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Boundary Layer Flow and Heat Transfer over a Permeable Stretching/Shrinking Sheet with a Convective Boundary Condition

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ABSTRACT

This paper concerns with the boundary layer flow and heat transfer over a permeable stretching/shrinking sheet in a viscous fluid, with the bottom surface of the plate is heated by convection from a hot fluid. The partial differential equations governing the flow and heat transfer are converted into ordinary differential equations using a similarity transformation, before being solved numerically. The effects of the suction, convection and stretching/shrinking parameters on the skin friction coefficient and the local Nusselt number are examined and graphically illustrated. Dual solutions are found to exist for a certain range of the suction and stretching/shrinking parameters. The numerical results also show that suction widens the range of the stretching/shrinking parameter for which the solution exists.

Keywords: Boundary layer; Stretching/shrinking sheet; Permeable surface; Convective boundary condition; Fluid Mechanics.

NOMENCLATURE

a, b	constants
C_f	skin friction coefficient
f	dimensionless stream function
h_f	heat transfer coefficient
k	thermal conductivity
Nu_x	local Nusselt number
Pr	Prandtl number
Re_x	local Reynolds number
S	suction parameter
T	fluid temperature
T_w	surface temperature
T_∞	ambient temperature
u, v	velocity components along the $x-$ and $y-$ directions, respectively
U_w	stretching/shrinking velocity
V_w	mass flux velocity
x, y	Cartesian coordinates along the surface and normal to it, respectively
α	thermal diffusivity
γ	convection parameter
η	similarity variable
θ	dimensionless temperature
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
σ	stretching/shrinking velocity
τ_w	surface shear stress
ψ	stream function

1. INTRODUCTION

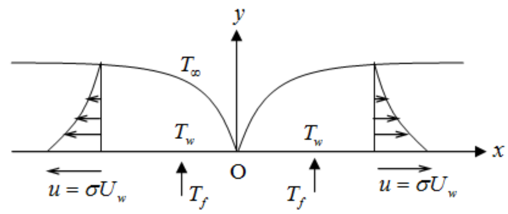
The flow and heat transfer over a stretching surface has many applications in manufacturing processes such as the aerodynamic extrusion of plastic sheets, glass and fiber production, manufacture of foods and polymer extrusion. Crane (1970) initiated the study of two-dimensional flow over a stretching surface in a quiescent fluid. The three-dimensional case was considered by Wang (1984). Thereafter, a number of investigations on this problem have been continued by many researchers who incorporated different physical conditions (see for example Lok et al. 2011; Yacob *et al.* 2011; Ishak *et al.* 2011; Bachok *et al.* 2012; Mahapatra and Nandy 2013; Malvandi *et al.* 2014; Sharma *et al.* 2014; Shit and Majee 2014).

In recent years, the investigation of the flow and heat transfer under a convective boundary condition has become a new interest. The use of the convective boundary condition is more general and realistic with respect to several engineering and industrial processes like the transpiration cooling process, material drying, etc. (Makinde and Aziz 2010). Usually, the boundary condition applied in the modelling of boundary layer flow and heat transfer is either prescribed surface temperature or prescribed surface heat flux. In the present paper, we consider the situation when the bottom surface of the plate is heated by convection from a hot fluid. This results in the heat transfer rate through the surface being proportional to the local difference in the temperature with the ambient conditions. This type of boundary condition was applied quit recently by Aziz (2009), Bataller (2008), Ishak (2010), Makinde and Aziz (2011) and Abu Bakar *et al.* (2012), among others. They reported that similarity solution exists if the convective heat transfer associated with the hot fluid on the lower surface of the plate is proportional to $x^{-1/2}$ where *x* is the distance from the leading edge of the solid surface.

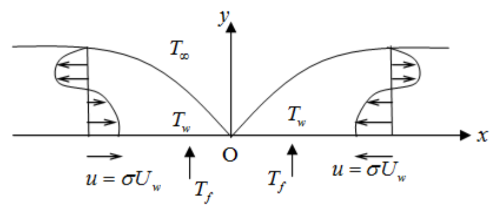
Different from Aziz (2009), who considered the problem of laminar thermal boundary layer flow over a flat plate with a convective surface boundary condition, in the present paper we investigate the boundary layer flow and heat transfer over a stretching/shrinking sheet with the same surface heating condition, and show that dual solutions exist for both stretching and shrinking cases.

2. MATHEMATICAL FORMATION

Consider a steady two-dimensional laminar boundary layer flow over a permeable stretching/shrinking sheet of temperature T_w immersed in quiescent viscous fluid as shown in Fig. 1. It is assumed that the sheet moves with a linear velocity $U_w = ax$ and the mass transfer velocity at the surface of the stretching/shrinking sheet is $v = V_w$, where a is a positive constant. It is also assumed that the bottom surface of the solid surface is heated by convection from a hot fluid of temperature $T_f = T_\infty + bx$, which provides a heat transfer coefficient h_f and b is a positive constant. The boundary layer equations describing the flow problem are as follows (Bejan, 2004).



(a) Stretching case ($\sigma > 0$)



(b) Shrinking case ($\sigma < 0$)

Fig. 1. Physical model and coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where *u* and *v* are velocity components in the *x* (along the stretching/shrinking sheet) and y (normal to the stretching/shrinking sheet) directions, respectively, ν is the kinematic viscosity, T is the temperature and α is the thermal diffusivity of the fluid.

The boundary conditions are (Aziz, 2009)

$$u = \sigma U_w, \quad v = V_w, \quad -k\frac{\partial T}{\partial y} = h_f (T_f - T_\infty) \ \text{at} \ y = 0$$

$$u \to \infty, \quad T \to T_\infty \ \text{as} \ y \to \infty \tag{4}$$

where σ is the stretching/shrinking parameter with $\sigma > 0$ for stretching and $\sigma < 0$ for shrinking and *k* is the thermal conductivity.

We seek for a similarity solution of Eqs.(1)-(3) subject to the boundary conditions (4) by introducing the following transformation (Aziz, 2009; Ishak, 2010)

$$\eta=\left(\frac{U_w}{\nu x}\right)^{1/2} y, \quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}, \quad \psi=\left(\nu x U_w\right)^{1/2} f(\eta) \tag{5}$$

where $f(\eta)$ and $\theta(\eta)$ are the dimensionless velocity and temperature, ψ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ which identically satisfies Eq. (1). Using (5), we get

$$u=axf'(\eta), \quad v=-\sqrt{a\nu}\, f(\eta) \tag{6}$$

where prime denotes differentiation with respect to η. From Eq. (6), the mass flux velocity can be defined as $V_w=-\sqrt{a\nu}S$, where S is a constant.

Substituting (5) into Eqs.(2) and (3), we obtain the following system of nonlinear ordinary differential equations

$$f'''+ff''-f'^2=0 \tag{7}$$

$$\frac{1}{\Pr}\theta''+f\theta'-f'\theta=0 \tag{8}$$

The transformed boundary conditions (4) can be written as

$$f(0)=S, \quad f'(0)=\sigma, \quad \theta'(0)=-\gamma\left[1-\theta(0)\right]$$
$$f'(\eta)\to 0, \quad \theta(\eta)\to 0 \text{ as } \eta\to\infty \tag{9}$$

where $S=f(0)>0$ is the suction parameter, Pr is the Prandtl number and γ is the convection parameter (Biot number), which are defined as

$$\Pr=\frac{\nu}{\alpha}, \quad \gamma=\frac{h_f}{k}\sqrt{\frac{\nu}{a}} \tag{10}$$

The quantities of physical interest in the present study are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as

$$C_f=\frac{\tau_w}{\rho U_w^2}, \quad q_w=\frac{xq_w}{k(T_w-T_\infty)} \tag{11}$$

Here, ρ is the fluid density, τ_w and q_w are the surface shear stress and the surface heat flux, respectively, which are given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

where μ is the dynamic viscosity. Using (5), (11) and (12), we get

$$C_f \mathrm{Re}_x^{1/2}=f''(0), \quad Nu_x/\mathrm{Re}_x^{1/2}=-\theta'(0) \tag{13}$$

where $\mathrm{Re}_x=U_w x/\nu$ is the local Reynolds number.

3. Results and Discussion

The nonlinear ordinary differential Eqs. (7) and (8) subject to the boundary conditions (9) were solved numerically using a shooting method with the help of Maple software. The description of this method can be found in Bhattacharyya *et al.* (2011), Aman and Ishak (2012) and Mohamed *et al.* (2013). The results were obtained for some values of the governing parameters involved, namely suction parameter *S*, stretching/shrinking parameter σ, convection parameter γ and Prandtl number Pr. Particular attention was given to the effect of the suction parameter and the stretching/shrinking parameter on the skin friction coefficient $f''(0)$ and the local Nusselt number (heat transfer rate at the surface) $-\theta'(0)$ as well as the velocity and temperature profiles. Using the shooting method, the dual solutions are obtained by setting two different initial guesses for the values of $f''(0)$ and $-\theta'(0)$, where all velocity and temperature profiles reach the infinity boundary conditions (9) asymptotically but with different shapes and boundary layer thicknesses. To conserve space, we restrict our attention to unit Prandtl number, taking *Pr* = 1. We expect our findings to be qualitatively similar for other values of *Pr* of *O*(1). Table 1 presents the values of $f''(0)$ for different values of σ when $S=0$ (impermeable surface), which shows a good agreement with those reported by Crane (1970) and Ishak *et al.* (2006).

Table 1 Values of $f''(0)$ for different values of σ when $S=0$ (impermeable surface)

σ	Crane (1970)	Ishak et al. (2006)	Present results
0.1			-0.031623
0.5			-0.353553
1	-1	-1.0000	-1.000000
2			-2.828427

Figure 2 displays the variation of the skin friction coefficient with the stretching/shrinking parameter σ when $S=1$, while Fig. 3 shows the local Nusselt number for different values of γ when $\Pr=1$. It is found that dual solutions exist for both stretching ($\sigma>0$) and shrinking ($\sigma<0$) cases. We term these solutions as first and second solutions in the following discussion, based on how they appear in Fig. 2, i.e. the first solution has a higher value of $f''(0)$ compared to that of the second solution. We note that the parameters γ and Pr give no effect to the flow field, which is clear from Eqs. (7)-(9). As discussed by Merkin (1985), Weidman *et al.* (2006), Paullet and Weidman (2007), Harris *et al*. (2009) and Rosca and Pop (2013), the first solution is stable and physically realizable while the second solution is not. We expect that the same behavior holds for the present solutions.

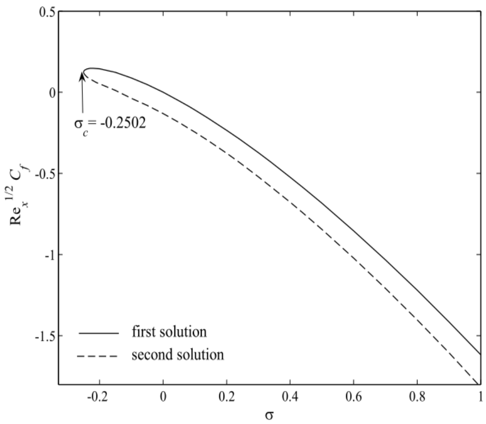


Fig. 2. Variation of the skin friction coefficient $C_f \, \mathrm{Re}_x^{1/2}$ with σ when $S = 1$.

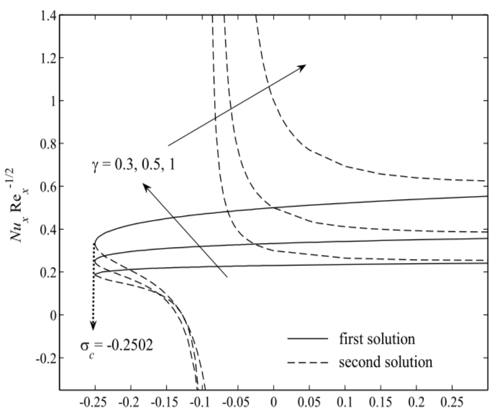


Fig. 3. Variation of the local Nusselt number $Nu_x / \mathrm{Re}_x^{1/2}$ with σ for different values of γ when $S = 1$ and $\mathrm{Pr} = 1$.

Figure 4 elucidates the variation of the skin friction coefficient as a function of the stretching/shrinking parameter σ for different values of *S*, while that of the local Nusselt number is presented in Fig. 5, for $\gamma = 1$ and $\mathrm{Pr} = 1$. It is seen from these figures that there are two solutions when $\sigma > \sigma_c$ (except at $\sigma = 0$), where σ_c is the critical value of σ for which the solution exists. A unique solution is obtained when $\sigma = \sigma_c$ and $\sigma = 0$, and no solution exists for $\sigma < \sigma_c$. The values of σ for different values of *S* are given in Figs. 4 and 5, which show that increasing *S* is to increase the range of σ for which the solution exists. For the first solution, which we expect to be the physically relevant solution, the skin friction coefficient increases (in absolute sense) as the suction parameter *S* increases. The values of the skin friction coefficient are positives for $\sigma < 0$, but are negatives for $\sigma > 0$. Physically, positive value means the fluid exerts a drag force on the solid surface, while negative value means the opposite. From Fig. 5, the local Nusselt number which represents the heat transfer rate at the surface increases as S increases. This is due to the fact that suction increases the surface shear stress, and thus increases the skin friction coefficient, in consequence increases the local Nusselt number. For the second solution, the local Nusselt number presented in Fig. 5 suggests that $-\theta'(0)$ becomes unbounded as $\sigma \to 0^+$ and as $\sigma \to 0^-$.

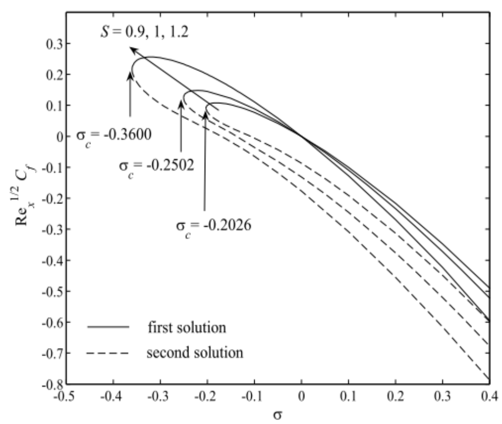


Fig. 4. Variation of the skin friction coefficient $C_f \, \mathrm{Re}_x^{1/2}$ with σ for different values of *S*.

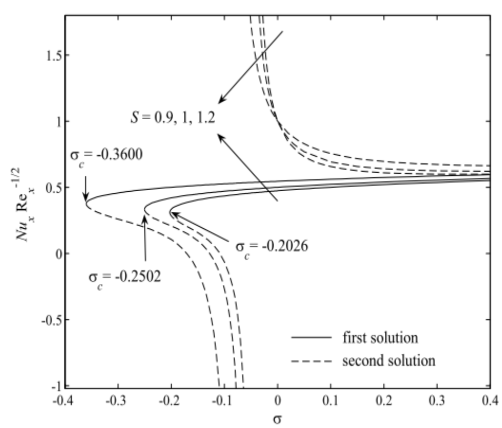


Fig. 5. Variation of the local Nusselt number $Nu_x / \mathrm{Re}_x^{1/2}$ with σ for different values of *S* when $\gamma = 1$ and $\mathrm{Pr} = 1$.

Figure 6 illustrates the effects of the convection parameter γ on the temperature profiles. It is noted from this figure that the temperature in the boundary layer increases with the increasing values of γ for both solutions. It is seen that increasing γ is to increase the magnitude of the temperature gradient at the surface $|\theta'(0)|$. As discussed by Aziz (2009), the parameter γ at any location *x* is directly proportional to the heat transfer coefficient associated with the hot fluid *h*. The thermal resistance on the hot fluid side is inversely proportional to *h*. Thus, the hot plate side convection resistance decreases as γ increases and in turn increases the surface temperature $\theta(0)$. As a result, the local Nusselt number increases with γ, as shown in Fig. 3.

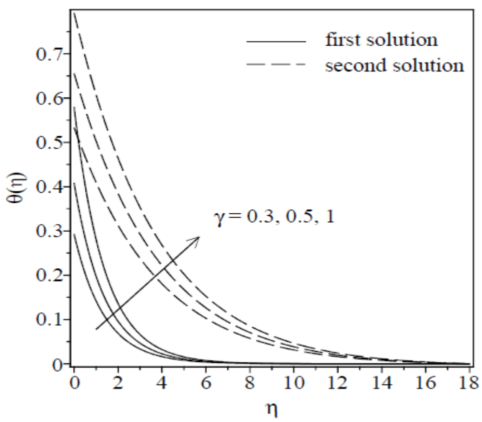


Fig. 6. Effect of the convection parameter γ on the temperature profiles $\theta(\eta)$ when $\Pr = 1$, $S = 1$ and $\sigma = -0.2$ (shrinking case).

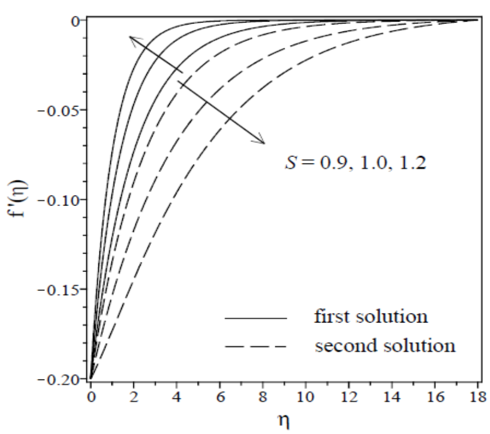


Fig. 7. Effect of the suction parameter *S* on the velocity profiles $f'(\eta)$ when $\Pr = 1$ and $\sigma = -0.2$ (shrinking case)

Figure 7 shows the graphical representation of the velocity profiles for different values of *S* while the others parameters are fixed. It is noticed from Fig. 7 that for the first solution, the fluid velocity in the boundary layer decreases (in absolute sense) as *S* increases. This is due to the fact that suction increases the surface shear stress which retards the flow, implying an increasing velocity gradient at the surface. Also, there would be a significant reduction in the velocity boundary layer thickness when *S* increases. Thus, the skin friction coefficient increases with suction at the boundary, which agrees with the results presented in Fig. 4. The opposite behavior is observed for the second solution.

Figure 8 is drawn to see the effect of suction on the temperature. It is clear that the temperature decreases for the first solution as *S* increases, but the opposite behavior is observed for the second solution. All velocity and temperature profiles presented in Figs. 6-8 satisfy the far field boundary conditions (9) asymptotically and hence, supporting the validity of the numerical results obtained.

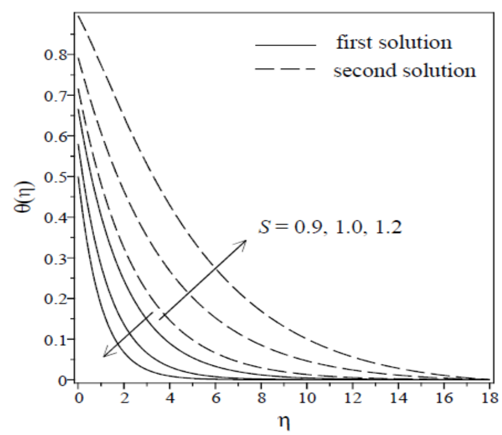


Fig. 8. Effect of the suction parameter *S* on the temperature profiles $\theta(\eta)$ when $\Pr = 1$, $\gamma = 1$ and $\sigma = -0.2$ (shrinking case).

4. CONCLUSION

The steady laminar boundary layer flow over a permeable stretching/shrinking sheet immersed in a viscous fluid under a convective surface boundary condition was numerically studied. The effects of suction, convection and stretching/shrinking parameters on the flow and the thermal fields were graphically illustrated and discussed. It was found that

- dual solutions exist for a certain range of the suction and stretching/shrinking parameters,
- suction widens the range of the stretching/shrinking parameter for which the solution exists,
- the magnitude of the skin friction coefficient increases as the suction as well as the stretching/shrinking parameter increases,
- the heat transfer rate at the surface increases with increasing values of both convection and suction parameters

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