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Far Field Evolution of Momentum Driven and Scalar Dominated Flow Field

V. Ilangovan

Department of Mechanical Engineering, Francis Xavier Engineering College, Tirunelveli, Tamilnadu, 627003, India.

Email: ilaadhi@rediffmail.com

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ABSTRACT

To capture the effect of initial conditions in far field evolution of momentum driven and scalar dominated flow field, Witze scaling has been used for collapsing vector and scalar data to attain asymptotic state at self-preserving region of the jet. It incorporates the initial mass, momentum, energy to capture the effect of heating level on both near and far field development of strongly heated coaxial turbulent round air jets entering into quiescent ambient. This paper compares the effectiveness of potential core length and jet effective diameter as the length scales to collapse both mean and fluctuating components of velocity vector and temperature scalar. Similarity considerations with Witze length scale using the initial momentum flux and buoyancy flux gives a good collapse at all levels of heating.

Key Words: Coaxial; Turbulent; Self-Similarity; Buoyancy; Length Scale**.**

NOMENCLATURE

d	diameter
T	mean Temperature
H	heat Flux
m	mass Flux
M	momentum Flux
u, v	mean velocity in axial, radial directions
u' ,v'	fluctuating velocity in axial, radial directions
θ	excess temperature
η	normalized radial coordinate
λ	velocity ratio
β	area ratio
δ	density or Temperature ratio
α	coefficient of thermal expansion

ΔT	temperature difference between hot and cold jets
Gr	Grashof number,
Re	Reynolds number
Ri	Richardson number
Fr	densimetric Froude number

Subscripts

i	inner
o	outer
c	centerline
O	jet exit
e	effective or equivalent
t	turbulent
∞	ambient

1. INTRODUCTION

A jet represents an example of free shear flows, which is produced when the fluid exits a nozzle with initial momentum. A strongly heated air jet is a typical example for momentum driven and scalar dominated flow. Strongly heated turbulent coaxial round air jets are used in many technical systems, such as burners, jet engines and chemical reactors, in which mixing of different fluids are required. The mixing is controlled by different passive and dynamic arrangements. Heating the jet is one of many passive flow control techniques used for jet noise reduction.

After a certain development distance from the jet exit, the flow is said to have reached a self-similar or Self-preserving state when the mean and turbulent profiles of the flow become independent of the downstream distance with proper length, velocity and temperature scaling. The vector data are scaled with the local velocity half-width and local centre-line velocity whereas the scalar data are scaled with the local temperature half-width and local centre-line temperature. This simple scaling is not adequate to capture the effect of initial conditions in far field evolution of momentum driven and scalar dominated flow field. Hence a new scaling should be used for collapsing vector and scalar data to attain asymptotic state at self-preserving region of the jet. The new length scale has to incorporate the initial mass, momentum, energy to capture the effect of heating level on both near and far field development of coaxial turbulent jets. Similarity considerations with Witze length scale using the initial momentum flux and buoyancy flux gives a good collapse of both axial mean velocity and radial mean velocity fields at all levels of heating.

Thring and Newby (1953) introduced the concept of 'effective diameter' based on momentum flux conservation for normalizing the axial coordinate of the jet in the far field, which was used by several researchers Becker *et al.* (1967), Dahm and Dimotakis (1987), Dowling and Dimotakis (1990) and Pitts (1991). When the self-similar behavior of this strongly heated jet is investigated, this scaling appears to be less efficient in collapsing the centerline velocity and temperature data (mean and rms) along a unique self-similar curve for all temperature ratios, as it ignores buoyancy flux. Sautet and Stepowski (1996) proposed a modification in the effective diameter by replacing the ambient density with local average density to capture the initial values of variable density jets. While the effective diameter collapsed the velocity data fairly well in the near field, it was inadequate in the intermediate and far fields of the jet. Papadopoulos and Pitts (1998) used dynamic length scale based on local turbulence intensity for normalizing axial coordinate to account the effect of virtual origin shift and Reynolds number dependence. They concluded that the initial turbulence intensity flux was the main controlling parameter of centerline decay of mean velocity and temperature. They showed that the initial turbulence intensity influences the flow through shear layer breakup of the jet potential core and transition of the jet into self-preserving flow in the far field region of jet.

Witze (1974) used potential core length of the jet as the scale for normalizing the axial coordinate. By accounting the effect of initial values of velocity and temperature of the jet, the length of the potential core was calculated. Similarity considerations with Witze length scale using the initial momentum flux and buoyancy flux gives a good collapse of the mean and turbulent evolutions both velocity and temperature fields at all levels of heating. Bridges (2006) studied the effect of heat addition on space-time correlations of jets that were normalized appropriately with potential core length. He showed that mean and standard deviation of the velocity field in both cold and hot jets collapsed fairly well with this scaling.

2. SCALAR DOMINATED FLOW FIELD

2.1 Fluxes in Hot-Jets

The characteristic of the environment into which the hot jet is released plays a role in determining the balance of inertia and buoyant forces. In pure jet, the initial momentum flux through a high velocity injection causes the turbulent mixing. In a pure plume the buoyancy flux causes a local acceleration which leads to turbulent mixing. In case of a hot jet, a combination of initial momentum flux and buoyancy flux is responsible for turbulent mixing. In hot jet, the buoyancy affects both mean and turbulent quantities and buoyancy production of turbulent kinetic energy becomes important. In hot jet, the buoyancy force is much stronger than the inertia force due to sharp temperature difference between jet temperature and ambient temperature.

2.2 Richardson Number

In case of a hot jet, the jet fluid temperature is different from that of the medium into which it is discharged. The hot jets may be buoyant or non-buoyant. The fluid motion of buoyant jet is governed by inertial, buoyant and viscous forces, which are often characterized by Froude and Reynolds numbers. The buoyancy effects within a flow can be evaluated through the use of a non-dimensional number called Richardson number which is the ratio of gravity forces to inertial forces at any cross section normal to the direction of flow, is given by:

$$\mathrm{Ri_O} = \mathrm{Gr}/\mathrm{Re}^2 = \alpha\Delta\mathrm{T}/\mathrm{Fr} \qquad (1)$$

, where Gr is Grashof number, Re is Reynolds number, Fr is Densimetric Froude number and α is the coefficient of thermal expansion, which is defined as:

$$\alpha = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial\mathrm{T}}\right)_{\mathrm{p}} \qquad (2)$$

With this definition, the Richardson number takes on the form of the inverse of Densimetric Froude number, which is the ratio of inertial to buoyancy forces. When the Richardson number is very small$(\mathrm{Ri_O} < 0.001)$, the fluid temperature has the role of a passive scalar and buoyancy effects are insignificant. Kotsovinos.N.E, (1977) remarked a scalar as 'active' when the flow field is influenced by it; otherwise the scalar is considered as 'passive'.

In the present study, the values of initial Richardson number ($\mathrm{Ri_O}$) are 0.2799, 0.237, 0.176, 0.066 and 0 at temperature difference (ΔT) = 440,400,360,320 and 300K respectively. The value of $\mathrm{Ri_O}$ varies with buoyancy addition through heating. This can be the result of high temperature of hot jet, low flow velocity, or large differences in fluid density when evaluated over the range of temperatures within the flow. As the temperature as scalar is able to influence the flow field, it is termed as scalar dominated flow field.

3. SELECTION OF PROPER SCALING

A proper scaling is necessary for collapsing temperature and velocity data, to capture both far and near field developments by incorporating the initial conditions of jets. The selected scaling should further satisfy the similarity conditions and collapse both mean and fluctuating components of velocity and temperature introduced as a scalar in the flow field. It helps to understand whether the mean and fluctuating behavior of the jet in the far downstream from the origin, becomes independent of the initial conditions (nozzle geometry, velocity profile, boundary layer, swirl, turbulence level, and density ratio),or depends only on the amount of mass, momentum and energy added to jet at its exit.

3.1 Effective Diameter as a Length Scale

Mass, momentum, turbulence and temperature distributions at the exit plane influence flow development in the near and far fields of the jet. Conservation of mass, momentum and heat fluxes are applied to show the influence of initial temperature difference on the far-field flow by introducing the concept of effective jet. It is a hypothetical jet whose mass, momentum and heat fluxes are equal to that of inner and outer jets of coaxial streams. The effective diameter was used for normalizing the radial coordinate of the jet in the far field. The validity of effective jet concept holds good only in the far field region as the average velocity and bulk temperature of the jet asymptotically approach the ambient velocity and temperature, a condition that is not valid in the near field region of coaxial jet mixing where large difference in temperature exists between inner and outer jets.

The Effective Diameter $d_e = \frac{2m_0}{\sqrt{\pi M_0 \rho_\infty}}$ was used for incorporating the effect of density, non uniformity in velocity through the mass (m_0) and momentum fluxes (M_o) at the jet exit. They showed that momentum driven jets with different initial conditions have same centerline decay constant and spread rate. They also showed that normalized mean velocity and higher order moments attain self-similarity far downstream with this scaling. The above scaling was used for normalizing both axial and radial coordinates of jet. But this length scaling is not adequate when heat flux (H_0) is added at jet exit plane.

In a coaxial jet arrangement, the magnitude of mass, momentum and heat fluxes of inner and outer jets were different. By applying the procedure suggested by Fisher *et al*. (1998) and Antoine (2007), the Effective Diameter (d_e), Effective Velocity(u_e) and Effective Temperature (θ_e) of equivalent circular jet having same mass flux, momentum flux and heat flux as of coaxial jets were calculated through the conservations of mass, momentum an energy as follows:

Effective Diameter $$d_e = d_i\left[\frac{(1+\lambda\beta)(1+\lambda\beta\delta)}{(1+\lambda^2\beta\delta)}\right]^{\frac{1}{2}} \qquad (3)$$

Effective Velocity $$u_e = u_i\left(\frac{1+\lambda^2\beta\delta}{1+\lambda\beta\delta}\right) \qquad (4)$$

Effective Temperature $$\theta_e = \theta_i\left[\frac{(1+\lambda\beta)}{(1+\lambda\beta\delta)}\right] \qquad (5)$$

, where λ is the velocity ratio (u_o/u_i) , β is the area ratio(A_o/A_i), and δ is the density or temperature ratio(T_i/T_o). For the attainment of similarity, the jet flow should be dependent only on the total mass, momentum and energy and not on initial value of any specific characteristics of the jet, the concept of effective diameter fails to collapse the data at far field region.

3.2 Similarity Considerations with Witze Length Scale

Witze correlation parameter was used to collapse the experimental data taken at different temperature and velocity ratios. Witze (1974) correlation gives the length of the potential core in terms of initial velocity and temperature ratios. The advantage of using Witze parameter is that the density change due to heat addition is also accounted in scaling parameter. It also fulfills the requirement of far stream similarity condition besides capturing effect of initial condition for revealing both near and far field evolution of heated jet. When the mean centerline velocities at different temperatures were plotted versus an axial scale that was normalized by the potential core length through Witze parameter, it collapsed fairly well. It is defined as $x_w = (2x)(\mathrm{k})(\gamma)^{0.5}$, where $\mathrm{k} = 0.08[1 - 0.16(\lambda)]^{-0.22}$, where λ is the velocity ratio (u_o/u_i) and γ is the temperature ratio(T_i/T_o). These values were taken for normalization of spatial variation of hydrodynamic and thermal field in order to consider the effect of the momentum driven and temperature driven flow simultaneously.

3.3 Justification for Choosing Witz Length Scale

Recent measurements by Bridges and Wernet (2007), Brown and Bridges (2006) have shown that most velocity statistics do not vary significantly with temperature, when plotted against actual axial distance as length scale. The correlations were relatively insensitive to temperature. They also found that the spectral shapes and second order velocity correlations turbulence statistics such as turbulence intensity, velocity spectra, and spatial and temporal correlations, initially insensitive to temperature with axial distance as scale parameter became more pronounced, visible and sensitive to temperature when normalized with potential core length as length scale. In the absence of proper scaling, they showed that the turbulence spectra for the unheated and heated jets were similar; except for a small increase in overall turbulent kinetic energy due to heat addition. Improper scaling showed that the two-point correlation of turbulent velocity fluctuations in jets were not affected by density variation caused due to the heat addition. Hence, all axial locations are normalized with respect to the jet potential core length through the Witze (1974) parameter in the studies of Bridges and Wernet (2007), Brown and Bridges (2006).

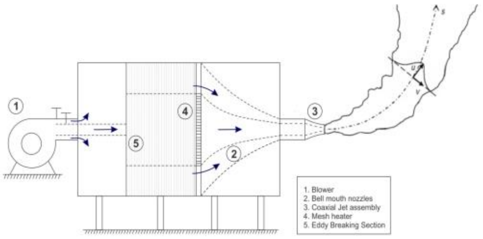

Fig. 1. Experimental Setup and Flow field.

4. EXPERIMENTAL METHOD

The main aim of the present work is to make a detailed study on the mean and turbulent flow characteristics of co-axial jet. The experimental setup and flow field is shown in Fig. 1. The case under considerations is a co-axial jet with velocity ratio, λ = 1, turbulent uniform nozzle exit velocity profile, hot inner jet with cold outer jet. The experiments were designed to study the non isothermal mixing of a strongly heated air jet issued from inner nozzle of coaxial assembly with outer cold jet released into a free ambient. The density ratio of the inner jet to the outer jet was varied by heating the inner stream. The measurements were obtained with three different outer jet exit constant exit Reynolds number. The Reynolds number of inner and outer jets was $3.0\text{X}10^4$ and $2.45\text{X}10^4$ respectively. Measurements of the instantaneous velocity and temperature in this heated jet have been performed with the aid of constant temperature and constant current Hotwire Anemometry. The use of X wire probe and of digital signal processing system represent special features of this experiment. Data were processed for obtaining the evolution of velocity and temperature fields such as the mean, statistical moments up to third-order and dissipation of the fluctuating temperature.

5. RESULT AND DISCUSSION

Attainment of self-similarity implies the existence of universal asymptotic behavior of the jet that enables to collapse the radial profiles in all downstream location into a single curve with suitable length and velocity scales. Self-similar behavior of a strongly heated jet was investigated and the effectiveness different length scales viz., effective jet diameter and potential core length as length scale to collapse all mean and turbulent flow statistics that accounts for the strong density variations in the far- field development was presented.

Figure 2 shows the similarity profile of axial mean velocity at various axial stations and temperature ratios. It shows the shape similarity of the radial profiles that remains invariant with axial distance. When the axial mean velocities at different temperatures were plotted versus an axial scale that was normalized by the potential core length through Witze parameter, it collapsed fairly well.

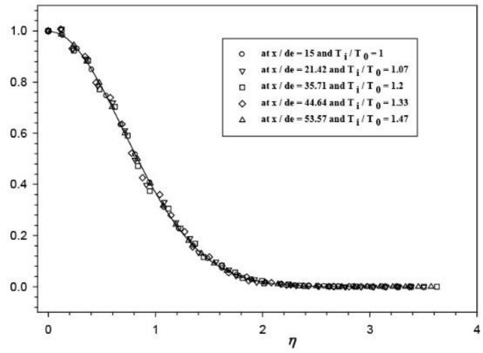

Fig. 2. Similarity profile of axial mean velocity at different temperature ratios.

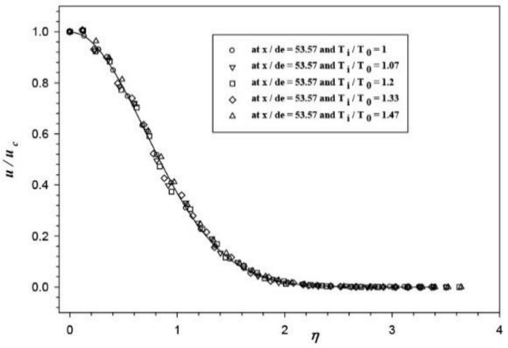

Fig. 3. Similarity profile of axial mean velocity at different temperature ratios at axial station x/de=53.57.

The effect of heat addition on the similarity profile of axial mean velocity at far field axial station x/de = 53.57 is given in Fig. 3. It is clear that the normalized mean flow remains invariant with different temperature ratios at the axial station x/de = 53.57. The physical requirements of constant flux of momentum across all sections of the jet and assumption of velocity profile similarity lead to the requirement of a hyperbolic decrease in the mean velocity along the jet axis. The plot of the normalized axial velocity distribution against the normalized length scale shows that the decrease approximates a nearly hyperbolic power law from 10 to 30 diameters, but deviates markedly at greater distances. This deviation may be either due to absence of power law decrease of velocity or improper length scaling which denies the existence of exact similarity.

Figure 4 shows the radial mean velocity variation across a flow cross section normalized with ‘effective diameter’. When the ‘effective diameter ‘scaling collapses the far field data of cold jet effectively, it fails to produce universal curve when the heat is added into the jet.

Figure 5 shows the similarity profile of mean radial velocity scaled with potential core length calculated with Witze correlation at different temperature and velocity ratios. It shows the shape similarity of the radial profiles that remains invariant with axial distance. When the radial mean velocities at different temperatures were plotted versus an axial scale that was normalized by the potential core length through Witze length scale, it collapsed fairly well.

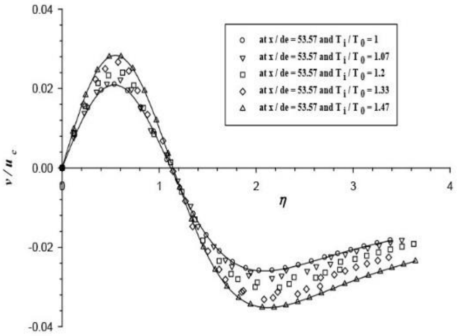

Fig. 4. Profile of radial mean velocity at different temperature ratios at axial station x/de=53.57 with 'effective diameter' scaling.

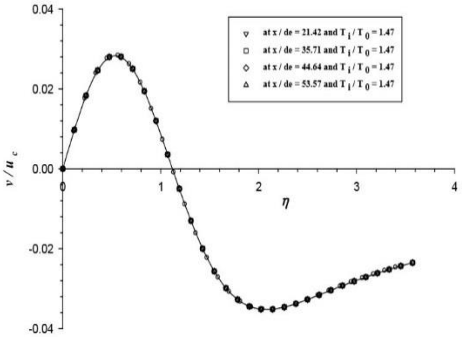

Fig. 5. Similarity profile of radial mean velocity scaled with Witze correlation at various levels of heating.

It was found that a strongly heated jet has more pronounced rate of centerline decay than a cold jet. This sudden decrease of velocity is due to buoyancy coming into play at large distances downstream. In the case of a cold jet, there is no transverse momentum that can alter the axial velocity, but in a strongly heated jet the transverse momentum of buoyancy lifts the axial velocity component from its centreline axis making it rise. This analysis is corroborated by Reichardt (1943) that the lateral transport of momentum is proportional to the transverse gradient of the horizontal component of momentum. The advantage of using Witze parameter is that the density change due to heat addition is also accounted in scaling parameter. It also fulfills the requirement of far stream similarity condition besides capturing effect of initial condition for revealing both near and far field evolution of heated jet. When the turbulent shear stress at different temperatures was plotted versus an axial scale that was normalized by the potential core length through Witze parameter, it collapsed fairly well.

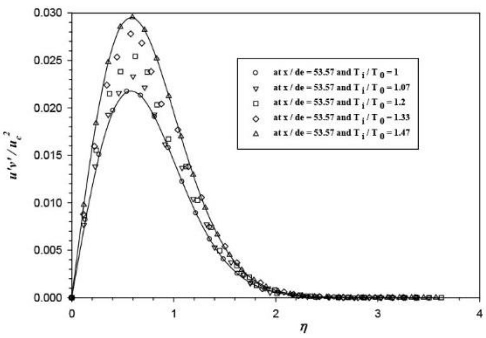

Fig. 6. Profile of Reynolds stress at different temperature ratios at axial station x/de=53.57 with 'effective diameter' scaling.

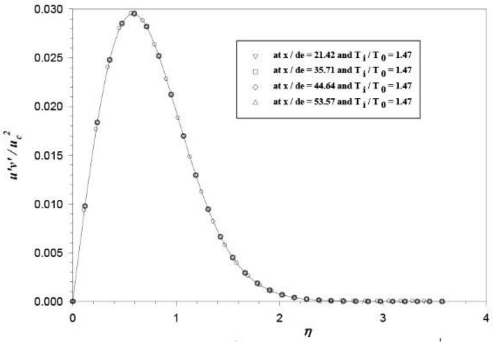

Fig. 7. Similarity profile of Reynolds stress scaled with Witze correlation at various levels of heating.

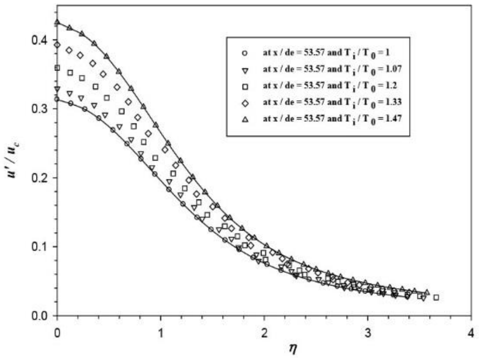

Fig. 8. Profile of axial component of turbulence intensity at axial station x/de=53.57 with 'effective diameter' scaling.

The radial profiles of Reynolds stress can be seen in the Fig. 6 wherein the distance coordinate was normalized with effective diameter. When effective diameter is used as the scaling, the flow which remains self similar at cold condition at axial station x/de = 53.57 turns into developing flow at x/de = 53.57 due to heat addition at jet exit. This is due to the reason that the effective diameter scaling obtained with momentum consideration does not account the density variation due to heat addition. When the value of turbulent Reynolds stress was analyzed, remarkable difference was seen at radial location close to $\eta = 0.5$ where the inner mixing layer exists between the inner hot and outer cold jet in the coaxial jet flow. The value u'v' / u_c^2 is 0.020 and 0.030 at cold jet condition and temperature ratio Ti/To = 1.47 respectively. The difference in the value of Reynolds stress increases with heat addition. Hence, the effect of heat addition is not captured by this scaling.

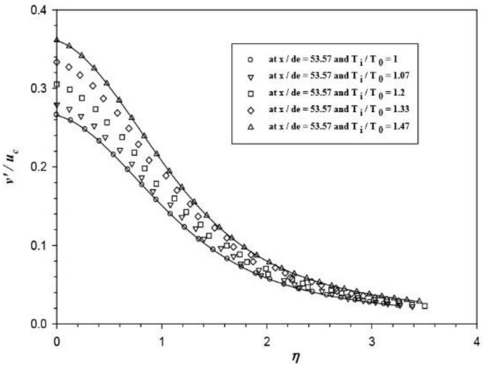

Fig. 9. Profile of radial component of turbulence intensity at axial station x/de=53.57 with 'effective diameter' scaling.

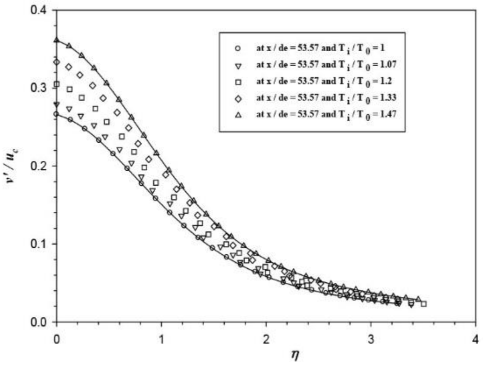

Fig. 10. Profile of azimuthal component of turbulence intensity at axial station x/de=53.57 with 'effective diameter' scaling.

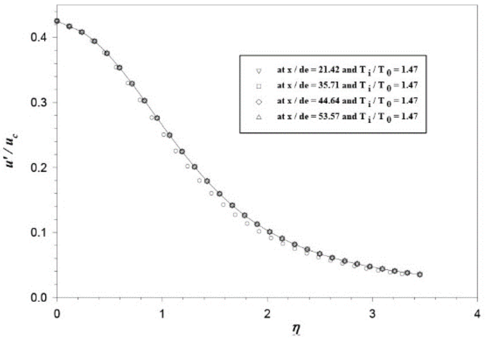

Fig. 11. Similarity profile of axial component of turbulence intensity scaled with Witze correlation at various levels of heating.

Similarity profile of Reynolds stress can be seen in the Fig. 7 wherein the distance coordinate was normalized with Witze length scale. The flow becomes self similar in the far field region of coaxial jet flow at different temperature ratios.

The profiles of axial, radial and azimuthal component of turbulence intensity at axial station x/de = 53.57 were shown in the Fig. 8, Fig. 9, Fig. 10 respectively. The distance coordinates were normalized with effective diameter. The radial profiles show variation with heat addition and universal similarity was not obtained with this normalization.

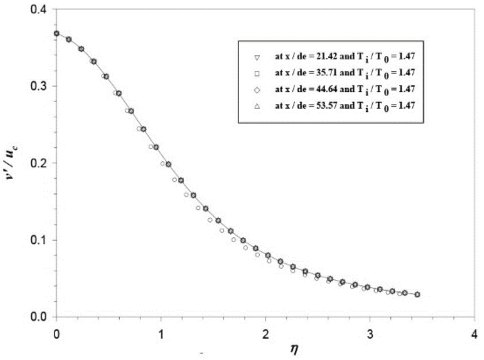

Fig. 12. Similarity profile of radial component of turbulence intensity scaled with Witze correlation at various levels of heating.

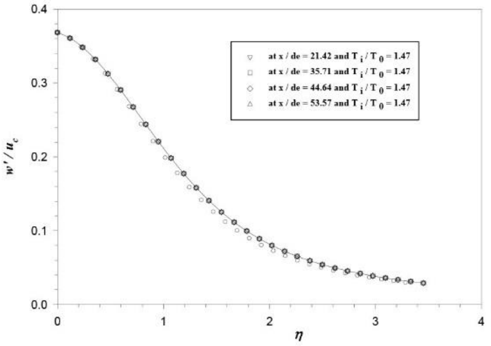

Fig. 13. Similarity profile of azimuthal component of turbulence intensity scaled with Witze correlation at various levels of heating.

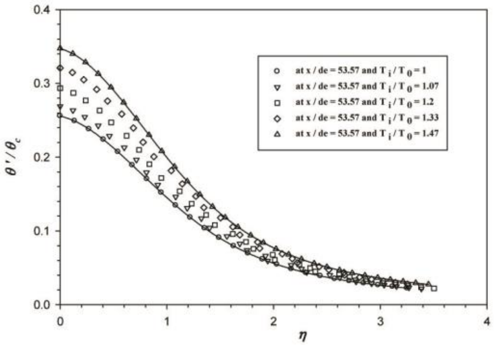

Fig. 14. Profile of temperature fluctuation at axial station x/de=53.57 with 'effective diameter' scaling.

Similarity profile of turbulent intensity can be seen in the Fig. 11, Fig. 12, and Fig. 13 respectively wherein the distance coordinate was normalized with Witze length scale. The turbulent velocity fluctuation of this heated coaxial jet flow becomes self similar in the far field region at different temperature ratios. The Witze correlation collapses both mean and turbulent flow field effectively at heated and unheated jet mixing.

Figure 14 shows the radial profiles of mean temperature at downstream axial stations normalized with 'effective diameter'. When the 'effective diameter 'scaling collapses the far field data of cold jet effectively, it fails to produce universal curve when the heat is added into the jet.

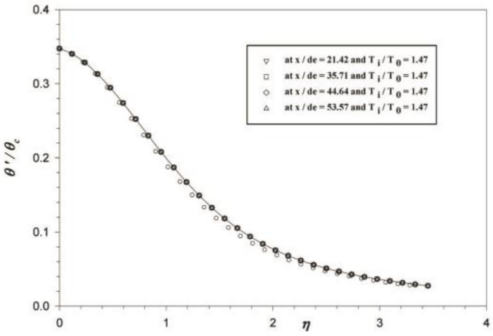

Fig. 15. Similarity profile of temperature fluctuation scaled with Witze correlation at various levels of heating.

Figure 15 shows the similarity profile of temperature fluctuation scaled with Witze correlation at various levels of heating. Similarity considerations with Witze length scale using the initial momentum flux and buoyancy flux gives a good collapse of mean temperature fields at all levels of heating.

6. CONCLUSION

Self-similar behavior of a strongly heated jet was investigated and its effectiveness using different length scales viz., effective jet diameter and potential core length as length scale to collapse all mean and turbulent flow statistics that accounts for the strong density variations in the far- field development was tested. Effective jet diameter scaling , based on momentum flux conservation appears to be less efficient for collapsing the mean and fluctuating part velocity and temperature data for all temperature ratios, as it ignores buoyancy flux. Similarity considerations with Witze length scale using the initial momentum flux and buoyancy flux gives a good collapse of the mean and turbulent evolutions both velocity and temperature fields at all levels of heating.

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