pr(a^2-1)}{8} - \frac{\Pr b}{3} \end{pmatrix}\eta^4 \qquad (22)$$

$$+ \begin{pmatrix} \frac{13(\Pr a^2 b - \Pr b) - \Pr^2 a^2 b}{40} \\ -\frac{3\Gamma\lambda\Pr b}{20} - \frac{\Pr a + 2\Pr^2 a}{6} \end{pmatrix}\eta^5 + \begin{pmatrix} \frac{\Pr + 2\Pr^2}{36} - \frac{11\Gamma\lambda\Pr^2 a}{72} \\ -\frac{4a\Pr b + \Pr b^2\Gamma\lambda}{40} \\ +\frac{11\Pr^2 a^3 - \Pr^2 a}{24} \end{pmatrix}\eta^6 + \cdots$$

For (21) and (22), iteration process continues sufficiently (as seen (11)).

In the series solutions (21) and (22), the unknown constant a,b which are denoted to velocity gradient and temperature gradient respectively, are found by applying very efficient approximation called Pade Technique (12-16) and infinite boundary conditions (6).

For numerical values of $\lambda=1$, $\Pr=1$ and $\Gamma=+1$ (opposing buoyancy force), we find unknown constants as

$$f'(0) = a = 1.412274065$$
$$\theta'(0) = b = -2.803862335 \qquad (23)$$

From (23) and put $\lambda=1$, $\Pr=1$ and $\Gamma=+1$ into (21-22), then numerical solution of (5) with MVIM is found as follow respectively,

$$f(\eta) = 1.412274065\eta + 0.3305923508\eta^3 - 0.1185514135\eta^4 + 0.02117728081\eta^5 - 0.00410565956\eta^6 - \frac{1}{2}\eta^2 \pm \cdots$$

$$\theta(\eta) = 1 - 2.803862335\eta + 3.530685162\eta^2 - 2.813244362\eta^3 + 1.763385808\eta^4 - 1.052008622\eta^5 + 0.7107541919\eta^6 \pm \cdots$$

5. RESULTS And DISCUSSION

The dimensionless form (5) and (6) of Marangoni boundary layer flow equations (1) and (2) are considered. These equations have been solved by modified variational iteration method (MVIM) and very efficient and accurate results are obtained by MVIM.

For both opposing and favorable buoyancy effected Marangoni flow with various values of Prandtl number (Pr) and Marangoni mixed convection parameter λ, our results are compared with well known asymptotic iteration method (Zhang and Zheng 2014) and fourth order Runge-Kutta method in Table 1,2,3.

Table 1 Comparison of velocity and temperature gradients values for buoyancy force effects

$\Gamma=+1$		MVIM		Ref (Zhang & Zheng)		Numerical[1]	
λ	Pr	$f'(0)$	$\theta'(0)$	$f'(0)$	$\theta'(0)$	$f'(0)$	$\theta'(0)$
1	1	1.412274065	-2.803862335	1.3770953	-2.9458683	1.4113699382952547	-2.78948714536900289
1	2	1.405165167	-3.976787667	1.3598498	-4.2130978	1.4002194673439794	-3.97480693986123512
0	0.5	1.332167618	-1.880343197	1.3287628	-2.0611599	1.3531950142211835	-1.90716896300225147
2	0.5	1.482416887	-2.056431944	1.4573970	-2.0884821	1.4889922387434242	-1.99825843240349954
$\Gamma=-1$		MVIM		Ref (Zhang & Zheng)		Numerical[1]	
λ	Pr	$f'(0)$	$\theta'(0)$	$f'(0)$	$\theta'(0)$	$f'(0)$	$\theta'(0)$
1	1	1.287203220	-2.640556677	1.2768744	-2.8990230	1.2917523393322974	-2.67069088539676658
1	2	1.315836672	-3.940255478	1.2981999	4.1678960	1.3040071102798589	-3.83725279754441573
1	3	1.334733660	-4.946965767	1.3153168	-7.9219190	1.3105560074629494	-4.7380888016884386
2	1	1.211855517	-2.528374125	1.2038062	-2.8498573	1.2264015411187497	-2.60354476159533110
3	1	1.122504874	-2.387794592	1.1230572	-2.8120538	1.1562579654494212	-2.52953330807238563

[1]Fourth order Runge Kutta

Table 2 Comparison of Temperature and Velocity gradients for various values of Pr **and** λ **for** $\Gamma=+1$

$\Gamma=+1$		MVIM		Numerical[1]	
λ	Pr	$f'(0)$	$\theta'(0)$	$f'(0)$	$\theta'(0)$
0.5	1	1.385624006	-2.739988206	1.3826578393120732	-2.76143002511236446
0.5	1.5	1.381628631	-3.392163066	1.37926876062386006	-3.40135895617698658
0.5	2	1.378716936	-3.925101657	1.37695951454900345	-3.94198486617161148
0	1	1.356249272	-2.649314461	1.35319501463357117	-2.7323451676649424
1.5	1	1.441773170	-2.858328650	1.43938712832371896	-2.8166003984916661
2	1	1.426551360	-2.829817768	1.46675860211244036	-2.84284309058585061

[1]Fourth order Runge Kutta

Table 3 Comparison of Temperature and Velocity gradients for various values of Pr **and** λ **for** $\Gamma=-1$

$\Gamma=-1$		MVIM		Numerical[1]	
λ	Pr	$f'(0)$	$\theta'(0)$	$f'(0)$	$\theta'(0)$
0.5	1	1.319306864	-2.686759875	1.32291768517141661	-2.70213587974090652
0.5	1.25	1.324084466	-3.053502073	1.32489599762663102	-3.03489176113386927
0.5	1.5	1.328280246	-3.385784692	1.32647661498123703	-3.3364729195401881
0	1	1.346509763	-2.72521503	1.35319502837390648	-2.73234497964024126
1.5	1	1.251247832	-2.587702322	1.25961368372507465	-2.63787843974779923
2	1	1.211855517	-2.528374125	1.22640155716212185	-2.60354451673469889

[1]Fourth order Runge Kutta

It is evident from Table 1 that our results better than the results in ref. (Zhang and Zheng 2014) with compared numerical method. Also from Table 2 and 3, it tells us that our presented method (MVIM) is efficient and powerful mathematical tool.

Fig. 2-3 demonstrate the variations of velocity profiles for both opposing $\Gamma=+1$ and favorable $\Gamma=-1$ buoyancy forces with different values of Prandtl Number (Pr).

Also, Fig. 4-7 show the variations of temperature profiles as the same meaning of Fig. 2-3 with various values of Pr and Marangoni mixed convection parameter λ respectively.

6. CONCLUSION

In this paper, we consider the nonlinear ordinary differential equations which corresponds to Marangoni boundary layer flow with buoyancy effects. These equations are solved by Modified variational iteration method analytically. The velocity gradient $f'(0)$, temperature gradient $\theta'(0)$ as well as the temperature and velocity profiles are analyzed and compared for buoyancy opposed and buoyancy favorable cases. Consequently, results show that MVIM is very powerful and convenient method for analytical and numerical solutions for nonlinear flow equations.

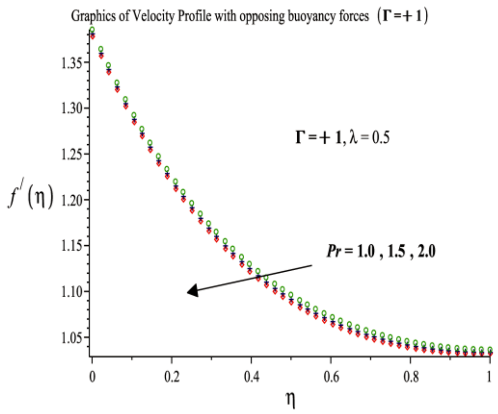

Fig. 2. Prandtl Number effects on velocity profile for eqn. (5) with opposing buoyancy forces.

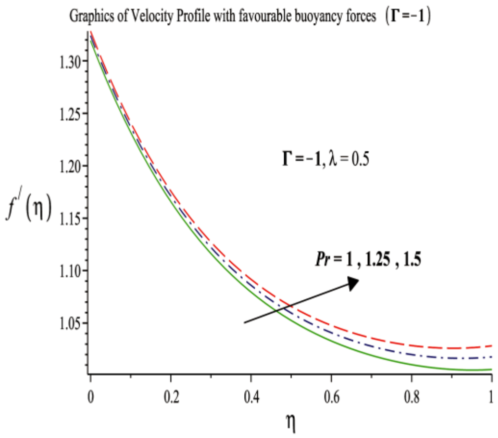

Fig. 3. Prandtl Number effects on velocity profile for eqn. (5) with assisting buoyancy forces

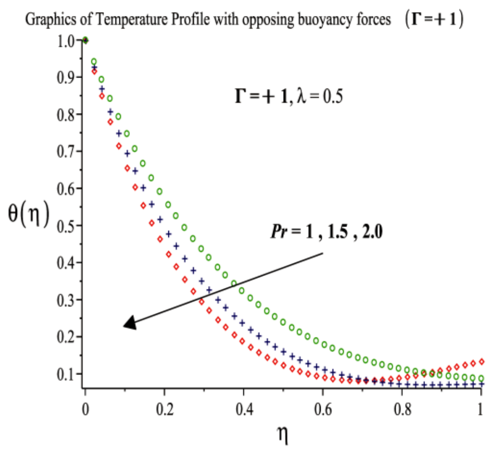

Fig. 4. Prandtl Number effects on temperature profile for eqn. (5) with opposing buoyancy forces.

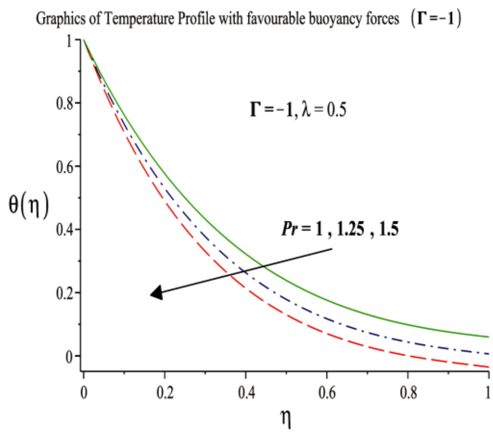

Fig. 5. Prandtl Number effects on temperature profile for eqn. (5) with assisting buoyancy forces.

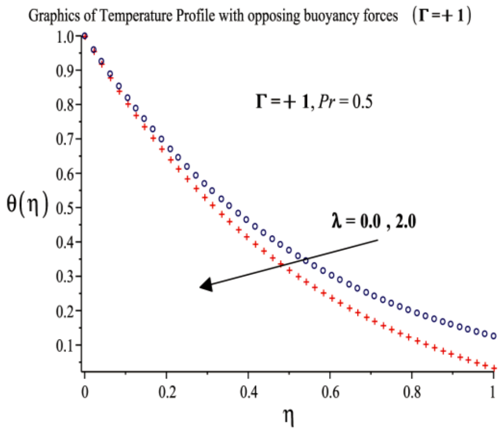

Fig. 6. Marangoni convection parameter effects on temperature profile for eqn. (5) with opposing buoyancy forces.

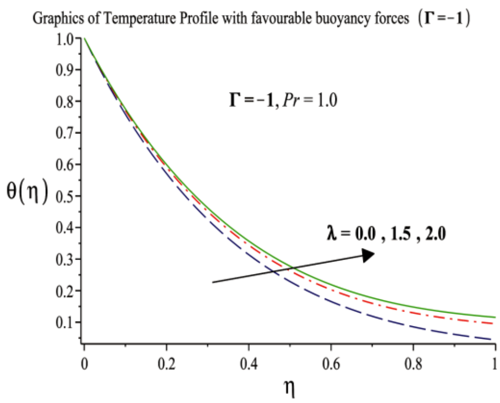

Fig. 7. Marangoni convection parameter effects on temperature profile for eqn. (5) with assisting buoyancy forces.

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REFERENCES

Aini, N. A., N. M. Arifin and F. Ismail (2012). Radiation effect on Marangoni convection boundary layer flow of a nanofluid. *Mathematical Sciences*, 6(21).

Al-Mudhaf, A. and A. J. Chamkha (2005). Similarity solutions for MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption effects. *Heat Mass Transfer* (42), 112-121.

Arafune, K. and A. Hirata (1999). Thermal and solutal Marangoni convection in In-Ga-Sb system. *J. Crystal Growth* (197), 811-817.

Baker, G. A., P. (1981). *Graves-Morris, Padé Approximants, Encyclopedia of Mathematics and Its Application 13, Parts I and II*, Addison-Wesley Publishing Company, New York.

Chamkha, A. J., I. Pop I. and H. S. Takhar (2006). Marangoni mixed convection boundary layer flow. *Meccanica* (41), 219-232.

Chien Hsin, C. (2007). Marangoni effects on forced convection of power-law liquids in a thin film over a stretching surface. *Physics Letters A* (370), 51-57.

Christopher, D. M. and B. X. Wang (2001). Similarity solution for Marangoni convection around a vapour bubble during nucleation and growth. *Int. J. Heat Mass Transfer* (44), 799-810.

Freidoonimehr, N, and M. M. Rashidi (2015). Dual Solutions for MHD Jeffery–Hamel Nano-Fluid Flow in Non-parallel Walls Using Predictor Homotopy Analysis Method. *Journal of Applied Fluid Mechanics* 8(4), 911-919.

Gelles, S. H. (1978). Microgravity studies in the liquid-phase immiscible system: aluminium–indium. *AIAA J.* 16(5), 431–438.

Golia, C. and A. Viviani (1985). Marangoni buoyant boundary layers. *L'Aerotechnica Missili e Spazio* (64), 29–35.

Golia, C. and A. Viviani (1986). Non isobaric boundary layers related to Marangoni flows. *Meccanica* (21), 200–204.

Hamid, R. A. and N. M. Arifin (2014). The effect of wall suction / injection an MHD Marangoni convection boundary layer flow in nanofluid. *AIP Conf. Proceedings* 1605, 386.

He, J. H. (1997). Variational iteration method for delay di¤erential equations. *Commun. Non-linear Sci. Numer. Simulation* 2(4), 235-236.

He, J. H. (1999). Variational iteration method-a kind of nonlinear analytical technique: some examples. *International Journal of Non-Linear Mechanics* (34), 699-708.

He, J. H. (2007). Variational iteration method-Some recent results and new interpretations. *Journal of Computational and Applied Mathematics* 3-17.

He, J. H. and W. Xu-Hong (2007). Variational iteration method: New development and applications. *Compt. Math. with Appl.* (54), 881-894.

Inokuti, M. and *et al*. (1978). General use of the Lagrange multiplier in nonlinear mathematical physics. In: S. Nemat-Nasser (Ed.). *Variational Method in the Mechanics of Solids,* 156-162. Pergamon Press, Oxford.

Jhankal, A. K. (2015). MHD Boundary Layer Flow near Stagnation Point of Linear Stretching Sheet with Variable Thermal Conductivity via He's Homotopy Perturbation Method. *Journal of Applied Fluid Mechanics* 8(3), 571-578.

Magyari, E. and A. J. Chamkha (2008). Exact analytical results for the thermosolutal MHD Marangoni boundary layers. *Int. J. Therm. Sci.* (47), 848–857.

Napolitano, L. G. (1978). Microgravity fluid dynamics. In *2nd Levitch conference*, Washington.

Napolitano, L. G. (1979, June). Marangoni boundary layers. In *Proceedings of the 3rd European symposium on material science in space*, Grenoble, ESA SP-142.

Napolitano, L. G. (1982). Surface and buoyancy driven free convection. *Acta Astronautica* (9), 199–215.

Okano, Y, M. Itoh and A. Hirata (1989). Natural and Marangoni convections in a two-dimensional rectangular open boat. *J. Chem. Eng Japan* (22), 275–281.

Pop, I., A. Postelnicu and T. Grosan (2001). Thermosolutal Marangoni forced convection boundary layers. *Meccanica* (36), 555-571.

Remeli, A., N. M. Arifin, R. Nazar and F. Ismail (2013). Marangoni mixed convection boundary layer flow in a nanofluid. *Malaysian Journal of Fundamental and Applied Sciences* 9(2), 81-85.

Zhang, Y. and L. Zheng (2012). Analysis of MHD thermosolutal Marangoni convection with the heat generation and a first-order chemical reaction. *Chemical Engineering Science* (69), 449-455.

Zhang, Y. and L. Zheng (2014). Similarity solutions of Marangoni convection boundary layer flow with gravity and external pressure. *Chinese J. of Chem. Eng.* 22(4), 365-369.