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On the Multiple Steady Flow States in Spindle Shaped Geometry of Bridge Foundations

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ABSTRACT

Flow hysteresis in a channel with a supercritical flow in the vicinity of a spindle-shaped bridge foundation (SSBF) with a change in flow rate has been investigated in a laboratory for the first time. Two spindle-shaped bridge foundations with diameters of 9 and 6.5 cm, which are placed at distances of 1.5 and 1 meter from the gate, have been used. The critical depth ranges between 0.027 and 0.0528 meters, and the used flow rates range from 250 to 600 liters/min. The flow regimes in the vicinity of the bridge foundations are classified based on the relative depths and Froude numbers created in sections B and C according based on the Froude number at the *vena contracta*. Sections B and C are near the bridge foundation and the flow passing through the center of the bridge foundation, respectively. The increase and subsequent decrease in velocity leads to different flow states. In some cases, two different behaviors can be seen from the flow with similar laboratory conditions. Hysteresis manifests in the range of Froude number 2.697~5.0. As the bridge foundations approach the valve, the hysteresis area becomes wider. Changes in the flow regime under the same conditions, called hysteresis, should be considered in designs. Also, with hysteresis, the relative residual energy and the downstream Froude number increased by 57.36% and 72.31%, respectively.

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1. INTRODUCTION

Conducting this research is aimed at investigating the paradoxical states of the supercritical regime in dealing with bridge foundations. The geometry is intended to reduce the vortices, which will be explained in further detail below. The occurrence of hysterical behavior is one of the phenomena that has been less discussed in previous studies. Generally, hysteresis is expected when the flow meets the obstacle. For the same input flow, two different behaviors are observed which are dependent on the flow cycle. One of the basic problems in hydraulic structures is the existence of severe floods and changes in the increase and decrease of flood flow intensity. By examining previous sources, it seems that in hydraulic studies, this phenomenon, which is called hysteretic behavior, has been less studied. During a flow cycle, the flow can be increased and then subsequently decreased to a lower value. Hysteresis is one of the important things in hydraulic design that is less appreciated. Previous studies related to the present research will now be discussed

The first category of papers related to hysteresis examined the protrusion of the channel floor and originated with Abecasis and Quintela (1964). Other primary studies on the states of hydraulic hysteresis on a raised floor with a fixed width are: Mehrotra (1974); Muskatirovic and Batinic (1977); Baines and Davies (1980); Austria (1987); and Lawrence (1987). Among the mentioned research, Austria (1987) and Lawrence (1987) are important, but additional studies are related to Baines and Whitehead (2003) and Defina and Susin (2006).

Baines and Whitehead (2003) focused on hysteresis using both theoretical and experimental methods. Different stable states could be achieved with similar input flows – indicating the presence of hysteresis. Defina and Susin (2006) investigated this phenomenon on a bulge where they defined WR and SR (Weak and Stronge) reaction or the same discharge for the flow. A weak reaction occurs when the flow is supercritical and obstacles do not change the flow regime. On the other hand, a strong reaction indicates a condition where the hydraulic jump causes the flow regime to change to subcritical.

Another category of papers is related to the narrowing of the channel width, which was done for the first time by Akers and Bokhove (2008). They investigated the different states in a gradual contraction. They showed that waves that have a diagonal shape can be caused by various effects, including surface tension. Defina and Viero (2010) investigated hydraulic jumps in the constriction. The findings show the instability of the flow due to the effect of floor friction, which has caused the formation of a hysteresis loop. Sadeghfam et al. (2017) conducted the variation of flow history in the contraction of cross-section using cusp-catastrophe theory. Also, Daneshfaraz et al. (2022a) investigated the effect of hysteretic flow behavior with supercritical flow in sudden and gradual contractions.

The third category of studies in the field of bridge foundations is very limited and we return to the paper of Defina and Susin (2003). They used different types of bridge foundations with several diameters and conducted the behavior of the flow in front of the foundations. They presented a theoretical prediction of the onset of hydraulic hysteresis. They found that when the flow meets the foundation, there are two different reactions, weak and strong, for otherwise similar flow conditions.

The other category of research includes Defina and Susin (2003, 2003). They presented a theoretical equation based on experimental data to further investigate dual flow behaviors under the same conditions. They discovered hysteretic behavior for this category can be defined for a valve opening rate at different flow rates. Two states can be achieved with similar gate-opening rates. Viero and Defina (2018) studied the flow passing through a vertical sluice gate (both supercritical and subcritical regimes). They presented theoretical equations based on the Froude number and relative gate opening. The results obtained from the theoretical equations were consistent with experimental data and confirmed that the presented equations accounted for hysteresis. Daneshfaraz et al. (2022b) investigated hysteresis in flow through a gabion constriction. They showed that the conjugate depth of the flow increased by 69.36% and the Froude number by 69.15% with the appearance of hysteretic behavior. Also, Daneshfaraz et al. (2023a) conducted a study of hysteretic effects for flow over a smooth-to-rough bed. Daneshfaraz et al. (2023b) studied different sill geometry and the hysteresis effects on the flow regime. The stability of the flow is influenced by both friction and slope.

The literature review shows that despite the studies and published papers conducted in connection with hysteresis, there is a need to study the causes of multiple flow behaviors under the same laboratory conditions. By examining the previous sources, it was found that there is a gap for more recent research in order to determine the cause of the hysteresis phenomenon, and it is still necessary to conduct other research. Generally, the theoretical relationships are not fully consistent with experiments. Part of the reason is the limitations of one-dimensional models. Here, the hysteretic behavior of a certain shape of bridge foundation is evaluated; these foundations can be useful in scouring control.

2. MATERIAL AND METHODS

2.1. Laboratory Specifications

The present experiment has been carried out using a flume that is 5.0m (length) × 0.5m (height) × 0.3m (width). The slope of the experimental setup was zero. A vertical sluice gate, with an opening of 2 cm was positioned 1.5 m from the tank inlet. The was generated by two pumps, each with a capacity of 7.5 L/s. Rotatmeters were used to measure volumetric flowrate. The current research has been investigated in two cases and four models whose details are presented in Table 1. The bridge foundations have a special shape that is a combination of cylindrical tubes and a triangular glass box (Fig. 1). The diameter of the cylindrical tubes is equal to 9.0 and 6.5 cm and they are located 1.0 and 1.5 m from the gate. In addition to the diameter of the tube, the effect of distance has also been investigated. The purpose of using this form of bridge foundation is that these foundations can be useful in scouring.

Figure 1 shows the flume. The hydraulic jump profiles created by the bridge foundation are illustrated in Fig. 2.

Section [B] is upstream of the bridge foundations and section [C] is where the bridge foundations are located and the flow passes around the foundations. It should be noted that section [A] shows the location of the gate and the flow passing through the *vena contracta*. Figure 2 shows flow profiles under similar conditions. These two flow profiles are described in detail below:

- Profile 1 shows a hydraulic jump in sections [B] and [C]. Section [A] includes the supercritical regime. At [B], the regime is transformed to subcritical by

Table 1 Information of bridge foundation and range of measured parameters

Case	Q (L/s)	Foundation diameter (cm)	foundation center from sluice gate (cm)	Re	Model no.
Case 1	4.17 to 10.0	9.0	150	50000 to 112000	SSBF1
		6.5			SSBF2
Case 2		9.0	100		SSBF3
		6.5			SSBF4

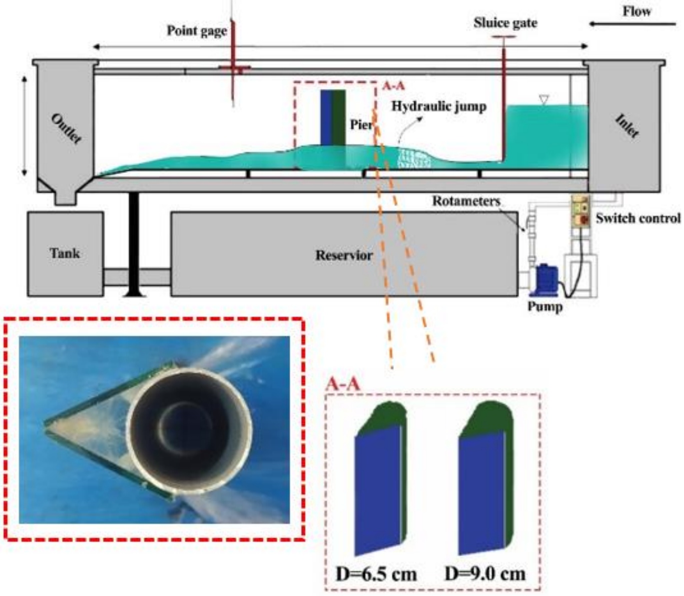

Fig. 1 Schematic of experimental flume and spindle shaped bridge foundations

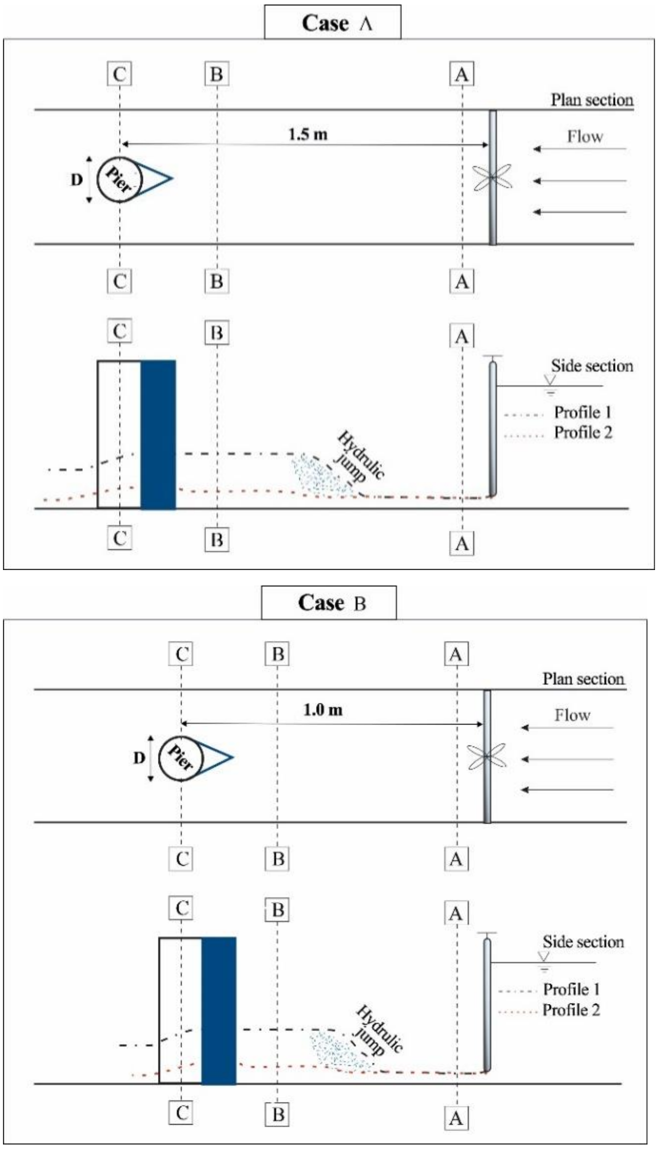

Fig. 2 Profiles in the hydraulic jump

the return flow. The flow regime in section [C] moves to the downstream side of the channel due to turbulence and interference when the flow meets the bridge foundation with a subcritical regime.

- ❖ Profile 2 shows the supercritical regime throughout the channel and the absence of a hydraulic jump. The regime created in sections [B] and [C] is supercritical and the bridge foundations placed do not cause a jump or change to the flow regime. The main goal and new approach of this research is focused on these two types of flow profiles and it will investigate the conditions responsible for determining whether a hydraulic jump will form in the flow as it comes into contact with the spindle-shaped bridge foundations (SSBF).

The input flow rates entering the laboratory flume are first increased and then decreased. The input flow is increased from 4.17 to 10 L/s and then decreased from 4.17 to 10 L/s in steps of 0.83 L/s. This progression of discharge is sufficient to demonstrate hysteresis.

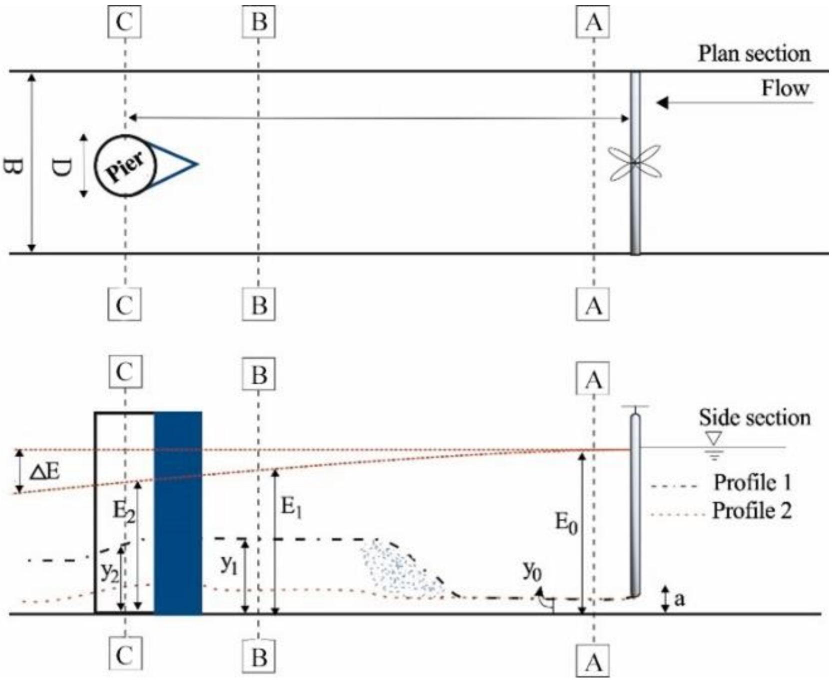

Fig. 3. Geometric and hydraulic parameter of channel equipped with spindle bridge foundation

2.2. Analysis of Parameters

Utilizing the nomenclature in Fig.3, the parameters for investigating hydraulic hysteresis flow is as follows initiated with reference to Eq. (1).

$$f_1(Q,\rho,g,a,D,B,b,y_A,y_B,y_C,y_{cr},\\ E_A,E_B,E_C,L,V_A,V_B,V_C)=0 \tag{1}$$

Q is flow rate, ρ is the density of the fluid, g is the acceleration of gravity, a is the gate opening. D is the diameter of the bridge foundation, B is the channel width, b is the narrowed width of the channel (b = B-D), y_A = depth of flow in section [A], y_B = depth of flow in section [B], y_C = depth of flow in section [C], y_{cr} is the critical depth of the flow, The E terms are the energy values in the respective sections, L is the distance between the bridge foundation and the valve and the V terms are the velocities in the various sections. By considering the parameters g, ρ and y_A as repeated parameters. The dimensionless parameters can be presented as an equation and since the range of Reynolds number is more than 2000, Re_0 can be omitted (Ghaderi & Abbasi, 2019; Daneshfaraz et al. 2020; 2021a, b, c). Also, the parameters a/y_A, B/D are held constant and thus eliminated as independent variables. Finally, the dependent parameters are a function of independent dimensionless parameters as shown in Eq. (2):

$$\frac{y_B}{y_A},\frac{y_C}{y_A},\frac{y_B}{y_C},\frac{E_B}{E_A},\frac{E_C}{E_A},Fr_B,Fr_C\\ =f_2(Fr_A,\frac{b}{B},\frac{D}{L}) \tag{2}$$

3. Results and Discussions

3.1. Longitudinal Profile of Flow

Flow profiles created in the laboratory at the collision of the supercritical flow with the foundations of the spindle-shaped bridge foundations are shown in Fig. 4. With flow rates of 4.17 to 10 L/s have been used in an increasing manner and then from 10 to 4.17 L/s in a decreasing manner. In the figure, with the increase and decrease of the flow rate, dual flow behaviors can be viewed for the identical flow rate. Figures 4 to 7 and

Tables 3 to 6 provide a summary of the experiments and the hysteretic flow behavior. It can be seen that in the incremental flow rate, a hydraulic jump is created due to the placement of bridge foundations in the flow path, and during this period both sections [B] and [C] are in the subcritical regime. With increasing flow, the jump disappears. With reductions in the discharge, flow has different scheme at a specific flow rate, but this does not occur for all flow rates. The decreasing flow rate created in the channel puts sections [B] and [C] entirely in the supercritical regime.

Table 1 shows hydraulic parameters and longitudinal profiles created in SSBF1 model. When the flow rate reaches 7.5 L/s (Fig. 4a-c), sections [B] and [C] are in the subcritical regime. By increasing the discharge and bringing it to the value of 8.33 L/s, the flow regime in sections [A] and [B] changes. Then, with the gradual decrease of the flow rate to 6.67 L/s (Fig. 4e-h), sections [B] and [C] have supercritical regimes until the discharge is 5.83 L/s. Thereafter, sections [B] and [C] are subcritical.

Table 3 shows hydraulic parameters and longitudinal profiles created in the SSBF2 model. With increasing flow to 6.67 L/s (Fig. 5a and 5b), sections [B] and [C] are in the

Table 2 Hydraulic parameters for SSBF1

Figure	Fr_A (*vena contracta*)	Regime (sub/super critical) Sec [B]	Sec [C]	Profile	Flow history
Fig. 4a	3.630	Sub	Sub	P 1	Flow increasing
Fig. 4b	4.269	Sub	Sub	P 1	Flow increasing
Fig. 4c	4.978	Sub	Sub	P 1	Flow increasing
Fig. 4d	5.717	Super	Super	p 2	Increased flow caused supercritical hysteresis
Fig. 4e	4.978	Super	Super	p 2	Decreasing flow
Fig. 4f	4.269	Super	Super	p 2	Decreasing flow
Fig. 4g	3.630	Sub	Sub	p 1	Decreasing flow caused subcritical hysteresis
Fig. 4h	2.697	Sub	Sub	p 1	Decreasing flow

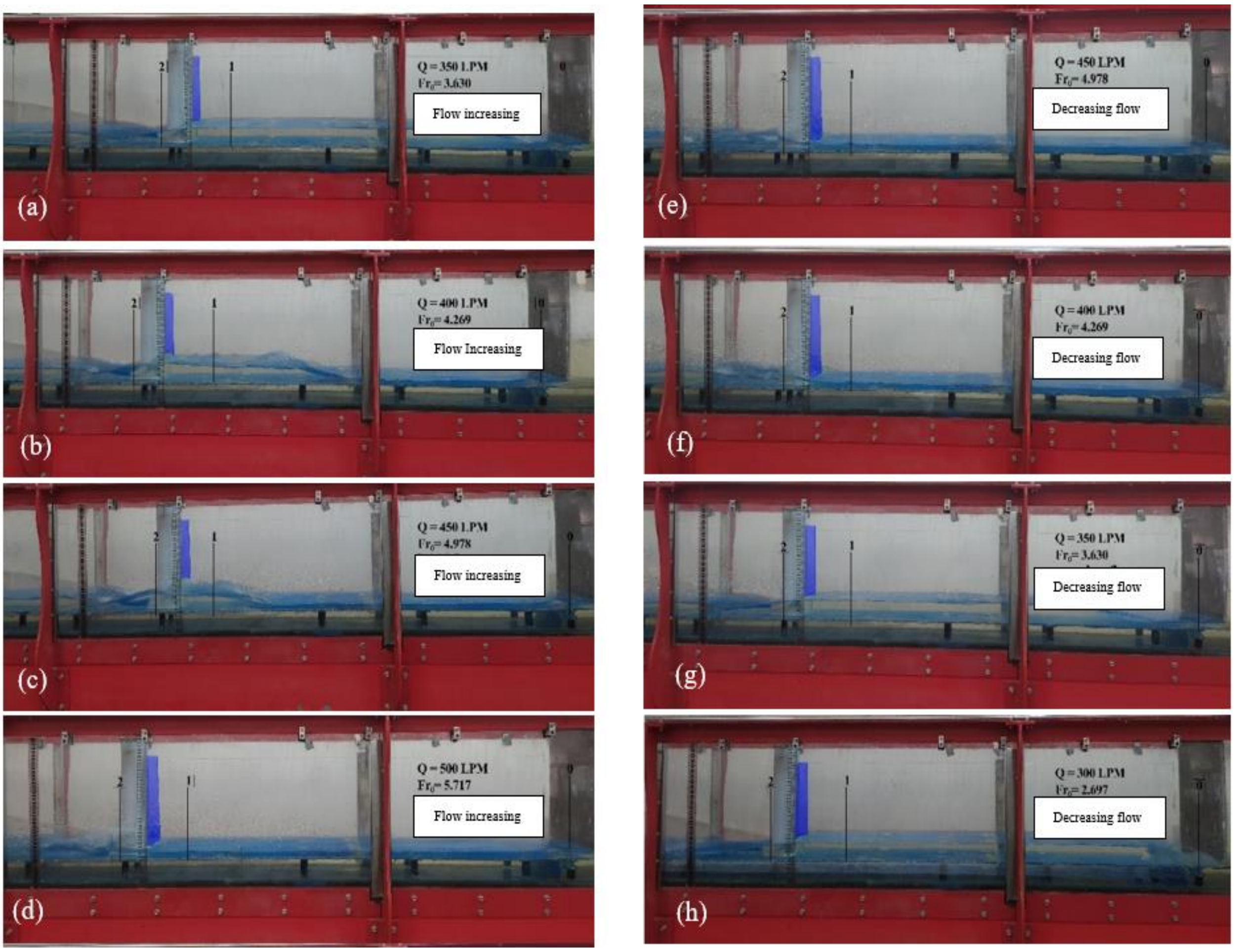

Fig. 4 Experimental test runs (case 1) for SSBF1

Table 3 Hydraulic parameters for SSBF2

Figure	Fr_A (*vena contracta)*	Regime (sub/super critical)		Profile	Flow history
		Sec B	Sec C		
Fig. 5a	3.630	Sub	Sub	P 1	Flow increasing
Fig. 5b	4.269	Sub	Sub	p 1	Flow increasing
Fig. 5c	4.978	Super	Super	p 2	Increased flow caused supercritical hysteresis
Fig. 5d	5.717	Super	Super	p 2	Flow increasing
Fig. 5e	4.978	Super	Super	p 2	Decreasing flow
Fig. 5f	4.269	Super	Super	p 2	Decreasing flow
Fig. 5g	3.630	Super	Super	p 2	Decreasing flow
Fig. 5h	2.697	Sub	Sub	p 1	Decreasing flow caused subcritical hysteresis

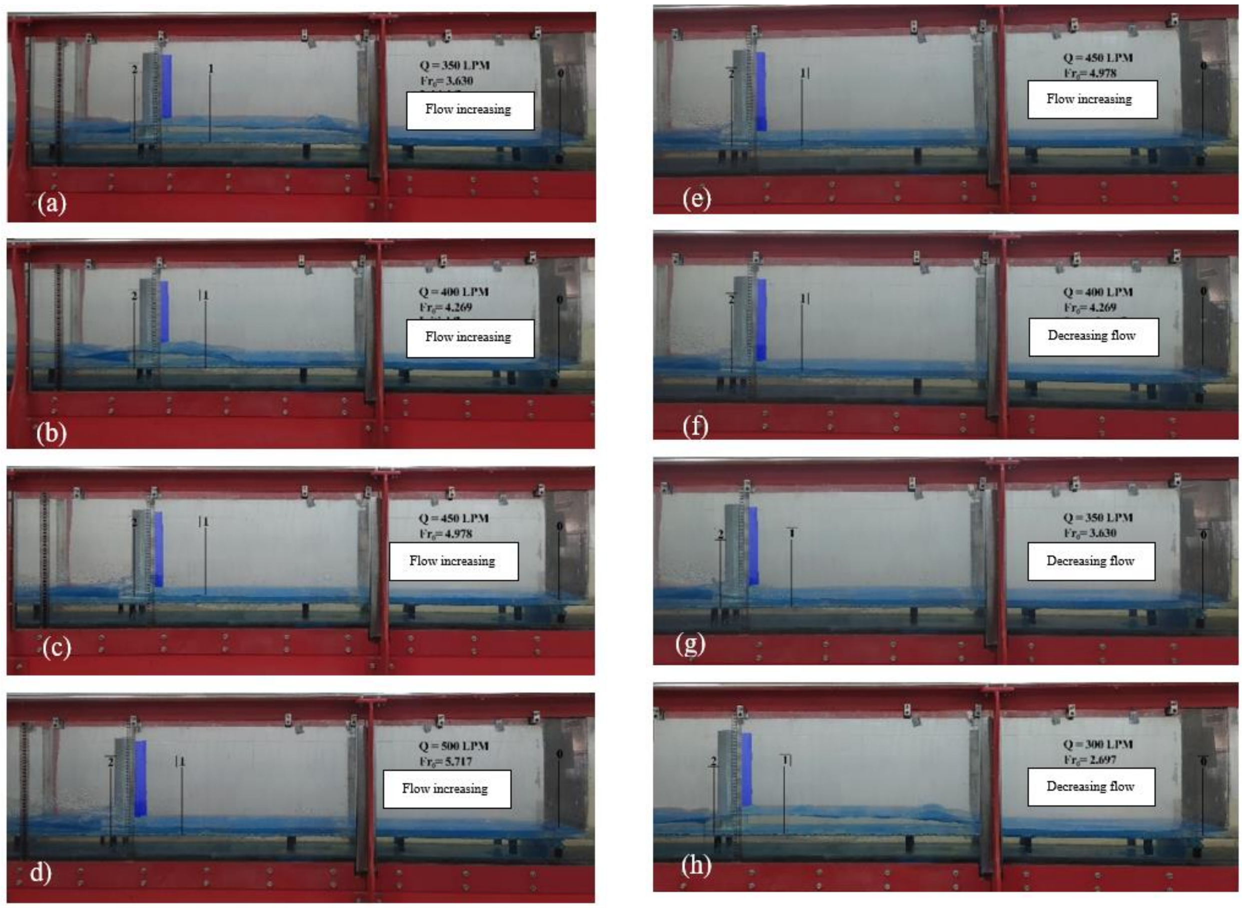

Fig. 5 Experimental test runs (case 1) for SSBF2

Table 4 Hydraulic parameters for SSBF3

Figure	Froude number (vena contracta)	Regime (sub/super critical)		Profile	Flow history
		Sec B	Sec C		
Fig. 6a	3.630	Sub	Sub	P 1	Flow increasing
Fig. 6b	4.269	Sub	Sub	P 1	Flow increasing
Fig. 6c	4.978	Super	Super	P 2	Increased flow caused supercritical hysteresis
Fig. 6d	4.269	Super	Super	P 2	Decreasing flow
Fig. 6e	3.630	Super	Super	P 2	Decreasing flow
Fig. 6f	2.697	Super	Super	P 2	Decreasing flow
Fig. 6g	3.630	Super	Super	P 2	Increased flow caused subcritical hysteresis

subcritical regime. By changing the flow rate and reaching the value of 7.5 L/s, changes in the conditions of the flow regime cause it to become supercritical. Then, with the decreasing of the discharge up to 5.83 L/s gradually (Fig. 5e-h), sections [B] and [C] have the supercritical regime, until the discharge is 5 L/s, sections [B] and [C] are subcritical.

Table 4 shows hydraulic parameters and longitudinal profiles created in the SSBF3 model. As flow increases to 6.67 L/s (Fig. 6a and b), sections [B] and [C] are placed in the subcritical regime. By continuing to increase the discharge and reaching 7.5 L/s, the flow encounters a regime change that turns into a supercritical state. Then, with the gradual decrease of the flow rate to 5.83 L/s (Fig. 6e), sections [B] and [C] are still in the supercritical regime.

Table 5 shows hydraulic parameters and longitudinal profiles created in the SSBF4 model. By changing the flow rate and reaching the value of 5.83 L/s (Fig. 7a and 7b), sections [B] and [C] are in the subcritical regime. By continuing to increase the discharge and reach to 6.67 L/s, the flow encounters a regime change that turns into a supercritical state. Then, by the decreasing of discharge to 5.83 L/s (Fig. 7e to h), sections [B] and [C] are still in the supercritical regime.

For the discharges that lead to the creation of supercritical flow (when the water collides with the foundations of the spindle-shaped bridge foundations), an area is created at its triangular sides, where the subcritical flow regime occurs (Fig. 8a). The lines crossing the bridge foundation and the channel wall are regular. If the Froude number near the bridge foundation is less than the *vena contracta* Froude number, the channel wall will exert its friction effect on the flow. In this case, due to the friction

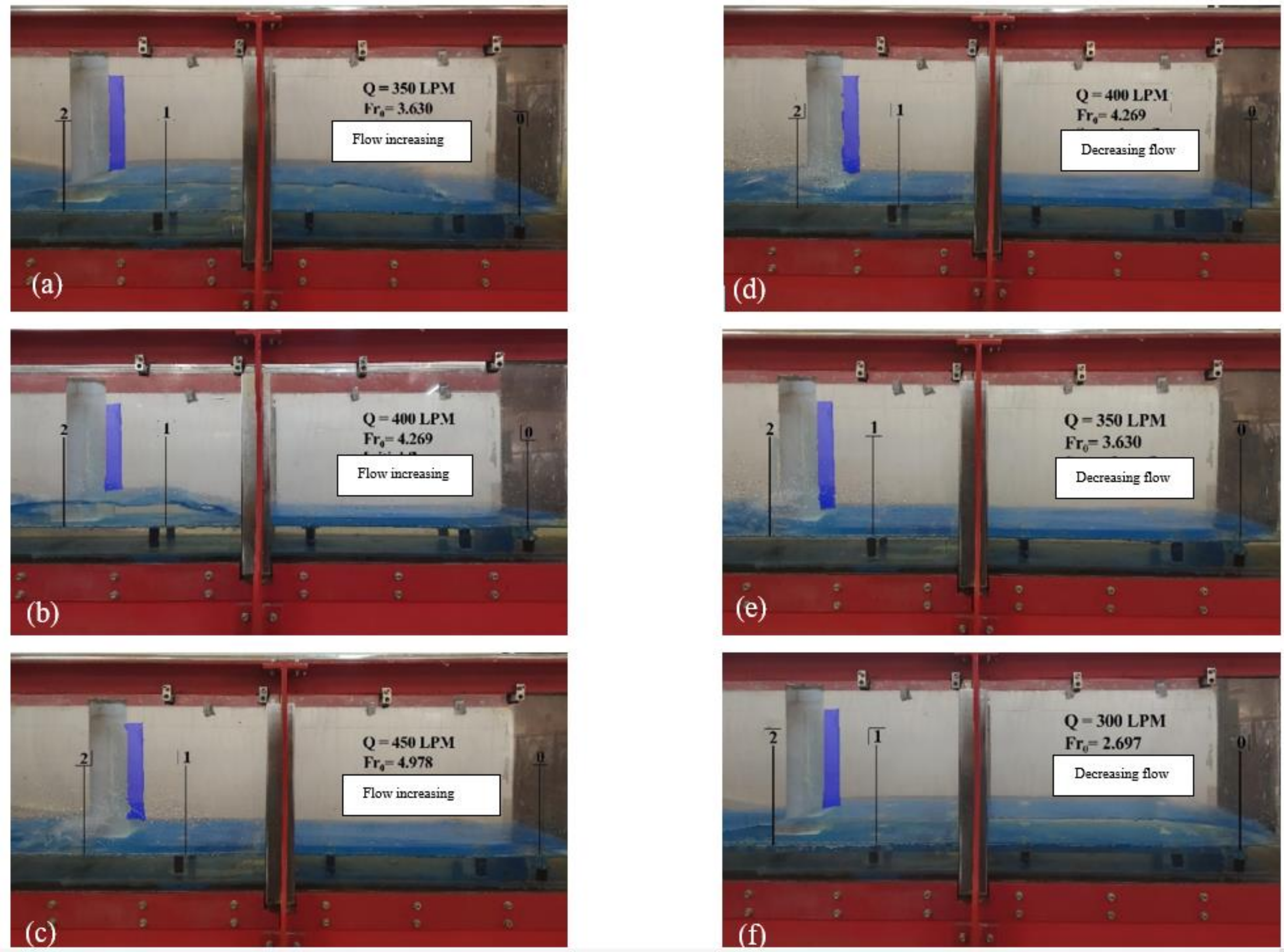

Fig. 6 Experimental test runs (case 2) for SSBF3

Table 5 Hydraulic parameters for SSBF4

Figure	Froude number (vena contracta)	Regime (sub/super critical)		Profile	Flow history
		Sec B	Sec C		
Fig. 7a	2.697	Sub	Sub	P 1	Flow increasing
Fig. 7b	3.630	Sub	Sub	P 1	Flow increasing
Fig. 7c	4.269	Super	Super	P 2	Increased flow caused supercritical hysteresis
Fig. 7d	4.978	Super	Super	P 2	Flow increasing
Fig. 7e	4.978	Super	Super	P 2	Decreasing flow
Fig. 7f	4.269	Super	Super	P 2	Decreasing flow
Fig. 7g	3.630	Super	Super	P 2	Decreasing flow
Fig. 7h	2.697	Super	Super	P 2	Decreasing flow

at the wall, the depth of the flow on the side of the walls increases and the depth in this area exceeds the depth in the middle of the channel. In this case, the flow regime near the walls is subcritical (Fig. 8b). The decrease in flow rate causes a hydraulic jump to be created upstream of the bridge foundation. The hydraulic jump causes the flow regime in the area of the bridge to be subcritical. Due to the fact that the Froude number of the flow is lower than the *vena contracta* Froude number, the flow lines show a regular behavior (Fig. 8c).

3.2. SSBF1 Findings

Hysteresis in sections [A], [B] and [C] is shown in Fig. 9 using dimensionless depth quantities. Figures 9a to 9c respectively of flow in sections [B] and [C] for the SSBF1 model. Figure 9a shows the variation of relative depth in section [B], fig. 9b is for section [C], and Fig. 9c shows the depth ratio of section [B] to section [C] versus upstream Froude number. With increasing and then decreasing flow, two equal flow rates cause different relative depths. Figure 4 clearly reveals this behavior. With SSBF1 model, hysteresis has appeared at flow rates of 6.67 and 7.5 L/s. At flow rates of 6.67 and 7.5 L/s, sections [B] and [C] are subcritical. In the decreasing flow rate, when the flow rate returns to 6.67 and 7.5 L/s, the hydraulic jump passes with a decrease in depth compared to the incremental flow rate. The resulting surface profile is profile 2. The result of changes in Froude numbers of sections [B] and [C] is shown in Fig. 10. Hysteresis is dependent on the current state of the flow as well as on its

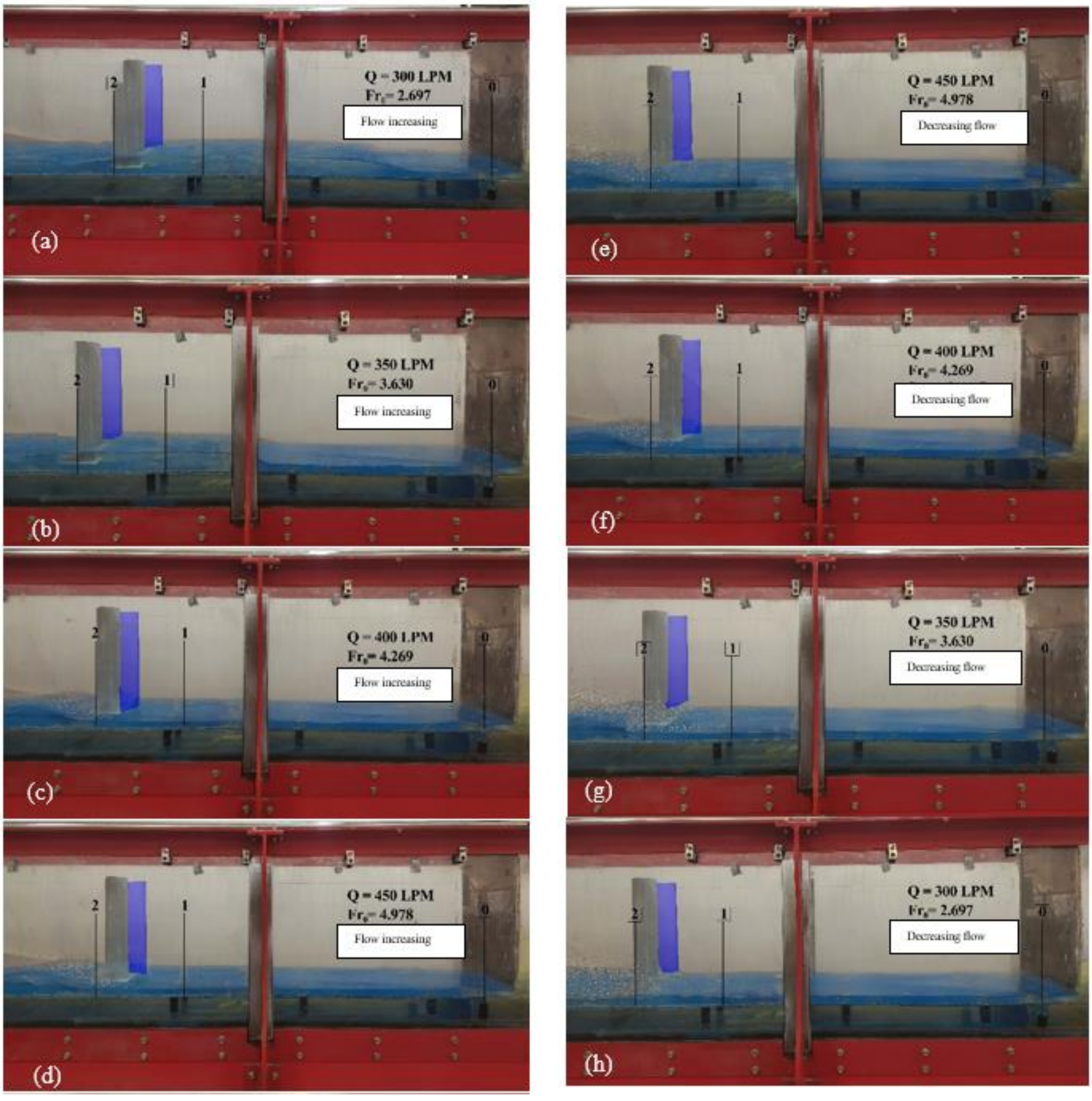

Fig. 7 Experimental test runs (case 2) for SSBF4

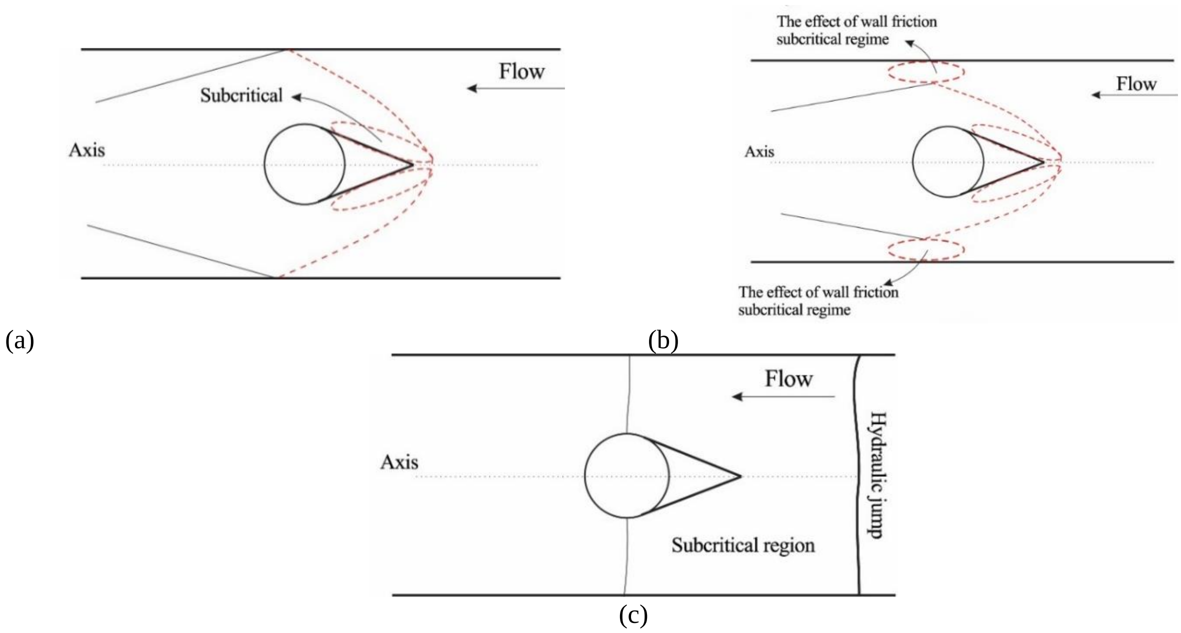

Fig. 8 Effect of wall friction in nearby bridge foundation

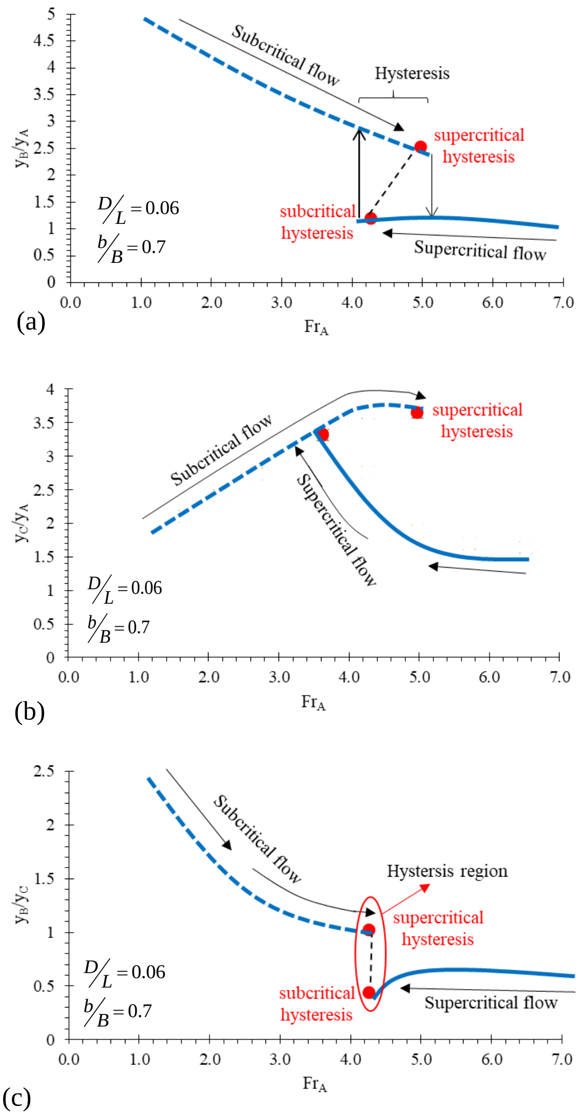

Fig. 9 Reproduction of hysteresis flags for model SSBF1. (a) Section [B]; (b) Section [C]; (c) Integration of [B] and [C]

previous state. With the increase in flow rate, the speed increases. On the other hand, with a decrease in depth, sections [B] and [C] (which were subcritical at low flow rates), become supercritical. As the flow is gradually decreased back to the incremental flow, both sections were in the subcritical regime.

Figures 10a and 10b shows the variation of Froude numbers in sections [B] and [C] versus the upstream Froude number. By first increasing and then decreasing the flow, the flow regime changes from subcritical to supercritical in sections [B] and [C] under the same conditions. In other words, in the SSBF1 model, the hysteresis is observed in the range of Froude number 4.2~5.0.

It is inferred from the above figure that with the increase of the inlet flow rate, the flow regime up to the *vena contracta* Froude number of 4.269 in both sections [B] and [C] is in the range of the subcritical regime. With the increase of the flow rate and for the upstream flow rate exceeding 4.269, the flow depth decreases and the flow regime becomes supercritical in both sections, so that the flow rates of sections [B] and [C] increase to 2.0 and 2.5, respectively. But by the decreasing of the discharge, when the *vena contracta* Froude number reaches 4.269, sections [B] and [C] are in the supercritical regime.

By examining the graphs more closely, it can be seen that in model SSBF1, the hysteretic behavior has increased the conjugate depth and downstream Froude number by 57.14 and 80 percent, respectively.

3.3. SSBF2 Results

Hysteresis in sections [A], [B] and [C] are shown in Fig. 11. Figures 11a to 11c respectively show hysteresis behavior in sections [B] and [C] from the SSBF2 model. Figure 11a shows flow in section B, Fig. 11b shows the behavior of flow in section [C], and Fig. 11c shows the depth ratio of section [B] to section [C]. With the gradual increase and decrease of flow rate, different relative

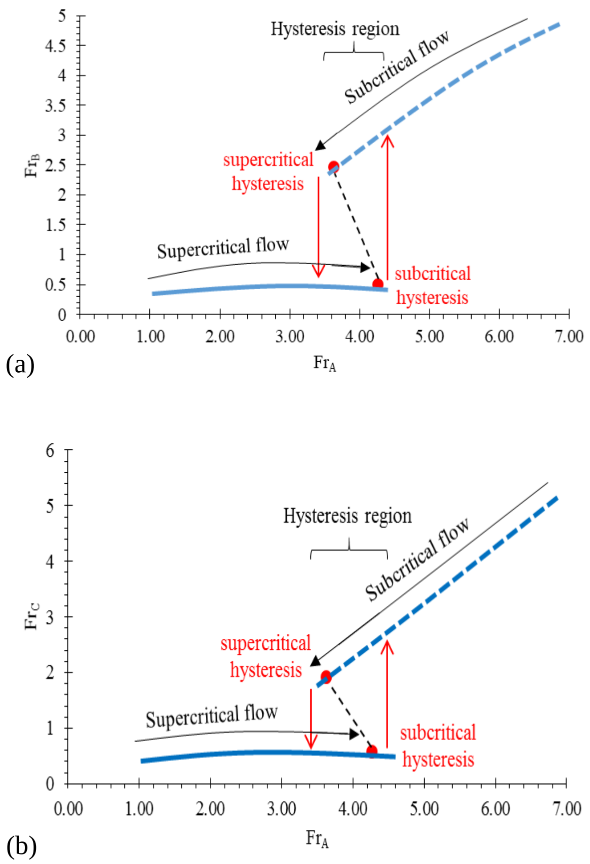

Fig. 10 Variation of Froude number Vs. *vena contracta* Froude number in model SSBP1. (a) Section B (b) Section C

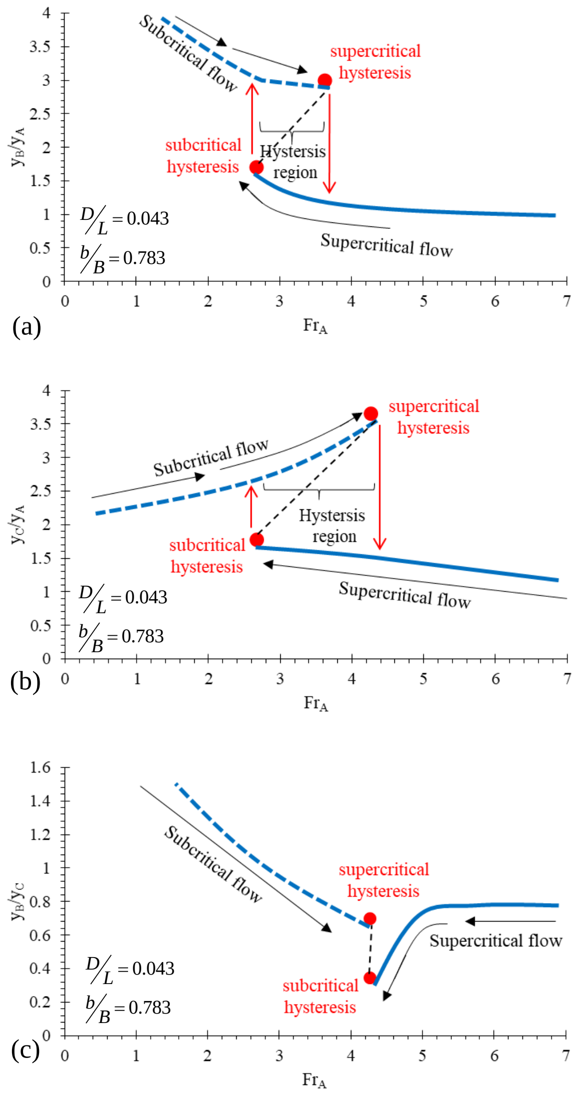

Fig. 11 Reproduction of hysteresis flags for model SSBF2.a) Section [B]; b) Section [C]; c) Integration of [B] and [C]

depths are created in both sections even though flow rates are the same. This behavior can be seen in Fig. 5. With an increase and the subsequent decrease in flow, hysteresis has appeared at flow rates of 5.83 and 6.67 L/s. Sections [B] and [C] have subcritical regime and that's longitudinal profile is type 1. By decreasing of discharge, when the flow rate is reduced to 5.83 and 6.67 L/s, the hydraulic jump passes through the obstacle, and both sections [B] and [C] are supercritical with surface profile 2. The effect of changing Froude numbers is shown in Fig. 12.

Figures 12a and b shows the variation of Froude numbers in sections [B] and [C] versus the upstream Froude number, respectively. By first increasing the flow rate and then subsequently decreasing it, the flow regime changes from subcritical to supercritical at [B] and [C]. With the SSBF2 model, the range in which hysteretic behavior is observed 3.63 < Fr < 4.269. Thus, sections [B] and [C] are subcritical. With model SSBF1, the hysteretic behavior has increased the conjugate depth and downstream Froude number by 48.57 and 80 percent, respectively.

3.4. SSBF3 Results

Figures 13a-13c show hysteresis sections [B] and [C] with the SSBF3 model. Figure 13a shows the flow in section [B], Fig. 13b shows the behavior of flow in section [C], and Fig. 13c shows the depth ratio of section [B] to section [C]. From these figures, it is seen that in an increasing and decreasing flow, two discharges with the same laboratory conditions can yield different relative depths. In the SSBF3 model, with an increase and then a decrease of flow, hysteresis has appeared at flow rates of 5.83 and 6.67 L/s; Sections [B] and [C] are subcritical and the surface profile created is type 1. With a decreasing flow rate, the hydraulic jump passes through the obstacle, and in both sections [B] and [C] it is supercritical and type 2. The impact of Froude numbers in sections [B] and [C] are shown in Fig. 14.

Figures 14a and b show the changes of Froude numbers in sections [B] and [C] versus upstream Froude number, respectively. By increasing and then decreasing flow, the flow regime changes from subcritical to supercritical in sections [B] and [C] for two *vena contracta* Froude numbers that are the same. Thus, in the SSBF3 model, the range in which hysteretic behavior is observed 3.63 < Fr < 4.269. When the flow rate decreases to less than 350 liters/min, the flow returns to the incremental state, i.e., the subcritical regime and sections [B] and [C] are subcritical.

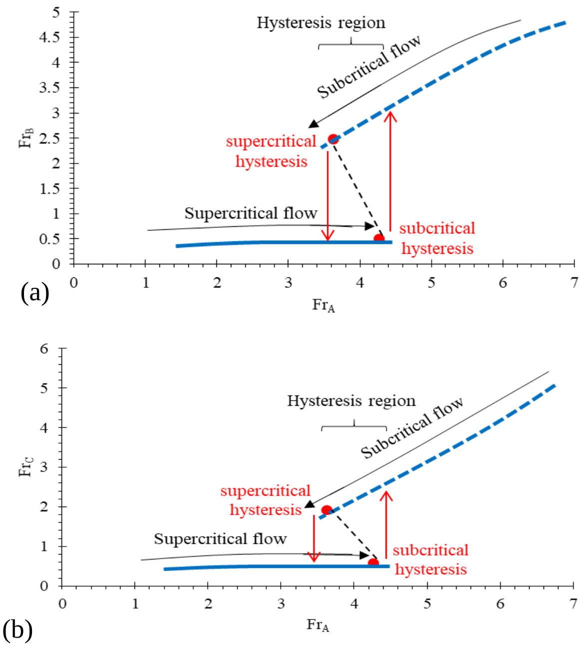

Fig. 12 Variation of Froude number with vena contracta Froude number for SSBF2: (a) Section [B]; (b) Section [C]

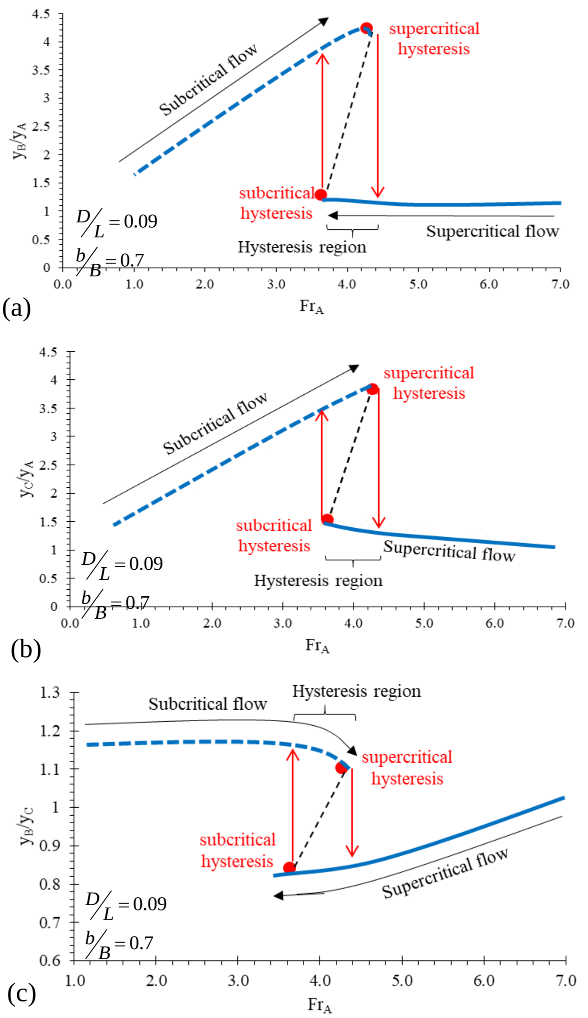

Fig. 13 Reproduction of hysteresis flags in model SSBF3. a) Section [B]; b) Section [C]; c) Integration of [B] and [C]

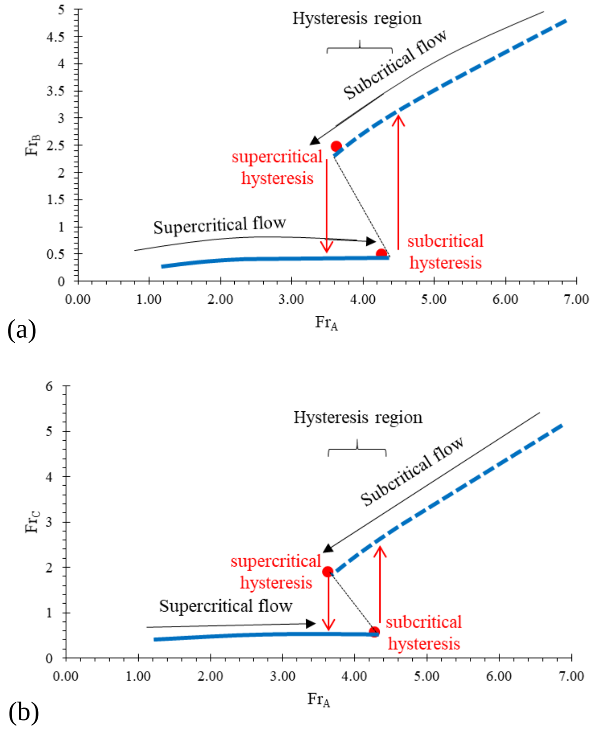

Fig. 14. Variation of Froude number with vena contracta Froude number for model SSBF3. (a) Section [B]; (b) Section [C]

3.5. SSBF4 Results

The hysteresis in sections [A], [B] and [C] are shown in Fig. 15. Figures 15a-15c respectively represent hysteresis in sections [B] and [C] for the SSBF4 model. Figure 15a shows the behavior in section [B]. Next., Fig. 15b shows the behavior of flow in section [C]. Finally, Fig. 15c shows the depth ratio of section [B] to section [C]. These figures how that for a stream with increasing and then decreasing flow rate, for similar conditions, different relative depths are created in both sections (see Fig. 7). In the SSBF4 model, hysteresis has appeared with increasing and decreasing flow rates of 5 and 5.83 L/s. For 5 and 5.83 L/s, sections [B] and [C] are subcritical and the surface profile created is type 1. With a decreasing flow, when the flow rate returns to 5 and 5.83 L/s., the hydraulic jump passes through the obstacle and the profile is type 2. The result of the changes in Froude numbers of sections [B] and [C] is shown in Fig. 16.

Figures 16 a and b show Froude numbers at [B] and [C] against the *vena contracta* Froude number. By first increasing and then decreasing the flow, the regime changes from subcritical to supercritical under similar conditions. When the flow rate decreases to less than 5 or is more than 5.83 L/s, the flow returns to the incremental flow rate state and sections [B] and [C] are subcritical.

A comparison among the models are shown in Fig. 17. The figure shows a comparison of the relative depth of section [B] against the Froude number of the flow passing under the valve. With a general and more detailed examination of the present study, it is concluded that as the distance between the center of the bridge base and the supercritical flow generator valve decreases, the range of

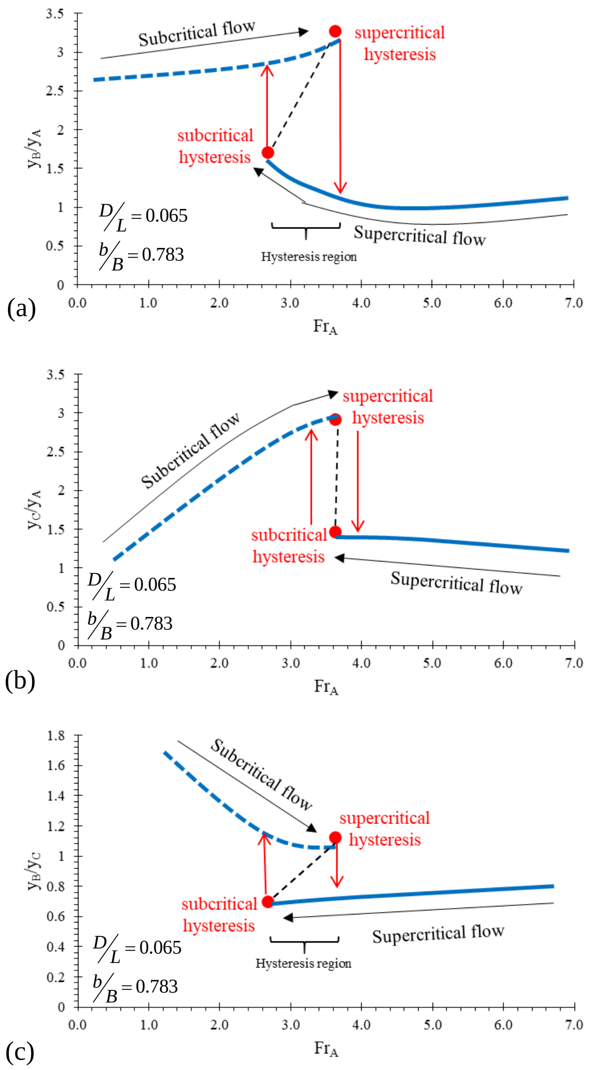

Fig. 15 Reproduction of hysteresis flags for SSBF4. a) Section [B]; b) Section [C]; c) Integration of [B] and [C]

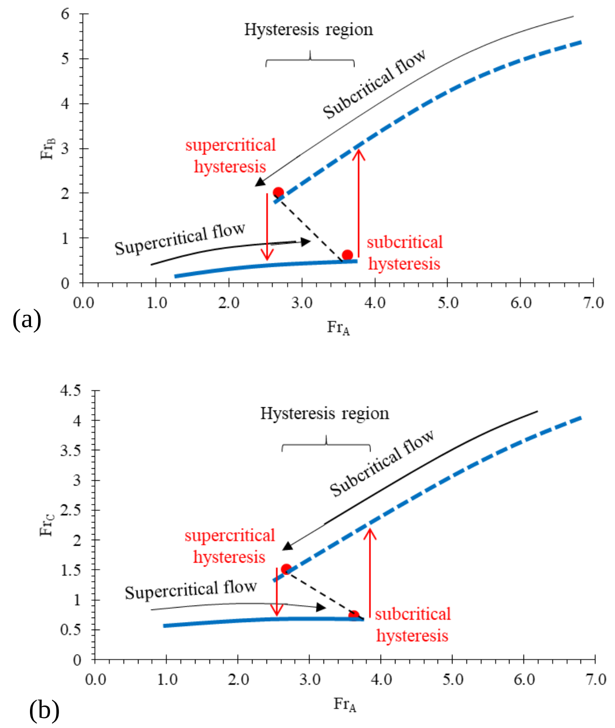

Fig. 16 Variation of Froude number with vena contracta Froude number for SSBF4: (a) Section [B]; (b) Section [C]

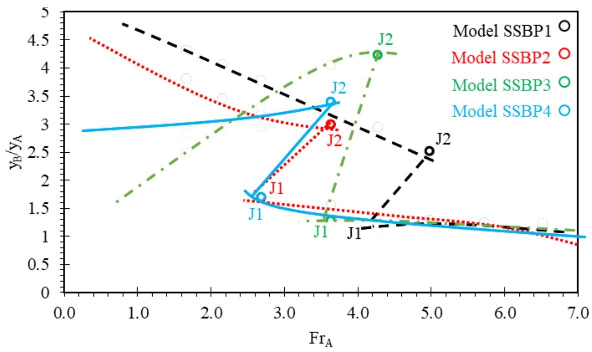

Fig. 17 Comparison of relative depths of section [B] of models with *vena contracta* Froude number

the hysteresis area becomes wider. Also, the increase in the diameter of the bridge base widens of the hysteresis area so that the hysteresis range of the SSBF3 model exceeds that of the SSBF4 model and the SSBF1 model exceeds the SSBF2 model.

3.6. Results Related to Residual Energy

To investigate the influence of the geometry two bridge foundations with cylindrical and spindle shapes have been used. In Fig. 18, the effect of changing the geometry of these elements on the relative residual energy are shown.

Diagrams a, b, c, and d, in Fig. 18 shows the relative residual energy versus the upstream Froude number in cylindrical and spindle-shaped bridge foundations with diameters of 6.5 and 9 cm at a distance of 1 meter from sluice gate. In other words, these figures compare the base model of cylindrical and spindle-shaped bridge. Carefully in these figures, it can be seen that the relative residual energy changes remaining in the spindle-shaped model have been more affected by the hysteretic behavior. The reason for this is the existence of a triangular area in front of the bridge foundation (spindle-shaped part) which affects the shear stress of the base wall. In fact, the hysteresis area at the base of the spindle-shaped bridge is relatively larger than that of the cylindrical model.

By examining the above figures, it can be seen that the relative energy remaining in spindle-shaped bridge foundations is higher than that of cylindrical (simple) foundations. It can be concluded that the hysteresis in cylindrical (simple) bridge foundations has appeared only in one flow input flow rate, but with spindle-shaped foundations, hysteretic behavior has been observed for two or three input flow rates. Considering that the absence of hysteresis or if this phenomenon is observed, the small Froude numbers in which this phenomenon appears is of great importance. Consequently, the use of cylindrical foundations in this case is recommended in terms of hysteretic behavior compared to spindle-shaped bridge

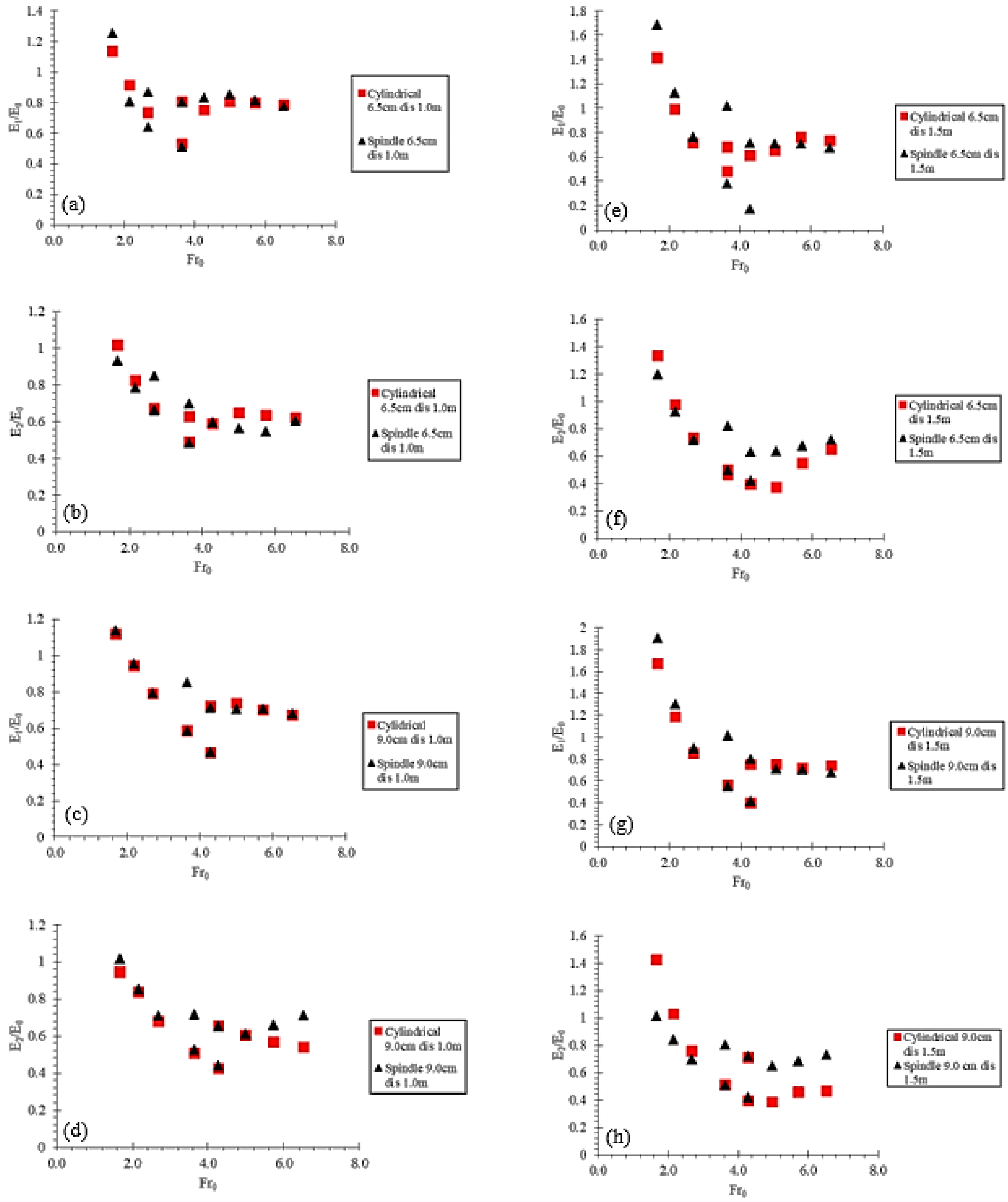

Fig. 18 Comparison of relative residual energy in present study. a, b, c, d, e, f, g, and h) The explanation is in the diagram legend

foundations with all their advantages. One of the most important advantages of using bridge foundations with a spindle-shaped geometry is the reduction of scouring depth around the bridge foundation or foundation group.

The hysteretic behavior causes the relative residual energy remaining in the spindle-shaped model to increase by 33.33%, 15.66%, 23.57%, and 21.22% in graphs a, b, c, and d respectively, compared to the cylindrical base, by converting the subcritical to supercritical regime.

By further examining the graphs, it is clear that the comparison of the mentioned models shows that the hysteresis area becomes more intense with the increase in the distance between the bridge foundation and the sluice gate.

4. Conclusion

A hydraulic jump forms in hydraulic structures when obstacles are in the flow path. Due to their non-linearity, the equations governing the flow in this phenomenon do not have exact and specific solutions. The hysteresis of supercritical flow is an issue that is not well known so far and it is formed in most cases near the water supply structures. Hysteresis creates different states in the flow; this phenomenon should be considered by designers. The goal of this research was to investigate the conditions and reasons for creating and not creating a hydraulic jump and the formation of two types of profiles. Two bridge foundations with diameters of 9 and 6.5 cm which were located at distances of 1.5 and 1 meter from the supercritical flow generating valve, have been used. The primary flow is a flow in which the flow rate increases from 4.17 to 10 L/s with a flow rate increase step of 50 liters/min. The decreasing flow rate is a flow in which the flow rate decreases from 10 to 4.17 L/s with a flow rate step of 50 liter/min. Two surface profiles are observed for the same laboratory system. The two profiles are: Profile 1: in which sections [B] and [C] are in the subcritical regime and Profile 2: in which sections B and C are in the supercritical regime.

Hysteresis occurs in supercritical flow in all the models of the current research, the results of which are briefly presented below.

- ❖ In the SSBF1 model, where the base of the bridge with a diameter of 9 cm is located at a distance of 1.5 meters from the valve, hysteresis is formed for Froude numbers ranging from 4.2-5.0. When the flow rate exceeds 7.5 L/s, the flow regime becomes supercritical and the hydraulic jump disappears. When the flow rate decreases below 6.67 L/s, the flow regime returns to the subcritical regime and the hydraulic jump is created again.
- ❖ In the SSBF2 model, where the base of the bridge with a diameter of 6.5 cm is located at a distance of 1.5 meters from the valve, hysteresis is formed for 3.63 < Fr < 4.269. When the flow exceeds 6.67 L/s, the regime is supercritical, and when the flow rate decreases to less than 5.83 L/s, the flow regime returns to the subcritical regime.
- ❖ With SSBF3, where the base of the bridge with a diameter of 9 cm is located at a distance of 1 meter from the valve, hysteresis occurs when 3.63 < Fr < 4.269. When the flow rate increases, the flow regime is supercritical. When the flow rate decreases below 5.83 L/s, the subcritical regime is established.
- ❖ With SSBF4, where the base of the bridge with a diameter of 6.5 cm is located at a distance of 1 meter from the valve, hysteresis occurs when 3.63 < Fr < 4.269. When the flow exceeds 5.83 L/s, the flow is supercritical. When the discharge reaches to 5.0 L/s by decreasing of the flow discharge, the flow regime back to subcritical regimes.
- ❖ With the formation of hysteretic behavior, the relative residual energy and the downstream Froude number increases by 57.36% and 72.31%, respectively.
- ❖ It is suggested that as a future research focus, numerical methods and neural networks can be used and their results should be compared with the results of the present research.

It was seen that as the bridge foundations approach the supercritical flow-generating valve, the hysteresis range becomes wider. In other words, the hysteresis area created in SSBF3 and SSBF4 models is wider than the area created in SSBF1 and SSBF2 models.

CONFLICT OF INTEREST

There is no conflict of interest to disclose.

AUTHORS CONTRIBUTION

Conceptualization, **E. Aminvash**, **R. Daneshfaraz**, **S. Sadeghfam**; Methodology, **E. Aminvash**, **R. Daneshfaraz**, **S. Sadeghfam**; Experimental set-up, **E. Aminvash**; Writing-original draft preparation, **E. Aminvash**, **R. Daneshfaraz**, **S. Sadeghfam**, **V. Süme**, **J. Abraham**; Writing-review editing, **E. Aminvash**, **J. Abraham**, **R. Daneshfaraz**, **V. Süme**; Project administration, **R. Daneshfaraz**, **S. Sadeghfam**, **E. Aminvash**. All authors have read and agreed to the publish version of the manuscript.

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