Journal of Applied Fluid Mechanics, Vol. 11, No. 1, pp. 127-136, 2018.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.29252/jafm.11.01.27806

An Innovative Roof Shape in Liquid Storage Tanks to Reduce Dynamic Sloshing Effects

P. Nouraeidanesh, M. M. Kabiri and M. A. Goudarzi†

International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran

†*Corresponding Author Email: m.a.goodarzi@iiees.ac.ir*

(Received March 22, 2017; accepted September 12, 2017)

ABSTRACT

In this study, a new type of tank roof form is suggested to reduce the high impact forces caused by sloshing. Using this roof allows the tank designers to consider less freeboard, which is economically valuable. For this purpose, an experimental investigation has been implemented to evaluate the efficacy of the proposed roof to distribute the contained liquid impact forces in several time stages. In these experimental measurements, a series of shaking table tests are conducted for a partially filled tank under harmonic and various earthquake excitations for both typical and proposed tank roof forms. The liquid impact forces are reasonably evaluated and compared for both types of tank roof. The efficacy of the proposed roof design is validated by experimental results and it is shown that the sloshing loads can significantly be reduced up to an average of 50% for the dimensions considered in the experiments.

Keywords: Freeboard effects; Sloshing; Rectangular tank; Earthquake excitation; Shaking table test.

NOMENCLATURE

A	amplitude
d	displacement time history of harmonic excitation
exc	excitation
F_m	model force value
F_p	prototype force value
Fr	freeboard
H_w	liquid height
L	tank length
L_r	geometric ratio
$SIRF$	Sloshing Impact Roof Force
t	time
ω_n	primary natural frequency of harmonic Excitation
ρ_m	model density
ρ_p	prototype density

1. INTRODUCTION

Large amplitude sloshing waves and related loads caused by earthquakes are always one of the most important factors in seismic designing of liquid storage tanks. The sloshing phenomenon can be defined as a highly nonlinear motion of the free surface in a partially filled tank, which could be a moving tank such as liquefied natural gas vessels, or onshore liquid storage tanks. Codes and standards tackle this issue by providing a sufficient freeboard to prevent collision between contained liquid and the tanks roof during earthquakes. Whereas an insufficient freeboard could result in strong damages to the tank shell, providing a high freeboard is not economical. Therefore, the needed resistance against sloshing loads should be provided or the methods to reduce induced sloshing force should be developed; The latter is the main aim of the present study.

The subject that partially filled liquid tanks are prone to violent sloshing under certain dynamic conditions has been shown in previous studies (Lee *et al*. 2002; Goudarzi and Sabbagh-yazdi 2009;2010). When the frequency of a tank excitation is close to the natural frequency of liquid sloshing, the enhanced fluid motion creates high impact loads on the tank walls and ceiling which can cause several structural damages (Chen *et al*. 2009). On the other hand, liquid sloshing, as a subject into itself, has been attracting much attention over the past few decades. In marine industries, a large number of studies on the liquid vessels have been carried out to attain a better understanding of pressure distribution in the tanks structure under excitation caused by sea waves. The estimation of liquid sloshing loads in tanks of ships is important in the design of ship vessels such as liquefied natural gas carriers and double-hull tankers (Takemoto *et al.* 1994; Shinkai *et al.* 1995; Cariou *et al.* 1999). These studies confirmed that liquid impacts are still far more difficult to assess and need more development. The previous studies in this field can be mainly divided into three groups: a) analytical studies b) numerical approaches and c) experimental studies.

A proper analytical model simplifies the understanding of sloshing mechanics and relies on extensive parametric study. Analytical formulation of liquid equations is well documented by many researchers for tanks with various regular geometries. The general equation of liquid motion in closed containers is often simplified by assuming the rigid walls for a storage tank. Furthermore, the liquid is assumed ideal, namely it is inviscid, incompressible, and irrotational. Capillary or surface tension effects are generally ignored in analytical formulation of liquid sloshing (Mimi 2011). Early studies of sloshing focused on linear problems in two dimensional simple geometrical containers, which can be solved by analytical methods (Wei *et al.* 2012). Stolbetsov (1967) theoretically studied the nonlinear sloshing in a rectangular tank due to horizontal excitation. He used a perturbation technique and presented two types of steady-state solutions. Ockendon (1973) proposed an analytical scheme for resonant sloshing due to external horizontal and vertical excitations. A third order asymptotic solution was mathematically derived. Kim *et al.* (1996) also developed an analytical solution of a partially filled rectangular tank under horizontal and vertical ground excitation. Faltinsen *et al.* (2003) studied the liquid sloshing in a square-base tank in frequency domain. The tank was forced under 3D arbitrary motions with a frequency close to the lowest natural frequency.

Analytical methods can accurately predict the linear and nonlinear sloshing motion when the motion is not violent. However, the liquid impact which is generally followed by wave overturning and breaking is difficult to clarify by using the analytical approaches. In addition, analytical study of liquid sloshing is often performed for simple boundary conditions. Recent advances in computational methods enables researchers to use numerical methods for studying large motion of free surface flow. Traditional numerical approaches such as volume of fluid method were generally employed to simulate the sloshing motion in liquid containers. This technique has been used by Celebi, *et al.* (1998) and Celebi and Akyildiz (2002) to simulate two-dimensional viscous liquid sloshing in moving rectangular baffled and un-baffled tanks. However, if the liquid sloshing is violent, some local physical phenomena become difficult to analyze; and this technique may become inaccurate to capture large sloshing motion anymore. Other numerical methods, such as moving boundary finite element approach was developed and implemented by Greaves, *et al.* (1997) to simulate nonlinear sloshing waves in ship tankers.

Most experimental studies such as the study of liquefied natural gas carriers have been carried out in marine industries. At shallow liquid containers, a hydraulic jump can be generated during a large excitation motion in which the tank wall is exposed to large impact pressures. In most cases, the fluid moves up to the tank ceiling, resulting in sharp pressure peaks on the top or in the corner. In relatively high filling, the impact pressure occurs in a large area of the tank roof, the liquid runs up to the tank ceiling and moves along the top boundary. If the liquid motion is in a standing mode, it will hit a large part of the roof. These phenomena have also been studied by Abramson *et al.* (1974), Kim *et al.* 1994 and Kim (2001). One of the earlier experimental investigations of nonlinear, free-surface standing waves was reported by Taylor (1953) who focused on the wave crest in the center of a rectangular tank. Milgram (1969) studied the sloshing impact pressure on roofed liquid tanks through an empirical study. Kobayashi (1980) studied the effects of impulsive pressure due to sloshing impacts. Minowa *et al.* (1994) conducted a series of shaking table tests on a rectangular tank to measure the impact pressure on tank roofs, natural frequencies and modes of bulging vibrations. Chen *et al.* (1996) simulated large-amplitude sloshing motion of liquid subjected to harmonic and earthquake base excitations. They concluded that non-linear sloshing effects should be considered in seismic-resistant tank design. Akyildız (2006) conducted numerical and experimental investigation on pressure variations and three-dimensional effects on liquid sloshing loads in a moving and partially filled rectangular tank. Goudarzi *et al.* (2010) conducted a series of tests to investigate the hydrodynamic sloshing force on storage tank roofs. They developed an analytical solution to model sloshing impact force on tank roofs. Their analytical solution parameters were calibrated with the experimental measurements. Akyıldız *et al.* (2013) experimentally investigated liquid sloshing in a cylindrical tank with various fill levels and ring baffles under the excitations of roll motion. The primary objectives of this experimental study were to examine the relative effectiveness of various baffle arrangements. They found that ring baffle arrangements are very effective for reducing the sloshing loads. Jin *et al.* (2014) designed a horizontal perforated plate and incorporated into a rectangular liquid tank that was excited under different amplitudes and frequencies. The experimental results indicated that the horizontal perforated plate can significantly restrain violent resonant sloshing in tanks.

As noticed, in the case of sloshing impact during earthquakes, there are only a limited number of studies to clarify this phenomenon. Other than the liquid impact phenomenon, the prevention methods have not been appropriately developed in previous studies.

There are two types of tanks for liquid containers; floating roofs and fixed roofs. An external floating roof tank is a storage tank commonly used to store large quantities of petroleum products such as crude oil or condensate. It comprises an open- topped cylindrical steel shell equipped with a roof that floats on the surface of the stored liquid. The roof rises and falls with the liquid level in the tank. As opposed to a fixed roof tank there is no vapor space (ullage) in the floating roof tank (except for very low liquid level situations). A fixed roof tank is a type of storage tank, used to store liquids, consisting of a cone- or dome-shaped roof that is permanently affixed to a cylindrical shell. Newer storage tanks are typically fully welded and designed to be both liquid- and vapor-tight. Older tanks, however, are often riveted or bolted, and are not vapor tight.

The present study aims to propose a different form of fixed tank roof to reduce the total Sloshing Impact Roof Force (SIRF). A series of experiments have been conducted to evaluate the efficacy of the proposed roof form. Hereafter, the proposed roof is named as “stepped roof” owing to its stepped form. This roof diminishes SIRF by imposing the liquid to hit into the tank roof gradually.

2. EXPERIMENTS

We established several shaking table experiments to evaluate the validity and quantitative efficacy of the proposed tank roof shape. These experiments can be specialized by two important features. One of them is related to the parameters captured during the experiments. In almost all of the previous experimental studies on the subject of sloshing impact phenomenon, the impact pressure has commonly been considered as the main parameter because of its measurability during the tests. However, in the present study, the total SIRF as the main parameter for the design of tank wall-roof connections is measured. Measuring this parameter is more difficult due to the additional requirements for disconnecting the tank roof from all around. The second important feature of the present experiments is picking up a large number of data per second that is required for vigorous assessment of the peak value of liquid impact forces. The details of these features are explained in the following paragraphs.

A series of experiments including forty shaking table tests has been conducted in the structural lab of International Institute of Earthquake Engineering and Seismology (IIEES). The dimensions of the test tank are 100cm×30cm×100cm (length×width× height). The schematic view of considered rectangular tank model is shown in Fig.1. As can be seen, the tank is fixed on the table by angle beam profiles placed at the periphery of tank top and bottom. The upper angles restrain the shell lateral deformation when the liquid height is high and provide a platform to fix the rods used to hold the suspended roof of the tank. The roof is suspended by four rods to attain the net SIRF and provide the possibility of variable vertical position for it.

Fig. 1. Schematic view of the experimental rectangular tank.

The tank and its roof are made of acrylic glass with thickness of 1.5 cm. A small gap is provided between the roof and the shell of the tank in order to prevent friction forces. As a result, the total SIRF is transferred from the tank roof to the fixed platform only by the suspended rods. These rods as well as other details of roof system are shown in Fig.2. The force transducers are set to receive five thousand data per second in order to accurately recording the maximum SIRF which happens in a very short period during the liquid sloshing impaction. To calibrate the measuring instruments and to investigate the verification and repeatability of tests, a number of primary experiments were conducted under harmonic and earthquake excitation. Due to the fact that the results of the experiments were highly sensitive to the rate of recorded data, the frequency of data recording was changed from 800 to 6000 data per second in a series of consecutive tests. It was observed that capturing 4000-5000 data per second could lead to trustable results for the maximum value of impact force. By respecting this limit, the tolerance of the main parameters is less than 1%. Besides, variations in the amount of force recorded in several consecutive tests using different excitation were compared. In this group of experiments, the maximum variation of force to the average imposing force was extracted as about 5 N and pressure variations was about 100 Pa.

To construct the stepped roof, a steel plate with 1.5mm thickness is formed in desired shaped and attached to the lower side of the tank roof. Fig.3 illustrates the stepped roof system in detail.

To consider various aspects of sloshing impact forces on a tank roof, different input excitations are considered for shaking the experimental tank. These oscillations include both a harmonic excitation with 1cm amplitude and three different earthquake excitations. The applied displacement time history of harmonic excitation is described as:

$$d = A\times Sin(\omega_n t) \tag{1}$$

Where, A is the amplitude and (ω_n) is the frequency of a harmonic excitation. The frequency of harmonic input excitations is set to the primary natural frequency of liquid sloshing motion to

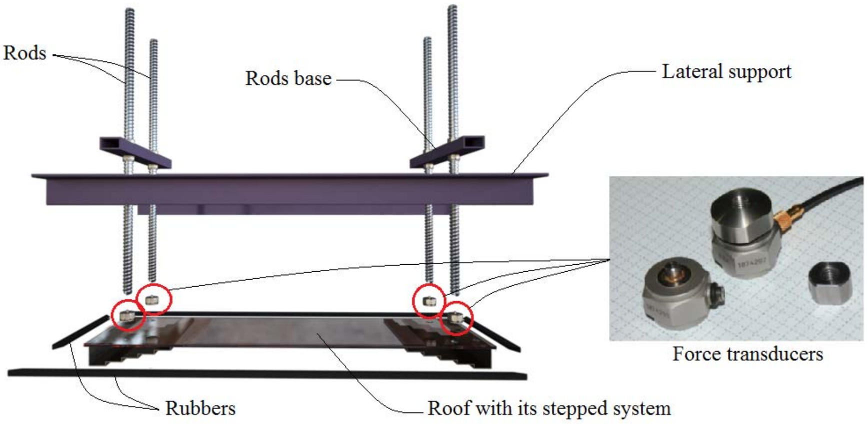
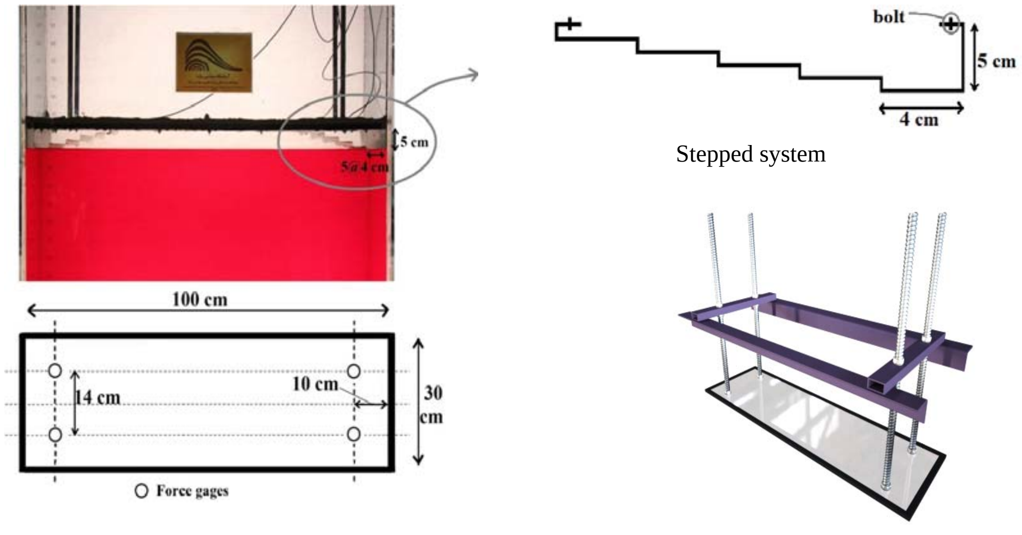


Fig. 2. Suspended roof and related supporting system.



Stepped system

Roof system

Fig. 3. Dimensions of stepped and flat roof systems and force transducers' locations.

intensify the SIRF during the liquid impact. The primary natural frequency (ω_n) of a rectangular tank depends on the tank geometries which can be obtained from the following equation:

$$\omega_n(rad/\sec)=\sqrt{\frac{\pi g}{L\times\tanh(\pi H_w/L)}} \qquad (2)$$

Where, L is the tank length and H_w is the liquid height of the storage tank.

Tabas, Chichi and Kobe earthquake records are selected as the input seismic excitations in the experiments. The scaled displacement time histories of these records, which are the input loads of shaking table, are shown in Fig.4. The displacement time histories of records were obtained from Pacific Earthquake Engineering Research (PEER) ground motion database. Due to the limitation of shaking table displacement, the input displacement records for selected earthquakes were firstly scaled so that their maximum values were adjusted to maximum displacement of the shaking table (30 mm). Then, the time interval scaling was performed for selected records, accordingly.

3. EXPERIMENTAL RESULTS AND DISCUSSION

The first objective of the experiments presented here is to measure the total liquid impact forces acting on the tank roof. Both harmonic and seismic records are considered as the input excitations of the shaking table. The tests are conducted for two different liquid heights of 50cm and 70cm and three various freeboards that are 1cm, 5cm and 10cm. Hereafter, the case studies are indicated by their specifications as H_w, Fr and exc which are liquid height, freeboard and excitation type of each test, respectively.

As an example, the time histories of SIRF for various freeboards with 50cm liquid height are presented in Fig.5. The time histories presented in this figure are the results of harmonic excitations.

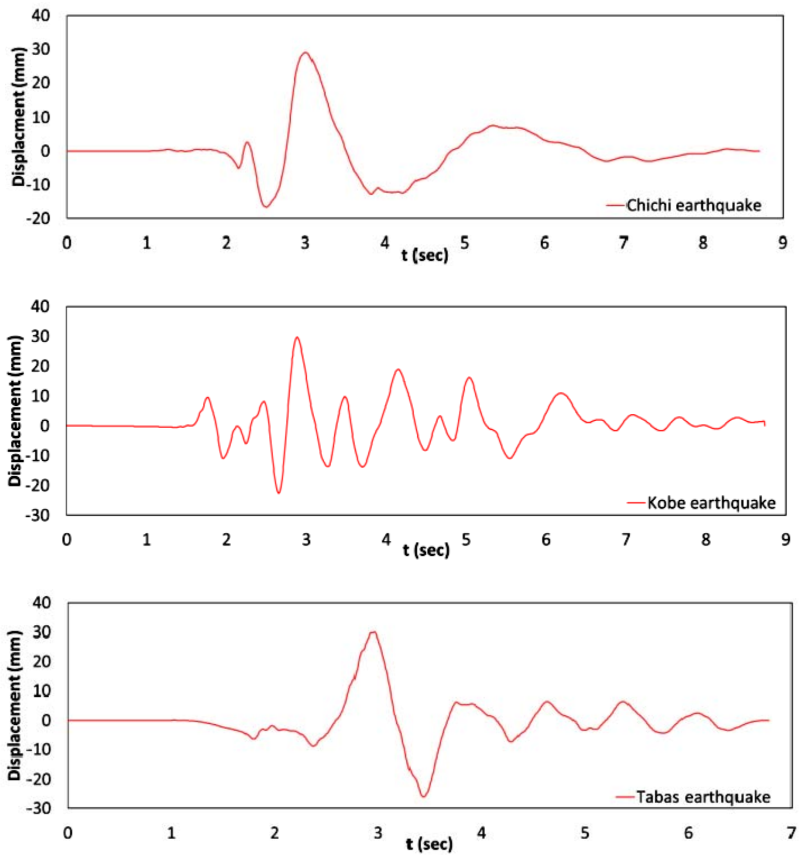


Fig. 4. Displacement time histories used as the input seismic excitations in experiments.

Table 1 Maximum SIRF values for different liquid level and freeboards (*N*)

		H_w = 50 cm	
	Fr = 1cm	Fr = 5cm	Fr = 10 cm
Harmonic	16.6	51.5	73.4
Chi-Chi	20.2	44.6	70.2
Kobe	32.1	33.3	67.1
Tabas	35.8	80.6	84.8
		H_w = 70 cm	
	Fr = 1cm	Fr = 5cm	Fr = 10 cm
Harmonic	14.1	70.3	93.8
Chi-Chi	44.8	36.5	70.9
Kobe	34.3	66.5	72.1
Tabas	30.8	100.5	86.9

As can be seen in Fig.5 and despite the general belief, increase in the freeboard heights not necessarily results in the reduction of SIRF. In the case of lower freeboard height (Fr=1cm), the maximum SIRF are negligible in comparison with other cases with higher freeboards. Although, the lower freeboards provide the higher contact area and volume of the liquid involved in the sloshing collision, it prevents the increment of liquid velocity, and consequently, the roof dissipates the sloshing wave energy. In addition, it can be seen from the results presented in Fig.5 that the sloshing frequencies do not considerably change due to the presence of tank roof. These graphs show that the main natural frequencies of contained liquid in the tanks with very small freeboard are slightly reduced with respect to corresponding tank with greater freeboard height. However, this reduction is generally too small and it seems that the natural sloshing period can be slightly affected by the presence of tank roof.

The maximum SIRF for all cases are extracted from the results of the experiments and tabulated in Table1. The results presented in this table prove the same above-mentioned trend for almost all cases.

From quantitative point of view, the experimental results indicate that SIRF values should be considered in the seismic design of tank roof and its connections. In other words, if the scaling laws are used to evaluate the maximum SIRF for real scale tanks, the values cannot be ignored. As an example,

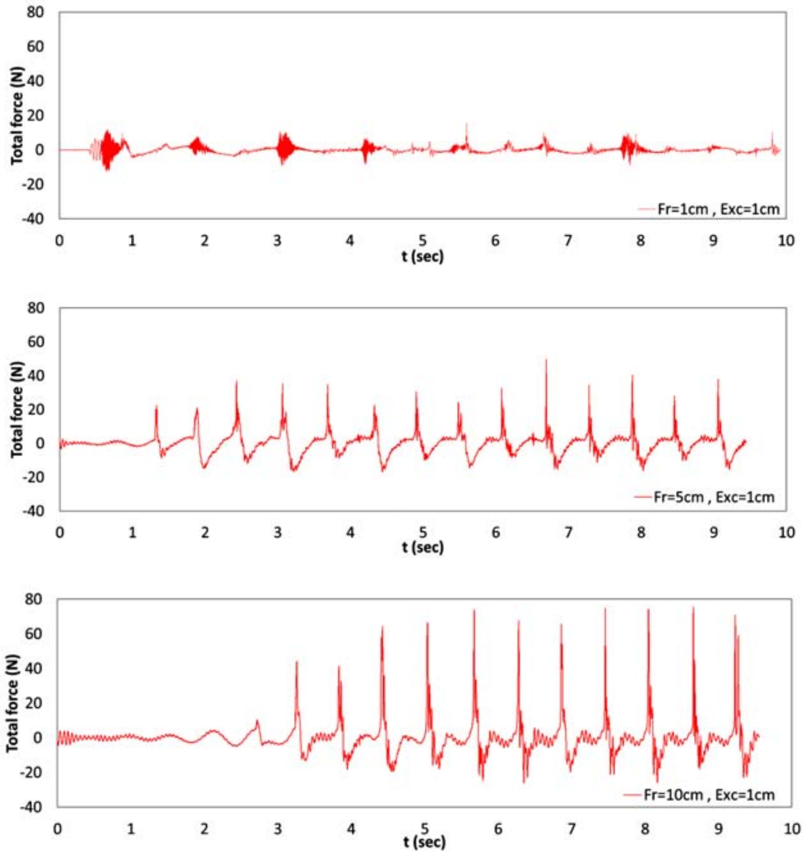


Fig. 5. Time histories of SIRF under harmonic oscillation with amplitude of 1cm for liquid height of $H_w = 50\ cm$ and different freeboards (1^{cm}, 5^{cm}, 10^{cm} respectively).

for a tank with dimensions of 10m length and 5m height, the estimated SIRF values can be calculated by Eq.3, which is extracted from fluid mechanics scaling laws.

$$\vec{F}_p = \frac{1}{L_r^3} \times \frac{\rho_p}{\rho_m} \times \vec{F}_m \tag{3}$$

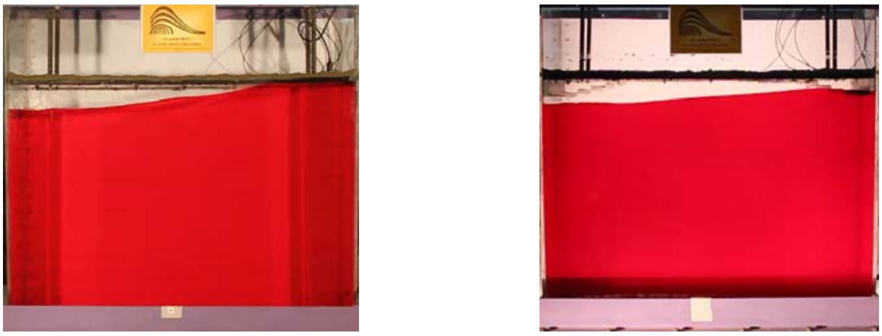
Where $\vec{F}_p$ and $\vec{F}_m$ are the force values for prototype and model, respectively. L_r is the geometric ratio of the scaled tank to the real one. For the example presented here, the maximum SIRF is:

$$\vec{F}_p = \frac{1}{(1/10)^3} \times 84.8 = 84800N = 84.8KN \tag{4}$$

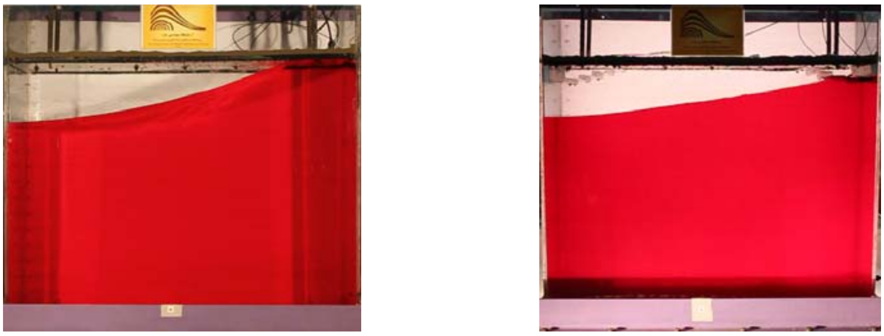
The tank wall-roof connections should be designed for this SIRF force. Moreover, the tank walls should resist against the tension loads caused by SIRF.

In liquid tanks, generally internal baffles are used to increase the damping ratio of contained liquid and reduce the sloshing wave height (Goudarzi 2010). However, the baffles exert extra force on the middle of the tank walls. Although, the baffles are efficient tools to suppress the sloshing motion of the liquid, the seismic designers do not have tendency to use them as an extra element inside the tank. Therefore, a new type of tank roof proposed here might be helpful to reduce the SIRF because despite having extra elements, it neither attaches to the tank wall, nor submerges to the contained liquid. In other words, the shape of the tank roof is simply formed as the stepped shape instead of a flat surface. Consequently, the proposed impact roof force is converted to a set of asynchronous smaller forces. Hence, the joints and plates at any height are designed to withstand a portion of the SIRF. In order to examine the validity of the proposed design, the above mentioned shaking table experiments conducted for the flat tank roof are repeated for the new stepped roof. Fig.6 shows several snapshots of primary wave collisions to both typical and stepped roof under harmonic oscillation.

The stepped plate is located at both sides of tank roof to dissipate the energy of sloshing waves. Sample force reduction time histories of considered cases are shown in Figs.7 and 8. The comparison of the seismic SIRF time histories for stepped and typical roof of the tank with Hw=50cm and Fr=5cm are presented in Fig.7. The same comparison under harmonic oscillations with Hw=50cm and 70cm with Fr=10cm are also presented in Fig.8. Both figures show remarkable force reduction for different excitations. These reductions in SIRF values are larger for harmonic oscillations.



Typical roof, Fr = 5cm Stepped roof, Fr = 5cm



Typical roof, Fr = 10cm Stepped roof, Fr = 5cm

Fig. 6. Sample snapshots of primary collisions to typical and stepped roof for, $H_w = 70\ cm$.

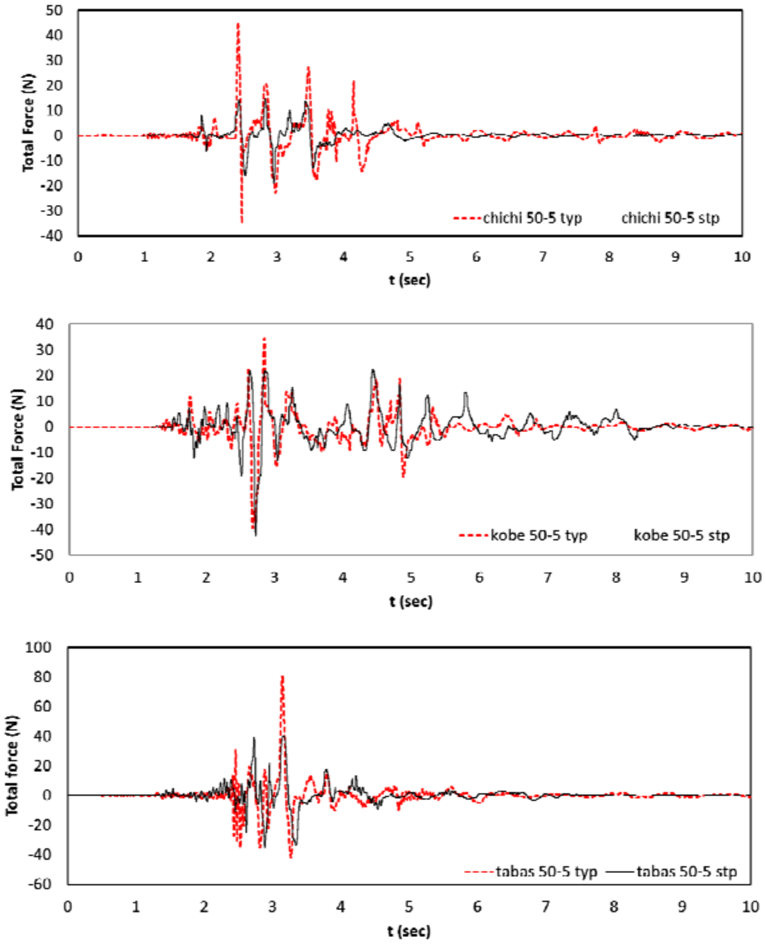


Fig. 7. SIRF comparison between typical flat and proposed stepped roof under seismic excitations.

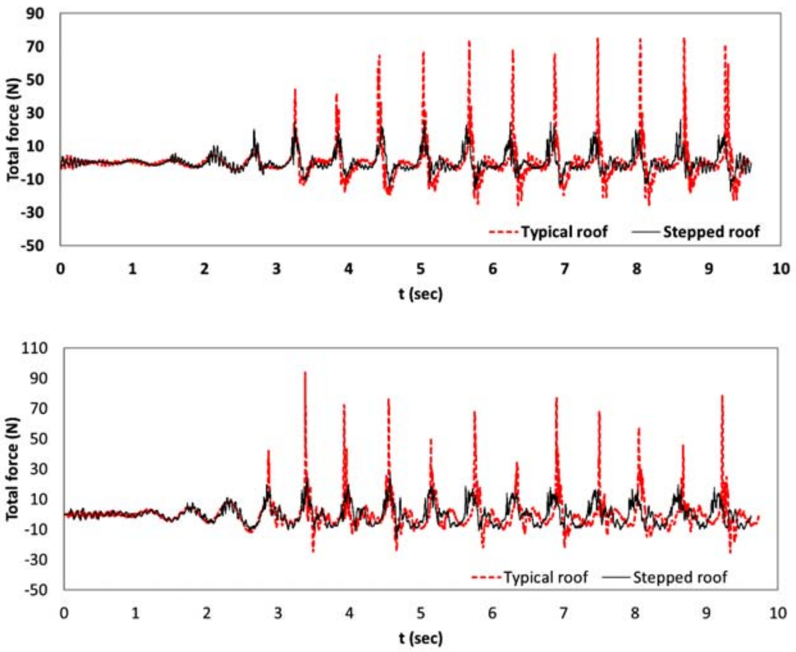


Fig. 8. SIRF comparison between typical and proposed stepped roof under harmonic excitations.

Table 2 Impact roof force values of both roof types and percentage of load reduction for each test

	H_w = 50 cm					
	Fr = 5cm			Fr = 10 cm		
	Typical roof	Stepped roof	Reduced values	Typical roof	Stepped roof	Reduced values
Harmonic	51.5	18.2	64.6 %	73.4	26.9	63.3 %
Chi-Chi	44.6	13.6	69.5 %	70.2	34.4	51 %
Kobe	33.3	20.6	38.1 %	67.1	60.3	10.1 %
Tabas	80.6	40.1	50.2 %	84.8	36.3	57.4 %
	H_w = 70 cm					
	Fr = 5cm			Fr = 10 cm		
	Typical roof	Stepped roof	Reduced values	Typical roof	Stepped roof	Reduced values
Harmonic	70.3	21.1	69.9 %	93.8	26.7	71.5 %
Chi-Chi	36.5	33	9.6 %	70.9	32.3	54.4 %
Kobe	66.5	25	62.4 %	72.1	61.4	14.8 %
Tabas	100.5	45	55.2 %	86.9	45.4	47.7 %

It is also noticeable from Fig.8 that the natural period (time between two peaks of impact force) is slightly reduced for stepped roof tank with respect to flat roof one. However, this reduction seems minor and it seems that the stepped roof will slightly affects the natural frequency of natural sloshing mode. To have an exact assessment of the presence of roof effects on natural frequency of contained liquid, further experimental measurements should be performed for this specific subject.

The maximum dynamic impact forces for both types of tank roofs are tabulated in Table 2 for all considered experimental cases. In this table, the percentage of reduced force values are also calculated and presented. As can be seen, the average reduction percentage caused by implementing the stepped roof for all cases are about 52 %, which shows remarkable efficacy of the proposed roof shape compared to regular design.

When the liquid oscillates in primary sloshing mode, the maximum wave height collides with the roof in both sides. Due to the cosine shape of free surface profile, the large area of the free surface is approximately flat when it hits the roof. Hence, considerable volume of liquid collides with the roof and generates the peak value of SIRF (Fig.9-a). On the other hand, when the roof is formed in a stepped shape, the total volume of moved liquid is divided into several smaller fragments. This makes the liquid to hit the roof asynchronously as presented in Fig.9-b. Therefore, it is expected that the peak value of SIRF is distributed into several smaller peaks by using the stepped roof.

(a) (b)

Fig. 9. Procedure of liquid collision to the flat and stepped roof types.

The example of SIRF for harmonic tank oscillation with 50cm liquid height and 10cm freeboard is shown in Fig.10. The details of one collision for the same case are shown in Fig.11. The SIRF reduction mechanism of stepped roof is clearly shown in these figures. As can be seen, the peak value of SIRF is divided into several peaks with minor peak values. This performance is considerably beneficial, especially considering the fact that the tank roof is designed based on the peak values.

4. CONCLUSIONS

In this study, the liquid sloshing impact phenomenon in a rectangular tank is investigated experimentally. The experiments are conducted using the shaking table of structural laboratory of International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran. The time histories of the total Sloshing Impact Roof Force (SIRF) are measured for 40 tests. Experimental results show that despite the general belief, the reduction of tank freeboard does not necessarily lead to the increase of SIRF. In most cases, the higher the freeboard is, the larger SIRF is. Therefore, the presence of roof to suppress the SIRF of the tank with insufficient freeboard during seismic loads should be considered in designing of liquid tanks. From quantitative point of view, extending the maximum SIRF for real scale tanks shows that the liquid impact force value cannot be ignored.

To reduce these sloshing effects, a new stepped shape tank roof is proposed to reduce the SIRF in this study. The mechanism of liquid collision with the typical and proposed roof has been discussed. Our experimental studies show that the proposed roof is quite efficient and it can significantly reduce the impact loads up to an average of 52% for the conditions explored in this study. Since this roof system has been studied on a laboratory scale, it is only presented as an idea of a proposed strategy in this paper. The description of the technical and executive issues, cost-effectiveness and design was ignored. After more studies and more precise numerical simulations, they will be discussed in detail in other articles.

ACKNOWLEDGEMENTS

This work has been supported by the International Institute of Earthquake Engineering and Seismology (IIEES) and Iranian National Science Foundation (INSF). The author wishes to express his gratitude to the IIEES and INSF for their supports.

REFERENCES

Abramson, H. N., R. L. Bass, O. Faltinsen and H. A. Olsen (1974). Liquid slosh in LNG carriers, 10th Symp. Naval Hydrodynamics, MIT, Cambridge, MA.

Akyildız, H. and N. E. Ünal (2006). Sloshing in a three-dimensional rectangular tank: numerical simulation and experimental validation. Ocean Engineering. 33(16), 2135-2149.

Akyıldız, H., N. Erdem Ünal and H. Aksoy (2013). An experimental investigation of the effects of the ring baffles on liquid sloshing in a rigid cylindrical tank. Ocean Engineering. 59(0), p. 190-197.

Cariou, A. and G. Casella (1999). Liquid sloshing in ship tanks: a comparative study of numerical simulation, Marine Struct. 12(3), 183–198.

Celebi, M. E., M. H. Kim and R. F. Beck (1998). Fully nonlinear 3-D numerical wave tank simulation, J. Ship Res. 42(1), 33–45.

Celebi, M. S. and H. Akyildiz (2002). Nonlinear modeling of liquid sloshing in a moving rectangular tank, Ocean Eng. 29(12), 1527–1553.

Chen, W., M. A. Haroun and F. Liu (1996). Large amplitude liquid sloshing in seismically excited tanks. Earthquake engineering & structural dynamics. 25(7), 653-669.

Chen, Y., K. Djidjeli and W. Price (2009), Numerical simulation of liquid sloshing phenomena in partially filled containers. Computers & fluids. 38(4), 830-842.

Faltinsen, O. M., O. F. Rognebakke and A. N.

Timokha. (2003) Resonant three-dimensional nonlinear sloshing in a square-base basin. Journal of Fluid Mechanics. 487, 1-42.

Goudarzi, M. A., S. Sabbagh Yazdi and W. Marx (2010). Seismic analysis of hydrodynamic sloshing force on storage tank roofs. Earthquake Spectra. 26(1), 131-152.

Goudarzi, M. A., S. R. Sabbagh Yazdi and W. Marx (2009). Investigation of sloshing damping in baffled rectangular tanks subjected to the dynamic excitation Bulletin of Earthquake Engineering 8(4), 1055-1072.

Goudarzi, M. A. and S. R. Sabbagh Yazdi (2008). Evaluating 3D earthquake effects on sloshing wave height of liquid storage tanks using finite element method. Journal of Seismology and Earthquake Engineering. 123-136

D. M. Greaves, A. G. L. Borthwick, G. X. Wu and R. Eatock Taylor (1997). A moving boundary finite element method for fully nonlinear wave simulations, J. Ship Res. 41(3), 181–194.

Jin, H., Y. Liu and H. J. Li (2014). Experimental study on sloshing in a tank with an inner horizontal perforated plate. Ocean Engineering. 82(0), 75-84.

Kim, Y. (2001). Numerical simulation of sloshing flows with impact load, Appl.Ocean Res. 23(1), 53–62.

Kim Y., Y. J. Park and H. R. Lee (1994). A numerical study on the prediction of sloshing impact pressure, Trans. Soc. Naval Arch. Korea 30(4), 61–73.

Kim, J. K., H. M. Koh and I. J. Kwahk (1996). Dynamic response of rectangular flexible fluid containers. Journal of Engineering Mechanics. 122(9), 807-817.

Kobayashi, N. (1980). Impulsive acting on the tank roofs caused by sloshing liquid, Proc. 7th World Conference Earthquake Engineering 5, 315–322.

Lee, T. h., Z. Zhou and Y. Cao (2002). Numerical simulations of hydraulic jumps in water sloshing and water impacting. Journal of fluids engineering. 124(1), p. 215-226.

Milgram, J. H. (1969). The motion of a fluid in a cylindrical container with a free surface following vertical impact. Journal of Fluid Mechanics. 37(03), p. 435-448.

Mimi. G. (2011). Numerical simulation of liquid sloshing in rectangular tanks using consistent particle method and experimental verification. PhD thesis, National University of Singapore.

Minowa, C., N. Ogawa, I. Harada and D. C. Ma (1994). Sloshing roof impact of a rectangular tanks, sloshing, fluid-structure interaction and structure, response due to shock and impact loads, ASME Pressure Vessel and Piping Conference PVP- 272, 13–21.

Ockendon, J. and H. Ockendon. (1973). Resonant surface waves. Journal of Fluid Mechanics. 59(02), 397413.

Shinkai A., S. Tamia and M. Mano (1995). Sloshing impact pressure induced on cargo oil tank walls on the middle-sized double hull tanker, Trans. Soc. Naval Arch. West. Japan 90, 91–98.

Stolbetsov, V. (1967). Oscillations of liquid in a vessel in the form of a rectangular parallelepiped. Fluid Dynamics. 2(1), 44-49.

Takemoto, H., S. Oka and *et al*. (1994). Experimental study on sloshing impact loads of middle sized tankers with double hull, J. Soc. Naval Arch. Japan 176, 399–410.

Taylor, G. (1953). An experimental study of standing waves. in Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. The Royal Society.

Wei, Z., Q. Yue and S. Ruan (2012). An experimental investigation of liquid sloshing impact load in a rectangular tank. in The Twenty-second International Offshore and Polar Engineering Conference. International Society of Offshore and Polar Engineers.