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A Simple Nonlinear Eddy Viscosity Model for Geophysical Turbulent Flows

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ABSTRACT

Eddy viscosity models in turbulence modeling can be mainly classified as linear and nonlinear models. Linear formulations are simple and require less computational resources but have the disadvantage that, those can't predict actual flow pattern in complex geophysical flows where streamline curvature and swirling motion are predominant. A constitutive equation of Reynolds stress anisotropy is adopted for the formulation of eddy viscosity including all the possible higher order terms quadratic in the mean velocity gradients and a simplified model is developed for actual oceanic flows where only the vertical velocity gradients are important. The simplified formulation is used for the study of natural convection flow in a vertical water column and the results are compared with the observational data and predictions of other existing turbulence models. The developed formulation can be incorporated in other computational fluid dynamics codes for the flow analysis in various engineering applications. The model predictions of marine turbulence and other related data (e.g. sea surface temperature, surface heat flux and vertical temperature profile) can be utilized in determining the effective siting for the Ocean Thermal Energy Conversion (OTEC) plants and in particular for the development of tidal energy projects.

Keywords: CFD; Turbulence modeling; Eddy viscosity; Geophysical flows; Natural convection.

NOMENCLATURE

A^*	Lumley flatness parameter
b_{ij}	Reynolds stress anisotropy
C_p	heat capacity
d_{ij}	diffusive transport
k	turbulence kinetic energy
P	pressure
P_{ij}	shear production
R_{ij}	vorticity tensor
S	salinity
S_{ij}	strain rate tensor
T	temperature
U	velocity
ν	kinematic viscosity
u	fluctuating velocity in x-direction
v	fluctuating velocity in y-direction
ν_t	eddy/turbulent viscosity
ν'	molecular diffusivity
w	fluctuating velocity in z-direction
ε	turbulence dissipation
ζ	elevation of the free surface
II	second invariant of Reynolds stress anisotropy
III	third invariant of Reynolds stress anisotropy
ϕ_{ij}	pressure strain correlation

1. INTRODUCTION

Turbulence modeling in computational fluid dynamics and geophysical modelling can be classified into four major approaches as direct numerical simulation (DNS)(Moin and Mahesh 1998), large eddy simulation (LES)(Piomelli 1999), Reynolds stress model (RSM)(Panda 2020; Mishra and Girimaji 2010; Mishra and Girimaji 2017) and eddy viscosity model (EVM)(Pope 2000). DNS consists of solving the Navier-Stokes equations resolving all the scales of motions. Computational cost of such simulations is very high for complex flows and for flows with higher Reynolds numbers. However, in LES Largest scales are resolved and the smaller scales are modelled. The computational cost is comparatively less than DNS. These two methods of turbulence simulations are not practically viable with limited computational resources. The Reynolds stress models (Mishra 2014; Panda and Warrior 2018) are more accurate than the eddy viscosity models, but those make the solver unstable, since those models solve equations for all the components of the Reynolds stresses.

In contrast to above mentioned three approaches, eddy viscosity models are simple and require less computational resources and provide a stable solver for performing different simulations and are based

on the Reynolds averaged Navier Stokes (RANS) equations in which Reynolds stresses appear as a result of time averaging of momentum conservation equations, The RANS equations can be written as:

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{DU_i}{Dt} = \frac{-1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\nu\left(\frac{\partial U_i}{\partial x_j}\right) - \frac{\partial}{\partial x_j}\left(\overline{u_i u_j}\right) \quad (2)$$

where, *U* and *P* are velocity and pressure respectively. In the above equation, there are four equations but ten unknowns. The additional unknowns result from the averaging of Navier-Stokes equations and are termed as Reynolds stresses. (Boussinesq 1877) was the first to postulate the assumption that the Reynold stress tensor is proportional to the strain rate tensor and can be written as

$$\overline{u_i u_j} = -2\nu_t S_{ij} + \frac{2}{3}k\delta_{ij} \quad (3)$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \quad (4)$$

Similarly the vorticity tensor has the form:

$$R_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \quad (5)$$

The eddy viscosity models can be further classified as linear and non-linear models. Most of the geophysical modellers have used linear version of the eddy viscosity models (Dijkstra *et al.* 2017; Smyth *et al.* 2012; Klingbeil *et al.* 2018; Yamazaki *et al.* 2014). The linear models are unable to predict the Reynolds stress anisotropy accurately when the flow is complex because of swirl and curvature effects, since those have not any higher order terms in the definition of anisotropy. However, the non-linear eddy viscosity models has quadratic terms in terms of the strain and vorticity, which can overcome such difficulties and can accurately predict the high Reynolds stress anisotropy resulting from the complex flow physics (Yang and Ma 2003; Shih *et al.* 1998). Very few researchers have used non-linear eddy viscosity models for modelling geophysical flows (Sasmal *et al.* 2014; Sasmal *et al.* 2015). They mainly have used the lower order terms in the definition of the Reynolds stress anisotropy.

In the standard k-epsilon model (Jones and Launder 1972) ν_t is defined as

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} \quad (6)$$

The equations for k and ò can be written as(Pope 2000),

$$\frac{Dk}{Dt} = D_k - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - \nu_t \overline{\frac{\partial u_j}{\partial x_i}\frac{\partial u_j}{\partial x_i}} \quad (7)$$

$$\frac{D\varepsilon}{Dt} = D_\varepsilon - \frac{C_{\varepsilon 1}P_k - \varepsilon C_{\varepsilon 2}}{k/\varepsilon} \quad (8)$$

The coefficients appearing in the dissipation equation were chosen by referring to the measured rate of decay in grid turbulence and local equilibrium turbulence respectively (Launder and Spalding 1983). To obtain ν_t, the transport equations of turbulence kinetic energy *k* and dissipation rate ε are need to be solved in the k-epsilon model. (Mellor and Yamada 1982) solved equations for *k* and *kl* , where *l* is the length scale. c_μ is the structure parameter. (Wilcox 1988) replaced dissipation by ω (which is the ratio of dissipation and kinetic energy) . In k-epsilon model the structural parameter is a constant value, that can be defined by referring to local equilibrium shear layers but in the geophysical turbulence model of (Mellor and Yamada 1982) the structural parameter was taken as function of shear and buoyancy. The effects of buoyancy, vorticity and Reynolds stress anisotropy were included in the structural parameter of (Kantha and Clayson 1994).

Reynolds stress anisotropy can be defined as

$$b_{ij} = \frac{\overline{u_i u_j} - 2/3k\delta_{ij}}{2k} \quad (9)$$

(Maity and Warrior 2011) proposed an eddy viscosity model based on a transport equation of second invariant of Reynolds stress anisotropy and studied the natural convection flow and mixing in a vertical water column. For non equilibrium shear flows (Craft *et al.* 1996) proposed a non-linear eddy viscosity model and took Reynolds stresses as more general function of vorticities and mean velocities. Considering the above formulation of anisotropy tensor (Sasmal *et al.* 2014) proposed an eddy viscosity formulation for geophysical turbulent flows. They didn't consider the cubic terms in the definition of Reynolds stress anisotropy. Later, (Sasmal *et al.* 2015) used the same formulation of eddy viscosity to model dissipation of turbulence in geo-physical flows.

In this article, a simplified formulation for the eddy viscosity in terms of the invariants of the Reynolds stress anisotropy is proposed for the geophysical flows by adopting a non-linear constitutive equation for Reynolds stress anisotropy tensor that include all the possible higher order terms quadratic in the mean velocity gradients, which accounts for the streamline curvature and swirling effects (Craft *et al.* 1997). We have neglected the heat flux and wall reflection terms from the model of (Craft *et al.* 1997), since those are nor not relevant to our work. The proposed formulation is implemented in the 1D General Ocean Turbulence Model(GOTM) (Burchard *et al.* 1999), where, the velocity is considered to be varying only in the vertical direction. Numerical experiments were conducted for the Fladenground experiment 1976(FLEX'76) (Burchard *et al.* 1999) and the ocean whether station papa (OWS PAPA) (Burchard *et al.* 1999) for which large observational datasets are available for different meteorological conditions. The model predictions are compared against those observational data and contrasted against the k-epsilon model predictions. The proposed formulation can accurately predict flow parameters for complex geophysical flows, where complex strain field exists because of the swirling and streamline curvature.

2. FORMULATION OF THE PROBLEM

The one-dimensional form of the Reynolds-averaged Navier-Stokes equations, energy conservation equation and salt conservation equation are used for the study of natural convection flow and heat transfer in a vertical water column. Effects of the advection, internal pressure gradients and horizontal transport are neglected.

$$\partial_t U - \nu\partial_{zz}U + \partial_z \overline{uw} - fV = -g\frac{\bar{\rho}}{\rho_0}\partial_x\zeta \qquad (10)$$

$$\partial_t V - \nu\partial_{zz}V + \partial_z \overline{vw} - fU = -g\frac{\bar{\rho}}{\rho_0}\partial_y\zeta \qquad (11)$$

where, U and V are the velocities, g is the acceleration of gravity, ζ is the elevation of the free surface, $\bar{\rho}$ is the averaged density, and ρ_0 a constant reference density resulting from Boussinesq approximation (Umlauf and Burchard 2005).

The Energy and Salinity conservation equations can be written respectively as

$$\partial_t T - \nu'\partial_{zz}T + \partial_z \overline{wt} = \frac{\partial_z I}{C_p\rho_0} \qquad (12)$$

$$\partial_t S - \nu'\partial_{zz}S + \partial_z \overline{ws} = 0 \qquad (13)$$

where T and S are the temperature and salinity respectively. ν' denote the molecular diffusivity, C_p is the heat capacity. I denotes the short wave radiation and its vertical divergence is taken as the source term in the energy conservation equation (Umlauf and Burchard 2005).

2.1 Simplified Formulation of Eddy Viscosity

The Reynolds stress in terms of Boussinesq eddy viscosity can be written as

$$\overline{u_i u_j} = -2\nu_t S_{ij} + \frac{2}{3}k\delta_{ij} \qquad (14)$$

We have assumed that the turbulence is in equilibrium state. The constitutive equation of (Craft *et al.* 1997) for the Reynolds stress anisotropy tensor is considered for the present eddy viscosity formulation,

$$b_{ij} = \frac{-\nu_t}{k}S_{ij} + c_1\frac{\nu_t}{\varepsilon}(S_{ik}S_{kj} - \frac{1}{3}S_{kl}S_{kl}\delta_{ij}) + c_2\frac{\nu_t}{\varepsilon}(R_{ik}S_{kj} + R_{jk}S_{kl}) + c_3\frac{\nu_t}{\varepsilon}(R_{ik}R_{jk} - 1/3R_{lk}R_{lk}) + c_4\frac{\nu_t k}{\varepsilon^2}(S_{kl}R_{kj} + S_{kj}R_{li}) + c_5\frac{\nu_t k}{\varepsilon^2}(R_{il}R_{lm}S_{mj} + S_{il}R_{lm}R_{mj} - \frac{2}{3}S_{lm}R_{mn}R_{nl}\delta_{ij}) + c_6\frac{\nu_t k}{\varepsilon^2}S_{ij}S_{kl}S_{kl} + c_7\frac{\nu_t k}{\varepsilon^2}S_{il}R_{kl}R_{kl} = \nu_t\phi_1 \qquad (15)$$

The coefficients of Eq(15) are calibrated against test cases as, $c_1 = -0.04$, $c_2 = 0.1$, $c_3 = 0.02$, $c_4 = 0.1$, $c_5 = -0.8$, $c_6 = -0.6$ and $c_7 = 0.6$.

An expression for b_{ji} can be written by interchanging the indices i and j, and here will represent only the final expression,

$$b_{ji} = \nu_t\phi_2 \qquad (16)$$

Multiplying Eq (15) and (16), an expression for the second invariant of Reynolds stress anisotropy can be obtained,

$$II = b_{ij}b_{ji} = \nu_t^2\phi_1\phi_2 \qquad (17)$$

After rearrangement of the terms, An equation for the turbulent viscosity will be obtained as

$$\nu_t = \frac{II^{\frac{1}{2}}}{(\phi_1\phi_2)^{\frac{1}{2}}} \qquad (18)$$

In the oceans, because of the strong disparity between the horizontal and vertical dimensions, the strain and vorticity take a simplified form. Thus by considering only vertical gradients of velocity and neglecting variations of U and V in other directions, the strain and vorticity tensor acquire the form

$$S_{ij} = \frac{1}{2}\begin{pmatrix} 0 & 0 & \frac{\partial U}{\partial z} \\ 0 & 0 & \frac{\partial v}{\partial z} \\ \frac{\partial U}{\partial z} & \frac{\partial V}{\partial z} & 0 \end{pmatrix} \qquad (19)$$

$$R_{ij} = \frac{1}{2}\begin{pmatrix} 0 & 0 & \frac{\partial U}{\partial z} \\ 0 & 0 & \frac{\partial v}{\partial z} \\ -\frac{\partial U}{\partial z} & -\frac{\partial V}{\partial z} & 0 \end{pmatrix} \quad (20)$$

2.2 Transport Equation for the Second Invariant of Stress Anisotropy

An equation for the second invariant developed by (Craft *et al.* 1997) is taken into consideration. A transport equation for Reynolds stress anisotropy can be written as,

$$\frac{Db_{ij}}{Dt} = \frac{1}{k}(d_{ij} + P_{ij} + \phi_{ij} - \varepsilon_{ij}) - \frac{b_{ij}}{k}(d_k + P_k - \varepsilon) \quad (21)$$

The transport equation for II is derived by multiplying the above equation by $2b_{ij}$. The resulting equation for the stress invariant is written as

$$\frac{DII}{Dt} = -2\frac{II}{k}(d_k + P_k - \varepsilon) + 2\frac{b_{ij}}{k}(d_{ij} + P_{ij} + \phi_{ij} - \varepsilon_{ij}) \quad (22)$$

where d_{ij} represents the diffusive transport, P_{ij} is the shear production, ϕ_{ij} is the pressure strain correlation which is the summation of slow and rapid term and d_{ij} is the dissipation rate of Reynolds Stress.

In order to model the pressure strain correlation, the Poisson equation for fluctuating pressure should be solved for determining the pressure fluctuations

The second moment closure model of (Craft *et al.* 1996) has the form:

$$\phi_{ij} = \phi_{ij}^S + \phi_{ij}^R \quad (23)$$

$$\phi_{ij}^S = -c_1\varepsilon[b_{ij} + c_1^*(b_{ik}b_{kj} - 1/3II\delta_{ij}] - (A^*)^{0.5}\varepsilon b_{ij} \quad (24)$$

$$\phi_{ij}^R = -0.6(P_{ij} - 1/3\delta_{ij}P_{kk}) + 0.3b_{ij}P_{kk} \quad (25)$$

In the expression for ϕ_{ij}^S,

$c_1 = 3.1[1 - \exp - R_t / 80)^2 (A^*)^{0.5} \min(II^{0.5}, 0.5), c_1^* = 1.2\$,$

and $P_{ij} = -(\overline{u_i u_k}\frac{\delta U_j}{\delta x_k})$

$$P_k = 0.5P_{ii} \quad (26)$$

$b_{ij}\phi_{ij}^S$ works out to be,

$$b_{ij}\phi_{ij}^S = II(-C_1\varepsilon) + III(-c_1c_1^*\varepsilon) - II\varepsilon(A^*)^{0.5} \quad (27)$$

The transport equation for the second invariant of Reynolds stress anisotropy can be simplified as:

$$\frac{DII}{Dt} = -2/k(P_k - \varepsilon) + 2/k(-1.6k/3b_{ij}S_{ij} + 0.3IIP_k - c_1c_1^*\varepsilon III - II\varepsilon(A^*)^{0.5}) \quad (28)$$

After suitable modifications and assumption for the geophysical flows, the equation for II can be written as:

$$\frac{DII}{Dt} = 2/k\varepsilon II(1 - c_1 - (A^*)^{0.5}) - SII^{0.5}(2.8II - 1.067) - 2/kc_1c_1^*\varepsilon III \quad (29)$$

where, $II = b_{ij}b_{ji}$ and $III = b_{ij}b_{jk}b_{ki}$ are the second and third invariants of the Reynolds stress anisotropy tensor(in turbulence studies those are used for plotting the anisotropy invariant mapping for demarcating the different states of turbulence) and A^* is the Lumley flatness parameter (Lumley 1979), which is zero at the wall, where turbulence goes to two component limit.

$$A^* = 1 - \frac{9}{8}(II - III) \quad (30)$$

For preventing the model form blowing up during numerical simulations realizability constraints for the second invariant were considered. The values of second invariant can be larger than one, near the walls because of higher values of turbulent stresses at those regions, those were not considered in this study,

$$0 \le II \le 1 \quad (31)$$

3. NUMERICAL MODELING

The temperature decrease with depth in the ocean and an upward and downward movement of water occurs as a result of temperature difference between the layers of fluids, which can be termed as free or natural convection flow, is dependent on the temperature, salinity and depth of the water. In this work, an one dimensional water column model ”General Ocean Turbulence Model”(Burchard *et al.* 1999) is used to study the natural convection flow in a vertical water column. The simplified formulation is used to simulate the flow and the results obtained from the simulations are compared with the observational results of ocean weather station papa (OWS Papa) and a realistic ocean test case of the Fladenground experiment 1976 (FLEX 76). A detailed discussion on such experiments and data collection is available in (Burchard *et al.* 1999). The discretization of the domain is achieved by dividing the domain into required number of intervals. The vertical discretization were refined at the surface and bottom. The discrete values for the mean flow quantities such as x and y components of velocity, temperature and salinity represent interval means and are located at the centers of the interval and the turbulent quantities are positioned at the interfaces of the intervals. The staggering of the grid allows for a second order approximation of the vertical fluxes of

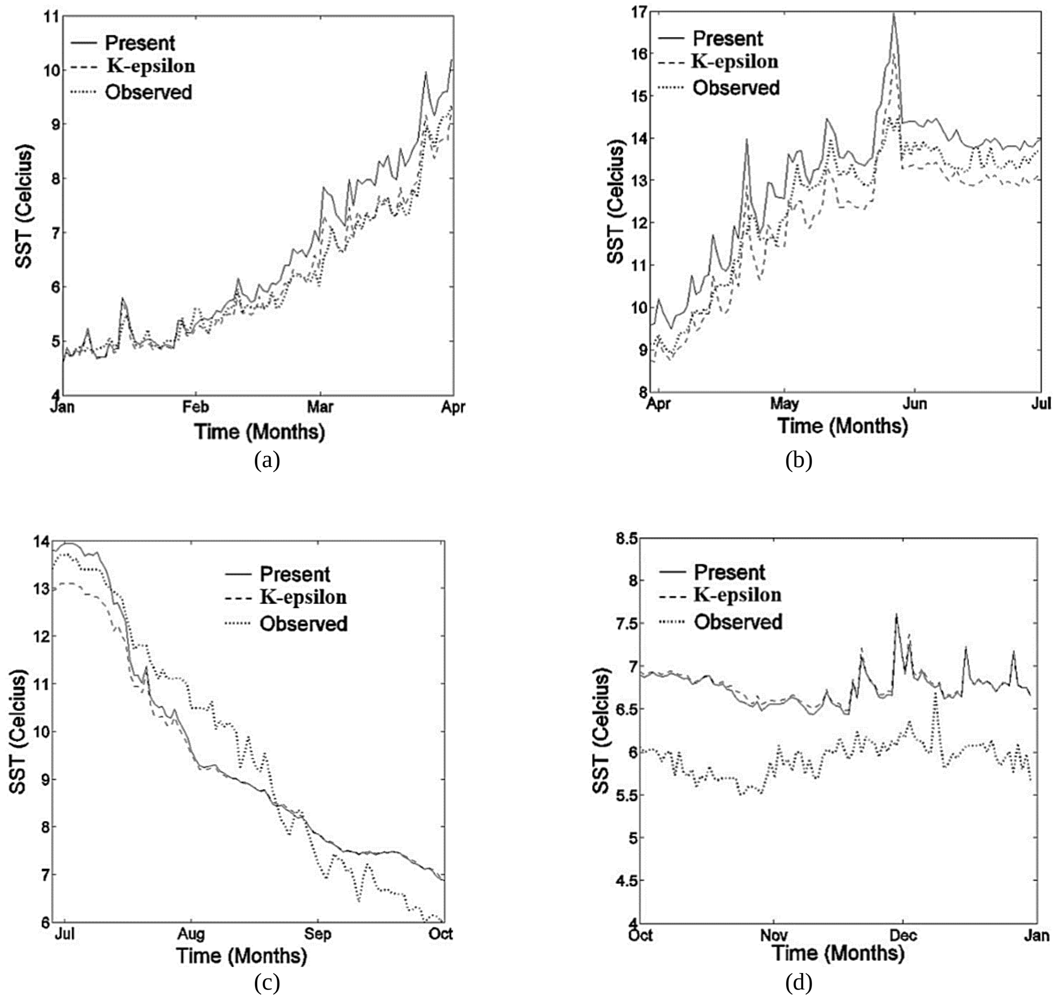


Fig. 1. Time series of temperature profiles for OWS papa.

momentum and tracers without averaging. Averaging of the eddy diffusivities is required for the vertical fluxes of kinetic energy, length scale and dissipation. Because of absence of advection and fully implicit treatment of diffusion the time stepping is equidistant, based on two time levels. For momentum and tracers a fully implicit discretization scheme is used, which results in a system of linear equations with tri-diagonal matrix for each transport equation. The resulting tri-diagonal matrix is solved by means of simplified Gaussian elimination (Burchard *et al.* 1999).

4. RESULTS AND DISCUSSION

The OWS Papa is located in the North Pacific, at 145W and 50 N, where sea temperature profiles and meteorological data have been collected from 1940s to the early 1980s. The current simulations of OWS Papa have been performed for the year 1961. It is situated in a region where horizontal advection of heat and salt is assumed to be small. for OWS Papa, meteorological data for sea surface temperature, air pressure, wind speed and direction are available. In station OWS Papa, horizontal advection of heat and salt is assumed to be small. Time series of SST profiles of OWS Papa are shown in Fig. 1. In four separate sub-figures the different model predictions are compared with observed data. OWS papa observational results are available up to 250 meter depth. The vertical profiles of temperature are shown in 2. The present model prediction matches well with the observational data and are comparatively better than the k-epsilon model predictions. The time series of eddy viscosity over a period of two months is shown in Fig. 3. Figures 4 and 5 show the variation of eddy viscosity and heat diffusivity respectively with depth. k-epsilon model predicts almost zero viscosity in lower layers but the present model shows nonzero viscosity in those layers, which can be considered as an improvement of the result because of the non linear terms in the formulation of the eddy viscosity.

The Fladenground experiment was performed at the northern North Sea at a water depth of 145 meter and a position $58^{\circ}.55'$ N and $0^{\circ}.55'$ E. Measurements of meteorological forcing and temperature profiles were carried out in spring 1976. Various turbulence modellers have validated their models and compared the performance of various turbulence models against FLEX 76 data (Sasmal *et al*. 2014) and (Burchard *et al*. 1999). Figure 6 represents the time series of temperature profiles during the Fladenground experiment 1976. From the present model predictions, it is observed that there is little improvement over the model predictions of the k-epsilon model for the sea surface temperature profiles. Time series of temperature profiles, at a depth of 100 meter are shown in Fig. 7. Since the present model properly represents the complex flow fields because of the addition of cubic nonlinear terms in the formulation of the Reynolds stress anisotropy, an improved prediction of temperature profiles is observed at a depth of 100 meter. On the Julian day 133, a storm occurred on that site, the storm can be noticed from the vertical temperature profiles in Fig. 7. Figures 7 (a) and 7 (b) represent Julian day 124 and 136 respectively. Both before and after storm, predictions of temperature are better than k-epsilon model and are matching with the trends of observational data.

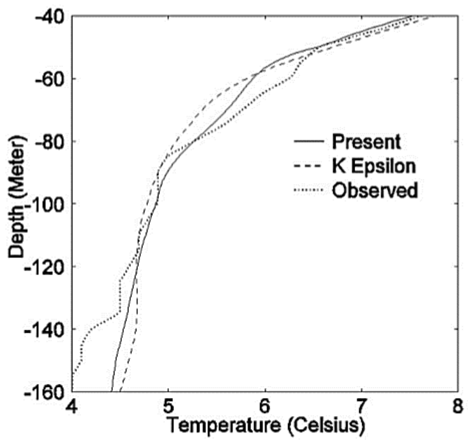


Fig. 2. Vertical profiles of temperature: comparison of the model predictions with the observed data for OWS papa.

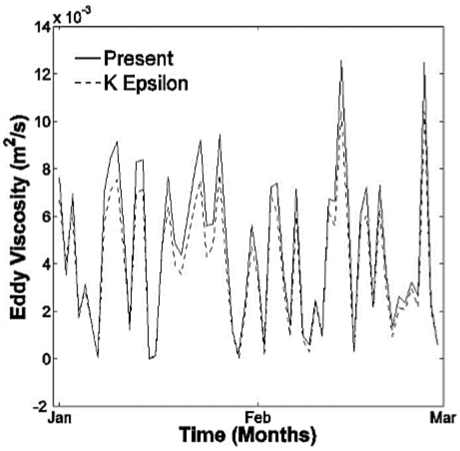


Fig. 3. Variation of eddy viscosity with time: comparison of the present and k-epsilon model predictions for OWS papa.

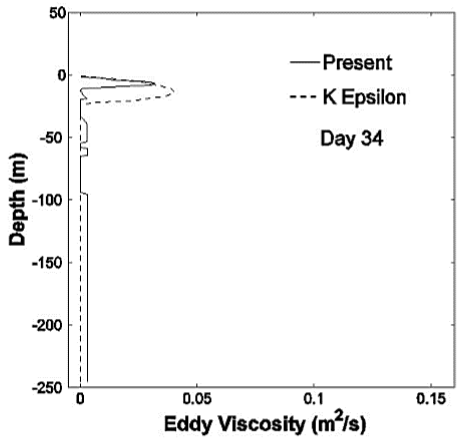


Fig. 4. Profile of eddy viscosity: comparison of the present and k-epsilon model predictions for OWS papa

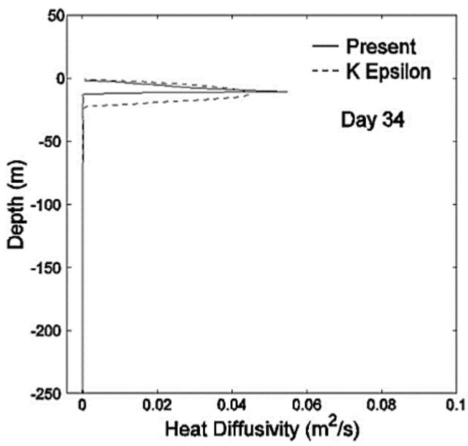
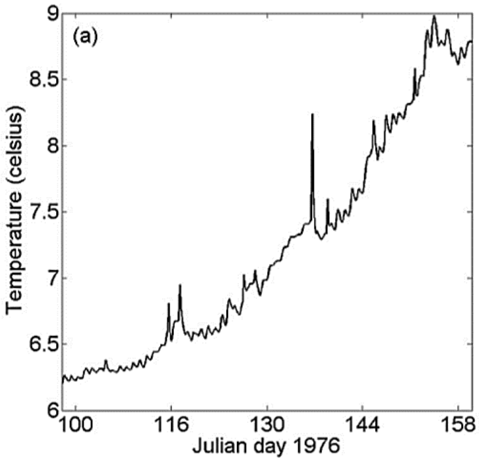


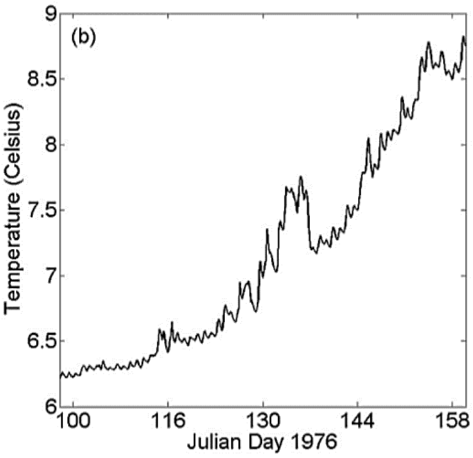
Fig. 5. Profile of heat diffusivity: comparison of the present and k-epsilon model predictions for OWS papa.

5. CONCLUSIONS

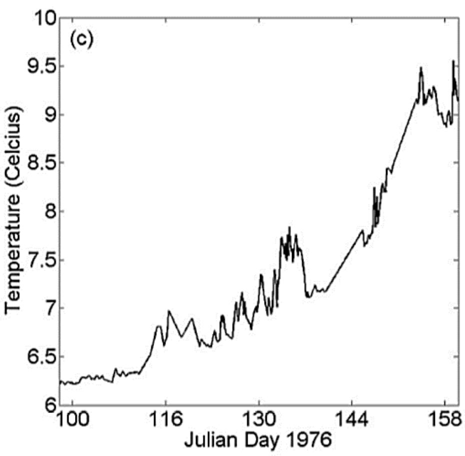
Effects of complex strain fields can be tackled by using nonlinear terms in the constitutive equation as done by other researchers. Addition of cubic terms to the Reynolds stress anisotropy constitutive equation ensures proper representation of flow field by mimicking streamline curvature and swirl effects in geophysical flows. The k-epsilon model predicts almost zero viscosity in the lower layers but the present model shows non-zero viscosity in those layers which is an improved prediction of the eddy viscosity field. Because of addition of swirling and curvature effects also there is an marked improvement of the temperature profiles. This simple model will be beneficial for flow prediction in coastal areas, where depth is less and the no-slip condition induces higher vorticity in the flow field. In future course of work focus can be placed on the modelling of the near wall geophysical turbulent flows by considering the near wall invariant parameters such as strain and stress invariants in the formulation of eddy viscosity or through incorporation of the pressure strain correlation representing the wall damping effects in the transport equation of the stress invariant.



(a)

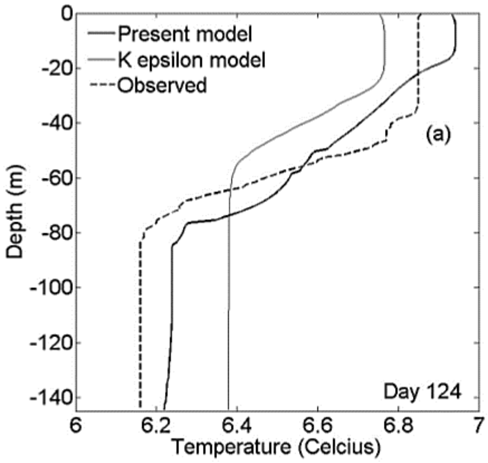


(b)

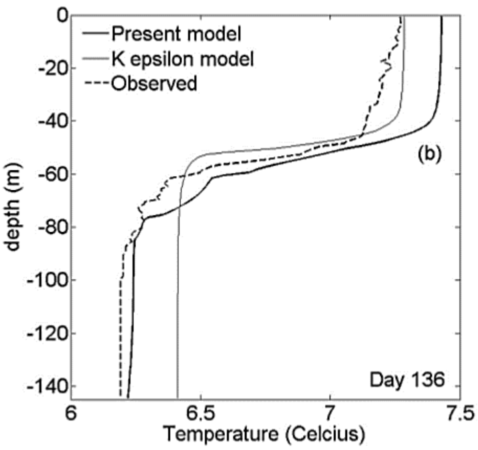


(c)

Fig. 6. Time series of temperature for the Fladen-ground experiment 1976. a) present model b) k-epsilon model c) observed data.



(a)



(b)

Fig. 7. Vertical temperature profiles during the Fladenground experiment 1976.a) Before storm b) After storm.

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